

# Interaction of spatial solitons in nonlinear optical medium

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The effects caused by nonresonant nonlinear interaction between noncollinear self-focusing beams are considered in 2D optical samples using multi-scale analysis. The analytical expression for the beams trajectories shift due to the mutual interaction is derived, and the range of parameters is given beyond which the mentioned consideration fails. We compare our results with the naive geometrical optics model. It is shown that these two approaches give the same results. This justifies the use of geometrical optics approach for description of elastic and almost-elastic collision processes both in Kerr and saturable nonlinear media.

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Nonlinear localized waves, or soliton-like excitations play important role in many branches of physics: nonlinear optics [1], plasma physics [2], hydrodynamics [3], magnetic systems [4] etc. In contrast with linear excitations such nonlinear creations may be exceedingly stable. That is, they can propagate over long distances without distortion. However most exciting feature of soliton phenomena is their interaction processes. In particular, when they collide with other ones, solitons exhibit partial-like behavior [5].

Despite the fact that there is a large diversity of nonlinear physical systems exhibiting soliton like excitations, due to universal properties of such creations, nonlinear localization dynamics can be described only within a few theoretical models [2]. This fact is of great importance. In particular, this allows one to study experimentally nonlinear phenomena in most convenient physical systems, while the direct experimental investigation of the particular system the subject of interest may be more difficult or even impossible. To date, such "model" experimental systems are nonlinear spin waves in ferromagnetic films [4] and spatial optical solitons [5]. However, magnetic envelope solitons can be observed only in (quasi) one dimensional samples and due to transverse instabilities they are unstable in higher space dimensions [4]. Thus recently suggested interaction effects [6] of noncollinearly propagating 1D envelope solitons in 2D magnetically ordered samples doubtfully will be easily realized experimentally. On the other hand nonlinear optical medium is most appropriate for the mentioned purpose (study 1D solitons noncollinear interaction in 2D samples). In particular, spatial optical solitons exhibit richness of characteristics not found for localized waves in other nonlinear media [5]. Indeed, much theoretical and experimental investigations have been performed for optical spatial solitons: elastic interaction of Kerr solitons [7, 8, 9, 10], almost elastic and inelastic collisions of solitons in saturable media including fusion, fission,

annihilation, and spiraling occurrences [11] (see also [5]).

In the present paper we consider theoretically the problem of nonresonant interaction of Kerr spatial optical solitons. The method used here has been suggested for analytical description of interaction of noncollinearly propagating 1D envelope solitons of magnetization in 2D magnetically ordered samples [6]. This method itself is a generalization of well-known 1D multiple scale perturbation theory [12, 13] for higher space dimensions. Since this approach allows us to study the case of interaction of two spatial solitons with different carrier wave frequencies, for the nonresonant interaction of solitons the results presented here are more general compared to ones obtained in [7], where the exact solutions are found but they concern only the case of spatial solitons interaction with the same carrier wave number. Later these analytical results has been used for suggesting different applications of spatial soliton interactions, e.g. producing nonlinear photonic switching [14] and all optical logic elements [15]. Optical solitons dragging and collisions in the presence of absorptions has been also studied [16] applying numerical methods. In the present paper we obtain analytical results for collisions of spatial solitons with different carrier wave number and point out possible relevance of this study to the above-mentioned applications.

For simplicity we consider the interaction of spatial optical solitons in isotropic thin optical films and assume that the electric field is normal to the film plane. If the medium is off-resonant with respect to the optical signal and the optical film is thin enough, then the dispersion can be neglected and modulations develop only along (single) transverse direction [9, 10]. As a result, in the case of appropriate sign of the nonlinear coefficient, so called 1D spatial solitons (self-focusing beams) are formed in 2D samples. Obviously one can consider the crossing of two beams and study analytically the influence of one self-focusing beam on the other one using the above mentioned method [6].

One could try to understand the nonlinear effect of spatial solitons interaction from the naive geometrical optics model: the propagation of the intense beam through the sample locally causes the increasing of the refraction index due to the nonlinear reaction of the medium (Kerr effect). This in turn generates wave-guiding area and as a consequence the beam becomes self-focusing [5]. At the same time, as long as the refraction index within the beam area is bigger than outside it is natural to expect that the second beam will be bent during crossing the first beam area and eventually its trajectory will be shifted after the interaction as is shown schematically in Fig. 1. From the same geometrical optics consideration it follows that this shift should be zero for perpendicular beams. However, the interaction process is much more complicated. Actually the second beam affects the induced waveguide of the first beam (the first (wide) beam in Fig. 1 is slightly shifted as well). This gives rise to "self-action" of the beam through other one during the interaction. In addition, interference effects may take place between the carrier waves of the interacting beams. For Kerr spatial solitons it is well established that for large enough converging input angles the solitons pass through each other unaffected. The only effect of such nonresonant interaction is the trajectory shift of the interacting beams [5]. Thus, qualitatively, the effect is the same as it comes out from the naive geometrical optics consideration. Surprisingly, in this paper we find that the results are the same even quantitatively.

In nonlinear Kerr medium polarization  $P$  depends non-

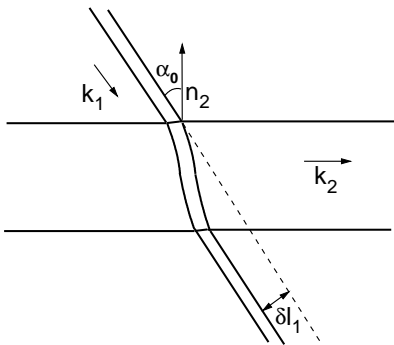


FIG. 1: Schematic picture of the interaction process between self-focusing beams in off-resonant optical medium. Solid lines indicate "borders" of the beams.  $\alpha_0$  defines an angle between first (narrow) beam and normal vector  $n_2$  of the second (wide) beam;  $\vec{k}_1$  and  $\vec{k}_2$  are their carrier wave vectors, respectively;  $\delta l_1$  stands for a shift of trajectory of the first beam, which is caused by the nonlinear interaction effects. Let us mention that the second beam trajectory is also slightly shifted.

linearly on electric field  $E$  as follows:

$$P = \chi^{(1)}E + \chi^{(3)}E^3, \quad (1)$$

where  $\chi^{(1)}$  and  $\chi^{(3)}$  are linear and nonlinear polarisability constants respectively. For simplicity we consider here centrally symmetric materials and therefore from the symmetry reasons the second order term in Exp. (1) is identically zero. The wave equation for the electric field reads (see for more details e.g. [2]):

$$\nabla^2 E - \beta E_{tt} = \gamma(E^3)_{tt} \quad (2)$$

with coefficients  $\beta = (1 + 4\pi\chi^{(1)})/c^2$  and  $\gamma = 4\pi\chi^{(3)}/c^2$ . The nabla operator acts in 2D space as long as film samples are considered in the present paper.

Let us consider the weakly nonlinear case i.e. when the cubic term is much smaller than the linear one. We do not repeat all the steps of calculations to obtain the 1D spatial soliton solution, let us just mention that a weakly nonlinear wave with a slowly varying envelope is sought in the following way [12]:

$$E = \sum_{\alpha=1}^{\infty} \varepsilon^\alpha \sum_{l=-\infty}^{\infty} \varphi_l^{(\alpha)}(\vec{\xi}, \vec{\tau}) \cdot e^{il(\vec{k}\vec{r} - \omega t)}, \quad (3)$$

where frequency  $\omega$  and wave vector  $\vec{k}$  of carrier wave are connected via simple dispersion relation  $\omega = k/\sqrt{\beta}$ ; the envelope  $\varphi_l^{(\alpha)} = (\varphi_{-l}^{(\alpha)})^*$  is a function of slow variables  $\vec{\xi} = \varepsilon(\vec{r} - \vec{v}t)$  and  $\vec{\tau} = \varepsilon^2\vec{r}/2k$ ;  $\vec{v} = d\omega/d\vec{k}$  is a group velocity of linear wave  $\vec{v} \parallel \vec{k}$  and  $\varepsilon$  is a formal parameter defining the smallness or "slowness" of the physical quantity before which it appears. Then building the ordinary perturbation scenario [2, 12] in the third approximation over  $\varepsilon$  one gets 1D nonlinear Schrödinger (NLS) equation:

$$i\frac{\partial \varphi_1^{(1)}}{\partial \tau} + \frac{\partial^2 \varphi_1^{(1)}}{\partial \xi^2} + 3\gamma\omega^2 \varphi_1^{(1)} |\varphi_1^{(1)}|^2 = 0, \quad (4)$$

which has well-known spatial soliton (self-focusing beam) solution if  $\gamma > 0$ . Physically, the spatial soliton formation is the result of balance between defocusing diffraction and focusing nonlinearity. It is worth to note that, since diffraction in general is strong, the nonlinearities involved in spatial solitons are much stronger compared to nonlinearities involved in temporal solitons. As we see from (4) the soliton envelope in the leading approximation is a function of variables  $\xi \equiv \vec{\xi}_\perp = \varepsilon(\vec{n}\vec{r})$  and  $\tau \equiv \vec{\tau}_\parallel = \varepsilon^2(\vec{k}\vec{r})/2k^2$  where  $\tau$  plays a role of "time" and  $\vec{n}$  is a unit vector perpendicular to  $\vec{k}$ . Let us emphasize once again that we have got 1D NLS due to fact that the longitudinal dispersion in off-resonant optical medium could be neglected, in other words wave group velocity  $v \equiv d\omega/dk$  does not depend on wave number  $k$ . In case of presence of longitudinal dispersion the physical

process would be described [2] by 2D NLS for which 1D solitonic solution would be unstable.

Now we shall start a main task of the paper, particularly, the analytical investigation of interaction between noncollinear self-focusing beams. For that purpose we are seeking for the solution following the general method developed in Refs. [6]:

$$E = \sum_{\alpha=1}^{\infty} \varepsilon^{\alpha} \sum_{l_1 l_2 = -\infty}^{\infty} \varphi_{l_1 l_2}^{(\alpha)} e^{i(\vec{k}_{l_1 l_2} \vec{r} - \omega_{l_1 l_2} t + \varepsilon \Omega_{l_1 l_2})}, \quad (5)$$

where  $\omega_{l_1 l_2} = l_1 \omega_1 + l_2 \omega_2$ ;  $\vec{k}_{l_1 l_2} = l_1 \vec{k}_1 + l_2 \vec{k}_2$ ;  $\omega_1$ ,  $\vec{k}_1$  and  $\omega_2$ ,  $\vec{k}_2$  are carrier frequencies and wave vectors, respectively;  $\varphi_{l_1 l_2}^{(\alpha)}$  and  $\Omega_{l_1 l_2}$  are functions of slow variables ( $p=1, 2$ )

$$\xi_p = \varepsilon \left[ (\vec{n}_p \vec{r}) - \varepsilon \psi_p(\xi_1, \xi_2, \tau_1, \tau_2) \right], \quad \tau_p = \varepsilon^2 \frac{(\vec{k}_p \vec{r})}{2k_p^2}, \quad (6)$$

where  $\vec{n}_1$  and  $\vec{n}_2$  are unit vectors perpendicular to carrier wave vectors  $\vec{k}_1$  and  $\vec{k}_2$ , respectively. Proceeding further with the similar calculations as it was done in Refs. [6] we come in the leading approximation to the following solution:

$$E = \varphi_{10}^{(1)} e^{i(\vec{k}_1 \vec{r} - \omega_1 t + \Omega_{10}^{(1)})} + \varphi_{01}^{(1)} e^{i(\vec{k}_2 \vec{r} - \omega_2 t + \Omega_{01}^{(1)})} + c.c., \quad (7)$$

where "c.c." denotes complex conjugated terms;  $\varphi_{10}^{(1)}$  and  $\varphi_{01}^{(1)}$  are the solutions of 1D NLS (see Eq. (4)) with derivatives over set of slow variables  $\xi_1$ ,  $\tau_1$  and  $\xi_2$ ,  $\tau_2$ , respectively. For example, in the simplest case of one-soliton envelopes  $\varphi_{10}^{(1)}$  and  $\varphi_{01}^{(1)}$  we have:

$$\begin{aligned} |\varphi_{10}^{(1)}| &= |A_1| \cdot \text{sech} \left\{ |A_1| \sqrt{6\gamma\omega_1^2} [(\vec{n}_1 \vec{r}) - \psi_1^{(1)}] \right\}, \\ |\varphi_{01}^{(1)}| &= |A_2| \cdot \text{sech} \left\{ |A_2| \sqrt{6\gamma\omega_2^2} [(\vec{n}_2 \vec{r}) - \psi_2^{(1)}] \right\}, \end{aligned} \quad (8)$$

and  $A_1$  and  $A_2$  are amplitudes of the first and the second self-focusing beams, respectively. Besides that, phase and position shifts of the first self focusing beam induced by weakly nonlinear interaction with the second beam are defined by the following formulas:

$$\frac{\partial \psi_1^{(1)}}{\partial \xi_2} = \frac{(\vec{n}_1 \vec{n}_2)}{(\vec{k}_1 \vec{n}_2)} \frac{\partial \Omega_{10}^{(1)}}{\partial \xi_2} = 3\gamma\omega_1^2 |\varphi_{01}^{(1)}|^2 \frac{(\vec{n}_1 \vec{n}_2)}{(\vec{k}_1 \vec{n}_2)^2}. \quad (9)$$

As far as according to perturbative approach we have a following scaling  $\partial \psi_1^{(1)} / \partial \xi_2 \sim \varepsilon^2$  and taking into consideration the dispersion relation ( $\omega_1^2 = k_1^2 / \beta$ ) the following restriction upon the soliton parameters is derived:

$$|A_2| \sqrt{\frac{3\gamma |\sin \alpha_0|}{\beta \cos^2 \alpha_0}} \ll 1, \quad (10)$$

where  $\alpha_0 = (\pi/2) - \theta_0$ , and  $\theta_0$  is an angle between the self-focusing beams. From Exp. (9) it is easy to get

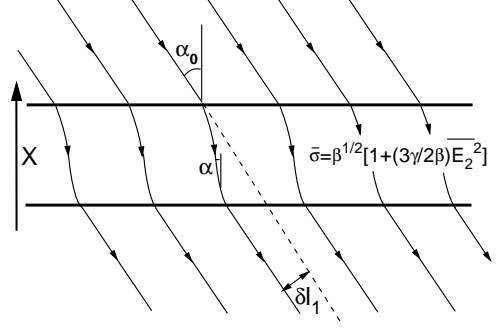


FIG. 2: Optical ray refraction through the self-focusing beam area with borders denoted by horizontal solid lines.

analytical expression for trajectory shift of the first beam caused by the nonresonant interaction with other one:

$$\begin{aligned} \delta l_1 &= \psi_1^{(1)}(\infty) - \psi_1^{(1)}(-\infty) \\ &= \frac{3\gamma \sin \alpha_0}{\beta \cos^2 \alpha_0} \int_{-\infty}^{\infty} |\varphi_{01}^{(1)}(\xi_2)|^2 d\xi_2. \end{aligned} \quad (11)$$

Particularly, in case of one-soliton envelopes (8) one gets from (11) the analytical expression for the trajectory shift of the first spatial soliton:

$$\delta l_1 = |A_2| \frac{\sqrt{6\gamma} \sin \alpha_0}{\beta \omega_2 \cos^2 \alpha_0} \quad (12)$$

In addition we want to emphasize that beyond the limit given by (10) the dynamics is governed by a set of two coupled NLS type equations, which in general is not integrable model and gives qualitatively different behavior of interacting solitons (see [17] and discussion there).

Now let us compare the results obtained above with the picture given by the naive geometric optics consideration. This will give us more deep insight to the problem. First suppose that one has only one self-focusing beam (particularly, the second (wide) beam) and let us calculate how it changes refraction index (see Fig. 2). In view of dependence of polarization upon the electric field (1) we can write down the expression for refraction index as  $\sigma = \sqrt{\beta + 3\gamma E_2^2}$ , where  $E_2$  denotes electric field in the self-focusing beam area and it has a form of envelope spatial soliton (8). Averaging refraction index over fast variables  $\vec{r}$  and  $t$  in the weakly nonlinear limit (the term proportional to  $E_2^2$  is small) we get the following approximate formula for slowly varying (along axis  $x$ ) averaged refraction index  $\bar{\sigma}(x) = \sqrt{\beta} [1 + (3\gamma/2\beta) \overline{E_2^2}]$ .

Let us now solve the following geometrical optics problem, particularly, how the optical rays refract propagating through the area of the second beam with slowly changing refraction index. For that purpose we note that

an angle  $\alpha$  between the ray and normal vector (with respect to the beam) at any point could be calculated via simple refraction formula:  $\sigma(x)\sin\alpha = \sigma_0\sin\alpha_0$ , where  $\sigma_0 \equiv \sqrt{\beta}$ . Taking into account the fact that the trajectory shift of the ray could be calculated as follows:  $\delta l_1 = \cos\alpha_0 \int_{-\infty}^{\infty} dx(\tan\alpha_0 - \tan\alpha)$ , we come exactly to

the formula (11) which we have obtained from multi-scale analysis [6]. However, to avoid any misunderstanding it has to be stressed that the geometrical optics approach to the spatial solitons interaction problem is not self-consistent. Indeed, as we have pointed out already, in this model the "self-action" effects of the first soliton through other one are neglected. Physically this means that the first beam is linear. But the diffraction in the nonlinear problem considered here is not negligible. Thus the first beam will diffract and the interaction picture given by the geometrical optics model is not meaningful under realistic experimental situations of the spatial solitons interaction.

Although the geometrical optics approach in some particular cases gives full understanding of the solitons interaction processes [18], in general one expects that this approach is valid only for qualitative description of the incoherent spatial solitons interaction [5]. As is mentioned above the nonlinear "self-action" of the beam through other one is neglected without justification in the geometrical optics model of solitons collisions. However, the analysis presented here shows that this additional nonlinear "self-action" during the interaction process does not affect the soliton dynamics asymptotically. This is why such naive geometrical optics model gives correct results even for almost-elastic collision processes between solitons in saturable media [5].

In summary, in the present paper we have considered the problem of nonresonant interaction of Kerr spatial solitons (self-focusing beams) with different carrier wave frequencies using multi-scale analysis. It is shown that the beams trajectories are shifted due to mutual interaction. The analytical expressions for these shifts are obtained as well. Surprisingly the naive geometrical optics model of the solitons collision is in full agreement with the results of general theory. In particular, this shows that the "self-action" of the soliton caused by nonresonant interaction process does not changes it's asymptotical behavior after the collision. This in turn justifies the use of the geometrical optics model for description of elastic and almost-elastic collision processes both in Kerr and saturable media.

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