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A Backstepping Approach for an Active Suspension System

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***Abstract*— In this paper, the control design problem for an active suspension system to maintain the comfort and safety of the vehicle body is considered. The road disturbance is modelled as a finite sum of sinusoidal functions with unknown frequencies, amplitudes and phases. The disturbance is parameterized and an adaptive controller is designed by using the backstepping technique. It is proven that the equilibrium of the closed loop system is stable and the vertical acceleration of the vehicle body tends to zero despite road disturbances. The effectiveness of the controller is illustrated with a simulation of a road test.**

I. Introduction

A suspension system is one of the most important component of a vehicle to maintain the comfort and safety by isolating the vehicle body from road induced vibration and shocks [1]. Suspension systems can be categorized into three main parts; passive suspensions [2], semi-active suspensions [3], and active suspension systems [4]. Active suspensions have been developed for achieving the vehicle required performance by applying an external force between the car body (sprung mass) and the wheel (unsprung). The illustration of an active suspension of a quarter car body is given in Figure 1. The effectiveness of the active suspension system intensely depends on the success of the control strategy that is employed for the applied force. Therefore, the control design for an active suspension has been attracted the attention of many researchers.

Various approaches have been employed for the control design of an active suspension system such as sliding mode [6] and well known LQR [5]. Fuzzy-logic control is also used in [13]–[16]. In [7]–[12], the control strategies with H_∞ approach are proposed to optimize the performance requirements such as ride comfort (*i.e.*, acceleration of car body), and suspension deflection (*i.e.*, the displacement between the sprung mass and unsprung mass). In these designs, the effect of the disturbance to the vehicle body is not rejected completely but attenuated while maintaining the other performance requirement.

The effective way of representing the road disturbance is the sum of sinusoidal functions with different amplitudes, frequencies and phases. This representation allows to approach the problem as the cancellation of sinusoidal disturbances. The common method to approach this problem is the internal model principle for which a general solution is given in [17] in the case of linear systems. The approaches for nonlinear systems are proposed in [18]–[20]. Disturbance cancelation designs also exist for LTI systems [21], [22].

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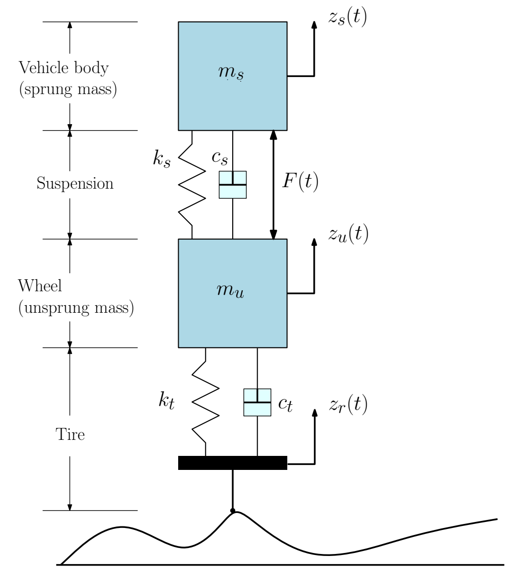


Fig. 1: The illustration of a quarter car model with an active suspension system.

Furthermore, rejection algorithms are given by state derivative feedback for both known [23] and unknown [24], LTI systems.

In this note, an adaptive controller is designed to cancel the effect of the unknown road disturbance disturbances on the vehicle body. The unknown road disturbance is modelled as a finite sum of sinusoidal functions with unknown frequencies, amplitudes and phases. Then, the road disturbance is represented in a parameterized form by using the technique given in [26]. This representation enables us to approach the problem as an adaptive control design. The essence of the approach is the backstepping procedure which has been shown in [25] for handling the unmatched uncertainties. Finally, it is proven that the equilibrium of the closed loop system is stable and the vertical acceleration of the vehicle body tends to zero despite road disturbances.

In Section II, the problem is introduced. The representation of unknown sinusoidal disturbances is given in Section III. In Section IV, the main design is presented and the stability theorem with its proof are given. The results of the simulation of a road test with various road disturbances is presented in Section V.

II. Problem Statement

A quarter car model is widely used to design a control law for an active suspension system [8]. The two degree of freedom quarter-car model is given in Figure 1. The equations of motion is given by

$$m_s\ddot{z}_s(t) = -c_s(\dot{z}_s(t)-\dot{z}_u(t)) - k_s(z_s(t)-z_u(t)) + F(t), \quad (1)$$

$$m_u\ddot{z}_u(t) = -c_s(\dot{z}_u(t)-\dot{z}_s(t)) - k_s(z_u(t)-z_s(t)) - k_t(z_u(t)-z_r(t)) - c_t(\dot{z}_u(t)-\dot{z}_r(t)) - F(t), \quad (2)$$

where m_s and m_u are masses of the car body (sprung mass) and wheel (unsprung mass), respectively. The parameters k_s and c_s are the coefficients for spring and damper of the suspension, respectively. The tire is modelled as a spring and damper where coefficients are given as k_t for spring and c_t for damper. The displacement of the car body and the wheel are given by z_s and z_u, respectively. The road disturbance and the control force are given by z_r and F, respectively.

We make the following assumptions regarding the measurement of states and the road disturbance:

Assumption 1: The measurements of relative displacements, $(z_s - z_u)$, tire deflection $(z_u - z_r)$ and the velocities; $\dot{z}_s$, $\dot{z}_u$ are available for control design.

Assumption 2: The road disturbance is represented as $z_r(t) = \sum_{i=1}^{q} g_i \sin(\omega_i t + \phi_i)$ where the amplitude, g_i, the frequency, ω_i and the phase, ϕ_i, are unknown. The number of maximum distinct frequency, q, is known.

The main aim of an active suspension is to keep the vertical acceleration of the car body almost zero despite bad road conditions in order to maintain the safety and comfort of passengers and loads in the car. To this end, we design a control law for $F(t)$ to satisfy the convergence of $\ddot{z}_s(t)$ to zero despite the road disturbance as given in Assumption 2 and maintain the stability of the equilibrium.

III. Road Disturbance Representation

Since the tire deflection, $(z_u(t) - z_r(t))$, is assumed to be measured, the main disturbance that affects the system is $\frac{c_t}{m_u}\dot{z}_r(t) = \sum_{i=1}^{q} \frac{c_t}{m_u}\omega_i g_i \cos(\omega_i t + \phi_i)$ that can be represented as the output of a linear exosystem,

$$\dot{w}(t) = Sw(t), \quad \dot{z}_r(t) = h^T w(t), \quad (3)$$

where $w(t) \in \mathbb{R}^{2q}$, The matrix S depends on the unknown frequencies of the road disturbance $\dot{z}_r(t)$, while the uncertainty of amplitude and phase is related to the unknown initial condition of (3).

The road disturbance is parameterized by following [26]. Let $G \in \mathbb{R}^{2q\times 2q}$ be a Hurwitz matrix with distinct eigenvalues and let (G,l) be a controllable pair. Since (h^T, S) is observable and the spectra of S and G are disjoint the unique solution $M \in \mathbb{R}^{2q\times 2q}$ of the Sylvester equation

$$MS - GM = lh^T. \quad (4)$$

is invertible [27]. The change of coordinates $\xi = Mw$ transform the exosystem (3) into the form

$$\dot{\xi} = G\xi + l\frac{c_t}{m_u}\dot{z}_r, \quad (5)$$

$$\frac{c_t}{m_u}\dot{z}_r = \theta^T\xi, \quad (6)$$

where $\theta^T = h^T M^{-1}$.

The unknown road disturbance $\frac{c_t}{m_u}\dot{z}_r(t)$ are represented as the product of an unknown constant and the vector $\xi(t)$ in (6). However, $\xi(t)$ can not be used in a control design, since it can not be measured. To overcome this problem, a conceptual observer is designed. The following lemma establishes the properties of the observer.

Lemma 1: The inaccessible disturbance $\frac{c_t}{m_u}\dot{z}_r$ can be represented in the form

$$\frac{c_t}{m_u}\dot{z}_r = \theta^T\hat{\xi} + \theta^T\delta, \quad (7)$$

where

$$\hat{\xi} = \eta + l\dot{z}_u, \quad (8)$$

$$\dot{\eta} = G(\eta + l\dot{z}_u) - l\frac{1}{m_u}\Big(-(c_s+c_t)\dot{z}_u + c_s\dot{z}_s - k_s(z_u - z_s) - k_t(z_u - z_r) - F\Big), \quad (9)$$

with the estimation error $\delta \in \mathbb{R}^{2q}$ obeys the equation

$$\dot{\delta} = G\delta. \quad (10)$$

Proof: Define an estimation error

$$\delta = \xi - \hat{\xi}. \quad (11)$$

The equation (10) is obtained by differentiating δ with respect to time and using (2) and (5). Substitution of (11) into (6) yields (7). ■

IV. Control Design and Stability

In order to design a control law that achieves the main aim, we employ a backstepping method. Firstly, we consider only the car body dynamics and propose a control strategy for F. Secondly, we represent the system with an error term and substitute the road disturbance representation, given in (7) in order to approach the problem as an adaptive control problem. Eventually, a final control law is designed and the overall closed loop stability is stated.

A. Backstepping Design

Considering the dynamics of the car body given in (1), we propose a desired value for F, as $-c_s\dot{z}_u + k_s(z_s - z_u)$ to achieve the convergence of $\ddot{z}_s$ to zero. The error term between the desired F and the actual F is given by

$$e = F - (-c_s\dot{z}_u + k_s(z_s - z_u)). \quad (12)$$

By taking the time derivative of e and using (7) for the representation of $\frac{c_t}{m_u}\dot{z}_r$, we obtain,

$$\dot{e} = H(t) + c_s\theta^T\hat{\xi} + \theta^T\delta - \frac{c_s}{m_u}F + \dot{F}, \quad (13)$$

where

$$H(t) = -\frac{c_s}{m_u}\Big(c_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s) + k_t(z_u - z_r) + c_t\dot{z}_u\Big) - k_s(\dot{z}_s - \dot{z}_u). \quad (14)$$

B. Main Controller and Stability Statement

By considering $\dot{F} - \frac{c_s}{m_u}F$ as a control input in (13) and using the certainty equivalence principle for unknown constant θ, the adaptive controller is given by

$$\dot{F} = \left(\frac{c_s}{m_u} - b\right)F - H(t) - c_s\hat{\theta}^T\hat{\xi} - b\left(c_s\dot{z}_u - k_s\left(z_s - z_u\right)\right) - \dot{z}_s, \tag{15}$$

$$\dot{\hat{\theta}} = \gamma c_s \hat{\xi} e, \tag{16}$$

where $b > \frac{c_s}{m_u} + 1$ and $\gamma > 0$.

In order to state the stability theorem, we define the following states and signals.

By differentiating (11) with respect to time and using (5) and (10), we obtain

$$\dot{\hat{\xi}} = G\hat{\xi} + l\frac{c_t}{m_u}\dot{z}_r, \tag{17}$$

and we define

$$\widetilde{\hat{\xi}} = \hat{\xi} - \overline{\hat{\xi}}, \tag{18}$$

where

$$\overline{\hat{\xi}} = \int_0^t e^{G(t-\tau)} l\frac{c_t}{m_u}\dot{z}_r(\tau)d\tau. \tag{19}$$

Taking derivative of (18) and using (17) yield

$$\dot{\widetilde{\hat{\xi}}} = G\widetilde{\hat{\xi}}. \tag{20}$$

The closed loop wheel (unsprung mass) dynamics is represented in state-space form as follows,

$$\dot{x}_u = A_u x_u + B_u\left(c_s\dot{z}_s - e\right) + B_r\begin{bmatrix} z_r & \dot{z}_r \end{bmatrix}^T \tag{21}$$

where

$$A_u = \begin{bmatrix} 0 & 1 \\ -\frac{k_t}{m_u} & -\frac{c_t}{m_u} \end{bmatrix}, \tag{22}$$

$$B_u = \begin{bmatrix} 0 & \frac{1}{m_u} \end{bmatrix}^T, \tag{23}$$

$$B_r = \begin{bmatrix} 0 & 0 \\ \frac{k_t}{m_u} & \frac{c_t}{m_u} \end{bmatrix}, \tag{24}$$

$$x_u = \begin{bmatrix} z_u & \dot{z}_u \end{bmatrix}. \tag{25}$$

We define

$$\widetilde{x}_u = x_u - \overline{x}_u, \tag{26}$$

where

$$\overline{x}_u = \int_0^t e^{A_u(t-\tau)} B_r \begin{bmatrix} z_r(\tau) & \dot{z}_r(\tau) \end{bmatrix}^T d\tau. \tag{27}$$

Taking derivative of (26) and using (21) yield

$$\dot{\widetilde{x}}_u = A_u\widetilde{x}_u + B_u\left(c_s\dot{z}_s - e\right). \tag{28}$$

The signal $\widetilde{x}_u(t), \widetilde{\hat{\xi}}(t)$ represent the dynamics of the unsprung mass and the observer for the case where $z_r(t) = \dot{z}_r(t) = 0$, respectively. In other words, the case where the car is moving on a flat surface. Finally, the estimation error for unknown constant parameter is given by

$$\widetilde{\theta}(t) = \theta(t) - \hat{\theta}(t). \tag{29}$$

Theorem 1: Consider the closed-loop system consisting of the plant (1),(2) forced by the unknown road disturbance (3), the disturbance observer (8), (9), and the adaptive controller (15), (16). Under Assumptions 1 and 2, the followings hold;

(a) The equilibrium $\dot{z}_s = e = 0, \widetilde{x}_u = 0, \widetilde{\theta} = \delta = \widetilde{\hat{\xi}} = 0$ is stable and the signals $\dot{z}_s(t), \ddot{z}_s(t), e(t), \delta, \widetilde{\hat{\xi}}, \widetilde{x}_u$ converge to zero as $t \to \infty$.
(b) The signals $x_u(t), F(t) - k_s z_s$ are bounded for all initial conditions and $F(s) - k_s z_s(s)$ converges to zero as $t \to \infty$ in the absence of disturbance.

C. Stability Proof

In this section, the proof Theorem 1 is given.

Proof of Theorem 1: Substituting (12) into (1) and (15) into (13) and using (29), we obtain the following system,

$$\ddot{z}_s = -\frac{c_s}{m_s}\dot{z}_s + \frac{1}{m_s}e, \tag{30}$$

$$\dot{e} = -be + c_s\widetilde{\theta}^T\hat{\xi} + c_s\theta^T\delta - \dot{z}_s. \tag{31}$$

The stability of the equilibrium of the closed loop system is established with the use of the following Lyapunov function,

$$V = \frac{1}{2}\left(\varepsilon_{\widetilde{x}_u}\left(m_s\dot{z}_s^2 + e^2 + \gamma\widetilde{\theta}^T\widetilde{\theta} + \varepsilon_\delta\delta^T P_G\delta\right) + \widetilde{\hat{\xi}}^T\widetilde{\hat{\xi}} + \widetilde{x}_u^T\widetilde{x}_u\right). \tag{32}$$

where

$$G^T P_G + P_G G = -2I, \tag{33}$$

$$A_u^T P_{A_u} + P_{A_u}A_u = -2I, \tag{34}$$

$$\varepsilon_{\widetilde{x}_u} = B_u^T P_{A_u} P_{A_u} B_u, \tag{35}$$

$$\varepsilon_\delta = c_s^2\lambda_{\max}(\theta\theta^T). \tag{36}$$

Taking time derivative of $V(t)$, in view of (10), (16), (20), (28), (30) and (31), we obtain

$$\begin{aligned}\dot{V} = &-c_s\varepsilon_{\widetilde{x}_u}\dot{z}_s^2 - \varepsilon_{\widetilde{x}_u}be^2 + \varepsilon_{\widetilde{x}_u}c_s\theta^T\delta e - \varepsilon_{\widetilde{x}_u}\varepsilon_\delta\delta^T\delta \\ &- \widetilde{\hat{\xi}}^T\widetilde{\hat{\xi}} - \widetilde{x}_u^T\widetilde{x}_u + \widetilde{x}_u^T PB_u\left(c_s\dot{z}_s - e\right).\end{aligned} \tag{37}$$

Using Young's inequality for the cross term, we get

$$\dot{V} \leq -\frac{\varepsilon_{\widetilde{x}_u}c_s}{2}\dot{z}_s^2 - \varepsilon_{\widetilde{x}_u}(b-1)e^2 - \varepsilon_{\widetilde{x}_u}\frac{\varepsilon_\delta}{2}\delta^T\delta - \widetilde{\hat{\xi}}^T\widetilde{\hat{\xi}} - \frac{1}{2}\widetilde{x}_u^T\widetilde{x}_u. \tag{38}$$

Using the fact that $b > \frac{c_s}{m_u} + 1$, from (38), we conclude

$$V(t) \leq V(0). \tag{39}$$

Defining

$$\Xi(t) = \left[\dot{z}_s(t), e(t), \widetilde{\theta}^T, \delta^T, \widetilde{\hat{\xi}}^T, \widetilde{x}_u^T\right]^T, \tag{40}$$

and using (32) and (39), we get

$$|\Xi|^2 \leq M_1|\Xi(0)|^2, \tag{41}$$

for some $M_1 > 0$. For all Ξ, the right hand side of (10), (16), (18), (26), (30) and (31) are continuous in Ξ and t, which implies that the right hand side of (38) is continuous in Ξ and t. Furthermore, the right hand side of (38) is zero at $\Xi = 0$. By the LaSalle-Yoshizawa theorem, (38) ensures that $\dot{z}_s(t), e(t), \widetilde{\hat{\xi}}, \widetilde{x}_u$ and δ converge to zero as $t \to \infty$. From the boundedness of $\Xi(t)$ and the convergence of $\dot{z}_s(t), e(t)$, it follows from (30) that $\ddot{z}_s(t)$ is bounded and converges to zero $t \to \infty$. This proves part (a) of Theorem 1.

From (21), (25), (26), (40), (41) and noting that A_u is Hurwitz and the road disturbance $z_r(t), \dot{z}_r(t)$ given in Assumption 2 is bounded, we conclude that $z_u(t)$ and $\dot{z}_u(t)$ are bounded. Recalling that it has already been established that $\widetilde{x}_u$ converges to zero, from (25) and (26), we conclude that $z_u(t), \dot{z}_u(t)$ converge to zero as $t \to \infty$ in the absence of the road disturbance (*i.e.* $z_r(t) = \dot{z}_r(t) = 0$).

From (7), (15), (18), (29), the dynamics of $F - k_s z_s$ is written as

$$\frac{\mathrm{d}(F - k_s z_s)}{\mathrm{d}t} = \left(\frac{c_s}{m_u} - b\right)(F - k_s z_s) + d(t), \qquad (42)$$

where

$$d(t) = \frac{c_s}{m_u}\Big(c_s(\dot{z}_u - \dot{z}_s) + k_s(z_u + k_t(z_u - z_r) + c_t(\dot{z}_u - \dot{z}_r)\Big) + k_s\dot{z}_u + c_s\widetilde{\theta}^T\left(\widetilde{\hat{\xi}} + \overline{\hat{\xi}}\right) + c_s\theta^T\delta - b(c_s\dot{z}_u + k_s z_u) - \dot{z}_s \qquad (43)$$

From the boundedness of $\Xi(t), z_u, \dot{z}_u(t), z_r(t)$ and $\dot{z}_r(t)$, and noting that G is Hurwitz it follows from (19), (43) that $d(t)$ is bounded. Recalling $\dot{z}_s(t), e(t), \widetilde{\hat{\xi}}, \widetilde{x}_u, \delta$ converge to zero as $t \to \infty$ and G, A_u are Hurwitz matrices, from (18), (25), (26) it implies that $d(t)$ converges to zero as $t \to \infty$ in the absence of the road disturbance. Noting that $b > \frac{c_s}{m_u}$, it follows from (42) and (43) that $(F - k_s z_s)$ is bounded. Furthermore, recalling $d(t)$ converges to zero as $t \to \infty$ in the absence of the road disturbance, it is concluded that $(F - k_s z_s)$ converges to zero as $t \to \infty$ in the same case. ■

V. Simulations

We perform a simulation to test the performance of the designed controller on a road that contains various road disturbance. The system parameters are given by $m_s = 320kg, m_u = 40kg, c_s = 1000Ns/m, ks = 18000N/m, kt = 200000N/m, ct = 60Ns/m$. The control parameter, $b = 525$ and the update gain, $\gamma = 8$. It is assumed that the road disturbance has at most three distinct frequencies. Therefore, the controllable pair (G, l) for the observer is chosen as $l = \begin{bmatrix} 0_5^T & 1 \end{bmatrix}^T$, with $0_n = [0, \ldots, 0]^T \in \mathbb{R}^n$ and $G = \begin{bmatrix} 0_5 & I_5 \\ 0_5^T & 0 \end{bmatrix} + l\begin{bmatrix} -127.5 & -342.0 & -381.7 & -227.0 & -75.8 & -13.5 \end{bmatrix}$. In order to simulate the road with various disturbances, the road disturbance $z_r(t)$ is chosen as follows

$$z_r(t) = \begin{cases} 0 & 0 \le t \le 2 \\ z_{r1}(t) & 2 < t < 8 \\ 0 & 8 \le t \le 12 \\ z_{r2}(t) & 12 < t < 14 \\ 0 & 14 \le t \le 20 \end{cases} \qquad (44)$$

where

$$z_{r1}(t) = 0.04\sin(2\pi t) + 0.05\sin(4\pi + \pi/8), \qquad (45)$$

$$z_{r2}(t) = 0.01\sin(30\pi t) + 0.02\sin(20\pi t + \pi/2) + 0.03\sin(4\pi), \qquad (46)$$

The function, $z_{r1}(t)$, represents the low frequency road disturbances whereas $z_{r2}(t)$ represents the road condition which is

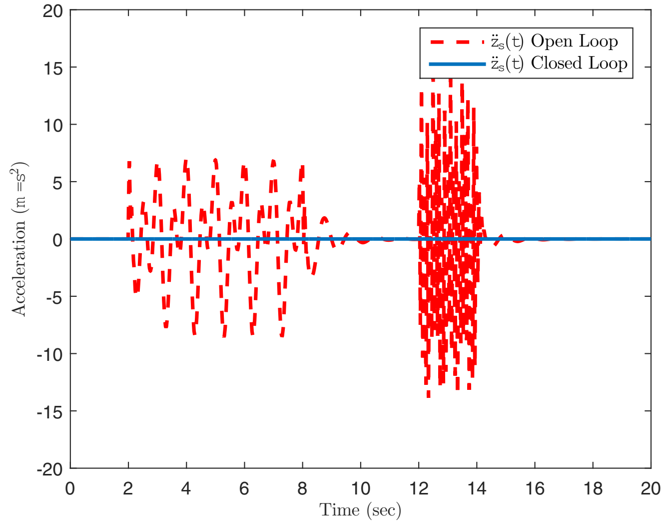


Fig. 2: The simulation results of the vertical acceleration of the body vehicle for the closed loop (solid) and the open loop case (dashed).

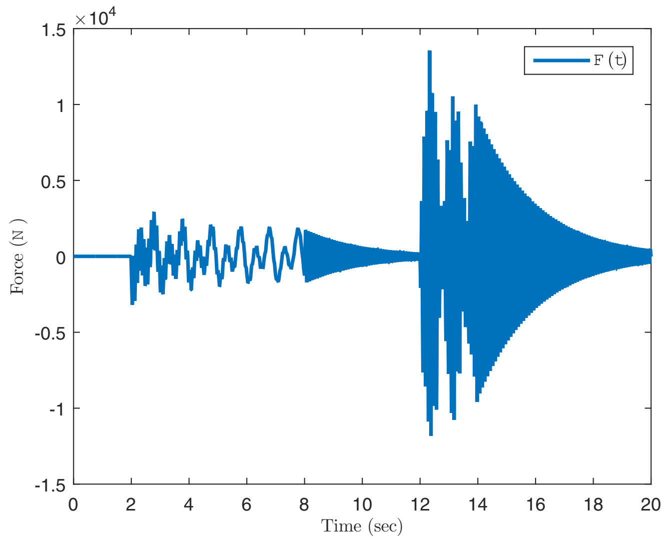
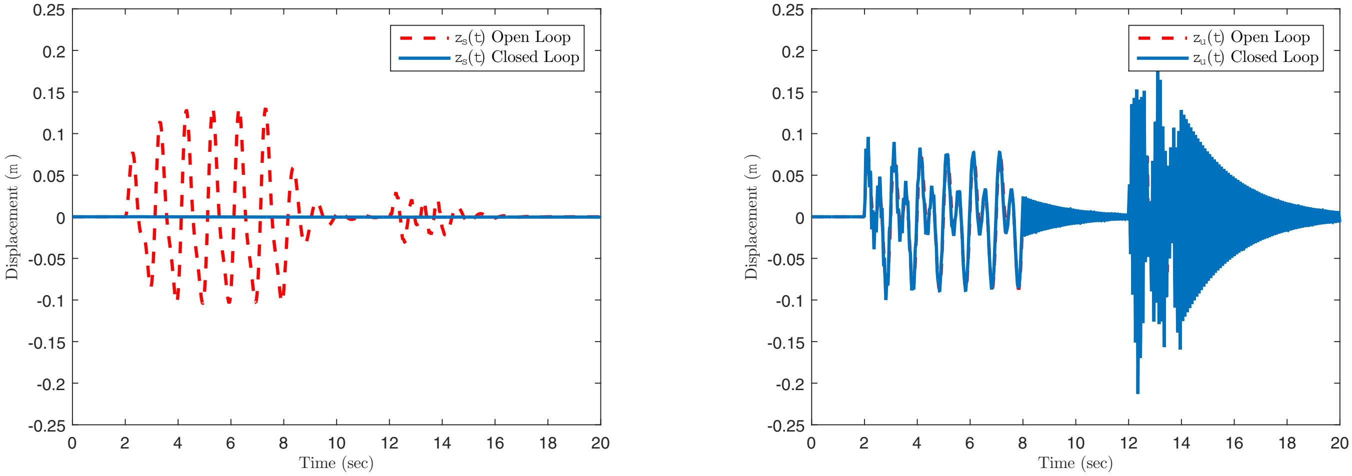


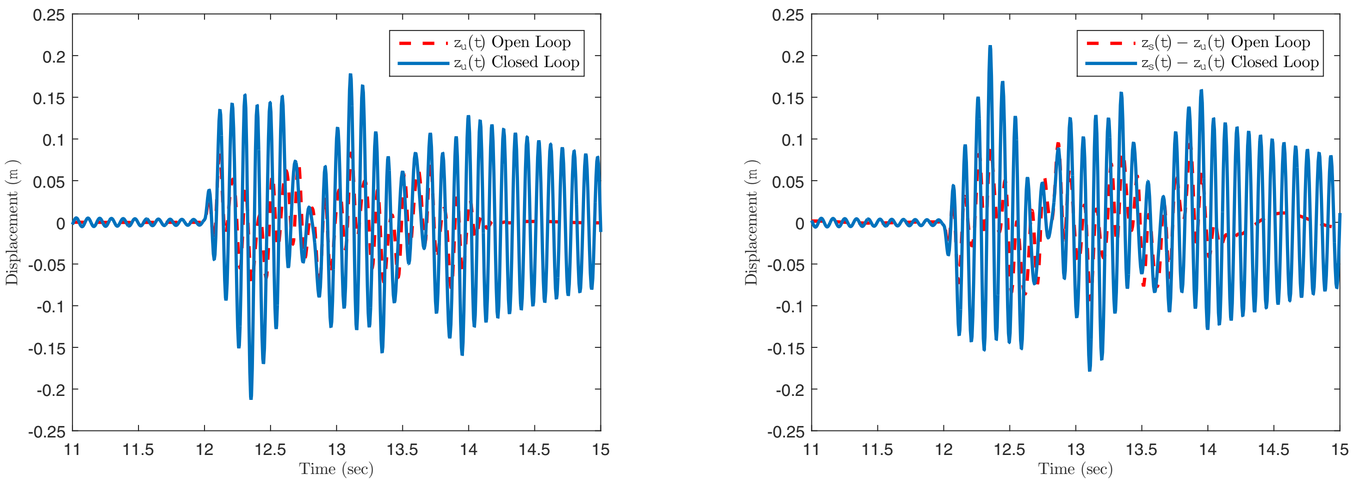
Fig. 3: The amount of force applied when the active suspension system is on.



(a) The displacement of the vehicle body (sprung mass) for the closed loop (solid) and the open loop case (dashed).

(b) The displacement of the wheel (unsprung mass) for the closed loop (solid) and the open loop case (dashed).

Fig. 4: The simulation results of displacement of the sprung mass (4a), the displacement of the unsprung mass (4b).



(a) The displacement of the wheel (unsprung mass) for the closed loop (solid) and the open loop case (dashed) when the road disturbance has high frequency signals.

(b) The relative displacement (change of suspension deflection) of the wheel (unsprung mass) and the vehicle body (sprung mass) for the closed loop (solid) and the open loop case (dashed) when the road disturbance has high frequency signals.

Fig. 5: The zoomed portion of the simulation for the unsprung mass position (5a) and the suspension deflection (5b) when the high frequency road disturbance is applied.

more rough and contains high frequency disturbances. A 20-second simulation is performed for two cases where the only passive suspension is available (open loop) and the active suspension system is on (closed loop). The results for two cases are given together in the figures. Since it is not easy to see the details of the high frequency responses, the results of the displacement of unsprung mass and the change in the suspension deflection (relative displacement of sprung and unsprung masses) are given in Figure 5.

As it is seen from Figure 2, the controller achieves to isolate the effect of the road conditions from the vehicle body and maintain the comfort of the passenger by keeping the vertical acceleration around zero. The amount of force that is applied to the system during the simulation is given in Figure 3. The displacements of the sprung and unsprung masses with their relative displacements are given in Figure 4. The displacement of vehicle body oscillates with a very low amplitude for all conditions when the active suspension is on. On the other hand, the displacement of the body oscillates with a high amplitude for low frequency disturbances when only passive suspension is available. As it is seen from Figures 4b, the displacement of the unsprung mass for the closed loop case is similar to the open loop case for low frequency disturbances. For the case where the frequency of the road disturbance is high, the controller makes the unsprung mass oscillate around a larger amplitude than the open loop case. This situation can be seen more clearly in Figure 5a. The displacement of the unsprung mass converges to zero when the road disturbance disappears and the boundedness of all signals are maintained as Theorem 1 stated.

VI. Conclusions

The problem of maintaining the comfort and safety of the vehicle body despite bad road condition is considered. An adaptive backstepping controller is designed for an active suspension system. A finite sum of sinusoidal functions with unknown frequencies, amplitudes and phases is employed as the model of a road disturbance. The disturbance is parameterized and an observer is developed by using the available state measurements. The observer allows to approach the problem as an adaptive control design. Finally, an adaptive controller is designed by using the backstepping technique. it is proven that the equilibrium of the closed loop system is stable and the vertical acceleration of the vehicle body tends to zero despite road disturbances. A road test that contains high and low frequency disturbances, is planned and the effectiveness of the controller is illustrated with a numerical simulation.

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