Multiuser Detection in Multipath Environments for Variable Spreading-Factor CDMA Systems

Adrian Boariu, *Member, IEEE*, and Rodger E. Ziemer, *Fellow, IEEE*

***Abstract*—A way to allow variable data rates in CDMA is by varying the spreading factor in accordance with the data rate requested by the user. A detection scheme suitable for multirate transmission is required in addition to one that combats multiple access interference of other users and intersymbol interference from multipath. In this letter, an energy-add multiuser detection method is combined with a scheme called Cholesky-iterative detection to cope with these challenges. It allows the users to be estimated on a symbol-by-symbol basis, making receiver complexity independent of data package length. Simulation results correspond closely to a single-user lower bound. The results also show that the bit-error probability performances of the various users for the Cholesky-iterative detector are closely clustered, while those of a previously published scheme, decorrelation feedback detection, are more widely spread.**

***Index Terms*—CDMA, intersymbol interference, multiuser detection, variable rate.**

I. Introduction

A FEATURE of third-generation (3G) mobile communication systems is the ability to handle multiple-rate users, thereby allowing mixed traffic, including voice, video, and data. Code-division multiple access (CDMA) is a leading candidate access method for 3G mobile radio communication systems. It is envisioned that some form of multiuser detection (MUD) will be required in order to provide acceptable system performance.

The pioneering work of Verdú [1] on optimum MUD resulted in a detector complexity growing exponentially with the number of users. This motivated extensive research on simpler, suboptimum detectors, which was conducted first for additive white Gaussian noise (AWGN) channels, both for synchronous and asynchronous CDMA systems. Some well-known multiuser detectors are the decorrelating receiver introduced by Lupas and Verdú [2], the multistage detector of Varanasi and Aazhang [3], the decorrelating decision-feedback (DDF) detector originated by Duel-Hallen [4], and the minimum mean-square-error detector studied by Xie, Short, and Rushforth [5]. Performance enhancements over these multiuser detectors can be obtained by using soft decision outputs, as in the suboptimum trellis-search detector introduced by Lei and Schlegel [6], or the multistage detector of Varanasi and Aazhang [3]. Also, combinations of soft output and soft bit detectors have been examined [7], [8]. Mitra [9] analyzed two multirate [multicode and variable spreading factor (SF)] access schemes in AWGN, providing asymptotic efficiency performance comparisons.

The multipath radio channel introduces intersymbol interference (ISI). Thus, the received energy of a given symbol spreads over several adjacent symbols. Whether the CDMA system is considered synchronous or asynchronous, an approach to designing a multiuser detector is to estimate an entire package of data in order to overcome the ISI. Klein and Baier [10] addressed MUD for synchronous systems using this approach. For asynchronous systems, Fawer and Aazhang [11] employed the multistage detector [3] for data estimates and the expectation–maximization algorithm for channel estimation. Their work has been extended to multirate CDMA systems by Hottinen, Holma, and Toskala [12].

This letter presents a multiuser detector for multirate CDMA systems that estimates the data on a symbol-by-symbol basis. Thus, the size of cross-correlation matrix employed in the detection process is proportional to the number of users and is independent of the data package length. Performance for multiple users with mixed rates is characterized through simulation and compared with an analytical single-user lower bound.

In this letter, Section II presents the system to be analyzed, Section III discusses the multiuser detector proposed for the system under consideration, simulation results are given in Section IV, and conclusions are summarized in Section V.

II. The System Model

Due to space constraints, please refer to [8] and [13] to aid in the understanding of the system model discussed later.

A. Communication System Model

The system to be analyzed (see [13, Fig. 1]) is typical for a reverse link CDMA system. Thus, we target the base station receiver. There are K users, the spreading code (SC) for the kth user is given by $\mathbf{c}_k = [c_{1,k} c_{2,k} \dots c_{Q,k}]$ with Q, the SC length, being a power of two, $Q = 2^m$, m integer. Thus, the SCs can be the extension by one bit of well-known sequences, e.g., Kasami, Gold, etc. The kth user is processed by a channel with the impulse response $\mathbf{h}_{c,k}$. All K users arrive synchronously at the receiver, although the asynchronous situation may be partially accommodated by setting some of the beginning values of the channel impulse response to zero with some performance degradation.

B. The Data Format

The system is assumed to have fixed chip duration T_c. A time slot interval includes a fixed number of chips dedicated for

Paper approved by G. E. Corazza, the Editor for Spread Spectrum of the IEEE Communications Society. Manuscript received May 12, 1999; revised January 24, 2000, August 15, 2000, January 18, 2001, and March 1, 2001. This work was supported by the BMDO and managed by the Office of Naval Research under Contract N00014-920-J01761/P0004.

A. Boariu is with Nokia Inc., Irving, TX 75039 USA (e-mail: adrian.boariu@nokia.com).

R. E. Ziemer is with the Electrical and Computer Engineering Department, University of Colorado, Colorado Springs, CO 80907 USA (e-mail: ziemer@eas.uccs.edu).

Publisher Item Identifier S 0090-6778(01)08158-2.

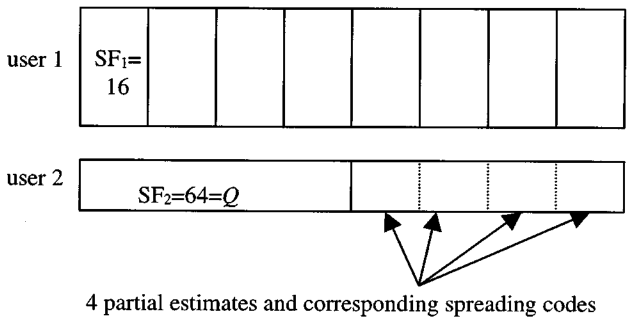

Fig. 1. Synchronous CDMA system with variable SF for two users.

transmission, proportional to the SC length $N_c = C_c Q$ where C_c is a constant. Also, a guard interval or pilots (known chip values) having the duration $N_g T_c$, N_g being a constant, is assumed available in each time slot/frame. Appropriate values of N_g will be addressed in Section III-A. The SF (number of chips per data symbol) for a given user varies according to the data rate; a high/low data rate implies a low/high SF value. The allowed SF values are powers of two and the maximum SF value is equal to the SC length, Q. The ratio Q/SF is assumed to be a power of two. Of course, there is a minimum value allowed for the SF. Data rates that are not multiples of two can be accommodated by combining the allowed rates in parallel, although this introduces a penalty in the peak-to-average power ratio due to summing of the codes that provide a multiple of the basic data rate. The adjustment of SF in accordance with the data rate has an impact in designing the MUD receiver. First, the users should be detected based on the minimum SF in the system (the maximum data rate). We call this minimum value the integration basis I_b. Thus, the system behaves as an equivalent one with SCs of length I_b but using $N_{\mathrm{sc}} = Q/I_b$ different SCs for each user. Soft output multiuser detectors are needed because partial estimates are employed for detection of the users with higher SFs. Second, the transmitted power of a given user should be increased in proportion to its data rate in order that all users have the same value of signal-to-noise ratio (SNR) per bit.

Consider the example in Fig. 1 with two users in the system, one having $\mathrm{SF}_1 = 16$ and the other having $\mathrm{SF}_2 = 64$, with user 1 transmitting with higher power. Assume, also, that $Q = 64$. The MUD scheme should use $\mathrm{SF}_1 = 16$ for detection, which means that $I_b = 16$. There are four estimates $(\mathrm{SF}_2/I_b = 4)$ that are employed for the detection of user 2, and soft outputs are required to combine separate detection statistics (e.g., if there are two hard partial estimates equal to 1 and the other two equal to -1, the final result is zero). Although the SC length is $Q = 64$, the system can be viewed, equivalently, as a system with *fixed* SF given by $I_b = 16$, but employing $N_{\mathrm{sc}} = Q/I_b = 4$ different SCs for each user cyclically.

C. The Channel Model

Recall that the received signal is detected based on the minimum SF value I_b available in the system. Thus, the system is considered as equivalent to one having fixed SF employing, cyclically (modulo N_{sc}), a number of $N_{\mathrm{sc}} = Q/I_b$ different SCs of length I_b for each user. Therefore, the SC for a given user, say k, can be represented as the concatenation of N_{sc} SCs of length I_b:

$$\begin{aligned}\mathbf{c}_k &= \left[\mathbf{c}_k^{[0]}\mathbf{c}_k^{[1]}\dots\mathbf{c}_k^{[N_{\mathrm{sc}}-1]}\right]\\ \mathbf{c}_k^{[j]} &= [c_{k,jI_b+1}c_{k,jI_b+2}\cdots c_{k,(j+1)I_b}]\\ \forall\, j &= 0,1,\dots,N_{\mathrm{sc}}-1.\end{aligned} \quad (1)$$

From now on $[\cdot]$ will denote the modulo N_{sc} operation, i.e., $[j] = j$ modulo N_{sc}.

We assume that the channel is estimated perfectly via a pilot sequence and for the kth user, at one time instant, is given by $\mathbf{h}_{c,k} = \left[h_{c,k,1}h_{c,k,2}\dots h_{c,k,W}\right]$, where W is the maximum delay spread of the channel. Finally, for the kth user, there are N_{sc} overall equivalent impulse responses $\mathbf{h}_k^{[j]} = \mathbf{h}_{c,k} * \mathbf{c}_k^{[j]}\ \forall\, j$, each having length $W + I_b - 1$, with $*$ denoting the convolution operator. Let $\mathbf{h}^{[j]} = \left[\mathbf{h}_1^{[j]\mathrm{T}}\mathbf{h}_2^{[j]\mathrm{T}}\dots\mathbf{h}_K^{[j]\mathrm{T}}\right]^{\mathrm{T}}\ \forall\, j$, where the superscript T is the transposition operation.

III. The Detector

A chain of equivalent transformations is applied to reduce solving the system under consideration—a multirate synchronous CDMA system in a multipath mobile channel—to a one-rate synchronous CDMA system in a flat fading channel. As already mentioned, the integration basis concept equivalently transforms a multirate system into a one-rate system. Section III-A describes a detection method that reduces the multipath problem to a single path one; thus the equation to be solved is the same as for a synchronous CDMA system in AWGN. For the final data estimates, the detector described in Section III-B is employed.

A. Energy-Add Multistage Detection Method

The energy-add detection method has been introduced in [13] and its applicability to a variable-SF synchronous CDMA system in multipath channel is straightforward using the integration basis concept. Also, the relations provided in [13, Sec. III-B] that describe the detection process can be easily adapted to include this cyclical model for the SCs (see Section II-B).

The initialization step required by the energy-add detector can be accomplished if the first symbol in the data stream (slot/frame) is without interference in the first detection stage. Here, the assumption that a time interval is available occupied only by a guard interval or pilots comes into play. Clearly, the duration of this time interval should be at least as large as the delay spread of the channel $N_g \geq W$ in order that data in the current time slot/frame not be influenced by the data transmitted in the previous time slot/frame.

Like any multistage detector, the first stage is crucial for providing reliable estimates for the next stages. For example, a channel power-delay profile that is exponentially decaying concentrates the signal energy at the beginning of the symbol, which is beneficial for the detection method. Also, less signal energy interferes with other symbols the larger the SF value.

Another undesirable effect for the variable-SF synchronous CDMA system related to this detection method is illustrated by

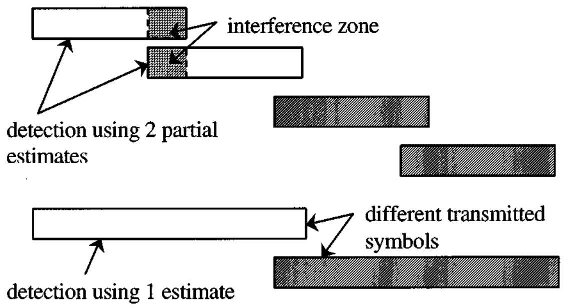

Fig. 2. Detection process showing two different integration bases.

the example shown in Fig. 2, where two different integration bases are used for detection. In the first case, a symbol is split in two and detected using two partial estimates. In the second case shown, the symbol is detected directly as one estimate. Note that signal energy embedded in the interference zone is, in the first case, interference energy that should be cancelled while, in the second case, it is useful energy. Both cases would perform similarly if the energy embedded in the interference zone were negligible (i.e., small delay spread W relative to integration basis I_b). Simulations verify this statement.

B. Cholesky-Iterative Detector

The energy-add detection method reduces the MUD problem of a multipath channel to an AWGN channel, and the relations given in [13, Sec. III-B] are generically given as

$$\mathbf{y}_i = \mathbf{R}\mathbf{x}_i + \mathbf{z}_i \tag{2}$$

where at the iI_bT_c time instant,

- $\mathbf{y}_i$ matched filter output vector after the interference cancellation is performed;
- $\mathbf{x}_i$ data vector including all users;
- $\mathbf{z}_i$ noise vector; and
- $\mathbf{R}$ correlation matrix.

Thus, any multiuser detector for the AWGN channel can be applied without modification to the multipath problem at hand in order to estimate the data of interest $\mathbf{x}_i$. However, as previously noted, soft output detectors should be used in combining the partial estimates of the low data-rate users.

The Cholesky-iterative detector discussed in [8] for the AWGN channel, as discussed earlier, can be applied to solve (2). The soft information employed in the detection process for each user is the log-likelihood ratio. Because soft information is associated with a given user, the partial estimates can be easily combined. Assume that for the kth user at time iI_bT_c the first partial value is estimated. Given the log-likelihood ratio denoted as $L_{k,i}$, the data decisions are based on $\mathrm{sign}(L_{k,i})$. The next partial value of the log-likelihood ratio for the same user at time $(i+1)I_bT_c$ is $L_{k,i+1}$ so the updated soft information is $L_{k,i} + L_{k,i+1}$ and the updated data decision is $\mathrm{sign}(L_{k,i} + L_{k,i+1})$.

Cholesky factorization of the correlation matrix $\mathbf{R}$ cannot be performed numerically for any number of users K in the system, and for any value of the integration basis I_b. Taking into account that one can write $\mathbf{R} = \mathbf{C}^*\mathbf{C}^{\mathrm{T}}$ (see [13, eq. (4)]), where $\mathbf{C}$ is a $K \times I_b$ matrix that has the first I_b columns of the $\mathbf{h}^{[i]}$ matrix, the following relation holds: $\mathrm{rank}(\mathbf{R}) \leq \mathrm{rank}(\mathbf{C}) \leq \min(I_b, K)$. Thus, Cholesky factorization over the $K \times K$ matrix $\mathbf{R}$ can be performed if $I_b \geq K$, i.e., $\mathbf{R}$ is full rank. To partially solve the problem of detecting the users when their number is greater than the minimum SF, the users are split into two disjoint sets and each set is detected successively. Each set of users behaves as interference for the other set. Thus, a parallel interference cancellation detector with MUD of each set is used for detection. Good results can be obtained if several iterations are performed (typically five or six). Using more then two sets is also a possibility.

Two intuitively obvious rules for efficient parallel interference cancellation are given below:

1) Create the sets of users such that the signal energy represented by each set is unbalanced. The set of users having greatest total signal energy can be detected first followed by the lower-energy set of users. This is beneficial at the start of the iteration process when the data estimates are less reliable.
2) In each set the number of users must be less than the minimum SF of the set, thereby making Cholesky factorization numerically possible. Recall that as the ratio of the number of users to minimum SF increases, the performance degrades.

According to the above rules, there is a tradeoff in splitting the users into sets; a large number of users in a set is less influenced by the interference due to the other set, but the ratio of the number of users to SF increases so that the MUD performance is affected.

IV. Simulation Results

Simulation results for the synchronous CDMA system with variable SF for constant chip duration are presented. The multiuser detector implemented is the one discussed in Section III. The SCs employed in the simulations are one-bit extensions of the $S(1)$ family [14] length 255, so $Q = 256$ chips. The total number of chips available for data per slot interval is $N_c = 2560$ chips and the slot duration is 0.625 ms. The SF ranges from 16 to 256. The carrier frequency is $f_0 = 2$ GHz and the speed of the mobiles is $\nu = 50$ km/h. The channel power-delay profile is uniformly distributed over $[0,\ 2\ \mu\mathrm{s}]$, which means that the maximum delay spread is equivalent to $W = 8$ chips, so the ISI is over two symbol intervals. The Doppler profile is the traditional Clarke spectrum. The system has a maximum load equivalent to 160 users with $\mathrm{SF} = 256$, the users having the same average SNR. We assume perfect knowledge of the channel, estimated via a pilot sequence, no coding scheme or antenna diversity employed, and perfect power control.

Derivation of an exact expression for the average bit error probability is difficult if all users are present in the system. For comparison of the simulation results with theory, a lower bound can be obtained if one user is considered active in the system. Assuming perfect knowledge of the channel, i.e., coherent detection, and with N_{sc} equivalent SCs, it can be proven that the lower bound is given by

$$\bar{P}_b = \frac{1}{N_{\mathrm{sc}}} \sum_{j=0}^{N_{\mathrm{sc}}-1} P_{\mathrm{low}}^{[j]} \tag{3}$$

with $P_{\mathrm{low}}^{[j]}$ given by [13, eq. (16)].

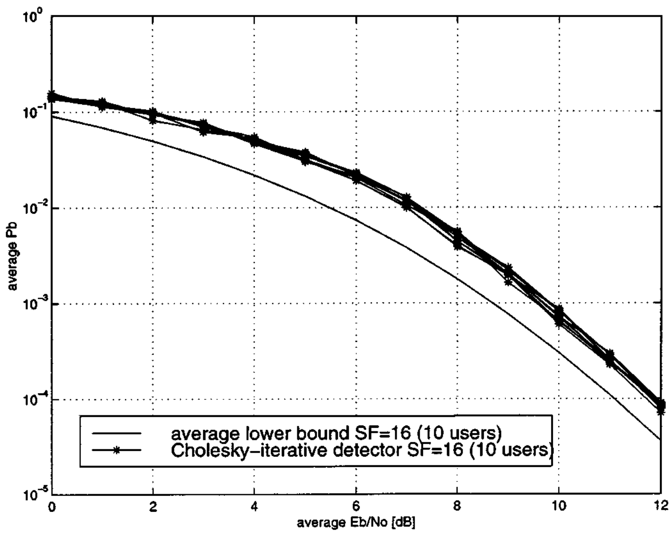

Fig. 3. Average P_b of each user for the Cholesky-iterative detector: 10 users with $\mathrm{SF} = 16$.

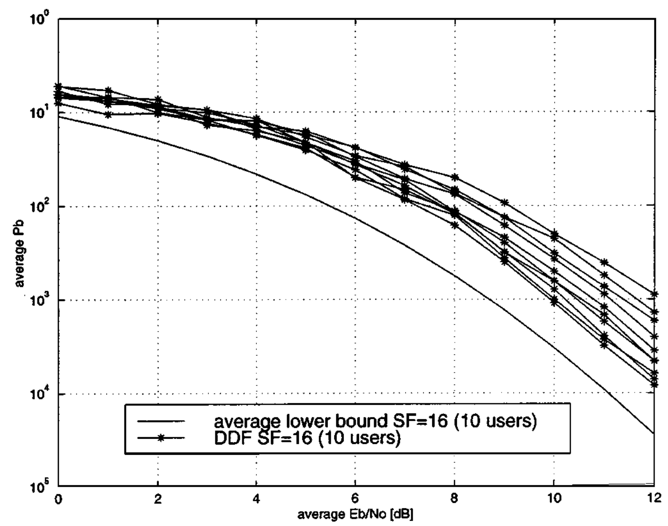

Fig. 4. Average P_b of each user for the DDF detector: 10 users with $\mathrm{SF} = 16$.

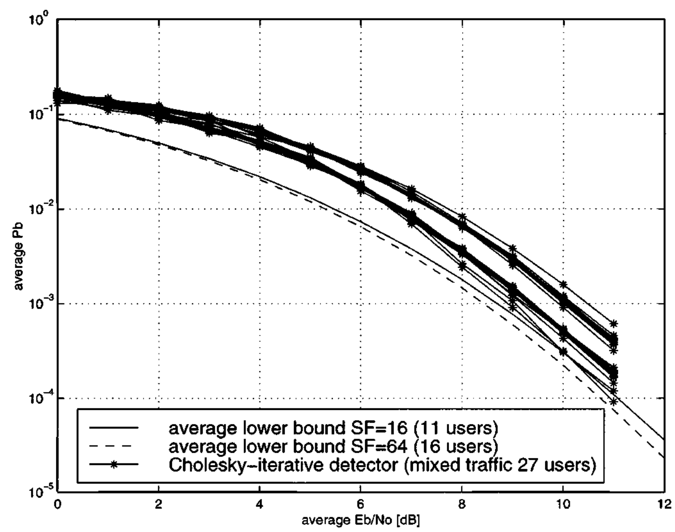

Fig. 5. Average P_b of each user for the Cholesky-iterative detector and mixed traffic: 1 user with $\mathrm{SF} = 16$, 10 users with $\mathrm{SF} = 32$, 14 users with $\mathrm{SF} = 64$, and 2 users with $\mathrm{SF} = 128$.

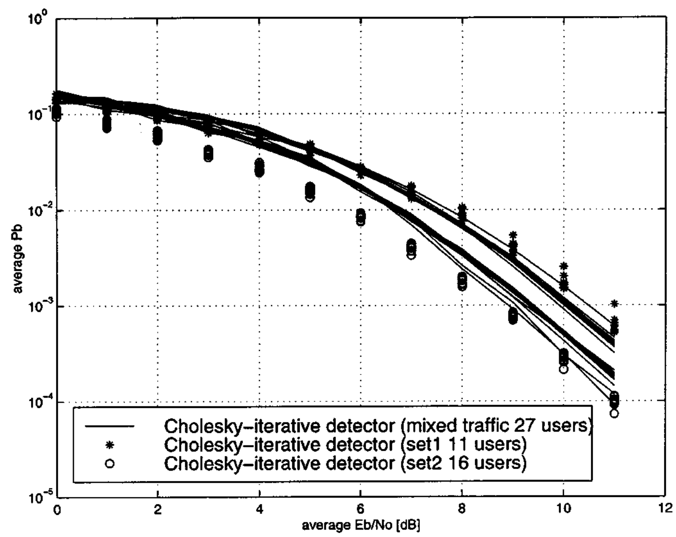

Fig. 6. Average P_b of each user for Cholesky-iterative detector: mixed traffic (27 users), *Set1* (11 users), and *Set2* (16 users).

Figs. 3 and 4 give simulation results for the Cholesky-iterative and the DDF detectors, respectively, for a system with 10 users with $\mathrm{SF} = 16$. It is unnecessary to split the users into sets in this case because the value of the SF is always greater than the number of users in the system. This allows Cholesky factorization to be performed. The improvement given by the Cholesky-iterative detector is evident; the average P_bs of the users are clustered compared with the DDF detector, where the user performances are widely spread. For high SNR, the user performances are less than 1 dB from the theoretical lower bound. Simulations for 80 users with $\mathrm{SF} = 128$ using a Cholesky-iterative detector have exhibited the same good behavior.

Simulation results for a system with mixed traffic (different SF for different active users) are given next. The system is the following: 1 user with $\mathrm{SF} = 16$, 10 users with $\mathrm{SF} = 32$, 14 users with $\mathrm{SF} = 64$, and 2 users with $\mathrm{SF} = 128$. The signal energy present in this system is the same as for a system with 156 users and $\mathrm{SF} = 256$, which is very close to the maximum load of 160 users with $\mathrm{SF} = 256$ assumed at the beginning of this section. There are 27 users in the system and the minimum SF is 16. Thus, Cholesky factorization cannot be performed. The users are split into two sets chosen as follows: *Set1* consists of the users with SF 16 and 32; *Set2* has all the other users. The parallel interference cancellation detector performs six iterations using *Set1* and *Set2*. This experimentally determined number gives zero errors in a noiseless channel.

The performance results are presented in Fig. 5. *Set2* uses $I_b = 64$ for detection, while *Set1* uses $I_b = 16$. For *Set1*, $K/I_b = 11/16$ (number of users to integration basis) which is larger than $K/I_b = 16/64$ of the *Set2*. Thus, better performance is expected for *Set2*, as verified in Fig. 5, although the performances of the sets are close. The explanation is the interdependence between detection of *Set1* and *Set2*. It can be observed better in Fig. 6. In addition to mixed traffic results, performance results for only one set present are given. *Set1* performs worse by itself than when present with *Set2* (for relatively high SNR) because of the interdependence between the estimates of the sets.

V. Conclusion

A synchronous CDMA system with variable SF and fixed chip duration in a delay-Doppler spread channel is simulated in this paper. As a MUD scheme, energy-add multistage detection is used with a Cholesky-iterative detector. The energy-add method is versatile in simultaneously handling MUD, ISI canceling, and variable data rates. Also, it allows detection on a symbol-by-symbol basis independently of the data package length. The average bit error probabilities of the users for the Cholesky-iterative detector are clustered and not as widely spread compared with the DDF detector. When the number of users is greater than the minimum SF in the system, splitting the users into sets and use of parallel interference cancellation allows Cholesky factorization. The tradeoff in dividing the users in various ways is also characterized.

Acknowledgment

We appreciate the helpful comments of the reviewers.

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