

Game Theory based Framework for Synthetic Aperture Radar Image De-noising and Change Detection

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Abstract

In this paper, we propose a novel game theory based framework for synthetic aperture radar image de-noising and segmentation based change detection. We find out the balance of the two aspects. The Nash game theory helps us find out the balance of segmentation accuracy and overall de-noising performance. In the de-noising part, we adopt the multi-diagonal matrix filter based algorithm to undertake the de-noising mission. Segmentation and change detection are finalized by the state-of-the-art methodologies in which the segmentation procedure transfers the difference map into the change map. As far as time-consuming is concerned, we compare the different methods for generating difference map. Fusion map is selected to be our difference map for image segmentation using fuzzy clustering. The experimental analysis shows the effectiveness and robustness of our propose framework with the comparison of other well-known change detection algorithms under the outer environment of noisy and noise-free. Finally, some potential optimization methods are discussed for future research.

Keywords: *Image Segmentation, Game Theory, Change Detection, Fuzzy Clustering, Synthetic Aperture Radar (SAR), Image De-noising*

1. Introduction

SAR (Synthetic Aperture Radar) image de-noising and segmentation based change detection (SCD) are two crucial research branches in the GIS (Geographic Information System) community. Compared with optical image, synthetic aperture radar (SAR) images have been the movement and the complicated and changeable weather interpretation of SAR image noise and difficult to handle. SCD includes using multi-temporal image to distinguish the regional land cover change which could be divided into two categories: changed or unchanged on the time scale.

As studied and well researched in the previous literatures, we can conclude to divide the unsupervised change detection algorithms into three chief steps: (1) Image preprocessing operation, this step is made up of some aspects such as image registration [1], illumination adjustment [2], image de-noising [3-8] and representation of compressed images [9-11]. (2) Generating the differencing map between images which are captured at the same places but on different time, popular differencing maps are: difference map, ration map and fusion map (such as wavelet fusion and pyramid fusion). (3) Image segmentation, in this step, differencing map is segmented to black and white, black represent the unchanged region and the white means the changed area. Fuzzy clustering is often used for segmentation. The noise in SAR images makes the process of detecting changes much more complex, we may be mysterious of the dot-like small areas especially when we are trying to distinguish the property of the areas, changed area or noise? Therefore, removing the noise in the initial step is crucial. As for the second step, the difference between the image need important role in change detection project, therefore, choose the appropriate method to generate different map becomes meaningful. Generating different maps of the most advanced algorithm can be divided into three types: difference

map, ratio map and fusion map. Although the multi-source image fusion method can take advantage, it consumes a lot of time for operation process, and makes them unable to cope with change detection project in real time condition. As discussed in the literature [12], in difference, change is minus the pixel intensity value measured by considering between two temporal image pixels whereas in ration, change is by applying the pixel than operators consider sequential images. However, in the case of SAR images, the log operator is often used instead of the subtraction operator since the image difference technology does not adapt to the SAR image and statistical non-robust to calibration error. The ratio operator can suppress the noise to a certain extent; however, it will enlarge the change information when the values in the two SAR images of the same position are all very small. Therefore, we adopt the wavelet fusion map to stand for our difference map. The third step is to use classification and segmentation methods to distinguish the map into binary image. The white represents the changed and the black unchanged, respectively. Traditional image segmentation methods includes: Modified artificial bee colony based computationally efficient multilevel thresholding [13], hybridization of fuzzy clustering and optimization algorithms [14], and context sensitive thresholding Technique [15].

In order to overcome the mentioned drawbacks, a novel game theory based framework for synthetic aperture radar image de-noising and segmentation based change detection is proposed in this paper. In general, de-noising and segmentation for change detection projects are separated, however, the efficiency and robustness of SCD algorithm is defined by both of the procedure. To optimize the traditional methodology, we combine the de-noising procedure and segmentation step based on Nash-game theory. We are intending to get the balance of segmentation and de-noising.

2. The Segmentation and De-noising

2.1. The Image Segmentation

The image segmentation problem can be generally dealt with through snakes [16] model, this model is popular adopted and deeply studied. To achieve the segmentation goal, the initiation of the contour ζ_0 is conducted to embed it into the contour of the objects. The evolution of the initial curve is designed so that, "energy" (local) minimum looking for contour. It should be paid attention that the energy of the snake is not inherence, whereas it depends on the parameterized contour. Some state-of-the-art research has found optimized and modified solution for the problem, such as geodesic model is a good example.

In this paper, we adopt the methodology of Mumford and Shah's research [17]. In their paper, the ultimate goal is to get the minimization of the formula 1.

$$\mathfrak{S}(I, \xi) = \int_{\Omega} (I - I_0)^2 d\vec{x} + \mu \int_{\Omega \setminus \xi} |\nabla I|^2 d\vec{x} + \nu |\xi| \quad (1)$$

In the formula 1, $|\xi|$ stands for the total length of the set ξ of discontinuities of I , and μ, ν are positive constants. The $\int_{\Omega} (I - I_0)^2 d\vec{x}$ represents the term of data fidelity, the $\mu \int_{\Omega \setminus \xi} |\nabla I|^2 d\vec{x}$ denotes the regularization in the regions of Ω , $\nu |\xi|$ is the regularization of the set ξ 's property of discontinuous. The formula 1 involves the problem of minimizing two parts: $I : \Omega \rightarrow \square$ and a set that $\xi \subset \Omega$.

We are able to conclude that the formula 1 is not convex, therefore, the existence of minimized solution for the function is not ensured. Nevertheless, Mumford and Shah proved the existence of the solution. There is a large amount of literature dealing with other forms of the Mumford-Shah energy, one of the solutions is to make identification of

the ξ denoted as the discontinuous set with the jump sets S_I of I to solve the issue. The procedure can be formulated as:

$$\mathfrak{R}(I) = \int_{\Omega} (I - I_0)^2 d\vec{x} + \mu \int_{\Omega} |\nabla I|^2 d\vec{x} + \nu H(S_I) \quad (2)$$

In the formula, Hausdorff measure $H(S_I)$ is adopted to denote the total length of ξ . Based on this type of approximations, the numerical solutions can be achieved by using a finite difference scheme or finite element method. Till now, we have introduced the basic knowledge for image segmentation which will be used later.

2.2. The Image De-noising with Multi-diagonal Matrix Filter

Image de-noising as a kind of restoration is becoming a hotspot in image processing community, to deal with this issue, a large number of state-of-the-art algorithms have been proposed. In order to generate the mathematical formulation of the de-noising issue, we firstly define an image in a mathematical approach. A mathematical image is a real-valued function which assigns to any point $\vec{x} = (x, y) \in \Omega$, where Ω is a subset of \mathbb{R}^2 , the gray level (for color images, there are three channels) is defined as $I(\vec{x})$. Define the image with noise component as: $I_0 : \Omega \rightarrow \mathbb{R}$. The restoration procedure of images can be then formulated as finding the minimization of the following formula:

$$\varepsilon_1(I) = \int_{\Omega} (I - I_0)^2 d\vec{x} + \mu \int_{\Omega} |\nabla I|^2 d\vec{x} \quad (3)$$

In the formula, μ represents the controlling parameter defined in [6] and the minimize is sought in the Sobolev space $H^1(\Omega)$. The prior satisfactory for the minimum is the Euler-Lagrange equation:

$$I - I_0 - \mu \Delta I = 0 \quad , \text{in } \Omega \quad (4)$$

In addition, we introduce a novel matrix filter based approach. In the approach, a squared symmetric matrix is defined and denoted as $\psi_{k \times k}(\xi)$. The property of the matrix is that on the ξ diagonals the value of the unit will be 1 and the others to be 0. The definition is shown in formula 5.

$$\psi_{k \times k}(\xi) = \begin{cases} 1 & |i - j| \leq \xi \\ 0 & \text{others} \end{cases} \quad i, j = 1, 2, \dots, k \quad (5)$$

As an explanation, $\psi_{7 \times 7}(5)$ is shown in the following equation 6. When we take into consideration of the $m \times n$ matrix of binary images, $A_{m \times n}$ and $\psi_{n \times n}(\xi)$ could be defined following the formula 6.

$$\psi_{7 \times 7}(5) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (6)$$

$$A_{m \times n} = \begin{bmatrix} a_{11} & \dots & a_{1j-1} & a_{1j} & a_{1j+1} & \dots & a_{1n} \\ \vdots & & & & & & \vdots \\ a_{i-11} & \dots & a_{i-1j-1} & a_{i-1j} & a_{i-1j+1} & \dots & a_{i-1n} \\ a_{i1} & \dots & a_{ij-1} & a_{ij} & a_{ij+1} & \dots & a_{in} \\ a_{i+11} & \dots & a_{i+1j-1} & a_{i+1j} & a_{i+1j+1} & \dots & a_{i+1n} \\ \vdots & & & & & & \vdots \\ a_{m1} & \dots & a_{mj-1} & a_{mj} & a_{mj+1} & \dots & a_{mn} \end{bmatrix}, \psi_{n \times n}(\xi) = \begin{bmatrix} \frac{\xi+1}{2} & & & & & & \\ 1 \dots 1 & 0 \dots & & & & & 0 \\ \vdots & & & & & & \vdots \\ 0 \dots & \dots & 0 & \underbrace{1 \dots 1}_{\xi} & 0 \dots & 0 & \\ \vdots & & & & & & \vdots \\ 0 \dots & & & & & \frac{\xi+1}{2} & \\ & & & & & 1 \dots 1 & \end{bmatrix}$$

The Figure 1 shows the flowchart of our proposed matrix filter.

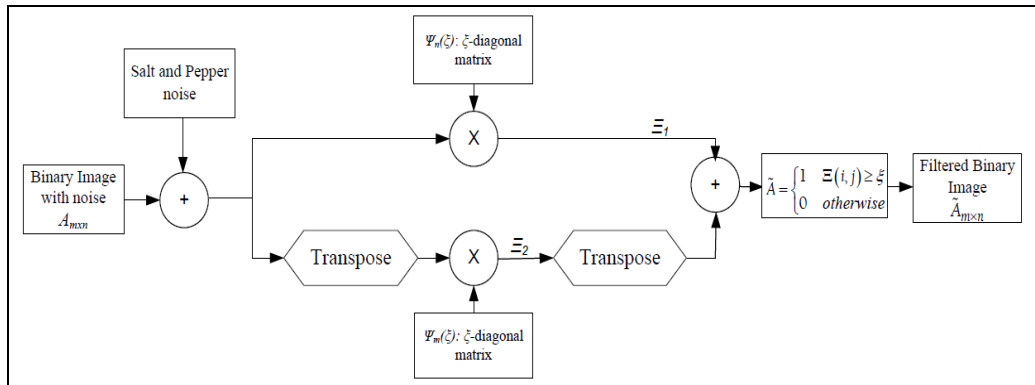


Figure 1. The Diagram of Proposed Filter

3. Balance Segmentation and De-noising using Game Theory

3.1. The Theoretical Analysis

In general, game theory is a framework for optimizing the multi-objective and multi-disciplinary problems. It substitutes the notion of optimum, irrelevant when more than one criteria is under consideration with the introduction of equilibrium. There are many definitions of game equilibria, depending on the nature of the game and the most known is the Nash equilibrium, classically accepted as solution to static with complete information games. Game theory was originally used in economic field for a long time, let's just mentioned the application of distributed control system by the partial differential equation is a novel field. Game theory is very suitable for processing against standards (including a zero-sum game is the most know). We consider that there are two objective function on behalf of the two players in the game theory: \mathfrak{J}_1 and \mathfrak{J}_2 defined over the same space of control variables defined as x . We can find out the detailed discussions and theoretical analysis of x in the literature [18].

In this step, we propose a novel framework based on game theory to join the segmentation and de-noising problem together. Inage de-noising is defined to be the player 1 and the de-noising is defined to be player 2. It provides us with the intuition that the de-noising could generally take in control of the intensity I and the segmentation could control the discontinuity set ξ .

To be more specific, we could find that the player 1's ultimate goal in to obtain the global minimization of the following formula:

$$\mathfrak{J}_1(I) = \int_{\Omega} (I - I_0)^2 d\bar{x} + \mu \int_{\Omega \setminus \xi} |\nabla I|^2 d\bar{x} \quad (7)$$

On the contrary, the player 2's ultimate goal in to obtain the global minimization of the following formula:

$$\mathfrak{S}_2(I, \xi) = \sum_{i=1}^k \int_{\Omega} (I - I_0)^2 dx + v |\xi| \quad (8)$$

$$\text{where } I_i = \frac{1}{|\Omega_i|} \int_{|\Omega_i|} I(x) dx$$

To theoretically analyze our proposed method, we design and implement a two-player static of complete information game. In the game, de-noising part acts as the first player while segmentation represents for the other. The player one tries to minimize the cost function $\mathfrak{S}_1(I, \xi)$ on the intensity field I , the second player tries minimizing the function $\mathfrak{S}_2(I, \xi)$ on the discontinuity set ξ . Under this circumstance, dealing with this game falls to finding a Nash equilibrium which is defined as (I^*, ξ^*) , so that:

$$\begin{cases} I^* = \arg \min_I \mathfrak{S}_1(I, \xi^*) \\ \xi^* = \arg \min_{\xi} \mathfrak{S}_2(I^*, \xi) \end{cases} \quad (9)$$

In the formula 9, we can obtain the solution of I^* in the Sobolev space $H^1(\Omega \setminus \xi^*)$ and ξ^* is considered to be the set of the union of curves made of a finite set.

3.2. The General Steps of Proposed Method

It could be revealed that even if each of the two proposed minimization issue has specific solution, it is still not guaranteed the existence and solvability of a Nash equilibrium. Theoretically, we can put the Nash equilibrium problem as a fixed point. Traditional and classical Nash theory assumes that the standard convex, apparently the first, and may be considered under the framework of convex relaxation, such as literature [16]. At the same time, under the assumption that the policy set is compact in some topology, in addition, the relative standard lower semicontinuous the same topological structure. We suppose the existence and solvability of the Nash equilibrium in this research, iterative methods are adopted by us to find the solution. The description of the steps are shown as the following.

Step1: Initial guess: $s^{(0)} = (I^{(0)}, \xi^{(0)})$. Set $k = 0$.

Step2: Repeat

$$\text{Step3: } \begin{cases} \bar{I}^{(k)} = \arg \min_I \mathfrak{S}_1(I, \xi^{(k)}) \\ \bar{\xi}^{(k)} = \arg \min_{\xi} \mathfrak{S}_2(I^{(k)}, \xi) \end{cases}$$

$$\text{Step4: } s^{(k+1)} = (I^{(k+1)}, \xi^{(k+1)}) = \tau s^{(k)} + (1 - \tau)(\bar{I}^{(k)}, \bar{\xi}^{(k)}) \quad \{0 < \tau < 1\}$$

Step5: $k = k + 1$

Step6: **Until** $s^{(k)}$ converges.

4. Comparison of Three Difference Maps

We are able to obtain the difference map in three approaches. Firstly, we can directly minus one image from the other one; secondly, ratio image can also be treated as the differencing map; finally, we adopt the wavelet fusion technique to capture the map. In the following discussion, We consider x_1 and x_2 are co-registered intensity leded SAR images which are acquired at the same geographical position and similar atmosphere condition but at difference time t_1 and t_2 .

4.1. The Minus Map

Minus map can be denoted as the expression $x_m(i, j)$. The geological location stands for the (i, j) . Therefore, the definition of minus map is shown in formula 10:

$$x_m(i, j) = |X_1 - X_2| \quad (10)$$

4.2. The Ratio Map

Ratio map can be defined in the formula 11 and 12:

$$x_{r1}(i, j) = \left| \frac{X_1}{X_2 + \xi} \right| \quad (11)$$

$$x_{r2}(i, j) = \left| \log(X_2 / X_1) \right| = \left| \log X_2 - \log X_1 \right| \quad (12)$$

In the formula 11, we introduce ξ to the denominator to avoid the zero-denominator error. Evidently, we can observe that the ration map will be more time-consuming.

4.3. The Fusion Map

We use the wavelet fusion to generate the revised fusion map. The Figure 2 shows the basic flowchart of wavelet image fusion.

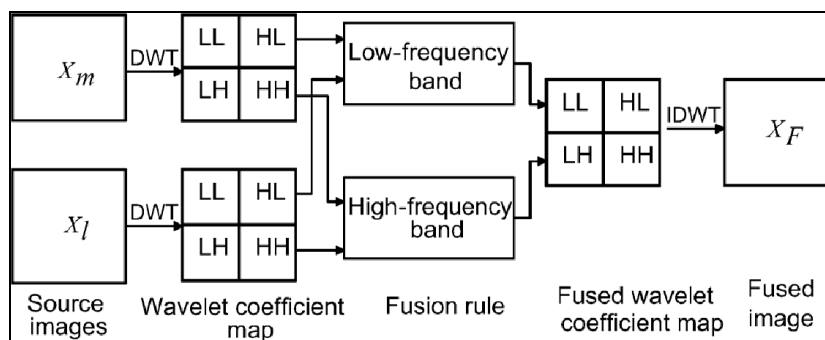


Figure 2. The Wavelet Fusion

The basic steps of generating fusion map are: First, we calculate the two source image DWT and obtain multi-resolution decomposition of each source image. We then corresponding approximation coefficients and detail sub-band decomposition of source images fusion in wavelet transform domain fusion rules using the development. In particular, the use of different fusion rules fusion wavelet coefficients of low frequency and high frequency band, respectively. Finally, the application of inverse DWT fusion for fusion results multiresolution image representation. The fusion map can not only preserve some tiny feature but also easier for clustering, therefore, we choose fusion map to be our differencing map.

5. Experiment and Analysis

In order to test the effectiveness or our proposed game theory based change detection algorithm, we use Kappa coefficient and some practical parameters to measure the overall performance compared with other state-of-the art algorithms. The dataset for experiment is two SAR images acquired by the European Remote Sensing 2 satellite SAR sensor over an area near the city of Bern, Switzerland, in April and May 1999, respectively [see Figure 3(a) and (b)]. The ground truth information is provided in Figure 3(c), the white area represents the real changed region while the black stands for the unchanged.

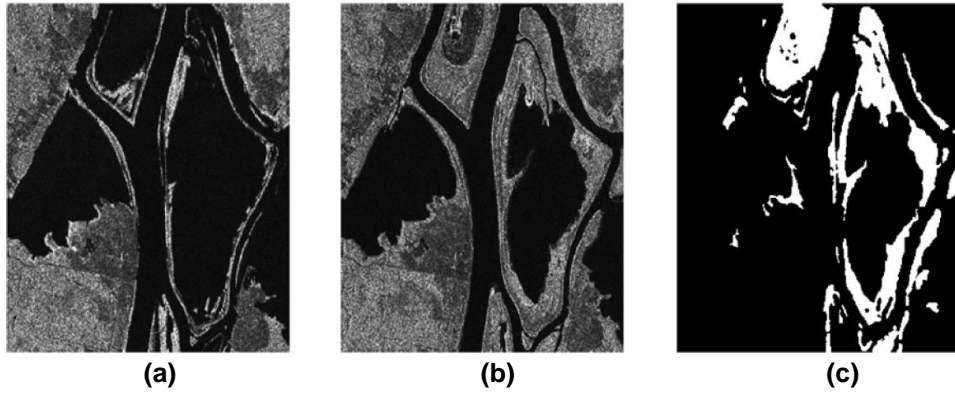


Figure 3. (a) SAR Image on Time 1 (b) SAR Image on Time 3 (c) Ground Truth

On the purpose of conducting fair experiment, we compare our proposed method to the state-of-the-art algorithms. They are listed as the following: 1) EM-based approach, 2) FCM-based approach, 3) FLICM-based approach, 4) RFLICM-based approach. We also introduce the evaluation matrix illustrated in the Table one th evaluate the effectiveness and correctness of our proposed method.

Table 1. Evaluation Matrix

	Ground Truth		
	<i>Unchanged</i>	<i>Changed</i>	<i>Sum</i>
<i>Unchanged</i>	X_{00}	X_{01}	X_{0+}
<i>Changed</i>	X_{10}	X_{11}	X_{1+}
<i>Sum</i>	X_{+0}	X_{+1}	X_{all}

Furthermore, we introduce False alarms, Missing alarms, Total error rate together with Kappa to evaluate the effectiveness of our method, they are defined as:

$$P_{False} = \frac{X_{10}}{X_{+0}} \times 100\% \quad (13)$$

$$P_{Miss} = \frac{X_{01}}{X_{+1}} \times 100\% \quad (14)$$

$$P_{Total} = \frac{X_{01} + X_{10}}{X_{all}} \times 100\% \quad (15)$$

$$Kappa = \frac{X_{all} (X_{00} + X_{11}) - \sum (X_{i+} \times X_{+i})}{X_{all}^2 - \sum (X_{i+} \times X_{+i})} \quad (16)$$

The Table 2 shows the Kappa evaluation standard. Generally, if the Kappa coefficient of an algorithm is larger than 0.75, we will regard the method to be robust and accurate. On the contrary, if the kappa coefficient is less than 0.5, we will conclude that the method is poor and not suitable for real applications. The Table 2 discusses the layers and ranks of Kappa coefficient, we can therefore find out the criterial for judging an algorithm. The Figure 4 shows the experimental result and Table 3 gives the quantitative comparison.

Table 2. The Kappa Coefficient

<i>Kappa Coefficient</i>	<i>Objective Quality</i>
<0	Worst
0.0-0.2	Poor
0.2-0.4	Reasonable
0.4-0.6	Good
0.6-0.8	Very Good
0.8-1.0	Excellent

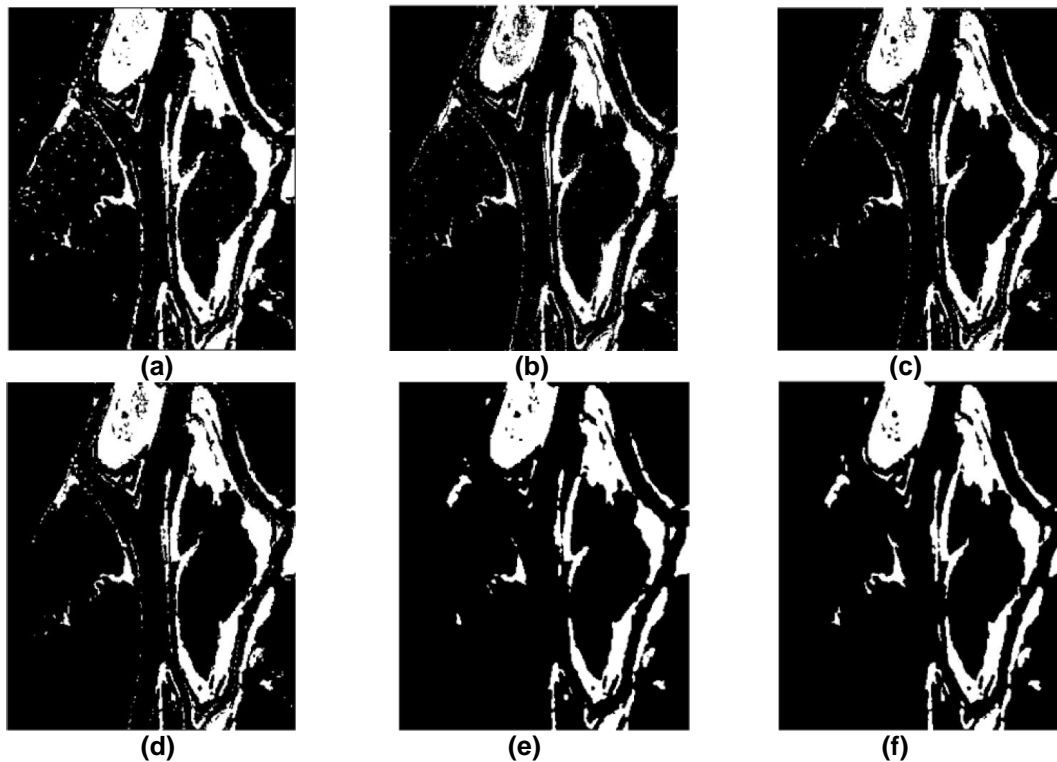


Figure 4. (a) EM-based (b) FCM-based (c) FLICM-based (d) RFLICM-based (e) Ours (f) Ground Truth

Table 3. The Quantitative Comparison

	<i>EM</i>	<i>FCM</i>	<i>FLICM</i>	<i>RFLICM</i>	<i>OURS</i>
<i>False(%)</i>	49.2	16.37	18.27	15.53	3.11
<i>Miss(%)</i>	23.9	13.89	17.52	13.27	4.14
<i>Total(%)</i>	38.5	15.59	17.91	14.12	3.75
<i>Kappa</i>	0.212	0.591	0.460	0.579	0.801

6. Conclusion and Summary

In this paper, we propose a novel game theory based framework for synthetic aperture radar (SAR) image de-noising and segmentation based change detection. We find out the balance of the two aspects. The Nash game theory helps us find out the balance of

segmentation accuracy and overall de-noising performance. The experimental result shows the robustness of our approach, in the future, we plan to use more image fusion techniques to do more research.

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