International Journal of Research and Applications
April - June © 2014 Transactions; 1(2): 63-68
ISSN (online): 2349-0020
http://ijraonline.com

ELECTRONICS RESEARCH ARTICLE

Reducing of EMSE Using Affine Combination of Adaptive Filters

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ABSTRACT

We propose an adaptive affine combination of two adaptive filters in that combination select one is fast and one slow. In this paper we proposed LMS, NMLS and CMA algorithm technique. By using these algorithm techniques we calculate the mixing parameter (η) at every instant and the performance of the measurement parameter that is excess mean square error EMSE is varied with the step size (μ) according to the variation the step size we achieve the good convergence rate and its adaptation is also taken into account in the transient analysis and steady state analysis. The proposed combination should acquire the good convergence properties for all kinds of stationary and non stationary environments. The resulting combination should profit than single filter technique.

Keywords: Adaptive filters, transient analysis, steady state analysis, EMSE, CMA.

I. INTRODUCTION

One of the most popular algorithms for adaptive filtering is the LMS algorithm. LMS adjusts the adaptive filter weights and modifying them by an amount proportional to the instantaneous estimate of the gradient of the error surface [1]. It neither requires correlation function and matrix inversions method, which makes it simple and easy when compared to other algorithms.

Minimization of MSE is achieved due to the iterative procedure incorporated in it[2].To make successive corrections in the direction of negative of the gradient vector of it .The adaptive filters that exhibit good convergence properties in stationary environments[3], do not necessarily present good tracking performance in non stationary environment. for combination of algorithms we calculate the convergence rate and EMSE in adaptive filters.

The rest of the paper is organized as follows In the next section II, we describe the affine combination of two adaptive filters, In Section III, analytical expressions for the optimum mixing parameter and the optimum EMSE, Section IV will discuss about the steady-state analysis two LMS filters, two NLMS filters and two CMA equalizers. Section V is about simulation results and discussion, some conclusion is given in Section VI.

II. COMBINATION OF SUPERVISED ALGORITHMS

The linear combination of two supervised adaptive filters is depicted in Fig.1, where the filter weights are adjusted to minimize the mean-square error cost function, obtaining at the output an estimate of the given "desired signal" d(n). The output of the overall filter is given by

$$y(n) = \eta(n) y_1(n) + [1-\eta(n)] y_2(n) \quad \text{.................(1)}$$

where $\eta(n)$ is the mixing parameter and $y_i(n)$, i=1,2 are the outputs of two transversal filters, i.e., $y_i(n) = u^T(n) w_i(n-1)$.The superscript T denotes transposition, $w_i(n-1)$, i = 1, 2 represent the length-M coefficient column-vectors characterizing the component filters, and $u(n)$ is their common input regressor column-vector

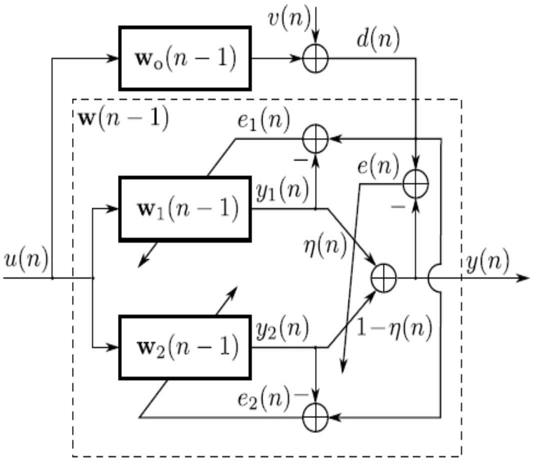

Fig1. Linear combination of two supervised adaptive filters

We focus on the affine combination of two algorithms of the following general class

$$w_i(n) = w_i(n-1) + \rho_i(n)u(n)e_i(n) \quad \text{……………..(2)}$$

where $\rho_i(n)$ is a step-size and $e_i(n)$ is the estimation error. Many algorithms can be written, by proper choices of $\rho_i(n)$ and $e_i(n)$. In supervised adaptive filtering, a "desired signal" $d(n)$ is available such

$$e_i(n) = d(n) - y_i(n) \quad \text{………………..(3)}$$

and a linear regression model holds, i.e.,

$$d(n) = u^T(n)w_o(n-1) + v(n) \quad \text{…………..(4)}$$

with $w_o(n-1)$ being the time-variant optimal solution and $v(n)$ a zero-mean random process uncorrelated with $u(n)$, whose variance is denoted by $\sigma_v^2 = E\{v^2(n)\}$ the sequences $\{u(n)\}$ and $\{v(n)\}$ are assumed stationary and that $v(n)$ is independent of $u(n)$ (not only uncorrelated). Defining the weight error Vectors $\tilde{w}_i(n) = w_o(n) - w_i(n)$ the a priori errors

$$e_{a,i}(n) = u^T(n)\tilde{w}_i(n-1) \quad \text{………………..(5)}$$

An important consequence of this model is that $v(k)$ will be independent of all $w_i(j)$, $\tilde{w}_i(j)$ and $e_{a,i}(k)$, i = 1,2 j < k.

For any particular time instant k. considering the combination of two LMS filters and the minimization of the overall instantaneous square error $e^2(n) = [d(n) - y(n)]^2$ proposed the following gradient based algorithm

$$\eta(n+1) = \eta(n) + \mu_n e(n)[y_1(n) - y_2(n)] \text{………..(6)}$$

To obtain a tradeoff between stability of this recursion and the algorithm's tracking capability in the initial phase of adaptation, $\eta(n) <= 1$

A) Combination of blind algorithms

It is a simplified communications system with a combination of two blind equalizers. In this case, the signal $a(n)$, assumed i.i.d. (independent and identically distributed) and non Gaussian, is transmitted through an unknown channel, whose model is constituted by an FIR filter and additive white Gaussian noise..

Algorithms based on the constant modulus cost function define the "estimation error" as

$$e_i(n) = [r - y_i^2(n)]y_i(n) \quad \text{………………(7)}$$

where $r = E\{a^4(n)\} / E\{a^2(n)\}$.

These assumptions were used in[10] obtain simple linear models that capture the behavior of CMA close to an optimum solution.

Thus, (7) was approximated by

$e_i(n) \approx \gamma(n)e_{a,i}(n) + \beta(n$ ….(8)

where

$\gamma(n) = 3a^2(nt_d)r$ ………..(9)

and

$\beta(n) = \gamma a(nt_d)a^3(nt_d)$ …....(10)

The variable $\beta(n)$ is identically zero for constant-modulus constellations, so the variability in the modulus of $a(n)$ (as measured by $\beta(n)$) plays the role of measurement noise for constant-modulus based algorithms. Model [5] was proposed to study convex combinations of constant-modulus based algorithms and extended to obtain explicit stability conditions for CMA to update the mixing parameter in order to combine two CMA equalizers, we could use a gradient rule to minimize the instantaneous constant-modulus cost $\tilde{J}_{cm}(n) = [r - y^2(n)]^2$ as considered in the convex combination.. Thus, we propose a stochastic gradient algorithm to minimize the instantaneous square decision error $\tilde{J}_d(n) = e_d^2(n)$,

where

$$e_d(n) = \hat{\hat{a}}(n - \tau_d) - y(n) \quad and \ \hat{a}(n - \tau_d)$$

the estimate of the transmitted signal at the output of the decision device. This results in the following update equation

$$\eta(n+1) = \eta(n) + \mu_\eta e_d(n)[y_1(n) - y_2(n)] \quad \text{....(11)}$$

In the presence of noise and/or when both component filters are far from convergence. assume $e_d(n) \approx e_a(n)$

B) A common formulation

Comparing (8) to (5), we can write the following general expression

$$e_i(n)=k(n)e_{a,i}(n)+\varphi(n) \quad ,i=1,2.. \quad \text{......(12)}$$

where $\kappa = 1$ and $\varphi(n) = v(n)$ for a supervised algorithm or $\kappa(n) = \gamma(n)$ and $\varphi(n) = \beta(n)$ for a blind one. In both cases $E\{\eta(n)\} = 0$. This model also holds for the overall scheme, i.e.,

$$e(n)=\kappa(n)e_a(n)+\eta(n) \quad \text{13)}$$

where e(n) represents the error of the combined filter: $e(n)=d(n)-y(n)$ for supervised algorithms or $e(n) = [r - y^2(n)]y(n)$ for constant-modulus-based algorithms, and ea(n) is the a priori error of the overall scheme. It should be noticed that (12) and (13) are approximations in the blind case. For the sake of simplicity, we use the equality sign here and in the expressions derived from (12) and (13).

The supervised LMS and NLMS algorithms and the blind CMA employ the step-sizes ϱi(n)and the estimation errors $e_i(n)$,where Q_n is a regularization factor and k. k represents the Euclidean norm. The models for the errors $e_i(n)$ of these algorithms are also shown in this table for convenient reference. The step-size interval which ensures the convergence and stability is different for each algorithm. For the LMS and NLMS algorithms, the step-size intervals are well-known in the literature whereas for CMA, the derivation of this interval was shown recently.

Using equation model (5) in the supervised case, and the fact that $e_d(n) \approx e_a(n)$ in the blind case, we can write a general expression for updating the mixing parameter, i.e.,

$$\eta(n+1)=\eta(n)+\mu_n e_g(n)[y_1(n)y_2(n)] \text{.........(14)}$$

where

$$e_g(n) = e_a(n) + b(n) \quad \text{......(15)}$$

and $b(n) = v(n)$ for the combination of supervised algorithms or $b(n) = 0$ for the combination of constant-modulus-based algorithms. In both cases, $\eta(n)$ is constrained to be less than or equal to 1 for all n . Algorithm (14) is denoted here by η-LMS.

III. THE OPTIMUM MIXING PARAMETER AND EMSE

An analytical expression for the optimum mixing parameter $\eta_0(n)$ can be obtained equating to zero the expected value of the gradient used to update $\eta(n)$ in (14), i.e., The error $e_g(n)$ in (16) can be rewritten as a function of the a priori errors ea,i(n),

i = 1, 2, as follows. Using (1), (12), and (13), the a priori error $e_a(n)$ of the overall scheme can be written as

$$\begin{aligned} e_a(n) &= \eta(n)e_{a,1}(n) + [1-\eta(n)]e_{a,2}(n) \\ &= e_{a,2}(n) + \eta(n)[e_{a,1}(n) - e_{a,2}(2)] \end{aligned} \quad \text{...(17)}$$

Replacing (17) in (15), and remarking that $y_1(n)-y_2(n)=e_{a,1}(n)-e_{a,2}(n)$ and

$$\begin{aligned} &E\{e_{a,2}^2(n) - e_{a,1}(n)e_{a,2}(n)\} \\ &- E\{\eta_o(n)[e_{a,2}(n) - e_{a,1}(n)]^2\} \\ &+ E\{b(n)[e_{a,2}(n) - e_{a,1}(n)]\} = 0 \end{aligned} \quad \text{...(18)}$$

In the blind case, $b(n) = 0$ and in the supervised case, $b(n) = v(n)$, which is assumed independent of $e_{a,i}(n)$, i = 1, 2. Hence, in both cases the third term on the L.H.S. of (18) is equal to zero.

To proceed, we remark that the EMSE of the component filters and the cross-EMSE can be calculated (18), respectively as

$$\varsigma_{ii}(n) \overset{\Delta}{=} E\{e_{a,i}^2(n)\}, \quad i=1,2 \quad and \quad \text{..............(19)}$$

$$\varsigma_{12}(n) = E\{e_{a,1}(n)e_{a,2}(n)\}. \quad \text{............(20)}$$

Introducing the differences

$$\Delta\zeta_{ii}(n) = \zeta_{ii}(n) - \zeta_{12}(n),\ i = 1,2, \quad (21)$$

and using (19)-(21) in (18), we arrive at

$$\eta_o(n) = \frac{\Delta\varsigma_{22}(n)}{\Delta\varsigma_{11}(n) + \Delta\varsigma_{22}(n)} \quad \text{.......(22)}$$

A similar expression was also obtained for the convex combination of two LMS filters at the steady-state. We should notice that (22) is more general: it holds for all $n \geq 0$ (not only at the steady-state) and the mixing parameter is not restricted to the interval [0, 1]. Defining the EMSE of the overall combined scheme as

$$\zeta(n)=E\{e^2_a(n)\} \quad \ldots\ldots(23)$$

We now obtain an analytical expression for its optimum value. By squaring both sides of (17) with $\eta(n) = \eta_o(n)$ and taking expectations, we arrive at

$$E\{e^2_a(n)\} = \eta^2_o(n)\, E\{e^2_{a,1}(n)\} + [1- \eta_o(n)]^2 E\{ e^2_{a,2}(n) + 2\eta_o(n)[1-\eta_o(n)]E\{e_{a,1}(n)e_{a,2}(n)\} \quad \ldots\ldots(24)$$

Using (19)-(22) in (24), we obtain

$$\zeta_o(n) = \zeta_{22}(n) - \eta_o(n)\Delta\zeta_{22}(n) \quad \ldots\ldots\ldots(25)$$

After some algebraic manipulations, (25) can be rewritten as

$$\varsigma_o(n) = \varsigma_{12}(n) + \frac{\Delta\varsigma_{11}(n)\Delta\varsigma_{22}(n)}{\Delta\varsigma_{11}(n)+\Delta\varsigma_{22}(n)} \quad \ldots\ldots\ldots(26)$$

This expression was obtained for the convex combination of two LMS filters at the steady state, but again it also holds for all $n > 0$.As already mentioned in (8) (22) (26) hold for the combination of any two algorithms that satisfy (12).The values of $\Delta\zeta_{ii}(n)$,i = 1,2 however do depend on the actual algorithms that are being combined .Thus provided approximations for $\zeta_{ij}(n)$ $i = 1,2$ are available,(22) and (26) can applied to the affine combination of different algorithms ,including combinations of algorithms of different families.

IV. STEADY-STATE ANALYSIS OF THE OPTIMUM COMBINER

In this section, the optimum mixing parameter and the optimum EMSE of the combination, given respectively by expressions (22) and (26), are particularized for the combination of two LMS filters, two NLMS filters, and two CMA equalizers in steady-state for stationary and non stationary environments. We assume that in a non stationary environment, the variation in the optimal solution wo follows a random-walk model, that is,

$$w_o(n) = w_o(n - 1) + q(n) \quad \ldots\ldots\ldots \quad (27)$$

In this model, $q(n)$ is an i.i.d (independent and identically distributed). vector with positive-definite autocorrelation matrix $Q=E\{q(n)q^T(n)\}$ independent of the initial conditions $\{w_o(-1),w(-1),\eta(-1)\}$ and of $\{u(l)\}$ for all l [4]. In supervised filtering, $q(n)$ is also assumed independent of the desired response $\{d(l)\}$ for all $l < n$. In blind equalization, $w_o(n)$ represents the zero-forcing solution and $q(n)$ models the channel variation. The analyses.

A) Transient Analysis of Realizable Schemes

In this section, we take into account the adaptation of $\eta(n)$ in the analysis By squaring both sides of (17) and taking expectations, we Obtain

$$E\{e_a^2(n)\} = E\{e_{a,2}^2(n)\} + E\{\eta^2(n)[e_{a,1}(n)-e_{a,2}(n)]^2\} + 2E\{\eta(n)[e_{a,2}(n)e_{a,1}(n)-e_{a,2}^2(n)]\} \quad \ldots\ldots (29)$$

To proceed, we assume that:

A1. The adaptation of $\eta(n)$ is slow so that the correlation between it and $e_{a,i}(n),e_{a,j}(n),i,j = 1,2$ can be disregarded.

This assumption follows from observations: simulations show that $\eta(n)$ converges slowly compared to variations in the input $u(n)$ and thus to variations on the a-priori errors. Using A1, (19)-(21) and (23), we can rewrite (29) as

$$\varsigma(n) \approx \varsigma_{22}(n) + E\{\eta^2(n)\}\alpha(n) - 2E\{\eta(n)\}\Delta\varsigma_{22}(n) \quad \ldots\ldots\ldots\ldots\ldots\ldots(30)$$

where we define

$$\alpha(n) \stackrel{\Delta}{=} E\{[y_1(n)-y_2(n)]^2\} = \Delta\varsigma_{11}(n) + \Delta\varsigma_{22}(n) \quad \ldots\ldots(31)$$

To estimate the EMSE of the combination for all $n \geq 0$ using (30), analytical expressions for $\zeta_{ij}(n)$, $i = 1,2$, $E\{\eta(n)\}$, and $E\{\eta^2(n)\}$ should be obtained. It is common in the literature to evaluate the EMSE as

$$\varsigma_{ij}(n) \stackrel{\Delta}{=} E\{e_{a,i}(n)e_{a,j}(n)\} \approx \mathrm{Tr}(RS_{ij}(n-1)) \quad \ldots.(32)$$

V. SIMULATION RESULTS

To verify the transient analysis in the supervised case, we consider the identification of a time invariant system. The optimum solution is formed with M = 7 independent random values between -1 an 1, and is given by w_o = [+0.90 −0.54 −0.03 +0.78 +0.52 −0.09].We assume white Gaussian input with variance 1/M so that Tr(R) = 1, and an average of 500 runs. Moreover, i.i.d. noise v (n) with variance σ^2 v = 0.01 is added to form the desired signal.

Fig. 2 shows the results of the EMSE and the mixing parameter for the affine combination of two LMS filters in the same situations considered in Fig. 3 and 4 in which the mixing parameter is updated with the N-LMS algorithm, where $\mu_n = 3$,. Similarly, with $\mu_n = 0.1$, the analysis can predict that the combination is not able to switch to the slow filter, We should notice that, due to the constraint imposed in the η-LMS algorithm $(\eta(n) \leq 1)$,.

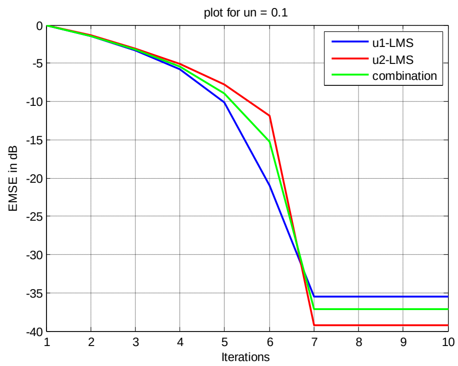

Fig. 2. Affine combination of two signals.

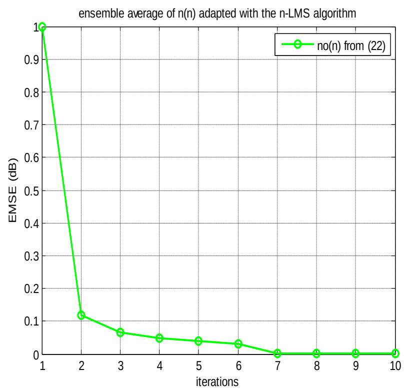

Fig. 3. Ensemble average of n(n) adapted with the N-LMS algorithm.

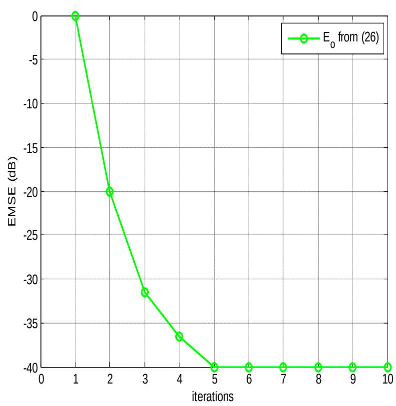

Fig. 4. Convex combination of two LMS filters at the steady state, but it holds for all n > 0.

VI. CONCLUSIONS

We proposed transient and steady-state analyses for the EMSE and the mixing parameter of the affine combination, based on the theoretical EMSE and cross-EMSE of the component filters and on the adaptation of the mixing parameter. This states the application to different combinations of algorithms of LMS, NLMS and CMA, considering white or colored inputs and stationary or non stationary environments. Good agreement between the analysis and the simulations was always observed. Moreover, we proposed and analyzed two normalized algorithms for updating the mixing parameter. The theoretical models can predict situations in which these algorithms can achieve a good performance results.

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