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Handwritten Mathematical Expressions Recognition

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Abstract

In this paper, we will present a dataset of handwritten mathematical expressions outcome of several mathematical disciplines (logic, analysis, algebra and probability). This paper also describes all the details concerning the necessary steps of our approach for handwritten mathematical expressions recognition. At the end of this paper, we will present all the results obtained by this system.

Keywords*: Mathematical expression; Radon transform; Connected component; Segmentation; Classification*

1. Introduction

Traditionally, transmission and information storage were performed using the paper documents. During the last thirty years, the creation of documents assisted by computer has continued to rise. This production capacity of digital documents would have allowed a useless of paper documents.

Actually, the treatment of the digital data flow and paper documents must be done in an efficient and integrated manner. The ultimate solution would be a computer deals with a paper document as effectively as it is able to do with other digital media. The objective of the analysis and document recognition is the identification of text, graphics or notations and the extraction of information.

There are two large sets in the field of document recognition: text analysis and graphical components analysis. Given the importance of mathematics in all branches of science (physics, engineering, medicine, economics, *etc.*), the recognition of handwritten mathematical expressions has become a very important area of scientific research. This importance is due to the large number of papers dealing with this subject.

This work presents our vision of handwritten mathematical expressions recognition which is based on four steps. The system begins by a set of preprocessing techniques in order to ameliorate the image quality and to make the features extraction efficient. The second step in the system process is the expression segmentation into individual symbols based on the connected component algorithm. To extract the features, we applied the Radon transform to exploit its ability to extract lines from the images. The latest step is the symbols classification by using Support Vector Machines (SVM).

This paper is organized as follows; Section 2 presents some related works. In Section 3, we present the system architecture. Section 4 describes the preprocessing techniques used in this system. Section 5 presents the expressions segmentation algorithm. In Section 6, we describe the proposed techniques for symbols recognition. The last section shows the experimental results.

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2. Related Works

The mathematical symbols recognition is a crucial step in handwritten mathematical expressions recognition; which makes the difference between the systems. For this reason, we present in the table below (Table 1) some techniques presented in the literature.

Table 1. Related Works of Mathematical Symbols Recognition

Paper	Method
Fateman et al [1]. Zhu *et al* [11]	Features extraction and nearest neighbor classification
Alvaro et al [2]. Zhu *et al* [11]	Euclidean distance between pixels and nearest neighbor classifier
Belaid and Haton [3]. Chan and Yeung [10]	Structural feature extraction and decision tree classification
Chan and Yeung [4] [10]	Flexible structural matching
Marzinkewitsch [5] . Chan and Yeung [10]	Three-layered backpropagation network
Suzuki *et al* [6]. Zhu *et al* [11]	Features extraction and cascading classifier
Malon *et al* [7]. Zhu *et al* [11]	Directions histogram of contour and bitmap-like feature and cascading classifier with binary SVM
Fateman *et al* [8] [9] . Chan and Yeung [10]	Template matching based on Hausdorff distance

3. System Architecture

The figure (Figure 1) summarizes the necessary steps in our system.

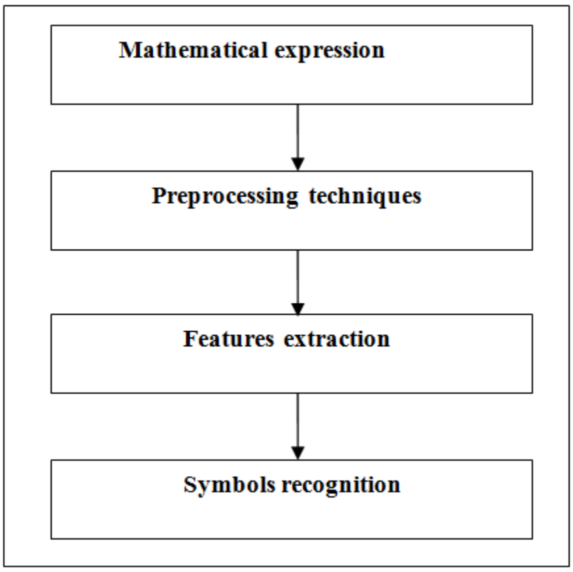


Figure 1. System Architecture

4. Pre-processing

Images preprocessing is a very important step in any system of recognition because of its ability to remedy the problems associated with the images acquisition. This results in a major improvement in the images quality. This image improvement allows extracting the features with an efficient manner and also plays a very important role in increasing the recognition rate.

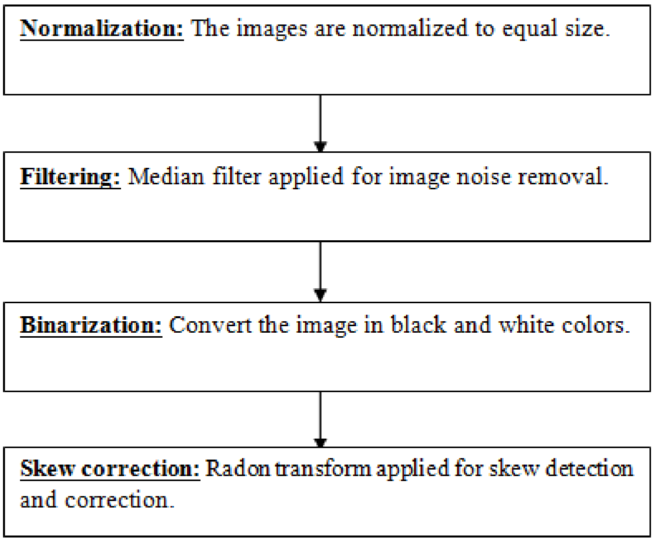


Figure 2. Preprocessing Techniques

5. Handwritten Mathematical Expressions Segmentation

The principle of the connected component labeling is to group under a unique label all adjacent pixels in an image in order to distinguish and extract different disconnected structures. This means that all pixels in a connected component share similar pixel intensity values.

The labeling is directly dependent on the connectivity considered (4-Connectivity and 8-Connectivity).

1	1	1
1	1	1
1	1	1

8-Connectivity

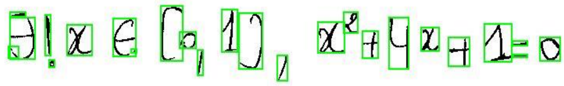
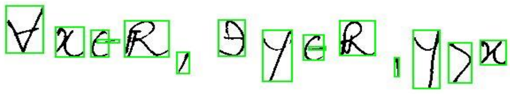
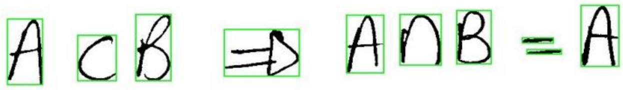
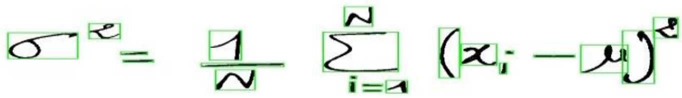
0	1	0
1	1	1
0	1	0

4-Connectivity

To label the different symbols in our mathematical expressions, we focus on scanning the image, pixel-by-pixel from top to bottom and left to right in order to identify connected pixel regions.

The table (Table 2) shows some mathematical expressions segmented by connected component algorithm (8-Connectivity).

Table 2. Mathematical Expressions Segmentation

Expression 1	
Expression 2	
Expression 3	
Expression 4	

6. Symbols Recognition

6.1. Radon Transform

The Radon transform has become an increasingly important area of research. It allows transforming two dimensional images with lines into a domain of possible line parameters where each line in the image will give a peak positioned at the corresponding line parameters [12]. That means, this transform converts a function (image) in a series of projections for each angle $\theta \in [0,\pi]$.

A projection at a given angle θ is obtained as the linear integration of the function on all parallel lines.

The Radon transform of a function $f(x,y)$ can be defined as [13]:

$$R(\rho,\theta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,\delta(\rho - x\cos\theta - y\sin\theta)\,dx\,dy \tag{1}$$

Where:

$$\rho = x\cos\theta + y\sin\theta \tag{2}$$

δ () is the Dirac delta function.

The Radon transform has some useful properties which are relevant for shape recognition (linearity, translation, rotation and scaling) [14, 15].

- Linearity:
  Let f, g two functions and α, β two constants:
  $R(\alpha f + \beta g) = \alpha R(f) + \beta R(g)$
  This property is the most important property of the Radon transform.

- Shifting:

If f is moved a distance (x_0 ,y_0) results in a shift of its transform in the variable ρ by a distance: d= $x_0 \cos\theta + y_0 \sin\theta$

- Rotation:

A rotation of the image by an angle θ_0 results a shift θ_0 of the transform in the variable θ.

- Scaling:

A scale of f implies the same in the ρ coordinates and the amplitude of the transform.

6.2. Features Extraction

The Radon transform is frequently used in different areas such as: tomography, astronomy and seismology. In this paper, we thought to use it for handwritten mathematical symbols recognition.

The basic idea which served us deeply in using this transform is to exploit its ability to extract lines from very noisy environment and its ability to represent the image lines in form of peaks.

To obtain the feature vector from the Radon transform matrix, we calculated the Mean of Radon Transform (MRT) based on the following formula:

$$\text{Vector}\ (\theta_i) = \frac{\frac{1}{n}\sum_{j=1}^{n} R_{ij}}{R_i} \tag{3}$$

Where R_i = MAX ($R_{i1}, R_{i2}, \ldots\ldots, R_{in}$); i=1,……, 180; n=77

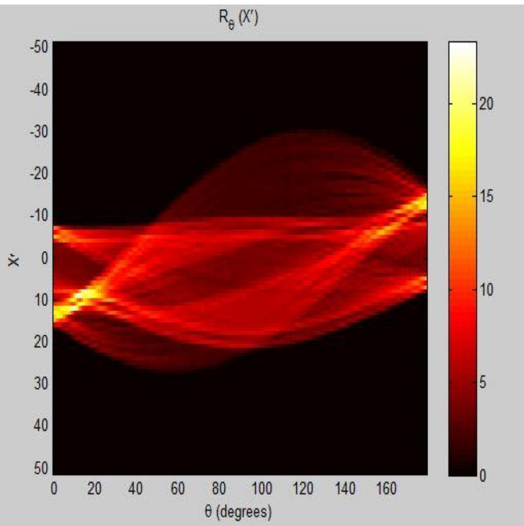


Figure 3. Radon Transform of Alpha symbol

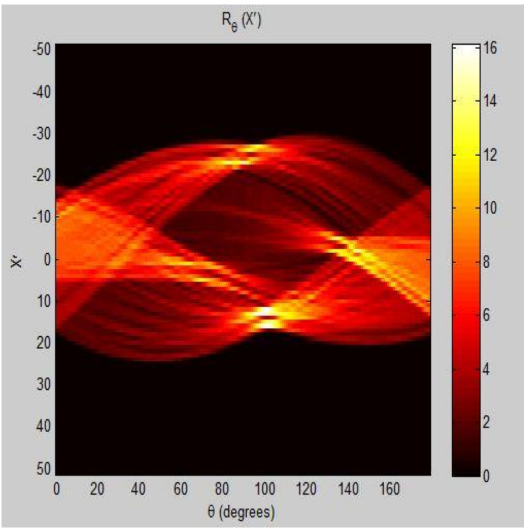


Figure 4. Radon Transform of Sigma Symbol

For the symbols recognition (classification), we applied Support Vector Machines (SVM) with Polynomial kernel function.

7. Experiments and Results

7.1. Datasets

To assess the performance and effectiveness of this recognition system, we used a dataset of mathematical expressions from the different disciplines of mathematics (Mathematical logic, Mathematical analysis, Mathematical algebra, Mathematical probability).

Table 3. Mathematical Expressions Dataset

Domain	Number of expressions
Mathematical logic	50
Mathematical analysis	50
Mathematical algebra	50
Mathematical probability	50

Concerning the symbols recognition, we used a dataset which we presented in [16].

7.2. Results

In this section, we present the results obtained by using the recognition system presented in this paper.

7.2.1. Mathematical Expressions Segmentation:

Table 4. Success Rate of Handwritten Expressions Segmentation

Domain	Expressions segmentation (Success rate)
Mathematical logic	100 %
Mathematical analysis	96 %
Mathematical algebra	100 %
Mathematical probability	98 %

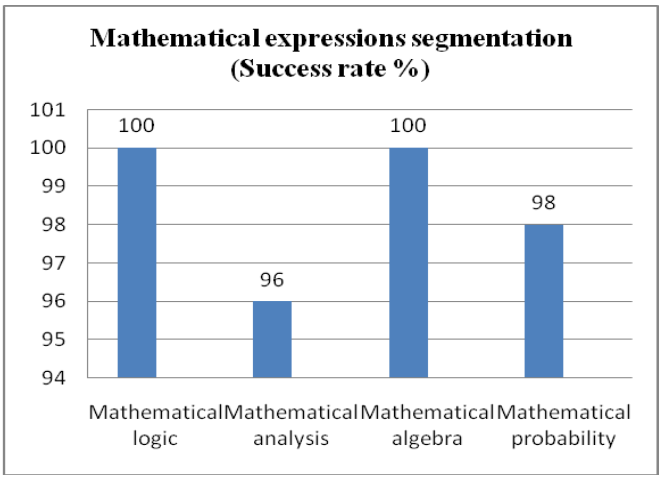


Figure 5. Graphical Representation of Success Rate of Handwritten Expressions Segmentation

7.2.2. Mathematical Symbols Recognition:

Table 5. Recognition Rate of Handwritten Mathematical Symbols

Classifier	Recognition rate
SVM	98 %
ANN	93 %

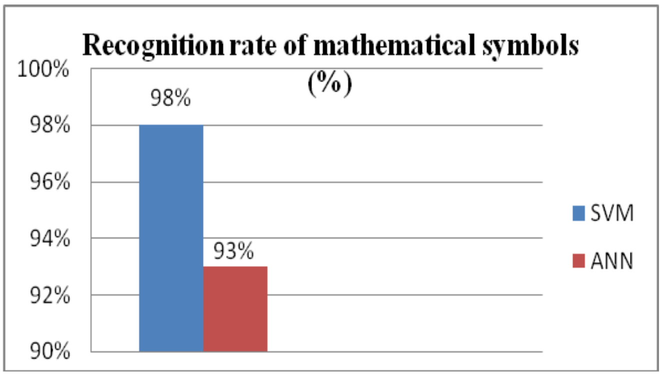


Figure 6. Graphical Representation of Recognition Rate of Handwritten Mathematical Symbols

7.2.3. Mathematical Expressions Recognition:

Table 6. Recognition Rate of Handwritten Mathematical Expressions

Domain	Recognition rate
Mathematical logic	90 %
Mathematical analysis	68 %
Mathematical algebra	82 %
Mathematical probability	72 %

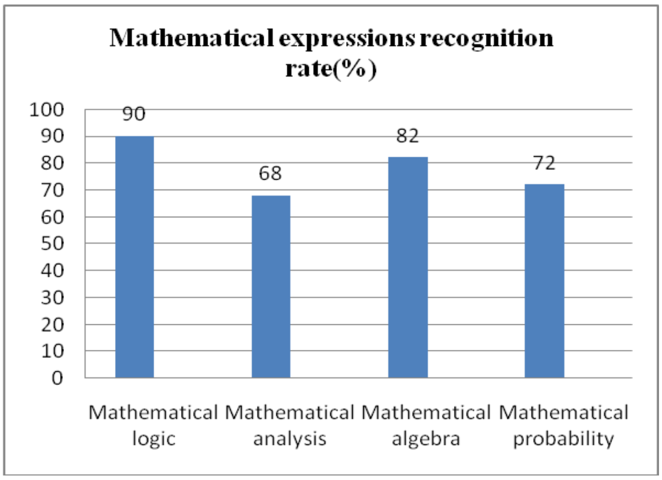


Figure 7. Graphical Representation of Recognition Rate of Handwritten Mathematical Expressions

8. Conclusion

The mathematical expressions recognition is a moving area of research which confronts several difficulties. For this, we presented in this paper a new system able to deal with these difficulties and capable to recognize in an efficient manner these expressions. This system is based on four necessary steps: The first concerns the preprocessing techniques (Normalization, Filtering, Binarization and Skew detection and correction). The second and third steps are respectively: the expressions segmentation (connected component algorithm) and the features extraction (Radon transform). The last step is the symbols classification (SVM). In this paper, we presented also the results obtained by using this system (expressions segmentation, symbols recognition and the expressions recognition).

References

[1] R. Fateman, T. Tokuyasu, B. P. Berman and N. Mitchell, "Optical character recognition and parsing of typeset mathematics," in Journal of Visual Communication and Image Representation, vol. 7, **(1996)**, pp. 2-15.

[2] F. Alvaro, J.-A. Sanchez and J. Benedi, "Recognition of printed mathematical expressions using two dimensional stochastic context-free grammars," in International Conference on Document Analysis and Recognition, **(2011)** September, pp. 1225-1229.

[3] A. Belaid and JP. Haton, "A syntactic approach for handwritten mathematical formula recognition" in Transaction on pattern analysis and machine intelligence, vol. 6, no. 1, **(1984)** Jan, pp. 105-11.

[4] K. F. Chan and D. Y. Yeung," Recognizing on-line handwritten alphanumeric characters through fexible structural matching". Pattern Recognition, vol. 32, no 7, **(1999)** July, pp. 1099-1114.

[5] R. Marzinkewitsch, "Operating computer algebra systems by handprinted input". In Proceedings of the 1991 International Symposium on Symbolic and Algebraic Computation, , Bonn, Germany, **(1991)**, pp. 411-413.

[6] M. Suzuki, F. Tamari, R. Fukuda, S. Uchida and T. Kanahori, "Infty: an integrated ocr system for mathematical documents," in ACM symposium on Document engineering, (New York, NY, USA), pp. 95-104, **(2003)**.

[7] C. Malon, S. Uchida and M. Suzuki, "Mathematical symbol recognition with support vector machines," in Pattern Recognition Letters, vol. 29, pp. 1326-1332, **(2008)** July.

[8] R. J. Fateman and T. Tokuyasu. "Progress in recognizing typeset mathematics". In Proceedings of the SPIE, San Jose, CA, vol. 2660, **(1996)**, pp. 37-50,.

[9] B. P. Berman and R. J. Fateman,"Optical character recognition for typeset mathematics". In Proceedings of the 1994 International Symposium on Symbolic and Algebraic Computation, pp.348-353, Oxford, UK, **(1994)** July.

[10] K.M. Chan and D.Y. Yeung, "Mathematical Expression Recognition: A Survey" in International Journal on Document Analysis and Recognition.

[11] S. Zhu, L. Hu and R. Zanibbi," Rotation-Robust Math Symbol Recognition and Retrieval Using Outer Contours and Image Subsampling" in Document Recognition and Retrieval. Proceedings of the SPIE, vol. 8658, no. 865805, **(2013)**, pp. 12.

[12] M. Miciak, "Character Recognition Using Radon Transformation and Principal Component Analysis in Postal Applications" in Proceedings of the International Multiconference on Computer Science and Information Technology, pp. 495 – 500.

[13] C. Hoilund, "The Radon Transform" Aalborg University, VGIS, 07gr721 November 12, 2007

[14] A. Hamouda, H. Rojbani and I. Eloudi. "A New Shape Descriptor Based on the Radon Transform: the Rθ-signature" in IEEE International Symposium on Signal Processing and Information Technology, ISSPIT **(2011)**.

[15] T.V. Hoang and S. Tabbone, "A geometric invariant shape descriptor based on the Radon, Fourier, and Mellin transforms" in International Conference on Pattern Recognition IEEE, **(2010)**, pp.2085-2088.

[16] Y. Chajri and B. Bouikhalene, "Handwritten mathematical symbols dataset" in Data in brief, vol. 7, June **(2016)**, pp. 432–436.