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An exponential-related function for decision-making in engineering and management

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Abstract: An intuitionistic fuzzy TOPSIS model, which is based on an exponential-related function (IF-TOPSIS) and a fuzzy entropy method, has been proposed in this study. The exponential-related function, which represents the aggregated effect of positive and negative evaluations in the performance ratings of the alternatives, based on the intuitionistic fuzzy set (IFS) data. Serves, as a computational tool for measuring the separation distance of decision alternatives from the intuitionistic fuzzy positive and negative ideal solution to determine the relative closeness coefficient. The main advantage of this new approach is that (1) it uses a subjective and objective based approach for the computation of the criteria weight and (2) its simplicity both in its concept and computational procedures. The proposed method has successfully been implemented for the evaluation of some engineering designs related problems including the selection of a preferred floppy disk from a group of design alternatives, the selection of the best concept design for a new air-conditions system and finally, the selection of a preferred mouse from a group of alternatives as a reference for a new design. Also, for each of the three case studies, the method has been compared with some similar computational approaches.

Keywords: Intuitionistic fuzzy TOPSIS; Exponential-related Function; Intuitionistic fuzzy entropy; MCDM

1 Introduction

The intuitionistic fuzzy set (IFS), which is an expansion of the traditional fuzzy set (FSs) theory was first proposed by Atanassov in 1986 [1]. It comprises of a membership and a non-membership function, which are used for the management of vagueness and uncertainty. As indicated by Wan and Li [2] and Aikhuele & Turan [3], the IFS are more adaptable, functional and capable than the traditional FS theory at handling uncertainty and vagueness in practices. The advantages of applying the IFS have been reported in [5] to include: (1) Its ability to model unknown information using hesitation degree, when the Decision-makers (DMs) are unsure about the preferences of an assessment. (2) It represents three grades of membership function, which include membership degree, non-membership degree, and hesitancy degree. Hence, the IFS can be said to consider opinions from three sides to arrive at the preferred one. (3) All the fuzzy numbers in the IFS theory can all be used to represent vagueness of “agreement”, although, they cannot be used to depict the “disagreement” of the DMs.

As a mathematical tool, the IFS has demonstrated the ability to deal with fuzziness and uncertainty in information and data in a real-life situation and this has resulted in its many applications in diverse fields of study mostly for solving multiple criteria decision making (MCDM) problems [3, 6–10]. However, among the numerous applications of IFS for MCDM, the technique for order preference by similarity to the ideal solution (TOPSIS) by Hwang and Yoon [11] has remained the most extensively used method. TOPSIS method is based on the concept that the most appropriate alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution and has remained one of the most reliable and practical decision-making tools which depend on preference information provided by the DMs [12, 13].

In matching up the preference information given by the DMs which are expressed in IFS, some metric methods were introduced, that is the score and accuracy functions as described in [14–17] and applied for solving MCDM problems. However, a recent investigation by Wu [17] suggests that the results obtained using the score and accuracy functions are not always consistent, while they also produce a negative priority vector in their applications. In addressing this issue, Wu [17], introduced the exponential score function. Although, the exponential score func-

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tion appears to address these shortcomings, the function is only effective for determining priority weight that involves pairwise comparison.

In this study, the exponential score function which have been extended in [18] (exponential-related function) is adopted in the intuitionistic fuzzy TOPSIS (IF-TOPSIS) model with intuitionistic fuzzy entropy method, for determining the criteria weight when the performance ratings are expressed in intuitionistic fuzzy number (IFN). The adoption of the new exponential-related function (*ER*) in the intuitionistic fuzzy decision-making method is undertaken to provide a flexible and a whole new approach to solving MCDM problems. In computing the weight of the criteria, the intuitionistic fuzzy entropy (IFE) originally proposed by Ye, [19] was adopted.

The main contribution and advantages of the new method and approach lies in the use of an objective approach for the computation of the criteria weight, which allows for complete assessment of the actual performance and value of each of the criteria. The application of the matrix method (i.e. the exponential-related function), which represent the aggregated effect of the positive and negative evaluations in the performance ratings of the alternatives based on the intuitionistic fuzzy set (IFS) data. The integration of the exponential-related function and the intuitionistic fuzzy entropy into the traditional intuitionistic fuzzy TOPSIS model, introduction of the MCDM method, which can be described as simple both in its concept and computational procedures, compared to other existing methods and finally. The exponential-related function, which serves as a parameter and a better alternative to the Euclidian distance that often has correlation issues, in the computation of the separation measures of each alternative from the intuitionistic fuzzy positive and negative ideal solution which is used in the determination of the relative closeness coefficient.

The rest of the paper is organized as follows; in section 2, the concept of the IFS, the intuitionistic fuzzy entropy, and the exponential-related function are presented. The algorithm of the Intuitionistic Fuzzy TOPSIS model based on the exponential-related function (IF-TOPSIS) and the intuitionistic fuzzy entropy (IFE) method are presented in section 3. In section 4, a numerical case study is presented to demonstrate the effectiveness of the model. While some come conclusions are presented in section 5.

2 The Basic Concept of IFS and the Exponential-Related Function

This section presents, the fundamental definitions and concepts of the IFS theory as described in [1] and the proposed exponential-related function with the IFE.

2.1 Intuitionistic Fuzzy Set

Definition 1

If the IFS A in $X = \{x$ is defined fully in the form $A = \{\langle x, \mu_A(x), \ v_A(x), \pi_A(x)\rangle \,|x \in X$, where $\mu_A : X \to [0,1]$, $v_A : X \to [0,1]$ and $\pi_A : X \to [0,1]$. The different relations and operations for the IFS are shown in Eq. (1) to (4).

$$AB = \{\langle x, \mu_A(x) .\ \mu_B(x),\ v_A(x) + v_B(x) - v_A(x) . v_B(x)\rangle \\ |x \in X \tag{1}$$

$$A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) .\mu_B(x), v_A(x) . v_B(x)\rangle \\ |x \in X \tag{2}$$

$$\lambda A = \{\left\langle x, 1 - (1 - \mu_A(x))^\lambda,\ (v_A(x))^\lambda |x \in X\right\},,\ \ \lambda > 0. \tag{3}$$

$$A^\lambda = \{\left\langle x, (\mu_A(x))^\lambda, 1 - (1 - v_A(x))^\lambda \right\rangle |x \in X\},\ \ \lambda > 0 \tag{4}$$

In the proceeding definition, comparisons between the IFS are presented, by introducing the score and accuracy functions as described in [14–16].

Definition 2

Let $A = (\mu, v)$ be an intuitionistic fuzzy number, a score function S and an accuracy function H of an intuitionistic fuzzy value can be represented as follow.

$$S(A) = (\mu - v),\ \text{where}\ S(A) \in [-1, +1] \tag{5}$$

$$H(A) = (\mu + v),\ \text{where}\ H(A) \in [0, 1] \tag{6}$$

Definition 3

Let $A = (\mu, v)$ be the intuitionistic fuzzy number, according to Wu (2015) the exponential score function S_e of the intuitionistic fuzzy number can be represented as:

$$S_e(A) = e^{(\mu - v)}\ \text{where}\ S_e(A) \in [1/e, e] \tag{7}$$

2.2 The Exponential Related Function (ER)

Definition 4 [18]

Let $A = (\mu, \, v)$ be the intuitionistic fuzzy number. The new exponential-related function ER of the intuitionistic fuzzy number can be defined as:

$$ER(A) = e^{\left(\frac{1-\mu^2-v^2}{3}\right)}, \text{ where } ER(A) \in [1/e, \, e] \quad (8)$$

Theorem 1: Let $A = (\mu, \, v)$ and $B = (\mu_1, \, v_1)$ be two intuitionistic fuzzy set, if $A \subseteq B$ then $ER(A) \le ER(B)$.

Proof. Assume that $A = (\mu, \, v)$ and $B = (\mu_1, \, v_1)$ are two comparable alternatives with intuitionistic fuzzy numbers based on some criteria c_i such that $A \subseteq B$ without loss of generality, let assume that $\mu_1^2 \le \mu^2$, and $v^2 \ge v_1^2$ such that $ER(A) \le ER(B)$

By Definition 4, we have that:

$$ER(A) = e^{\left(\frac{1-\mu^2-v^2}{3}\right)}$$

and

$$ER(B) = e^{\left(\frac{1-\mu_1^2-v_1^2}{3}\right)}$$

Then

$$ER(B) - ER(A) = e^{\left(\frac{1-\mu_1^2-v_1^2}{3}\right)} - e^{\left(\frac{1-\mu^2-v^2}{3}\right)}$$

$$= e^{\left(\frac{1-\mu_1^2-v_1^2}{3}\right)-\left(\frac{1-\mu^2-v^2}{3}\right)} = e^{\left(\frac{1-\mu_1^2-v_1^2-1+\mu^2+v^2}{3}\right)} = e^{\left(\frac{\mu^2-\mu_1^2+v^2-v_1^2}{3}\right)}$$

This can be rewritten as:

$$= e^{\left(\frac{(\mu^2-\mu_1^2)}{3}+\frac{(v^2-v_1^2)}{3}\right)}$$

Let assume the power of the exponential is multiply by 3, and then we have;

$$= e^{((\mu^2-\mu_1^2)+(v^2-v_1^2))}$$

Since, $A \subseteq B$, $\mu_1^2 \le \mu^2$, and $v^2 \ge v_1^2$. Hence $(\mu^2 - \mu_1^2) \ge 0$, and $(v^2 - v_1^2) \ge 0$.

Then it follows that $ER(B) - ER(A) \ge 0$. □

Theorem 2: Let $A = (\mu, \, v)$ and $B = (\mu_1, \, v_1)$ be two intuitionistic fuzzy set, from the above theorem (1), we can conclude:

(1) $ER(B) > ER(A)$, if and only if $B > A$

(2) $ER(B) > ER(A)$, if and only if $\mu^2 - v^2 > \mu_1^2 - v_1^2$

2.3 The intuitionistic fuzzy entropy (IFE)

Following the operations of the IFS, let us consider an intuitionistic fuzzy set A in the universe of discourse $X = \{x_1, x_2\, x_3, \ldots, x_n$.The intuitionistic fuzzy set A is transformed into a fuzzy set to structure an entropy measure of the intuitionistic fuzzy set by means of $\mu_{\tilde{A}}(x_i) = (\mu_A(x_i) + 1 - v_A(x_i))/2$. Based on the definition of fuzzy information entropy Ye (2010) proposes the intuitionistic fuzzy entropy as follows:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\left\{\sin\frac{\pi * [1+\mu_A(x_i)-v_A(x_i)]}{4} + \sin\frac{\pi * [1-\mu_A(x_i)+v_A(x_i)]}{4} - 1\right\} * \frac{1}{\sqrt{2}-1} \quad (9)$$

When the criteria weights are completely unknown, we can use the IFE to determine the weights. The criteria weight is given as:

$$W_j = \frac{1-H_j}{n-\sum_{j=0}^{n} H_j} \quad (10)$$

where $W_j \in [0, 1]$, $\sum_{j=1}^{n} W_j = 1$, $H_j = \frac{1}{m}E(A_j)$ and $0 \le H_j \le 1$ for $(j = 1, \, 2, \, 3, \ldots, \, n)$.

3 Algorithm of the IF-TOPSIS and Intuitionistic Fuzzy Entropy (IFE) Method

In this section, the algorithm for the IF-TOPSIS and the IFE Method is concisely expressed using the stepwise procedure. The schematic diagram of the proposed model is shown in Fig. 1.

Step 1: Set up a group of Decision Makers (DMs) and aggregate their evaluations using Intuitionistic Fuzzy Weighted Geometric (IFWG) operator [20]. Once the DMs has given their judgment using linguistic variables, the variables are then converted to the intuitionistic fuzzy number (IFNs), as shown in Table 1. The weight vector $\lambda = (\lambda_1, \lambda_2, \lambda_3, .., \lambda_l)^T$ is used to aggregate all the DMs individual assessment matrices $D^k\,(k = 1, 2, 3, \ldots, \, l)$ into the group assessment matrix (i.e. intuitionistic fuzzy decision matrix) $R_{yxz}(x_{ij})$.

$$IFWG\,(d_1 d_2 d_3, \ldots, d_n) = \left(\prod_{i=1}^{n}(\mu_{ij})^{w_j}, \; 1-\prod_{i=1}^{n}(1-v_{ij})^{w_j}\right) \quad (11)$$

Table 1: Fuzzy numbers for approximating the linguistic variable

Linguistic terms	Intuitionistic fuzzy number
Very low (VL)	(0.30, 0.40)
Low (L)	(0.50, 0.50)
Good (G)	(0.50, 0.60)
High (H)	(0.70, 0.80)
Excellent (EX)	(0.90, 0.90)

$$R_{mxn}(a_{ij}) = \begin{bmatrix} (\mu_{11}, v_{11}) & (\mu_{12}, v_{12}) & \cdots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (\mu_{22}, v_{22}) & \cdots & (\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}, v_{m1}) & (\mu_{m2}, v_{m2}) & \cdots & (\mu_{mn}, v_{mn}) \end{bmatrix} \quad (12)$$

Step 2: Determine the weight of each of the evaluating criteria w_j using the IFE method.

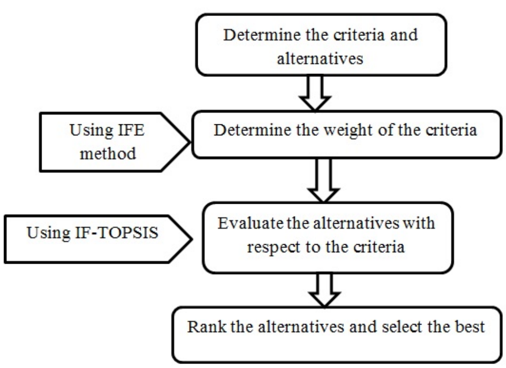

Figure 1: The schematic diagram of the proposed model

Step 3: Using the exponential related function ER (i.e. equation 8) convert the intuitionistic fuzzy decision matrix $R_{yxz}(x_{ij})$ to form the exponential related matrix $EM_{yxz}\left(ER_{ij}\left(a_{ij}\right)\right)$, which represents the aggregated effect of the positive and negative evaluations in the performance ratings of the alternatives based on the intuitionistic fuzzy set (IFS) data.

$$EM_{yxz}\left(E_{ij}\left(a_{ij}\right)\right) = \begin{bmatrix} ER_{11}(x_{11}) & ER_{12}(x_{12}) & \cdots & ER_{1n}(x_{1z}) \\ ER_{21}(x_{21}) & ER_{22}(x_{22}) & \cdots & ER_{2n}(x_{2z}) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ ER_{y1}(x_{y1}) & ER_{y2}(x_{y2}) & \cdots & ER_{yz}(x_{yz}) \end{bmatrix} \quad (13)$$

Step 4: Define the IFPIS $A^+ = (\mu_j, v_j)$ and IFNIS $A^- = (\mu_j, v_j)$ for the alternatives.

$$A^+ = \{\langle C_j, \ [1,\ 1]\rangle \,|C_j \in C\},$$
$$A^- = \{\langle C_j, \ [0, 0]\rangle \,|C_j \in C\},\ j = 1, 2, 3, \ldots., z$$

Step 5: Compute the exponential-related function-based separation measures in intuitionistic fuzzy environment $(d_i^+(A^+,\ A_i)$ and $(d_i^-(A^-,\ A_i)$ for each alternative for the IF-PIS and IFNIS.

$$d_i^+(A^+,\ A_i) = \sqrt{\sum\nolimits_{i=1}^{n} \left[w_j\left(1 - \left(EM_{yxz}(a_{ij})\right)\right]^2\right.} \quad (14)$$

$$d_i^-(A^-,\ A_i) = \sqrt{\sum\nolimits_{i=1}^{n} \left[w_j\ \left(EM_{yxz}(a_{ij})\right)\right]^2} \quad (15)$$

where w_j is the weight of the criteria.

Step 6: Compute the relative closeness coefficient, (CC_i), which is defined to rank all possible alternatives with respect to the positive ideal solution A^+. The general formula is given as;

$$CC_i = \frac{d_i^-(A^-,\ A_i)}{d_i^-(A^-,\ A_i) + d_i^+(A^+,\ A_i)} \quad (16)$$

where $CC_i\,(i = 1, 2, ..n)$ is the relative closeness coefficient of A_i with respect to the positive ideal solution A^+ and $0 \leq CC_i \leq 1$.

Step 7: Rank the alternatives in the descending order.

4 Illustrative Examples

Case 1: Let's consider a practical decision-making problem originally reported in [21]. In this case, the original problem is modified to make a new example, however, using the same decision matrices while the attributes weight are derived using the intuitionistic fuzzy entropy method.

Suppose a product manufacturing company want to select a preferred floppy disk from a group of candidates; S_1, S_2, and S_3 as a reference disk for a new design. A group of three experts with the following weights values $\lambda = \{0.35, 0.36, 0.28\}$ respectively, are to make a decision about the floppy disk with respect to the following criteria: Performance (C_1), Appearance (C_2) and Cost (C_3). The experts' preference judgments are given as shown in Table 2. Using the algorithm of the IF-TOPSIS and the IFE as given in section 3, the best floppy disk design from the three design alternatives with respect to the three criteria is selected.

In Steps 1&2, the individual expert's assessments for the three designs with respect to the criteria are aggregated

Table 2: The expert's individual preference judgments

		C_1	C_2	C_3
E1	S_1	(0.013, 0.129)	(0.028, 0.144)	(0.021, 0.136)
	S_2	(0.013, 0.107)	(0.038, 0.139)	(0.047, 0.155)
	S_3	(0.003, 0.042])	(0.018, 0.054)	(0.014, 0.150)
E2	S_1	(0.040, 0.161)	(0.034, 0.093)	(0.047, 0.199)
	S_2	(0.047, 0.127)	(0.040, 0.081)	(0.102, 0.206)
	S_3	(0.014, 0.113)	(0.016, 0.086)	(0.030, 0.187)
E3	S_1	(0.006, 0.118)	(0.004, 0.053)	(0.003, 0.174)
	S_2	(0.015, 0.046)	(0.001, 0.026)	(0.021, 0.157)
	S_3	(0.009, 0.034)	(0.005, 0.019)	(0.011, 0.103)

using the IFWG operator. The final comprehensive group assessment matrix for the expert's assessment, called the intuitionistic fuzzy decision matrix $R_{3x3}(x_{ij})$, is given in Table 3. The criteria weight is calculated from the intuitionistic fuzzy matrix using the IFE method which can be calculated by inputting the formula in a Microsoft excel program. The final result is given as $w = \{0.29, 0.23, 0.47$ respectively.

Table 3: Intuitionistic fuzzy decision matrix

	C_1	C_2	C_3
S_1	(0.01572, 0.137701)	(0.017381, 0.100663)	(0.016271, 0.169955)
S_2	(0.021565, 0.097692)	(0.01392, 0.087304)	(0.049624, 0.174406)
S_3	(0.007142, 0.06619)	(0.012031, 0.056134)	(0.017245, 0.150849)

In step 3–5, using the exponential-related function, the intuitionistic fuzzy decision matrix $R_{3x3}(x_{ij})$ is converted to form the exponential related matrix $EM_{3x3}\left(ER_{ij}\left(a_{ij}\right)\right)$, while the exponential related function-based separation measures $(d_i^+(A^+,\ A_i)$ and $(d_i^-(A^-,\ A_i)\quad (i = 1, 2, 3)$ is calculated using equation (14) and (15). In step 6–7, the relative closeness coefficient CC_i, $(i = 1, 2, 3)$ to the ideal solution is calculated using equation (16), the relative closeness coefficients for each of the alternatives are ranked in the descending order. The results are given in Table 4.

From the ranking result of the three floppy design alternatives, we can conclude therefore that the design concept S_2 is the best design based on the three evaluating criteria provided by the three Expert's preference judgments. Table 5 shows that the result is totally in agreement with the result in [21]. This proves the effectiveness and feasibility of the proposed model at handling uncertainty and for decision-making.

Case 2: Let's consider another decision-making problem originally reported by Joshi & Kumar [22]. In this case, the problem has been modified to make a new example using the same decision matrix, while the attributes weights are derived using the intuitionistic fuzzy entropy method.

Suppose a design company wants to select the best concept design for a new air-conditions system from the following alternatives S_1, S_2, S_3, and S_4. The DMs are to evaluate and select the best concept design with respect to Safety (C_1), Attractive design (C_2) and Reliability criteria (C_3) design cost (C_4) and compatibility design (C_5). The aggregated DMs preference judgments are presented in Table 6 (i.e. Intuitionistic fuzzy decision matrix). From these the best concept design for the new air-conditions system can be selected based on the IF-TOPSIS and IFE method.

Using the IF-TOPSIS algorithm we select the best concept design for an air conditions system, where the criteria weight is calculated from the intuitionistic fuzzy matrix using the IFE method. The result of the evaluation is given as:

$$w=\{0.161269,\ 0.144649,\ 0.14052,\ 0.40608,\ 0.147482\},$$

respectively.

Using the exponential-related function, the intuitionistic fuzzy decision matrix $R_{4x5}(x_{ij})$ is converted to form the exponential related matrix $EM_{4x5}\left(ER_{ij}\left(a_{ij}\right)\right)$, while the exponential related function-based separation measures $(d_i^+(S^+,\ S_i)$ and $(d_i^-(S^-,\ S_i)\quad (i = 1, 2, \ldots, 4)$ is calculated using equation (14) and (15). In step 6-7, the relative closeness coefficient CC_i, $(i = 1, 2, \ldots, 4)$ to the ideal solution is calculated using equation (16), the overall computational results as well as the ranking of the relative closeness coefficients for each of the alternatives are given in Table 7.

From the ranking result of the four air-conditions system design alternatives, we conclude that the S_3 is the best design with respect to the five evaluating criteria. The result is totally in agreement with the result in [22] (Table 8).

Case 3: Finally, Let us consider a practical MCDM problem originally reported by Ye [23] and adopted by Liu & Ren [24]. In this case, the original problem has been modified to make a new example, however, using the same decision matrix.

Suppose a computer manufacturing company wants to select a preferred mouse from a group of candidates; A_1, A_2, A_3 and A_4 as a reference mouse for a new design. Again, a group of experts is asked to make a decision with respect to Performance (C_1), Cost (C_2) and Appearance (C_3). The experts aggregated evaluations are given in Table 9. We select the preferred mouse using the IF-TOPSIS method.

Using the IFE method, the criteria weight is calculated from the intuitionistic fuzzy matrix and the result is

Table 4: The relative closeness coefficients for the three design alternatives

	C_1	C_2	C_3	d_i^+	d_i^-	CC_i	Ranking
S_1	1.387	1.391	1.382	0.2224	0.8011	0.7827	2
S_2	1.391	1.392	1.380	0.2223	0.8010	0.7828	1
S_3	1.394	1.394	1.385	0.2245	0.8032	0.7816	3

Table 5: Comparison of ranking results for the case 1

	Proposed Approach	Rank	Yue [21]	Rank
S_1	0.782693	2	0.3563	2
S_2	0.782756	1	0.3625	1
S_3	0.781552	3	0.2812	3

Table 6: Intuitionistic fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
S_1	(0.230, 0.587)	(0.610, 0.200)	(0.192, 0.630)	(0.220, 0.750)	(0.196, 0.620)
S_2	(0.260, 0.554)	(0.200,0.610)	(0.633, 0.192)	(0.094, 0.875)	(0.620, 0.196)
S_3	(0.620, 0.197)	(0.610, 0.200)	(0.259, 0.560)	(0.310, 0.660)	(0.227, 0.590)
S_4	(0.197, 0.620)	(0.360, 0.454)	(0.337, 0.484)	(0.150, 0.820)	(0.322, 0.500)

Table 7: The relative closeness coefficients for the four design alternatives

	C_1	C_2	C_3	C_4	C_5	d_i^+	d_i^-	CC_i	Ranking
S_1	1.5381	1.2494	1.5736	1.6565	1.5662	0.3056	0.8056	0.7250	2
S_2	1.5115	1.5590	1.2362	1.7961	1.2436	0.3467	0.8403	0.7079	4
S_3	1.2438	1.2494	1.5151	1.5628	1.5406	0.2582	0.7567	0.7456	1
S_4	1.5660	1.4317	1.4529	1.7332	1.4654	0.3311	0.8309	0.7151	3

Table 8: Comparison of ranking results for the case 2

	Proposed Approach	Rank	Joshi & Kumar [22]	Rank
S_1	0.7250	2	0.680	2
S_2	0.7079	4	0.257	4
S_3	0.7456	1	0.922	1
S_4	0.7151	3	0.446	3

Table 9: Intuitionistic fuzzy decision matrix

	C$_1$	C$_2$	C$_3$
A$_1$	(0.45, 0.35)	(0.50, 0.30)	(0.20, 0.55)
A$_2$	(0.65, 0.25)	(0.65, 0.25)	(0.55, 0.15)
A$_3$	(0.45, 0.35)	(0.55, 0.35)	(0.55, 0.20)
A$_4$	(0.75, 0.15)	(0.65, 0.20)	(0.35, 0.15)

given as $w = \{0.377, 0.311, 0.313$ respectively. Using the exponential-related function, just as in case 1&2, the intuitionistic fuzzy decision matrix $R_{4x3}\left(x_{ij}\right)$ is converted to form the exponential related matrix and the exponential related function-based separation measures $(d_i^+(A^+,\ A_i)$ and $(d_i^-(A^-,\ A_i)$ $\ (i = 1, 2, \ldots, 4)$ is calculated for each of the alternative, while the relative closeness coefficient CC_i , $(i = 1, 2, \ldots, 4)$ to the ideal solution is calculated using equation (14). The final results are shown in Table 10.

From the ranking result of the four alternative mouse designs, we conclude that the A_2 is the best design with respect to the three evaluating criteria, and the ranking result is in agreement with the result in [23, 24] as shown in Table 11.

Table 10: The relative closeness coefficients of the four candidates

	C_1	C_2	C_3	d_i^+	d_i^-	CC_i	Ranking
A_1	1.3589	1.3231	1.5232	0.2348	0.8111	0.7755	4
A_2	1.2378	1.2378	1.2712	0.1438	0.7233	0.8342	1
A_3	1.3589	1.3143	1.2787	0.1881	0.7671	0.8031	3
A_4	1.1657	1.2285	1.3499	0.1446	0.7188	0.8325	2

Table 11: Comparison of ranking results for the case 3

	Proposed Approach	Rank	Liu and Ren [24]	Rank	Ye [23]	Rank
A_1	0.7755	4	0.4989	4	0.6862	4
A_2	0.8342	1	0.6722	1	0.9375	1
A_3	0.8031	3	0.5901	3	0.8502	3
A_4	0.8325	2	0.6705	2	0.9311	2

5 Conclusion

In this paper, we have proposed a new matrix method (i.e. the exponential-related function (*ER*)) for comparing intuitionistic fuzzy sets, and as a replacement for the traditional exponential score function originally proposed by Wu [17], which have been found ineffective for solving some MCDM problems. The new exponential-related function (*ER*), which has been developed and adopted in the intuitionistic fuzzy TOPSIS model and intuitionistic fuzzy entropy is used for solving MCDM problems in which the weight of the evaluating criteria are completely unknown and the performance ratings of the alternatives are expressed in IFN. The criteria weight here, have been calculated using the intuitionistic fuzzy entropy method originally proposed by Ye [19].

The main advantage and contribution of the new method and approach is that (1) it uses an objective approach for the computation of the criteria weight, which allows for complete assessment of the actual performance of each of the criteria by assisting in the identification of the difference between the present situation (which is considered to be ideal) and the level of performance it intended to achieved in the future. (2) Simplicity in the MCDM method both in its concept and computational procedures as compared to other existing methods. (3) The application of the exponential-related function, which stands to represent the aggregated effect of the positive and negative evaluations in the performance ratings of the alternatives based on the intuitionistic fuzzy set (IFS) data and (4) finally, it serves as a parameter and a better alternative to the Euclidian distance that often has correlation issues, in the computation of the separation measures of each alternative from the intuitionistic fuzzy positive and negative ideal solution which is used in the determination of the relative closeness coefficient.

To validate the feasibility and effectiveness of the method, the IF-TOPSIS, model has been applied for the assessment of some engineering designs related problems including selection of a preferred floppy disk from a group of design alternatives, the selection of the best concept design for a new air-conditions system and finally, for the selection of a preferred mouse from a group of alternatives as a reference for a new design. In the future, we will continue working on the application of the proposed method in other domain, specifically for problems with more criteria and alternatives and to make some comparisons with the adaptive fuzzy control of strict-feedback nonlinear time-delay systems, which have recently found applications in the intuitionistic fuzzy environment.

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