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Linearized Analysis of (2+1)-Dimensional Einstein-Maxwell Theory

Jiro SODA

Research Institute for Theoretical Physics
Hiroshima University, Takehara, Hiroshima 725

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On the basis of previous result by Hosoya and Nakao that (2+1)-dimensional gravity reduces the geodesic motion in moduli space, we investigate the effects of matter fields on the geodesic motion using the linearized theory. It is shown that the transverse-traceless parts of energy-momentum tensor make the deviation from the geodesic motion. This result is important for the Einstein-Maxwell theory due to the existence of global modes of Maxwell fields on torus.

§ 1. Introduction

Since a paper by DeWitt,[1)] quantum gravity has been widely discussed. Although there remain serious difficulties in it, many progresses have been done. Especially Coleman recently proposed[2)] a very interesting story about a cosmological constant. We recognized the importance of topology changing phenomena in quantum gravity from his paper. Canonical approach to general relativity[3)] can be understood as a deformation theory of Riemannian manifold M. This deformation process may also be viewed in superspace S, which is the collection of Riemannian metrics modulo diffeomorphisms of M. Each point of S is a 3-geometry. As is well known in quantum cosmology, 3-geometry is the carrier of information about time. This extra variable is identified with conformal factor. Then what we would like to know is a deformation of conformal Riemannian manifolds which is characterized by $\tilde{g}_{\mu\nu} = g^{-1/3} g_{\mu\nu}$ in (3+1)-dimensions. This is the so-called "conformal superspace"[4)] which is defined by superspace modulo conformal mappings. As a deformation theory, topology changing effects are difficult to treat. Only we can say is that the global dynamics of gravity is important. In order to get some feelings about topology changing, we shall study the global mode of gravity at the classical level.

For this purpose we take (2+1)-dimensional gravity as a playground.[5)] In a recent paper, Witten has argued that the vacuum Einstein equations in (2+1)-dimensions are an exactly solvable system both classically and quantum mechanically.[16)] As Moncrief pointed out,[7)] however, his analysis is unusual as for gravity. More conventional approach to the initial value problem of the gravity has been done by Hosoya and Nakao[8)] and also by Moncrief himself. They consider a space-time as a direct product $M = R \times \Sigma$ where Σ is a compact orientable two manifold. Moncrief has analyzed initial value problem and concluded the existence of the solution of the constraint equations for genus $g \geq 1$. For $g=1$ case, Hosoya and Nakao pointed out that the motion of the moduli parameters follow the geodesic curve defined by the Weil-Petersson metric[9)] in the conformal superspace (moduli space). From our point of view, the present status of research of (2+1)-dimensional gravity is still unsatisfactory in two points. First except for genus $g=1$ case there is almost no concrete result (Moncrief proved the existence theorem). The second point is that previous study is concentrated solely on pure gravity. In this paper we would like to attack the second problem. So we shall concentrate on $g=1$ case in what follows.

§ 2. Canonical gravity

Although the part of analysis in this section is merely translation of the well-known results in (3+1)-dimensional gravity, we shall start here to make this paper self-contained. Given a Einstein-Hilbert action

$$S=\int\sqrt{-g}\,R^{(3)}d^3x\,, \tag{2·1}$$

there is a standard prescription for obtaining a Hamiltonian formulation. The canonical theory begins with the following decomposition of the metric tensor

$$\begin{aligned} ds^2&=g_{\mu\nu}dx^\mu dx^\nu \\ &=-N^2dt^2+h_{ij}(dx^i+N^idt)(dx^j+N^jdt)\,, \end{aligned} \tag{2·2}$$

where μ, ν range over 0, 1, 2 and i, j range over 1, 2. Using this (2+1)-decomposition of the metric, we obtain

$$S=\int N\sqrt{h}(K_{ij}K^{ij}-K^2+R^{(2)})\,, \tag{2·3}$$

where $K_{ij}=(1/2N)(h_{ij,0}-N_{i|j}-N_{i|j})$ is the extrinsic curvature and $K=K_{ij}h^{ij}$ is its trace. $R^{(3)}$ and $R^{(2)}$ denote the three and two dimensional scalar curvatures, respectively. The stroke indicates the covariant derivative defined by the spatial metric h_{ij}. Here we discard the surface term, because we shall concentrate on (2+1)-dimensional space-time $M=R\times\Sigma$ where Σ is a compact orientable two manifold. The canonical conjugate momentum π^{ij} to h_{ij} is given by

$$\pi^{ij}=\sqrt{h}(K^{ij}-h^{ij}K)\,. \tag{2·4}$$

The ADM action for Einstein's equations takes the form[10]

$$S=\int d^3x\{\pi^{ij}\dot{h}_{ij}-NH-N_iH^i\}\,, \tag{2·5}$$

$$H=\frac{1}{\sqrt{h}}(\pi^{ij}\pi_{ij}-\pi^2)-\sqrt{h}R^{(2)}\,, \tag{2·6}$$

$$H^i=-2\pi^{ik}_{|k} \tag{2·7}$$

with $\pi=\pi_i^i=-\sqrt{h}K_i^i$. Note that the Hamiltonian constraint can be rewritten as

$$H=G_{ijkl}\pi^{ij}\pi^{kl}-\sqrt{h}R^{(2)}\,, \tag{2·8}$$

$$G_{ijkl}=\frac{1}{2}h^{-1/2}(h_{ik}h_{jl}+h_{il}h_{jk}-2h_{ij}h_{kl})\,. \tag{2·9}$$

This tensor, G_{ijkl}, is the so-called supermetric on superspace. This metric has signature $(-,+,+)$ in 3-dimensional metric space Riem(Σ). This is different from superspace whose naive dimension is one. The evolution equations obtained from (2·5) are

$$\frac{\partial h_{ij}}{\partial t}=\frac{2N}{\sqrt{h}}(\pi_{ij}-h_{ij}\pi)+N_{i|j}+N_{j|i}\,, \tag{2·10}$$

$$\frac{\partial \pi^{ij}}{\partial t}=\frac{N}{2\sqrt{h}}h^{ij}(\pi^{kl}\pi_{kl}-\pi^2)-\frac{2N}{\sqrt{h}}(\pi^{ik}\pi_k{}^j-\pi^{ij}\pi)+\sqrt{h}(N^{|ij}-h^{ij}N^k_{|k})$$
$$+(N^k\pi^{ij})_{|k}-\pi^{ki}N^j_{|k}-\pi^{kj}N^i_{|k}\,. \tag{2·11}$$

Together with the constraint equations

$$H=0\,, \tag{2·12}$$

$$H^i=0\,, \tag{2·13}$$

we obtain Einstein's equations $R_{\mu\nu}=0$. Equations (2·12) and (2·13) are the Gauss-Coddazzi equations giving necessary and sufficient conditions for the embedding of hypersurface with second fundamental form K_{ij} in a space-time satisfying Einstein equations. The momentum contraints (2·13) have clear meaning that it is a generator of space diffeomorphism, Diff(Σ). Wheeler's superspace can be defined by

$$\mathrm{Riem}(\Sigma)/\mathrm{Diff}(\Sigma)\,. \tag{2·14}$$

The complexity of the Hamiltonian constraint (2·12) is the origin of difficulty for reducing the phase space to the physical space.

From now on we shall reveal the special features of (2+1)-dimensional gravity following Hosoya and Nakao. For technical reason it is further necessary to specialize our model to $g=1$ case, where g is the genus of Riemann surface. Using the traceless part of the extrinsic curvature $\tilde{K}^{ij}=k^{ij}-1/2h^{ij}K$ and its trace $\tau=-K$, the action becomes

$$S=\int d^3x\Big[\tilde{K}^{ij}\frac{\partial\tilde{h}_{ij}}{\partial t}+\tau\frac{\partial\sqrt{h}}{\partial t}-\sqrt{h}N\Big\{K_{ij}K^{ij}-\frac{1}{2}\tau^2+R^{(2)}\Big\}+2\sqrt{h}N^i\tilde{K}^{il}_{|l}\Big] \tag{2·15}$$

with $\tilde{h}_{ij}=h_{ij}/\sqrt{h}$. Here τ=const over the spatial surface is taken following York. The momentum constraint can be solved by expanding $\tilde{K}^{ij}$ in terms of the basis $\{\phi^{(\alpha)ij}\}$ of the quadratic differentials

$$\tilde{K}^{ij}=\sum_{(\alpha)}p_{(\alpha)}\phi^{(\alpha)ij}/2v \tag{2·16}$$

with $v=\int d^2x\sqrt{h}$. The deformation of $\tilde{h}_{ij}$ is represented as

$$\frac{\partial\tilde{h}_{ij}}{\partial t}=\sum_{(\alpha)}\frac{\partial\rho^{(\alpha)}}{\partial t}\mu^l_{(\alpha)i}\tilde{h}_{lj}+\text{diffeo}\,. \tag{2·17}$$

This equation defines the Teichmüler parameters $\rho^{(\alpha)}$ and the corresponding Beltrami differentials $\mu^l_{(\alpha)i}$. Substitute the expansion Eqs. (2·16) and (2·17) for $\tilde{K}^{ij}$ and $\partial\tilde{h}_{ij}/\partial t$

into the action Eq. (2·15) in the phase space. Due to the special gauge $N=N(t)$, the final form of the action becomes

$$S=\int dt\Big[\sum p_{(\alpha)}\frac{\partial\rho^{(\alpha)}}{\partial t}+\tau\frac{\partial v}{\partial t}-\tilde{N}(\sum p_{(\alpha)}p_{(\beta)}g^{(\alpha)(\beta)}-v^2\tau^2)\Big], \tag{2·18}$$

where

$$v=\int d^2x\sqrt{h}\,,$$

$$\tilde{N}=N/2v\,. \tag{2·19}$$

Here

$$g^{(\alpha)(\beta)}=\int d^2x\sqrt{h}\,\phi_{ij}^{(\alpha)}\phi_{kl}^{(\beta)}h^{ik}h^{jl}/2v \tag{2·20}$$

is the Weil-Petersson metric. From this result, the geodesic motion in the conformal superspace is apparent.

§ 3. Linearized analysis

The main purpose of this paper is to analyze the effects of matter fields on the geodesic motion. Our standing point is the following. We arbitrarily pick up some point in the conformal superspace and look at the infinitesimal neighborhood of this point. From this point of view, the geodesic motion is a straight line. The effects of matter fields can also be easily seen. To do this we shall start from linearized theory of gravity and then incorporate the matter fields perturbatively. Our strategy is to treat the approximation in which gravity is "weak". In the context of general relativity this means that the space-time metric is nearly flat. The criterion of the weak gravity does not seem to apply to our case, because the Einstein equation in (2+1)-dimensions implies that the space-time is locally flat. Globally, however, there are "topological degrees of freedom" to consider.

Let us start to analyze the pure linearized gravity. For the moment, we simply assume that the deviation, $h_{\mu\nu}$, of the actual space-time metric

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \tag{3·1}$$

from the flat metric $\eta_{\mu\nu}$ is "small". We mean by "linearized gravity" that approximation to general relativity which is obtained by substituting equation (3·1) for $g_{\mu\nu}$ in the Einstein-Hilbert action and retaining only the terms quadratic in $h_{\mu\nu}$. The result is given by

$$S=\int d^3x(\Gamma_\lambda{}^{\sigma\nu}\Gamma^\lambda_{\nu\sigma}-\Gamma_\nu{}^{\lambda\nu}\Gamma^\sigma_{\sigma\lambda})\,, \tag{3·2}$$

where $\Gamma_{\sigma\mu\nu}=(1/2)(h_{\sigma\mu,\nu}+h_{\sigma\nu,\mu}-h_{\mu\nu,\sigma})$. This action is also rewritten as

$$S=\int d^3x[K_{ij}K^{ij}-K^2+\Gamma_{ijk}\Gamma_{kji}-\Gamma_{ikk}\Gamma_{jji}+n/2(h_{kk,ii}-h_{ik,ik})]\,, \tag{3·3}$$

where $K_{ij}=\Gamma_{0ij}$, $n=h_{00}$ and $N_i=h_{0i}$. To cast this action into first order form, let us define the momentum

$$\pi^{ij}=K\eta^{ij}-K^{ij}. \tag{3·4}$$

Then, we get

$$S=\int d^3x\{\pi^{ij}\dot{h}_{ij}-[\pi_{ij}\pi^{ij}-\pi^2+\Gamma_{ijk}\Gamma_{kji}-\Gamma_{ikk}\Gamma_{jji}+2N_i\pi^{ij}_{,j}$$
$$+(n/2)(h_{kk,ii}-h_{ik,ik})]\}. \tag{3·5}$$

Our next task is to solve the constraint equations

$$\pi^{ij}_{,j}=0,$$
$$h_{kk,ii}-h_{ik,ik}=0. \tag{3·6}$$

For this purpose we use the following decomposition,

$$\pi^{ij}=\pi^{ij}_{TT}+(LV)^{ij}+(1/2)\eta^{ij}\pi,$$
$$h_{ij}=h^{TT}_{ij}+(LW)_{ij}+(1/2)\eta_{ij}h, \tag{3·7}$$

where $(LV)^{ij}=\partial^i V^j+\partial^j V^i-\eta^{ij}\partial_k V^k$. Here h^{TT}_{ij} and π^{ij}_{TT} represent transverse-traceless parts of h_{ij} and π^{ij}. The general solution of constraint equations is

$$\pi^{ij}=\pi^{ij}_{TT}-\partial^i\partial^j(\pi/\Delta)+\eta^{ij}\pi,$$
$$h_{ij}=h^{TT}_{ij}-(1/2)\eta_{ij}h+\epsilon_{jk}\partial_i\partial^k\phi+\epsilon_{ik}\partial_j\partial^k\phi, \tag{3·8}$$

where ϕ is an arbditrary function. Inserting Eq. (3·8) to the action, we obtain the well-known action[11])

$$S=\int d^3x\left\{\pi^{ij}_{TT}\dot{h}^{TT}_{ij}-\left[\pi^{TT}_{ij}\pi^{ij}_{TT}+\frac{1}{4}(h^{TT}_{jk,i})^2\right]\right\}. \tag{3·9}$$

In the topologically trivial space nothing happens. As we are considering the torus case, h^{TT}_{ij} and π^{ij}_{TT} are spatially constants. So we get

$$S=\int dt\int d^2x[\pi^{ij}_{TT}\dot{h}^{TT}_{ij}-\pi^{TT}_{ij}\pi^{ij}_{TT}]$$
$$=\int dt\left[p_\alpha\dot{q}^\alpha-\frac{g^{\alpha\beta}}{2v}p_\alpha p_\beta\right]. \tag{3·10}$$

Note that the Weil-Petersson metric $g^{\alpha\beta}$ does not depend on q^α in contrast to the full gravity. This fact is undersood as follows. As we regard the deviation from the background metric as small, we are on the tangent space at some point in the conformal superspace. As a consequence the geodesic motion is straight. It is natural, because any geodesic motion is locally straight. At this point we would like to emphasize that the clear-cut result for the case $g=1$ heavily depends on the constancy of h^{TT}_{ij}. We cannot expect the geodesic motion in the conformal superspace in the case $g\geq 2$.

We are now in a position to discuss the effects of matter fields on geodesic motion.

Our action to consider is

$$S=\int d^3x\sqrt{g}R-\frac{1}{4}\int d^3x\sqrt{g}\,g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}\,. \tag{3·11}$$

To analyze this system, we assume that the Maxwell fields are sufficiently small so that quartic term can be negligible. Then full action reduces to

$$\begin{aligned}S&=\int d^3x(\Gamma_{\lambda}{}^{\sigma\nu}\Gamma^{\lambda}_{\nu\sigma}-\Gamma_{\nu}{}^{\lambda\nu}\Gamma^{\sigma}_{\sigma\lambda})-\frac{1}{4}\int d^3xF_{\mu\nu}F^{\mu\nu}+\int d^3x\frac{1}{2}h^{\sigma\mu}T_{\sigma\mu}\\&=\int d^3x\Big\{\pi^{ij}\dot{h}_{ij}-[\pi_{ij}\pi^{ij}-\pi^2+\Gamma_{ijk}\Gamma_{kji}-\Gamma_{ikk}\Gamma_{jji}]\\&\quad+N_i(2\pi^{ij}_{,j}-T_{0i})+\frac{n}{2}(h_{kk,ii}-h_{ik,ik}+T_{00})+\frac{1}{2}h_{ij}T_{ij}\Big\}\\&\quad+\int d^3x\Big[\pi^i\dot{A}_i-\frac{1}{2}\Big(\pi^i\pi_i+\frac{1}{2}F^{ij}F_{ij}\Big)+A_0\partial_i\pi^i\Big].\end{aligned} \tag{3·12}$$

The small perturbation term can be written as

$$h_{ij}T_{ij}=h^{TT}_{ij}T^{TT}_{ij}+h^{L}_{ij}T^{L}_{ij}+h^{T}_{ij}T^{T}_{ij}\,. \tag{3·13}$$

Solving the constraint equations, we can obtain the longitudinal and the trace parts which are represented by matter fields. These are, however, higher order effects which we disregard in our approximation scheme. Finally retaining the relevant part only, we obtain

$$\begin{aligned}S&=\int dt\int d^2x\Big[\pi^{ij}_{TT}\dot{h}^{TT}_{ij}-\pi^{TT}_{ij}\pi^{ij}_{TT}+\frac{1}{2}h^{TT}_{ij}T^{ij}_{TT}\Big]\\&=\int dt\Big[p_\alpha\dot{q}^\alpha-\frac{g^{\alpha\beta}}{2v}p_\alpha p_\beta+q^\alpha F_\alpha\Big],\end{aligned} \tag{3·14}$$

where $F_\alpha=\int d^2x\mu_\alpha{}^{ij}T_{ij}$. Equation (3·14) is our main result. The geodesic motion in the conformal superspace is deviated by the transverse-traceless parts of the energy momentum tensor, i.e., its global mode part. Note that the final formula need not assume specific matter fields. The reason why we concentrate on the Maxwell fields is the existence of global modes of Maxwell fields on torus.

Let us recall the several elementary facts about the Riemann surface. In the complex notation, the Abelian differential ω satisfies the periodicity ;

$$\oint_a\omega=1\,,\qquad\int_b\omega=\tau=\tau_1+i\tau_2\,, \tag{3·15}$$

and the Riemann relation

$$\frac{i}{2}\int\omega\wedge\bar{\omega}=\mathrm{Im}\,\tau=\tau_2\,, \tag{3·16}$$

where a and b represent homology cycles for the different two directions of a torus. In terms of the real basis

$$\omega = \alpha + i * \alpha\,, \tag{3·17}$$

Eq. (19) means

$$\oint_a \alpha = 1\,, \quad \oint_a * \alpha = 0\,,$$

$$\oint_b \alpha = \tau_1\,, \quad \oint_b * \alpha = \tau_2\,, \tag{3·18}$$

and also Eq. (20) implies

$$\int \alpha \wedge * \alpha = \tau_2\,. \tag{3·19}$$

We shall take the basis as

$$\xi_{(1)} = \alpha\,, \quad \xi_{(2)} = * \alpha\,,$$

$$\eta^{(1)} = \frac{\alpha}{\tau_2}\,, \quad \eta^{(2)} = \frac{* \alpha}{\tau_2}\,. \tag{3·20}$$

The orthonormality relation, $\int \xi_{(\alpha)} \wedge {}^* \eta^{(\beta)} = \frac{1}{\tau_2} \delta^{(\alpha)(\beta)}$, holds and the metric becomes

$$g^{(\alpha)(\beta)} = \int \eta^{(\alpha)} \wedge {}^* \eta^{(\beta)} = \frac{1}{\tau_2} \delta^{(\alpha)(\beta)}\,. \tag{3·21}$$

Note that

$$\oint_a A = a^{(1)}\,, \quad \oint_b A = a^{(1)} \tau_1 + a^{(2)} \tau_2\,. \tag{3·22}$$

Using these bases, Eq. (3·20), the gauss law constraint is solved as

$$E^i = \epsilon^{ij} \partial_j \phi + \{\pi_{(1)} \eta^{(1)i} + \pi_{(2)} \eta^{(2)i}\}\,, \tag{3·23}$$

and the vector potential decomposes to

$$A_i = \partial_i \chi - \epsilon_{ij} \partial^j \left(\frac{B}{\Delta}\right) + \{a^{(1)} \xi_{(1)i} + a^{(2)} \xi_{(2)i}\}\,, \tag{3·24}$$

where Δ is the Laplacian. From the knowledge we can obtain the spatial part of energy momentum tensor as

$$\begin{aligned} T_{ij} = {} & \partial_i \phi \partial_j \phi - \frac{1}{2} \eta_{ij} \partial_k \phi \partial^k \phi + \frac{1}{2} \eta_{ij} \pi_\alpha \eta_l{}^\alpha \epsilon_{lk} \partial^k \phi + \frac{1}{2} \eta_{ij} \pi_\alpha \pi_\beta \eta_k{}^\alpha \eta_k{}^\beta \\ & - \pi_\alpha \eta_i{}^\alpha \epsilon_{ik} \partial^k \phi - \pi_\alpha \eta_j{}^\alpha \epsilon_{ik} \partial^k \phi - \pi_\alpha \pi_\beta \eta_i{}^\alpha \eta_j{}^\beta \\ & + 2 \partial_i \partial_j \left(\frac{B}{\Delta}\right) B - 2 \partial_i \partial^l \left(\frac{B}{\Delta}\right) \partial_j \partial^l \left(\frac{B}{\Delta}\right) + \eta_{ij} \partial_k \partial^l \left(\frac{B}{\Delta}\right) \partial_k \partial^l \left(\frac{B}{\Delta}\right) - \frac{1}{2} \eta_{ij} B^2\,. \end{aligned} \tag{3·25}$$

As we are interested in the global modes, we pick up them

$$T_{ij}=\frac{1}{2}\eta_{ij}\pi_\alpha\pi_\beta\eta_k{}^\alpha\eta_k{}^\beta-\pi_\alpha\pi_\beta\eta_i{}^\alpha\eta_j{}^\beta\,, \tag{3·26}$$

and put this expression into the formula (3·14). The result is

$$S=\int dt\left[p_\alpha\dot{q}^\alpha-\frac{g^{\alpha\beta}}{2v}p_\alpha p_\beta+q^\alpha\pi_\gamma\pi_\beta C_\alpha{}^{\beta\gamma}\right],$$

$$C_\alpha{}^{\beta\gamma}=\int d^2x\mu_\alpha{}^{ij}\left(\frac{1}{2}\eta_{ij}\eta_k{}^\beta\eta_k{}^\gamma-\eta_i{}^\beta\eta_j{}^\gamma\right). \tag{3·27}$$

In the case of the Einstein-Maxwell theory, the Wilson loop degrees bend the trajectory of the global degrees of gravity. This is understood as follows: The Wilson loop wind the non-trivial cycles of the torus, then the free motion of the torus is disturbed by its tension.

§ 4. Conclusion

We have succeeded in getting quantitative understanding for (2+1)-dimensional gravity in the case of linearized theory. Although analysis for full gravity is too difficult to make any quantitative statement, we expect that the qualitative features remain true for this case. Furthermore we might speculate that the global deformation of space is dictated by the global part of energy-momentum tensor in any dimensions. This is the motivation of our previous work[12] in which we analyzed the abelian gauge theory in topologically non-trivial space. In this case, however, the global dynamics may not be the geodesic motion in the conformal superspace due to the potential term.

Now we understand the qualitative behavior of (2+1)-dimensional gravity coupled with the matter fields in the case $g=1$. Next we would like to explain how to relate our analysis to the topology changing phenomena. Imagine the Riemann surface with genus $g=2$, pinching some cycle produces a degenerate Riemann surface which is the boundary of the superspace. Although we cannot go through this point classically,[13] there may be a chance to surmount this pinching point as a quantum mechanical tunneling process. If this is possible, topology changing effects can be formulated on the universal moduli space which has previously been studied in superstring theory.[14] At this point matter fields must play an important role. Of course we should study (2+1)-dimensional gravity in the case $g\geq 2$ before challenging this big problem.

Application of our method to (3+1)-dimensional gravity is interesting and tractable in the case of simple topology. We are also planning to study the Einstein-Maxwell theory in the case of special initial data. It will be helpful to understand our results.

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References

1) B. S. DeWitt, Phys. Rev. **160** (1967), 1113.
2) S. Coleman, Nucl. Phys. **B310** (1988), 867.
3) K. Kuchac, in *Quantum Gravity* 2, ed. C. J. Isham, R. Penrose and D. W. Sciama (Claredon Press, Oxford, 1981).
4) J. W. York, Jr., Phys. Rev. Lett. **28** (1972), 1082.
J. W. York, Jr., J. Math. Phys. **14** (1973), 456.
N. O. Murchadha and J. W. York, J. R., Phys. Rev. **D10** (1974), 428, 437.
5) S. Giddings, J. Abbott and K. Kuchac, Gen, Rel. Grav. **16** (1984), 751.
6) E. Witten, Nucl. Phys. **B311** (1988), 46.
7) V. Moncrief, 'Reduction of the Einstein Equations in (2+1)-dimensions to a Hamiltonian Systems over Teichmüler Space,' Yale University Preprint.
8) A. Hosoya and K. Nakao, RRK 89-11; RRK 89-16.
9) A. E. Fischer and A. J. Tromba, Trans. Amer. Math. Soc. **284** (1984), 319.
10) R. Arnowitt, S. Deser and C. W. Misner, in *Gravitation* ed. L. Witten (Wiley, New York-London, 1962).
11) R. Arnowitt and S. Deser, Phys. Rev. **113** (1959), 745.
12) A. Hosoya and J. Soda, RRK 89-13 (1989).
13) R. P. Geroch, J. Math. Phys. 8 (1967), 782.
14) D. Friendan and S. Shenker, Phys. Lett. **B175** (1986), 287.