Power Allocation in Wireless Relay Networks

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by

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Acknowledgments

Abstract

This thesis is mainly concerned with power allocation issues in wireless relay networks where a single or multiple relays assist transmission from a single or multiple sources to a destination.

First, a network model with a single source and multiple relays is considered, in which both cases of orthogonal and non–orthogonal relaying are investigated. For the case of orthogonal relaying, two power allocation schemes corresponding to two partial channel state information (CSI) assumptions are proposed. Given the lack of full and perfect CSI, appropriate signal processing at the relays and/or destination is also developed. The performance behavior of the system with power allocation between the source and the relays is also analyzed. For the case of non-orthogonal relaying, it is demonstrated that optimal power allocation is not sufficiently effective. Instead, a relay beamforming scheme is proposed. A comprehensive comparison between the orthogonal relaying with power allocation scheme and the non-orthogonal relaying with beamforming scheme is then carried out, which reveals several interesting conclusions with respect to both error performance and system throughput.

In the second part of the thesis, a network model with multiple sources and a single relay is considered. The transmission model is applicable for uplink channels in cellular mobile systems in which multiple mobile terminals communicate with the base station with the help of a single relay station. Single-carrier frequency division multiple access (SC-FDMA) technique with frequency domain equalization is adopted in order to avoid the amplification of the multiple access interference at the relay. Minimizing the transmit power at the relay and optimizing the fairness among the sources in terms of throughput are the two objectives considered in implementing power allocation schemes. The problems are visualized as *water-filling* and *water-discharging* models and two optimal power allocation schemes are proposed, accordingly.

Finally, the last part of the thesis is extended to a network model with multiple sources and multiple relays. The orthogonal multiple access technique is employed in

order to avoid multiple access interference. Proposed is a joint optimal beamforming and power allocation scheme in which an alternative optimization technique is applied to deal with the non-convexity of the power allocation problem. Furthermore, recognizing the high complexity and large overhead information exchange when the number of sources and relays increases, a relay selection scheme is proposed. Since each source is supported by at most one relay, the feedback information from the destination to each relay can be significantly reduced. Using an equal power allocation scheme, relay selection is still an NP-hard combinatorial optimization problem. Nevertheless, the proposed sub-optimal scheme yields a comparable performance with a much lower computational complexity and can be well suited for practical systems.

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List of Abbreviations

AF   Amplify-and-Forward

AWGN   Additive White Gaussian Noise

BPSK   Binary Phase-Shift Keying

CCI   Co-Channel Interference

CDMA   Code Division Multiple Access

CIR   Channel Impulse Response

CP   Cyclic Prefix

CSI   Channel State Information

CSMA   Carrier Sense Multiple Access

CSNR   Channel Signal-to-Noise Ratio

dB   decibel

DCF   DeCorrelate-and-Forward

DF   Decode-and-Forward

DFT   Discrete Fourier Transform

EPA   Equal Power Allocation

FDE   Frequency-Domain Equalization

FDMA   Frequency Division Multiple Access

IBP   Iterative Bisection Procedure

IDFT   Inverse Discrete Fourier Transform

IRI	Inter-Relay Interference
ISI	Inter-Symbol Interference
KKT	Karush-Kuhn-Tucker
LMMSE	Linear Minimum Mean Square Error
LOS	Line-Of-Sight
LTE	Long-Term Evolution
MAI	Multiple Access Interference
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Square Error
MRC	Maximal-Ratio Combiner
MSE	Mean Square Error
NLOS	Non-Line-Of-Sight
NP-hard	Non-deterministic Polynomial-time hard
NUM	Network Utility Maximization
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
OPA	Optimal Power Allocation
PAPR	Peak-to-Average Power Ratio
QoS	Quality-of-Service
QPSK	Quadrature Phase-Shift Keying
SC-FDMA	Single-Carrier Frequency Division Multiple Access

SER	Symbol-Error-Rate
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
TDMA	Time Division Multiple Access
WiMAX	Worldwide interoperability for Microwave Access

1. Introduction and Organization of the Thesis

1.1 Introduction

In recent years, a large number of wireless applications which require high data rate and high transmission quality have been introduced in order to meet the tremendously increasing demand in wireless communications. Given very limited network resources and a crowded wireless frequency spectrum shared by an increasing number of operators and services, the three fundamental design issues in digital wireless communication systems, namely capacity, coverage and interference, have become more intertwined. This problem can be readily faced in any incumbent cellular mobile networks as well as in other recently emerging networks such as wireless ad-hoc networks and wireless sensor networks [1].

As an illustrative example, let's examine the cellular networks. Due to a limited amount of available resources such as bandwidth and transmit power, users' requirements on the data rate and reliability cannot always be satisfied, especially when the number of users with high-speed data services is increasing rapidly in many commercial networks. The limit in the transmit power is also the main reason for the limit in transmission coverage. Such a limitation can be experienced by the users at the cell edge in any cellular network where an insufficient transmit power level could not overcome the attenuation over a long transmission distance. And finally when the available frequency spectrum is not enough to accommodate all users, some users have to share the same frequency band to transmit their signals. As a result, users at the cell edge not only experience insufficient power levels but also suffer from the increasing interference power from other users using the same frequency band.

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It is also well known that the cause of these fundamental design issues is rooted in the random quality of the wireless channels. Due to the scattering, reflection and diffraction of the transmitted energy caused by obstacles such as buildings, trees, etc., in the environment, multiple versions of a signal transmitted from a source may arrive at the destination via multiple paths with different attenuations, phase shifts and delays. The overall received signal could be a constructive or destructive superposition of these versions [2, 3]. In fact, severe destructive combinations may occasionally happen, causing a severe drop of the channel gain, which is called *deep fade*. Communication over a channel in deep fade results in a temporarily failure or discontinuity of the service.

An effective way to mitigate the adverse effect of fading is to employ *diversity* techniques. The basic idea behind all diversity techniques is to provide different replicas of the same information over multiple independently faded paths in order to decrease the probability that the received signal is in *deep fade*, thus increase the reliability and the probability of successful transmission. Depending on the characteristics of the channels as well as the transceiver structures, common diversity techniques that have been studied extensively in the literature and applied in practice include time diversity (e.g., channel coding, interleaving) [4], spatial diversity (e.g., multiple-input multiple-output (MIMO) systems) [5], combination of multi-path and frequency diversity (e.g., orthogonal frequency division multiplexing (OFDM)) [6].

Among the above-mentioned techniques, spatial diversity with multiple transmit antennas and/or multiple receive antennas (also known as MIMO techniques) has emerged as a promising solution since it does not suffer from a reduced bandwidth efficiency. However, due to the limitations on power and size, terminal devices in many wireless applications such as mobile terminals in cellular networks or sensor terminals in wireless sensor networks cannot support multiple co-located antennas in order to fully exploit spatial diversity.

In the field of microwave transmission, relaying transmission technique has been widely employed for several decades. Using repeaters in the middle of the transmis-

sion link can effectively extend the transmission range between the transmitter and receiver. Recently, its capability of providing spatial diversity in the form of cooperation among different users or nodes in the network has been explored [7–10]. Cooperative communication schemes can provide enhancements in terms of end-to-end throughput even if they require additional radio resources due to multi-hop transmissions.
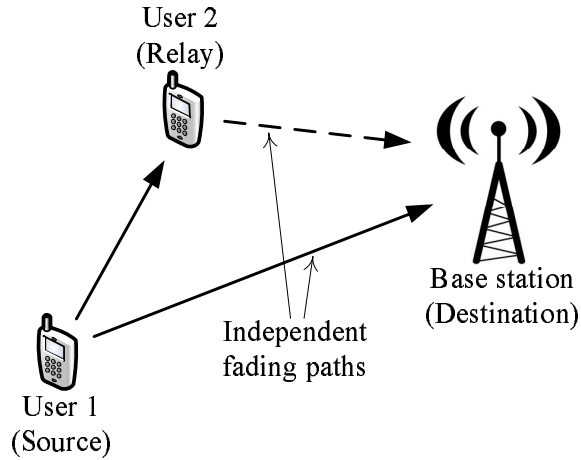
Figure 1.1   A wireless relay network.

The key idea of relay communications can be explained with the help of Fig. 1.1, which shows a simple wireless network with 3 nodes, i.e., User 1 (source), User 2 (relay) and the base station (destination). User 1 tries to communicate with the base station with the help of User 2. With only one transmit antenna User 1 cannot individually create spatial diversity. However, due to the broadcasting nature of wireless transmission, User 2 can receive the transmitted signal from User 1 and then tries to assist User 1's transmission by sending some version of its received signal to the base station. Because the two versions of the source signal experience independent fading paths, spatial diversity can be obtained in such a system. Since operating in the half-duplex mode, the relay cannot receive and transmit at the same time. Therefore, two time slots (or transmission phases) are needed to complete each transmission from the source to the destination[1]. Further improvement can be achieved by creating more

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[1]It is noted that User 1 can also use *repetition coding* scheme [2] such that it retransmits the signal in the second time slot provided that the fading characteristics of the two time slots are highly

independent fading paths with more relays deployed in the system.

Beyond the diversity capability to mitigate the fading effects, relaying transmission can also reduce the propagation attenuation to increase the capacity and/or coverage of the networks. However, to exploit those advantages, various design problems concerning wireless cooperative networks need be addressed. The works presented in this thesis focus on one of the problems, namely *the power control/allocation* problem.

In a conventional wireless network, power control is one of the main methods to handle the interference, improve the quality of the signal reception, thus increasing the coverage and/or capacity of the overall network. In cooperative networks with more terminals participating in each transmission, the power control of each source terminal as well as the power allocation issue at the relays becomes much more complicated. However, this also represents a potential venue to offer a significant improvement on the quality of transmissions in many situations. Methods to realize this improvement in several relay-assisted transmission models are the main subject of this thesis. Further detailed motivations and contributions of the current research will be given in each chapter.

1.2   Organization of the Thesis

This dissertation is organized in a manuscript-based style. The first part of the thesis discusses some relevant background and knowledge of wireless communications and power allocation. Included as the main content and contributions of the thesis are published or submitted manuscripts. There are also footnotes added to provide answers and/or add clarifications to the external examiner's questions and comments. Since these revisions are not in the original versions of the included manuscripts, they are formatted in sans serif font.

In Chapter 2, fundamental knowledge of wireless channels and methods to mit-

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independent. However, the quality of the channel from User 1 to the base station is usually poor and receiving the help from User 2 in a closer distance can improve the quality of the transmission.

igate the effects of fading are summarized. Major advantages and disadvantages of cooperative communications are then explained, which motivates the need of efficient resource allocation schemes in order to exploit the advantages and avoid the disadvantages. In each of the following chapters, a brief introduction precedes each manuscript in order to connect the manuscript to the main context of the thesis.

The first manuscript included in Chapter 3 considers an *orthogonal* amplify-and-forward relaying system with two different assumptions on the availability of the channel state information at the relays and destination. Corresponding to each assumption, an optimal power allocation scheme is proposed aiming at minimizing the symbol error rate of the overall transmission. The second manuscript included in Chapter 4 can be considered as an extension of the manuscript in Chapter 3, in which a *non-orthogonal* AF relaying model is studied. An interesting comparison between the two models is presented, which suggests some important signal processing operations at each terminal in the relay networks under consideration. It is noted that the manuscripts in Chapters 3 and 4 are restricted to a single source model in which several relays help the transmission between one source and one destination.

In the second part of the thesis, beginning from Chapter 5, a relaying model with multiple sources is investigated. To avoid the interference coming from the relay(s), orthogonal multiple access techniques are implemented. The manuscript in Chapter 5 develops a framework for a single-carrier FDMA based relaying transmission with multiple sources and a single relay and proposes two power allocation schemes related to signal-to-interference-plus-noise ratio and capacity. Further investigations of the transmission model with multiple sources and multiple relays are presented in Chapter 6 and Chapter 7. In Chapter 6, a joint beamforming and power allocation scheme is proposed. Considering the trade-off between performance improvement and information exchange overhead, a relay assignment scheme is proposed in Chapter 7, which can be applied in any relaying systems that employ orthogonal multiple access techniques.

Finally, Chapter 8 concludes this thesis by summarizing the contributions and suggesting potential research problems for future studies.

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2. Background and System Model

This chapter reviews some important background on statistical characteristics of wireless fading channels and methods to mitigate the effects of fading. It also describes fundamental concepts of relay communications and power allocation schemes. The purpose of this chapter is to familiarize the reader with the field of the study and establish the foundation for the rest of this thesis.

2.1 Statistical Model of Wireless Channels

In wireless communication systems, in order to send information/data over wireless channels a source/transmitter first digitally modulates a reference signal (i.e., changing its parameters corresponding to the information)[1], then up-converts the modulated signal to some frequency band at a carrier frequency f_c before radiating it over the air via an antenna system. The transmitted signal can be mathematically expressed by

$$s_{\mathrm{RF}}(t) = \Re\left\{s(t)\mathrm{e}^{j2\pi f_c t}\right\}, \tag{2.1}$$

where $\Re\{\cdot\}$ denotes the real part of the enclosed quantity, $s(t)$ is called the complex envelope (or the equivalent baseband representation) of the transmitted signal $s_{\mathrm{RF}}(t)$. Depending on the required transmission rate of the information, the modulated signal is usually designed to occupy an appropriate portion of radio frequency spectrum (i.e., the signal bandwidth B_s) which is inversely proportional to the symbol duration of the transmitted signal, T_s. At the destination/receiver side, a reverse procedure is

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[1]It is noted that other processing steps may be included in order to improve the robustness of the signal against the deterioration effects of the channel.

carried out to retrieve the desired information.
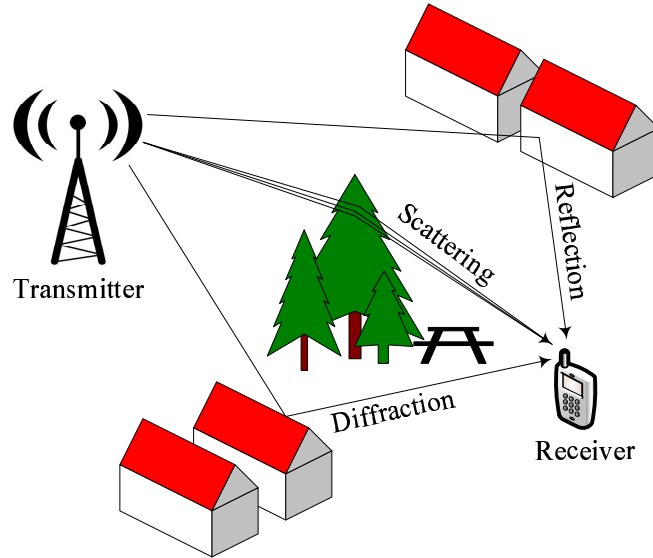
Figure 2.1   Multipath propagation in wireless channels.

When the signal is transmitted over a wireless channel, the destination usually receives a number of different copies of the transmitted signal over different propagation paths. They may include a signal traversing over a line-of-sight (LOS) path if no obstacle obscures the straight line between the source and destination. Obviously, this is a favorable transmission condition since the power of the received signal varies mainly according to the propagation distance while other randomness factors do not affect the transmitted signal significantly. However, an LOS signal is not always available, especially in cellular networks where there are a lot of obstacles surrounding both the source and destination. In such situations, different versions of the transmitted signal come from the non-line-of-sight (NLOS) paths. As illustrated in Figure 2.1, these components are the results of reflection, diffraction and scattering of the signal being an electromagnetic wave. When hitting objects whose sizes are much larger than the wavelength (e.g., walls, buildings), the electromagnetic wave may reflect into various directions before being coupled by the antenna system at the destination. The wave may also impinge irregular surfaces like sharp edges and thus be diffracted while the scattering happens when the wave traverses to a large number of objects much smaller than the wavelength, then being reflected off in multiple different directions.

The overall impairment of these phenomena can be characterized by the path-loss, shadowing and fading effects. While the path-loss is simply the attenuation incurred by the transmitted signal when propagating over a distance from a source to a destination, shadowing and fading are two types of variation in the transmitted signal power over time and frequency. Shadowing causes the fluctuation of the signal strength around the path-loss over distances up to a few hundreds of wavelengths. As a result, it is also called large-scale fading. On the other hand, another type of fading is caused by the constructive and destructive superpositions of multiple attenuated, phase-shifted and delayed versions of the transmitted signal traveling via multiple propagation paths. It occasionally results in a *severe* attenuation of the received signal power, which is called *deep fade* (see the illustration in Figure 2.2). When the channel is in a deep fade, it is very difficult to recover the transmitted information at the receiver. Because each multipath component can undergo a phase shift of 2π over a traveled distance as short as one wavelength, this type of power fluctuation occurs over a very small time-scale and, for this reason, it is also referred to as small-scale fading.
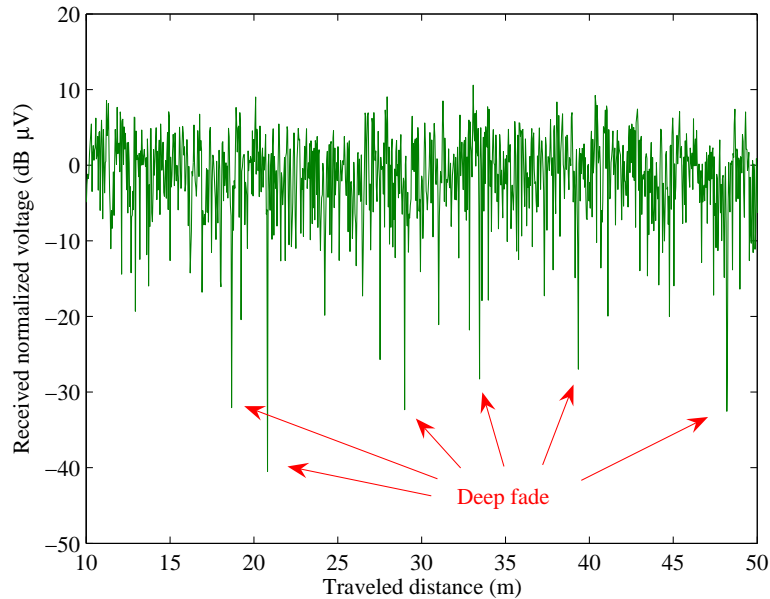
Figure 2.2   Deep fades due to multipath fading.

With multipath propagation, the characteristics of a wireless channel can be fully described by its time-varying channel impulse response (CIR) which is the response of the channel at time t to a Dirac delta function/impulse applied at time $t - \tau$, i.e., τ seconds before. Mathematically, the CIR can be written as [1]

$$h(\tau, t) = \sum_{\ell=1}^{N_p} \alpha_\ell(t) e^{j\theta_\ell(t)} \delta(\tau - \tau_\ell(t)), \qquad (2.2)$$

where N_p denotes the number of resolvable multipath components, $\alpha_\ell(t)$, $\theta_\ell(t)$, and $\tau_\ell(t)$ are the time-varying attenuation, phase shift, and propagation delay of the ℓth path, respectively. The characteristics of the multipath channel in the frequency domain also play an important role and are described by the channel frequency response at time t:

$$H(f, t) = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi f\tau} d\tau = \sum_{\ell=1}^{N_p} \alpha_\ell(t) e^{j\theta_\ell(t)} e^{-j2\pi f\tau_\ell(t)}. \qquad (2.3)$$

As mentioned earlier, a typical digital wireless communication system has several signal processing steps at different frequency bands. To focus on the issues related to the baseband processing steps (i.e., signal waveform designs), the equivalent baseband representations of the signals and channel are usually preferred. If the signal $s_{\mathrm{RF}}(t)$ in (2.1) is transmitted over a wireless channel, the equivalent baseband (or complex envelope) of the received signal can be mathematically represented as

$$r(t) = h(\tau, t) \star s(t) + z(t) = \sum_{\ell=1}^{N_p} \alpha_\ell(t) e^{j\theta_\ell(t)} s(t - \tau_\ell(t)) + z(t), \qquad (2.4)$$

where $\star$ denotes the convolution operation, $s(t)$ represents the equivalent baseband transmitted signal (or symbol), whose energy is E_s over the duration of T_s seconds and $z(t)$ is zero-mean additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$ (watts/Hz).

By correlating $r(t)$ with an appropriate waveform (which can be done with a correlator or a matched-filter), an equivalent discrete-time baseband input-output model of (2.4) can be represented by

$$r[n] = \sum_l a_l[n] s[n - l] + z[n], \qquad (2.5)$$

where the noise term $z[n]$ is modeled as a circularly symmetric complex Gaussian random variable, denoted as $\mathcal{CN}(0, N_0)$, $a_l[n]$ represents the lth tap of the channel filter at discrete time n. Roughly speaking, $a_l[n]$ is a function of the gains $\alpha_\ell(t)e^{j\theta_\ell(t)}$ of the propagation paths with $\tau_\ell(t)$ close to l/B_s where B_s is the sampling rate and also twice the bandwidth of the equivalent complex baseband signal.

An important observation from (2.4) is that compared to the transmitted signal which confines in a time duration T_s, the received waveform is now distorted and spread over a duration of $T_s + T_d$ where T_d is the time delay spread of the multipath channel, i.e., $T_d = \max_{\ell,k}|\tau_\ell(t) - \tau_k(t)|$. Beside causing distortion to the transmitted signal in each signaling duration, this time dispersiveness may cause the interference of the transmitted signals over several consecutive durations. This effect is called inter-symbol interference (ISI), which is illustrated in Figure 2.3.

Transmitted signal   Received signal
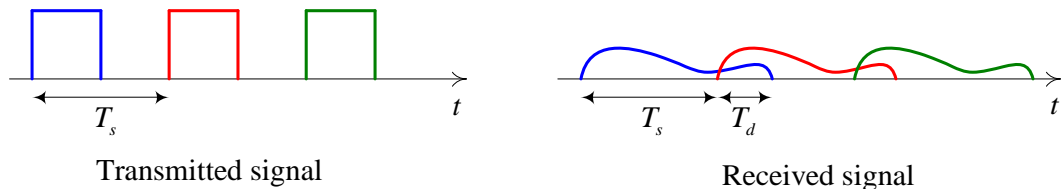
Figure 2.3   Inter-symbol interference due to multipath fading.

If the delay spread is much smaller than the symbol duration, i.e., $T_d << T_s$, then it is reasonable to set $\tau_\ell(t) \approx 0$, yielding

$$h(\tau, t) \approx \rho(t)e^{j\varphi(t)}\delta(\tau) \tag{2.6}$$

and

$$H(f, t) \approx \rho(t)e^{j\varphi(t)}, \tag{2.7}$$

where

$$\rho(t)e^{j\varphi(t)} = \sum_{\ell=1}^{N_p} \alpha_\ell(t)e^{j\theta_\ell(t)}. \tag{2.8}$$

Inspection of (2.6) and (2.7) reveals that at each time t, the channel is frequency-nonselective or flat since $H(f, t)$ is practically constant over the whole signal bandwidth, while $h(\tau, t)$ causes attenuation and phase rotation to the transmitted signal.

Intersymbol interference in this case is also negligible. When T_d increases, the ISI becomes more severe and the channel frequency response becomes more frequency-selective.

One can also observe from (2.6) and (2.7) the time-varying nature of wireless channels via the presence of the time variable t. This is due to the *relative* movements among the transmitter, receiver and/or the surrounding obstacles over time. If the channel impulse response varies at a rate much slower than the signaling rate, the channel is referred to as slow fading. On the other hand, if the channel impulse response changes quickly within a signal duration, the channel is considered to be fast fading. For frequency-flat and slow fading channels, the channel model (2.5) reduces to

$$r[n] = a[n]s[n] + z[n]. \tag{2.9}$$

As indicated in Eq. (2.8), the multiplicative factor $\rho(t)e^{j\varphi(t)}$ is the sum of N_p statistically independent multipath components. From the central limit theorem, for each time instant the real and imaginary parts of $\rho(t)e^{j\varphi(t)}$ can reasonably be approximated as two statistically independent Gaussian random variables with the same variance σ^2 and expected values η_R and η_I, respectively. In the absence of any line-of-sight (LOS) path between the transmitter and receiver, no dominant multipath component is present and $\eta_R = \eta_I = 0$. In such a case the phase term $\varphi(t)$ is found to be uniformly distributed over $[-\pi, \pi)$, while the amplitude $\rho(t)$ follows a Rayleigh distribution with probability density function (pdf)

$$p(\rho) = \frac{\rho}{\sigma^2}e^{-\frac{\rho^2}{2\sigma^2}}, \quad \rho \geq 0. \tag{2.10}$$

The Rayleigh fading channels are considered throughout this thesis. For other statistical channel models, the interested reader can refer to various references in the literature such as [2–4].

Corresponding to different types of channel fading, there are several suitable solutions to combat the detrimental effects of fading and/or exploit the source of diversity inherent in each type of fading.

In fast-fading channels, channel coding and interleaving techniques can be used to break up the error bursts which occur over a sequence of consecutive data symbols and protect the information symbols against additive noise when the SNR of the received signal drops due to deep fades. In frequency-selective fading channels, the received signal can be processed with a channel equalizer to compensate for the ISI or one can use the orthogonal frequency division multiplexing (OFDM) technique. Interestingly, frequency-selective fading channels offer a source of diversity that can be exploited by using appropriate signal processing techniques.

In slow and flat fading channels, there is no source of temporal or frequency diversity. For such channels, multiple version of the transmitted information symbol can be obtained by using multiple transmit and/or receive antennas (or multiple coordinated transmitters and/or receivers). In order to obtain independently faded versions of the information symbol, antenna elements should be placed with sufficient separation (polarized or spaced). Beside being a method to provide spatial diversity, multiple-input multiple-output (MIMO) techniques have been shown to achieve a higher throughput than the conventional single antenna transmission model. For this reason, MIMO techniques can also be employed in any type of fading channel.

As mentioned in Chapter 1, relay (or cooperative) transmission is extended from MIMO techniques to generate multiple versions of the information symbols. In fact, it can provide spatial diversity while overcoming the limits on the physical size and power that MIMO techniques may experience in applications using small terminals such as in cellular mobile or sensor networks. The next section describes wireless relay networks in more detail.

2.2   Wireless Relay Networks

A wireless relay network may consist of one or several source terminals/nodes communicating with one or several destination terminals/nodes with the help of one or several relay terminals/nodes. Note that one node can also play the role of both source node and relay node. From this point forward, source nodes, destination nodes,
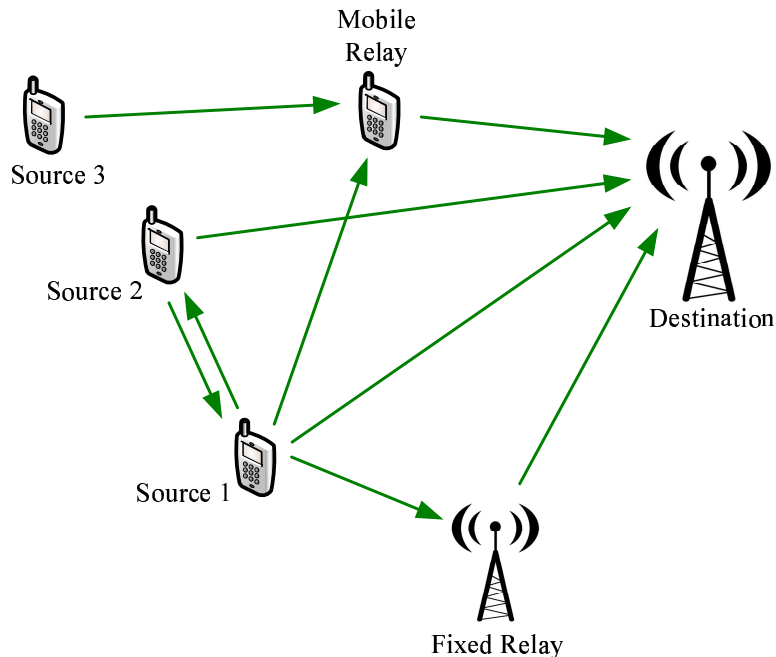
Figure 2.4   A general wireless relay network.

and relays nodes shall be referred to simply as sources, destinations, and relays.

Figure 2.4 shows a general wireless relay network. Depending on the propagation conditions, a direct link between a source and the destination may be useful (e.g., Source 1 and Source 2) or may be not (e.g., Source 3). Without the direct link, only propagation attenuation can be reduced. In contrast, with the direct link, the diversity benefit can also be obtained.

Figure 2.4 also shows different degrees of cooperation. The mobile relay node or fixed relay node just plays the role of supportive relaying since they have no demand to transmit their own data. A different situation applies to Source 1 and Source 2 where they both forward the data from each other to the destination. This scenario forms a cooperative relaying scheme. The kind of relay deployments as shown in Figure 2.4 potentially boost the maximum diversity and may also reduce the propagation attenuation if designed properly [5].

2.2.1 Advantages and Disadvantages of Cooperation in Wireless Networks

If the ultimate goal is to support long distance transmission, relaying can help users at the cell edge or in shadowed areas from insufficient capacity and/or coverage problems, hence maintaining the fairness among all users in a network. With dedicated relays, the number of base stations may be reduced or even eliminated in some infrastructure-less deployments such as mobile ad-hoc networks. These benefits come from the potential performance gains offered by relaying techniques. The first and most significant gain is the reduction in propagation attenuation. Since the path-loss is proportional to the propagation distance, placing a relay in the middle of the transmission path between one source and one destination, as illustrated in Figure 2.5, can significantly reduce the path-loss.
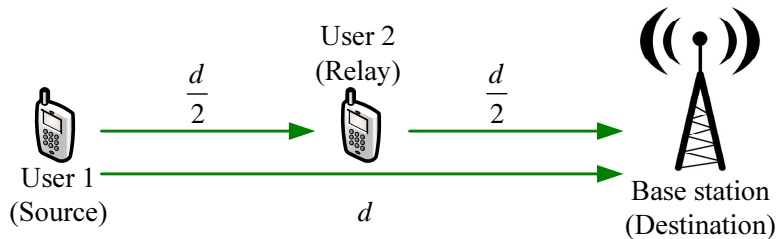
Figure 2.5 Reducing path-loss with relaying transmission.

For the direct link from a source to a destination, the signal-to-noise ratio is inversely proportional to the propagation distance, i.e., $\text{SNR}_{\text{direct}} \sim d^{-n}$ where n is the path-loss exponent. Let the signal-to-noise ratios of the links from the source to relay and from the relay to destination be SNR_1 and SNR_2, respectively. Then, the overall signal-to-noise ratio for an Amplify-and-Forward relay link can be expressed in the following form[2]:

$$\text{SNR}_{\text{AF}} = \frac{\text{SNR}_1\text{SNR}_2}{\text{SNR}_1 + \text{SNR}_2 + 1} \overset{\text{high SNR}}{\sim} \frac{\left(\frac{d}{2}\right)^{-n}\left(\frac{d}{2}\right)^{-n}}{\left(\frac{d}{2}\right)^{-n} + \left(\frac{d}{2}\right)^{-n}} \sim 2^{n-1}d^{-n}. \quad (2.11)$$

With a typical path-loss exponent $n = [2, \ldots, 6]$, the corresponding gain obtained

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[2]One can see this form in (2.26) or from the derivations given in [6].

from this relaying transmission is in the range of $(3-15)$dB. Beside this unique advantage, relaying could also offer a diversity gain similar to MIMO techniques [6].

Obviously, when more nodes participate in each transmission, more complicated scheduling procedures and synchronization techniques are required. The overhead information needed for those operations therefore increases. The increase in overhead information implies a decrease in the overall system throughput since extra frequency channels and/or time slots are needed in most cases. Furthermore, in order to obtain most of the potential gains, more channel coefficients need be estimated if the system employs coherent modulation/demodulation schemes. As such, a careful system design is needed to realize the full potentials of relaying techniques.

2.2.2 Important Design Problems

There are many problems that need be considered in designing a relay network. The following lists just a few of them:

- *Resource Optimization.* Since network resources are shared by many nodes, it is important to find out how to distribute those resources among the nodes in order to make sure that the network is optimized under some performance criteria. Among different network resources, frequency sub-channels, time slots and transmit powers are the most important ones.

- *Overhead Information/Feedback/Feedforward.* Information exchange plays a crucial role in implementing any resource optimization process or even be mandatory for many relay networks using coherent modulation/demodulation techniques. Useful information can be the full channel state information (for example, instantaneous channel realizations) or partial channel state information (for example, average channel realizations), or some other quantized/codebook-based limited-rate feedback information.

- *Interference Management.* The existence of multiple relays in a network naturally creates a multiuser scenario. In addition to the existence of multiple

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sources, interference among data streams generated by the sources and relays can potentially increase. Interference management is therefore closely related to resource optimization problems as well as multiple access techniques in relaying systems.

Many other issues such as the level of relay mobility, time and frequency synchronization require careful examinations but they are not in the scope of this thesis. The above listed issues are fundamental and relevant to relay networks with a single source or multiple sources.

2.2.3 A System Model with a Single Source

In this section, a cooperative transmission model with a single source and multiple relays is first introduced. The two relaying techniques, namely non-orthogonal and orthogonal, are then discussed together with the relay-beamforming and power allocation issues. The system model and discussion of two relaying techniques are basically based on the contents presented in [8, 10]. The notations used in this section are as follows. Italic, bold lower case and bold upper case letters denote scalars, vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$ stand for complex conjugate, transpose, Hermitian transpose, and statistical expectation operations, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a vector of complex Gaussian random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The notation $\mathbf{I}_L$ stands for an identity matrix of size $L \times L$.

For the wireless relay system illustrated in Figure 2.6, the source terminal, S, communicates with the destination terminal, D, with the help of L relay terminals, $R_1, \ldots, R_L$. All terminals are equipped with one antenna, which cannot be used to transmit and receive signals at the same time, i.e., each terminal operates in a half-duplex mode. The transmission for every information symbol, s, happens in two phases. In the first phase, the source transmits the signal to the destination via the direct channel, h_s, and to the relays via "uplink" (source-relay) channels, $\mathbf{h}_u = [h_{u1}, \cdots, h_{uL}]^T$, where h_{ul} is the coefficient of the channel between the source
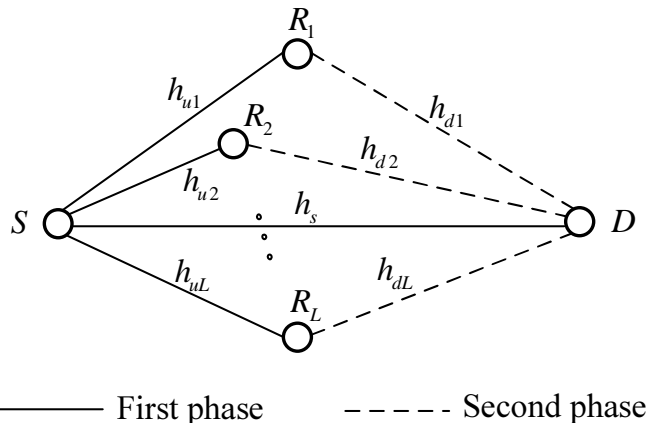
Figure 2.6 System model of a single-source wireless relay network.

and the lth relay. The signal received at the destination in this phase is

$$d_1 = \sqrt{P_S}h_s s + n_s, \tag{2.12}$$

where P_S is the average transmitted power of the source, $\mathbb{E}\{|s|^2\} = 1$, and $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ represents AWGN with variance σ_s^2. The signals received at the relays can be collectively written as

$$\mathbf{r} = \sqrt{P_S}\mathbf{h}_u s + \mathbf{n}_u, \tag{2.13}$$

where $\mathbf{r} = [r_1, r_2, \cdots, r_L]^T$, and $\mathbf{n}_u = [n_{u1}, n_{u2}, \cdots, n_{uL}]^T \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_L)$ accounts for AWGN at the relays with the same variance σ_u^2 at each relay.

As mentioned earlier, many approaches with different architectures can be implemented in the second phase depending on the design objective, the availability of the CSI and the characteristics of the channels involved. For example, repetition-based schemes, selection-based schemes or distributed space-time coding schemes may be used [7]. In terms of the signal processing at the relays, these schemes can be categorized into two common protocols: transparent relaying and regenerative relaying. In transparent relaying, the relay only amplifies the signal or performs some linear and/or non-linear operations in the analog domain before retransmitting it. The most basic form of transparent relaying is Amplify-and-Forward (AF). Regenerative relaying, on the other hand, requires the relay to change the waveform and/or the information contents by performing some processing in the digital domain. An example

is to have the relay receive the information from the source, decode, re-encode and finally retransmit the processed version of the information to the destination. The basic version of the regenerative relaying protocol is Decode-and-Forward (DF).

The transmissions from multiple relays can be carried out in an orthogonal or a non-orthogonal manner. In the following, we summarize the signal processing processes in the second phase of the orthogonal and non-orthogonal AF relaying protocols since they are the protocols adopted throughout the thesis.

Nonorthogonal AF Protocol

Nonorthogonal relay transmission is a bandwidth-efficient transmission scheme since all relays need only one channel use (or time slot) in the second phase to forward the signal to the destination. The signal received at the destination in the second phase can be expressed as

$$
\begin{aligned}
d_2 &= \sum_{l=1}^{L} h_{dl} \frac{w_l}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}} r_l + n_d \\
&= \sum_{l=1}^{L} \frac{h_{ul} h_{dl}}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}} w_l s + \sum_{l=1}^{L} \frac{h_{dl}}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}} w_l n_{ul} + n_d,
\end{aligned} \tag{2.14}
$$

where $\{h_{dl}\}_{l=1}^{L}$ are the coefficients of the "downlink" (relay-destination) channels and $n_d \sim \mathcal{CN}(0, \sigma_d^2)$ represents additive white Gaussian noise (AWGN) at the destination in the second phase. The set of weighting (or amplifying) factors, $\{w_l\}_{l=1}^{L}$, play a very important role in the relaying operation. These weighting factors are complex numbers and satisfy the transmit power constraint at the relays. The expression in (2.14) implies that the lth relay needs to know the instantaneous channel magnitude, $|h_{ul}|$, to normalize its received signal[3]. Also, the second term in (2.14) shows that the noise experienced by each relay is also amplified before being forwarded to the destination.

---

[3] An alternative scenario is when the lth relay knows only the first- and second-order statistics of the channel. In this case, the normalization factor $\left(P_S |h_{ul}|^2 + \sigma_u^2\right)^{-1/2}$ is replaced by $\left(P_S \mathbb{E}\{|h_{ul}|^2\} + \sigma_u^2\right)^{-1/2}$.

After two phases of transmission, the destination needs to make a detection decision on the transmitted symbol s based on received signals d_1 in (2.12) and d_2 in (2.14). To this end, let

$$\mathbf{a} = \left[\frac{h_{u1}h_{d1}}{\sqrt{P_S|h_{u1}|^2 + \sigma_u^2}}, \ldots, \frac{h_{uL}h_{dL}}{\sqrt{P_S|h_{uL}|^2 + \sigma_u^2}} \right], \tag{2.15}$$

$$\mathbf{w} = [w_1^*, \ldots, w_L^*], \tag{2.16}$$

$$\mathbf{A} = \mathrm{diag}\left(\frac{h_{d1}}{\sqrt{P_S|h_{u1}|^2 + \sigma_u^2}}, \ldots, \frac{h_{dL}}{\sqrt{P_S|h_{uL}|^2 + \sigma_u^2}} \right). \tag{2.17}$$

The received signals after two phases can be collectively written as:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_s \\ \mathbf{a}\mathbf{w}^H \end{bmatrix}}_{\mathbf{h}} s + \underbrace{\begin{bmatrix} n_s \\ \mathbf{n}_u^T \mathbf{A}\mathbf{w}^H + n_d \end{bmatrix}}_{\mathbf{n}}$$

$$= \mathbf{h}s + \mathbf{n}, \tag{2.18}$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega})$ with

$$\boldsymbol{\Omega} = \mathrm{diag}(\sigma_s^2, \sigma_u^2 \mathbf{w}\mathbf{A}^H\mathbf{A}\mathbf{w}^H + \sigma_d^2). \tag{2.19}$$

These signals are combined for the detection of the source symbol as

$$\hat{s} = f_1 d_1 + f_2 d_2, \tag{2.20}$$

where $\mathbf{f} = [f_1, f_2]$ is the receiver's filter at the destination. The instantaneous signal-to-noise ratio (SNR) at the output of the filter can be shown to be

$$\mathrm{SNR} = P_S \frac{\mathbf{f}\mathbf{h}\mathbf{h}^H\mathbf{f}^H}{\mathbf{f}\boldsymbol{\Omega}\mathbf{f}^H}. \tag{2.21}$$

It is important to emphasize that one of the benefits of using relays is to improve the detection performance at the destination. In a network with a single source, this criterion is equivalent to improving the SNR at the destination. Therefore, with the total transmit power of all the relays being P_R, it is of interest to find the optimal weighting factors at the relays and the optimal filter at the destination to maximize

the instantaneous SNR in (2.21). This optimization problem can be formulated as follows [8]:

$$\max_{\mathbf{w},\mathbf{f}} \quad \text{SNR}$$
$$\text{s.t.} \quad \mathbf{w}\mathbf{w}^H \leq P_R. \tag{2.22}$$

It can be established that, for a given weighting vector $\mathbf{w}$, the optimal filter $\mathbf{f}$ is the principle eigenvector of the matrix $\mathbf{\Omega}^{-H/2}\mathbf{h}\mathbf{h}^H\mathbf{\Omega}^{-1/2}$. That is,

$$\mathbf{f}_{\text{opt}} = \left[\frac{h_s^*}{\sigma_s^2}, \frac{\mathbf{w}\mathbf{a}^H}{\sigma_u^2\mathbf{w}\mathbf{A}^H\mathbf{A}\mathbf{w}^H + \sigma_d^2}\right], \tag{2.23}$$

which is essentially a maximal-ratio combiner (MRC).

Substituting $\mathbf{f}$ into (2.21), the SNR can be rewritten as

$$\text{SNR} = P_S\left(\frac{|h_s|^2}{\sigma_s^2} + \frac{\mathbf{w}\mathbf{a}^H\mathbf{a}\mathbf{w}^H}{\sigma_u^2\mathbf{w}\mathbf{A}^H\mathbf{A}\mathbf{w}^H + \sigma_d^2}\right). \tag{2.24}$$

The final step is to maximize (2.24) with respect to $\mathbf{w}$. The optimal weighting factors are found to be [8]

$$\mathbf{w}_{\text{opt}} = c\tilde{\mathbf{w}}, \tag{2.25}$$

where $\tilde{\mathbf{w}} = [\tilde{w}_1, \ldots, \tilde{w}_L]$ with $\tilde{w}_l = \frac{h_{ul}h_{dl}\sqrt{P_S|h_{ul}|^2+\sigma_u^2}}{P_S|h_{ul}|^2\sigma_d^2+P_R|h_{dl}|^2\sigma_u^2+\sigma_u^2\sigma_d^2}$ and $c = \sqrt{P_R/\|\tilde{\mathbf{w}}\|^2}$. The complex weighting vector $\mathbf{w}_{\text{opt}}$ implemented at the relays is called the optimal *relay beamforming* scheme. The resultant instantaneous SNR becomes:

$$\text{SNR} = \frac{P_S|h_s|^2}{\sigma_s^2} + \sum_{l=1}^{L}\frac{\frac{P_S|h_{ul}|^2}{\sigma_u^2}\frac{P_R|h_{dl}|^2}{\sigma_d^2}}{\frac{P_S|h_{ul}|^2}{\sigma_u^2} + \frac{P_R|h_{dl}|^2}{\sigma_d^2} + 1}. \tag{2.26}$$

Since the SNR in (2.26) can only be achieved with full and perfect CSI, it is useful and interesting to examine the designs when only partial CSI is available at the destination and/or relays. These designs are parts of the contributions of this thesis. In particular, as will be presented in Chapter 3 and Chapter 4, the joint design of relay beamforming and MRC filtering schemes may perform poorly under partial CSI assumptions. As such, modifications on the signal processing at the relays and destination are needed.

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It should be noted that simultaneous transmissions from all relays in the second phase require perfect timing and frequency synchronization among all the relays[4]. Another relaying scheme that has looser synchronization requirements is introduced next.

Orthogonal AF Protocol

In the orthogonal AF protocol, orthogonal transmissions from relays to destination can be carried out with time-division multiple access (TDMA) technique[5], in which each relay is assigned its own time slot in the second phase. As such, the received signal at the destination in the lth time slot of the second phase is

$$
\begin{aligned}
d_{2l} &= h_{dl}\frac{w_l}{\sqrt{P_S|h_{ul}|^2 + \sigma_u^2}}r_l + n_{2l} \\
&= h_{dl}\frac{w_l}{\sqrt{P_S|h_{ul}|^2 + \sigma_u^2}}h_{ul}s + h_{dl}\frac{w_l}{\sqrt{P_S|h_{ul}|^2 + \sigma_u^2}}n_{ul} + n_{2l}.
\end{aligned} \tag{2.27}
$$

Let $\mathbf{d}_2 = [d_{21}, d_{22}, \ldots, d_{2L}]^T$. Then the received signals over L time slots can be written in the vector form as:

$$
\mathbf{d}_2 = \mathbf{WFH}_d\mathbf{h}_u s + \mathbf{WFH}_d\mathbf{n}_u + \mathbf{n}_d, \tag{2.28}
$$

where $\mathbf{W} = \mathrm{diag}(w_1, \ldots, w_L)$ is a diagonal matrix with its diagonal elements being the weighting factors corresponding to the L relays, $\mathbf{H}_d = \mathrm{diag}(h_{d1}, \ldots, h_{dL})$ is the diagonal matrix of "downlink" (relay-destination) channel gains and $\mathbf{F} = \mathrm{diag}((P_S|h_{u1}|^2 + \sigma_u^2)^{-1/2}, \ldots, (P_S|h_{uL}|^2 + \sigma_u^2)^{-1/2})$ is the normalization matrix. The noise at the destination in the second phase is modeled as $\mathbf{n}_d = [n_{21}, n_{22}, \ldots, n_{2L}]^T \sim \mathcal{CN}(\mathbf{0}, \sigma_d^2\mathbf{I}_L)$.

---

[4]The issues of timing and synchronization among the relays are beyond the scope of this thesis. However, it can be seen that in non-orthogonal relay networks, the destination might not be able to synchronize a combination of multiple received signals in time and frequency domains itself. A possible approach is to have the destination send adjustment information obtained in training periods to all the relays so that each relay can adjust its transmit frequency and time instant. For timing synchronization only, using some guard intervals or cyclic prefix as in OFDM is also useful.

[5]It can also be implemented with frequency-division multiple access (FDMA) [6] or code-division multiple access (CDMA) technique [9].

To obtain the detection filter at the destination, write the received signals after two phases as follows:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \mathbf{d}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_s \\ \mathbf{WFH}_d\mathbf{h}_u \end{bmatrix}}_{\mathbf{b}} s + \underbrace{\begin{bmatrix} n_s \\ \mathbf{WFH}_d\mathbf{n}_u + \mathbf{n}_d \end{bmatrix}}_{\mathbf{n}}$$

$$= \mathbf{b}s + \mathbf{n}, \qquad (2.29)$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ with

$$\mathbf{R} = \mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \begin{bmatrix} \sigma_s^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix} \qquad (2.30)$$

and

$$\mathbf{R}_2 = \sigma_u^2 \mathbf{F}^2 \mathbf{W}\mathbf{H}_d\mathbf{H}_d^H\mathbf{W}^H + \sigma_d^2\mathbf{I}_L. \qquad (2.31)$$

If all the instantaneous channels are known at the destination, the optimal receiver (filter) is

$$\mathbf{f} = \mathbf{b}^H\mathbf{R}^{-1}, \qquad (2.32)$$

which is also a maximal-ratio combiner (MRC). In this case, the instantaneous SNR can be expressed as:

$$\begin{aligned} \text{SNR} &= P_S\mathbf{b}^H\mathbf{R}^{-1}\mathbf{b} \\ &= \frac{P_S|h_s|^2}{\sigma_s^2} + \sum_{l=1}^{L} \frac{\frac{P_S|h_{ul}|^2}{\sigma_u^2}\frac{|w_l|^2|h_{dl}|^2}{\sigma_d^2}}{\frac{P_S|h_{ul}|^2}{\sigma_u^2} + \frac{|w_l|^2|h_{dl}|^2}{\sigma_d^2} + 1}. \end{aligned} \qquad (2.33)$$

Similar to the non-orthogonal relaying scheme, in the orthogonal relaying scheme an optimal weighting matrix $\mathbf{W}$ is also to maximize the instantaneous SNR. However, different from the relay beamforming scheme $\mathbf{w}_{\text{opt}}$ given in (2.25), the SNR given in (2.33) depends on $\{|w_l|^2\}_{l=1}^L$, which simply constitutes a power allocation scheme.

Let $p_l = |w_l|^2$ for $l = 1, 2, \ldots, L$. Under the constraint on total transmit power at the relays being P_R, the optimal power allocation scheme that maximizes the instantaneous SNR given in (2.33) can be found to be [10]:

$$p_l(\mu) = \sqrt{\frac{P_S|h_{ul}|^2/\sigma_u^2 + 1}{|h_{dl}|^2/\sigma_d^2}}\left[\sqrt{\frac{P_S|h_{ul}|^2/\sigma_u^2}{\mu}} - \sqrt{\frac{P_S|h_{ul}|^2/\sigma_u^2 + 1}{|h_{dl}|^2/\sigma_d^2}}\right]^+, \qquad (2.34)$$

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where $[x]^+ = \max\{x, 0\}$, μ is chosen such that $\sum_{l=1}^{L} p_l(\mu) = P_R$. This optimal power allocation scheme can be illustrated as a *water-filling* procedure [2] as shown in Figure 2.7. First, define the following parameter:

$$f(h_{ul}, h_{dl}) = \sqrt{\frac{P_S|h_{ul}|^2/\sigma_u^2 + 1}{P_S|h_{ul}|^2|h_{dl}|^2/(\sigma_u^2\sigma_d^2)}}. \tag{2.35}$$

Equation (2.34) says that any relay with the parameter $f(h_{ul}, h_{dl})$ higher than the water level $1/\sqrt{\mu}$ will not be employed in assisting the transmission from the source to destination, i.e., $p_l = 0$. This behavior of the power allocation scheme is different from the relay beamforming scheme given in (2.25) where participation of all relays is *always* needed in assisting the transmission. Also in Figure 2.7, the parameter β_l is defined as

$$\beta_l = \sqrt{\frac{|h_{dl}|^2/\sigma_d^2}{P_S|h_{ul}|^2/\sigma_u^2 \left(P_S|h_{ul}|^2/\sigma_u^2 + 1\right)}}. \tag{2.36}$$
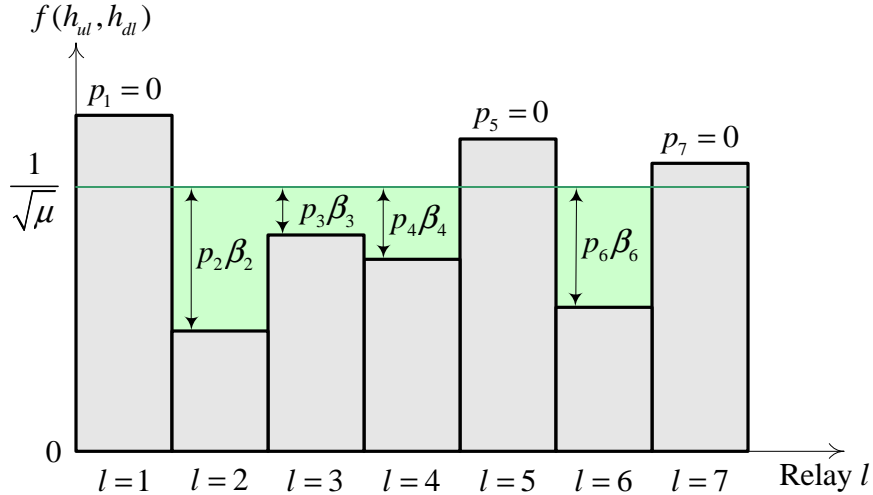
Figure 2.7 Illustration of a power allocation scheme with $L = 7$ relays.

The beamforming and power allocation schemes discussed above apply for a relay network model having a joint (i.e., total) relay power constraint. For relay network models with limited power budgets at the relays, the individual relay power constraints should also be taken into account. While applying a constraint on the total transmit power of all the relays helps to reduce the interference the network may cause (e.g., in multi-cell interference-limited systems), applying an individual relay power

constraint reflects the practical limitation of each relay's power consumption (e.g., in sensor networks). Furthermore, joint source-relay power constraint or a combination of both joint and individual power constraints can also be considered in wireless relay networks.

It should also be pointed out that both the non-orthogonal and orthogonal AF relaying schemes discussed in this section require that the relays and destination have full and perfect CSI of all channels involved in the transmission. However, obtaining perfect CSI at the transmitter(s) is a challenging task, even in the conventional transmissions (i.e., with no support from relays) [11]. As a result, investigating various signal processing techniques at the relays as well as different combining and detection techniques at the destination which can adapt to different amounts of available CSI has gained a great deal of interests [8,10,12–14]. Such investigation is also the focus of this thesis.

Limited feedback concepts have also been considered in wireless relay networks under various signal processing models. One simple relaying protocol that can be considered as a limited feedback case is *incremental* relaying [6]. In incremental relaying, the relay only forwards the source's signal if the destination fails to decode the desired information from the signal arrived in the first phase. Another protocol is called *relay selection* in which, based on the available CSI, the "best" relay will be chosen to assist the transmission of the source [10,15]. An efficient codebook-based beamforming is proposed in [16] while another type of codebook-based beamforming, named perturbation-based beamforming technique, is also proposed in [17,18]. These simple, yet effective schemes are attractive for practical implementation.

As contributions to the research area of wireless relay networks with partial CSI, in Chapter 3 and Chapter 4, the partial CSI in the form of the first- and second-order statistics is exploited in designing power allocation schemes for orthogonal relaying networks and a beamforming scheme for non-orthogonal relaying networks, respectively. A comparison between those two models is also conducted in Chapter 4, which results in several useful findings for practical system designs.

Although presenting an important evolution step over the point-to-point communications, relaying transmission with a single source does not cover a complete picture of the functionalities of a general relay network. Investigating signal processing issues in a relay network with multiple sources and/or multiple relays provides further understanding of the role of relays from a network viewpoint and this is another focus of this thesis.

2.2.4 A System Model with Multiple Sources

In a relay network with multiple sources, an efficient multiple access technique is required in order to avoid noise and interference amplification at each relay. A general system model is illustrated in Fig. 2.8 where the transmissions from K sources to the destination are supported by L relays.
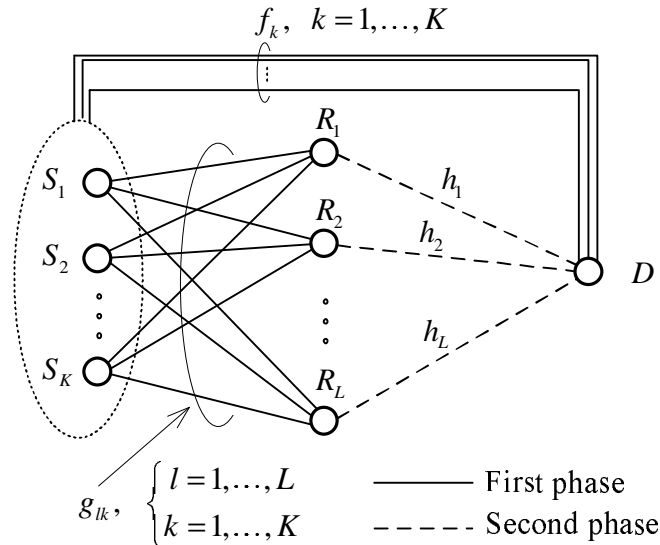
Figure 2.8 System model of orthogonal multiuser wireless relay networks.

Usually, orthogonal multiple access techniques such as code-division multiple access (CDMA), orthogonal frequency-division multiple access (OFDMA) or time-division multiple access (TDMA) are preferred so that multiple access interference is completely avoided and the multi-source relay-assisted transmission can be decomposed into multiple orthogonal single-source transmissions. However, these transmissions are still coupled because they share the available resources at the relays. Note that

these resources are constrained in each relay (e.g., maximal transmit power at each relay) and/or even in the whole network (e.g., total transmit power consumed in the network). Furthermore, the design of such a system must also take into account the amount of CSI required at the relays and destination since the amount of CSI increases with the number of sources and relays and certainly imposes a bandwidth burden on the feedback links.

Once the multiuser transmissions are decoupled, similar signal processing techniques as used in single source cases can be employed. The remaining, but most challenging task is how to efficiently allocate the constrained resources available in the network to enhance the quality of the transmission from all sources while keeping the amount of overhead information exchanged within the network at a reasonable level. Similar to conventional wireless networks, cooperative relaying networks also have several degrees of freedom such as time slots, frequency bands/subcarriers, multiple antenna elements, multiple codes and transmit power levels. With more nodes participating in each transmission, these degrees of freedom play an important role in boosting the overall system performance. In this thesis, we focus on the power allocation aspect. However, the interplays with other resources are also examined.

Similar to other techniques such as adaptive modulation and coding, or user scheduling, power control can exploit the availability of channel information to adjust the transmit power to combat fading and co-channel interference (CCI) in order to improve the performance of communication systems. In wireless relay networks, even when CCI is not present, for the constructive combining of the multiple versions of the transmitted signal, it still requires power control at each source and allocation of the transmit power at each relay for the signals from different sources.

The ultimate goal of power allocation is to minimize the bit- or symbol- or frame-error-rate. In interference-free system (or a single source system as introduced in Section 2.2.3), one can use the signal-to-noise ratio (SNR) or just the received signal power as a replacement of the error rate [19, 20]. However, in complex systems with interference it is difficult to measure the accurate error rates or it may requires a

large delay to accumulate a sufficient number of samples. A simpler parameter that can be used to determine the power allocation scheme is the signal-to-interference-plus-noise (SINR). Although the average SINR does not directly correspond to the average error rate, it is strongly related to other system performance measures such as Quality-of-Service (QoS) or capacity. Under the assumption of Gaussian interference, the capacity is approximately proportional to $\log_2\left(1 + \frac{\text{SINR}}{\Gamma}\right)$, where Γ is the SINR gap that depends on the specific modulation and coding scheme used in the system. In this thesis, Γ is set to unity for simplicity and also because the effects of different modulation and coding schemes are not taken into account. One issue of the SINR-based power allocation that requires a careful attention is that it may become infeasible in such a way that increasing the transmit power also increases the interference power.

A network utility maximization (NUM) based power allocation has been widely used (see e.g., [21, 22]). The NUM-based power allocation aims to maximize the overall system utility subject to individual user's QoS requirements (in terms of data rates). Let $\mathbf{p} = [p_1, p_2, \ldots, p_L]$ and $\mathbf{P}^{\text{max}} = [P_1^{\text{max}}, P_2^{\text{max}}, \ldots, P_L^{\text{max}}]$ be the actual and maximum transmit power vectors for a system with L transmitters. Also let the SINR and the minimum rate required for a link associated with the lth transmitter be $\gamma_l(\mathbf{p})$ and r_l^{min}, respectively. A NUM-based power allocation can be formulated as

$$
\begin{aligned}
\max_{\mathbf{p}} \quad & \sum_{l=1}^{M} U_l(r_l) \\
\text{s.t.} \quad & r_l = \log_2\left(1 + \frac{\gamma_l(\mathbf{p})}{\Gamma}\right) \geq r_l^{\text{min}}, \ \forall l, \\
& \mathbf{0} \leq \mathbf{p} \leq \mathbf{P}^{\text{max}},
\end{aligned}
\tag{2.37}
$$

where $U_l(r_l)$ is usually in the form of a generalized α-fairness utility function:

$$
U_l(r_l) = \begin{cases} \log(r_l) & \text{if } \alpha = 1 \\ \frac{(r_l)^{1-\alpha}}{1-\alpha} & \text{if } \alpha \geq 0 \text{ and } \alpha \neq 1 \end{cases}
\tag{2.38}
$$

Inspecting (2.38) reveals that if $\alpha = 0$ problem (2.37) will maximize the average total throughput. If $\alpha = 1$ it maximizes the proportional fairness and when $\alpha \to \infty$ it is a max-min fairness case. In other words, increasing α will increase the fairness.

The power allocation schemes considered in Chapters 5 and 6 will focus on the max-min fairness case. Power allocation schemes that minimize the total transmit power are also investigated in Chapter 5. A simple, yet efficient relay selection scheme for relay networks with multiple sources and multiple relays will also be introduced in Chapter 7.

2.3 Summary

This chapter has briefly reviewed the statistical characteristics of wireless fading channels and introduced relay communications as an emerging technique to combat the impairments of fading channels. The key advantages and disadvantages of cooperation in wireless networks have been discussed together with two basic relaying protocols: amplify-and-forward (AF) and decode-and-forward (DF). Focusing on the AF protocol and the system model with a single source, the issues of beamforming and/or power allocation to maximize the SNR at the destination have been explained. These issues will be examined in detail in the next two chapters, Chapter 3 and Chapter 4, under the partial CSI assumptions. For wireless relay networks with multiple sources, performance criteria for designing the networks have also been discussed. This sets the stage for our developments of several frameworks for cooperative multiuser wireless networks in Chapters 5, 6 and 7.

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3. Power Allocation in Orthogonal Wireless Relay Networks with Partial Channel State Information

In the previous chapter, a system model for a single-source wireless relay network has been presented. It was shown in [1] that the optimal scheme for the *non-orthogonal* relay networks with full channel state information (CSI) at the relays and destination is the combination of beamforming at the relays and maximal-ratio combining at the destination while under two other assumptions of partial CSI, relay selection was proven to be the best solution. The manuscript included in this chapter studies an *orthogonal* relay network under those partial CSI assumptions. Different from the non-orthogonal relaying scheme which requires beamforming at the relays, power allocation among the relays has been shown to be essentially optimal in orthogonal relay networks since the destination can handle the phase differences among the signals forwarded from different relays. It is pointed out that the reduction in path-loss offered by relaying transmission have also been taken into account when investigating the trade-offs with different simulation settings.

Power Allocation in Orthogonal Wireless Relay Networks with Partial Channel State Information

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Abstract

Wireless amplify-and-forward (AF) relay networks in which the source communicates with the relays and destination in the first phase and the relays forward signals to the destination in the second phase over orthogonal and uncorrelated Rayleigh fading channels are considered. Convex programming is used to obtain optimal and approximately optimal power allocation (OPA) schemes to maximize the average signal-to-noise ratios (SNRs) at the output of the receiver filters under two different assumptions of *partial* channel state information (CSI) . Analysis and simulation results demonstrate the superiority of the proposed power allocation schemes over the equal-power allocation scheme. Performance comparison to the extreme cases of (i) direct transmission between the source and destination and (ii) having full CSI is made to illustrate the gain and loss, respectively, of the proposed schemes. The impact of power allocation between the source and relays is also investigated by computer simulation.

Index terms

Wireless relay networks, amplify-and-forward relaying, beamforming, power allocation, convex programming.

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3.1 Introduction

Knowledge of channel state information (CSI) plays an important role in signal detection of many communication systems. In wireless relay networks, signal transmission from a source to a destination usually consists of two or more hops via some relays. Therefore, estimating channels' characteristics and/or their instantaneous values at the relays and destination is much more challenging and it typically requires a large amount of overhead. Thus, designing transmission methods for wireless relay networks that can adapt to partial knowledge of CSI has gained a significant interest in recent years [1–3].

In relay networks, whether the relays decode the signals received from the source (i.e., in decode-and-forward relaying), or just amplify the signals (i.e., in amplify-and-forward relaying) before forwarding the decoded or amplified signal to the destination, a certain amount of CSI of the source-relay and relay-destination links is desired in order to improve the system performance. In [1], jointly optimal designs of precoding at the relays and decoding at the destination are considered for the case that the source transmits to the relays and destination in the first phase, while all the relays forward signals to the destination in the second phase *over the same frequency band and at the same time*. It is shown that when the destination and relays know all the instantaneous channel information, the optimal precoder at the relays uses the cooperative transmit beamforming technique and the optimal decoder at the destination is essentially a maximum-ratio combining (MRC) receiver. Simulation results obtained with two relays in [1] show a significant performance improvement, approximately 5 dB at the symbol error probability of 10^{-3}, when compared to the conventional MRC receiver with equal-power allocation among the relays. However, when only partial CSI is available at the destination, due to the lack of coherent combining of signals received at the destination, a large performance degradation of the system considered in [1] can be observed.

In order to maintain the diversity order and hence improving the error performance, orthogonal transmissions from the relays to destination can be employed [4–7].

With orthogonal transmissions, the signals forwarded from the relays do not interfere, and hence they can be more effectively combined at the destination. It should be mentioned that orthogonal transmissions can be implemented with time-division multiple access (TDMA) , frequency-division multiple access (FDMA) or code-division multiple access (CDMA). Depending on what technique to be implemented, the trade-off for performance improvement compared to the system in [1] is either a decrease in system throughput, a larger transmission bandwidth, or a combination of both.

This paper studies the problem of power allocation in wireless amplify-and-forward (AF) relay networks. The networks under consideration are similar to those studied in [4–7] in which each terminal is equipped with a single antenna and orthogonal transmissions are conducted between the relays and destination in the second phase. In fact, a similar power allocation problem was investigated in [6], in which the authors propose an optimal power allocation that maximizes the source-destination channel capacity under the *full* CSI assumption. Different from the power allocation with full CSI proposed in [6], derived in this paper are power allocation schemes under the following two *partial* CSI assumptions:

- *Assumption A:* Every relay knows all the instantaneous source-relay channels, the first-order and second-order statistics of the source-destination and relay-destination channels, while the destination knows all the instantaneous channels.

- *Assumption B:* Every relay only knows the instantaneous channel from the source to itself and has no information of the channels from the source to other relays. The destination knows the instantaneous channels from the source and all the relays to itself, the first-order and second-order statistics of the channels from the source to all the relays.

The above two assumptions are the same as those considered in [1] (referred to as Assumptions II and III in [1]). Note that Assumption A requires that each relay terminal estimates and broadcasts its own "uplink" channel (from the source to it-

self) to the destination terminal and other terminals, while the destination estimates the channel coefficients of the "direct" path (from the source to the destination) and of the "downlink" channels (from all relays to the destination). More ideally, the destination can also send the instantaneous channel coefficients of the "direct" path and the "downlink" channels back to all the relays. However, such a full CSI assumption may not be practical due to a large feedback overhead. Nevertheless, the full CSI assumption is considered in both [1] and [6] and the results obtained in these papers can serve as benchmarks of the performance obtained with partial CSI in the corresponding system models. Instead of sending the instantaneous CSI of the "direct" path and the "downlink" channels, Assumption A requires that the destination sends the first-order and second-order statistics of the "downlink" channels back to the relays. Since the first-order and second-order statistics vary much slower than the instantaneous CSI, the feedback overhead is significantly reduced. Under Assumption B, the destination does not need to send any feedback information to the relays[1].

The rest of this paper is organized as follows. Section II presents the system model of the wireless relay networks under consideration. Section III provides the optimal power allocation scheme under Assumption A. Section IV presents a co-phasing process at the relays, an optimal filter and the optimal power allocation under Assumption B. Numerical results are presented and discussed in Section V. Finally, Section VI concludes the paper.

Notations: Italic, bold lower case and bold upper case letters denote scalars, vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$ stand for complex conjugate, transpose, Hermitian transpose, and statistical expectation operations, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a vector of complex Gaussian random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The notation $\mathbf{I}_L$ stands for an identity matrix of size $L \times L$.
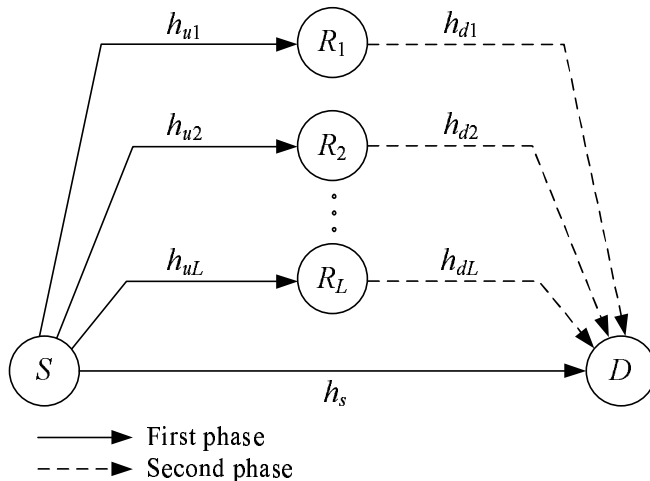
Figure 3.1   System model of an orthogonal wireless relay network.

3.2   System Model

Figure 3.1 illustrates a wireless relay system in which the source terminal, S, communicates with the destination terminal, D, with the help of L relay terminals, $R_1, \ldots, R_L$. All terminals are equipped with one antenna, which cannot be used to transmit and receive signals at the same time, i.e., each terminal operates in a half-duplex mode. The transmission for every information symbol, s, happens in two phases. In the first phase, the source transmits the signal to the destination via the direct channel, $h_s \sim \mathcal{CN}(\mu^{(s)}, \Sigma^{(s)})$, and to the relays via "uplink" (source-relay) channels, $\mathbf{h}_u = [h_{u1}, \cdots, h_{uL}]^T \sim \mathcal{CN}(\boldsymbol{\mu}^{(u)}, \boldsymbol{\Sigma}^{(u)})$, where h_{ul} is the instantaneous channel gain between the source and the lth relay. Typically, the uplink channels are uncorrelated (as long as the relays are sufficiently far apart), which means that the covariance matrix $\boldsymbol{\Sigma}^{(u)}$ is diagonal. The signal received at the destination in this phase is $d_1 = h_s s + n_s$, where $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ represents additive white Gaussian noise (AWGN) with variance σ_s^2. The signals received at the relays can be collectively written as $\mathbf{r} = [r_1, r_2, \cdots, r_L]^T = \mathbf{h}_u s + \mathbf{n}_u$, where $\mathbf{n}_u \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_L)$ accounts for AWGN at the relays with the same variance σ_u^2 at each relay.

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[1]It should be pointed out that Assumption B is more practical than Assumption A.

In the second phase, while the source is idle, each relay amplifies the received signal and forwards it to the destination. Without loss of generality, assume that orthogonal transmissions from relays to destination are carried out by TDMA[2], i.e., each relay is assigned its own time slot in the second phase. With perfect timing and synchronization, the received signal at the destination in the second phase (after L time slots) can be represented as

$$
\begin{aligned}
\mathbf{d}_2 &= \mathbf{G}\mathbf{H}_d\mathbf{r} + \mathbf{n}_d \\
&= \mathbf{G}\mathbf{H}_d\mathbf{h}_u s + \mathbf{G}\mathbf{H}_d\mathbf{n}_u + \mathbf{n}_d,
\end{aligned}
\tag{3.1}
$$

where $\mathbf{G} = \mathrm{diag}(g_1, \cdots, g_L)$ is the diagonal matrix of the amplification factors employed by L relays, $\mathbf{H}_d = \mathrm{diag}(h_{d1}, \cdots, h_{dL})$ is the diagonal matrix of "downlink" (relay-destination) channel gains. The noise at the destination in the second phase is modeled as $\mathbf{n}_d \sim \mathcal{CN}(\mathbf{0}, \sigma_d^2\mathbf{I}_L)$, while $\mathbf{h}_d = [h_{d1}, \cdots, h_{dL}]^T \sim \mathcal{CN}(\boldsymbol{\mu}^{(d)}, \boldsymbol{\Sigma}^{(d)})$.

In AF relaying, to limit the transmitted power of the lth relay to $|w_l|^2$, the relay gain is set as follows:

$$
g_l = \frac{w_l}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}}, \quad l = 1, \ldots, L,
\tag{3.2}
$$

where $P_S = \mathbb{E}\{|s|^2\}$ is the average transmitted power of the source. Note that in computing g_l, it requires that h_{ul} is available at the lth relay, which is the case under both Assumptions A and B. Let $\boldsymbol{\Psi} = \mathbf{G}\mathbf{H}_d = \mathrm{diag}(g_1 h_{d1}, \cdots, g_L h_{dL})$, one can write

$$
\boldsymbol{\Psi} = \mathrm{diag}\left(\varepsilon_1 w_1, \cdots, \varepsilon_L w_L\right),
\tag{3.3}
$$

where the parameter

$$
\varepsilon_l = \frac{h_{dl}}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}}
\tag{3.4}
$$

depends on the gains of the channels connected to the lth relay, while w_l is the parameter to be designed for the lth relay.

---

[2]The results obtained in this paper equally apply to orthogonal FDMA or orthogonal CDMA.

Combining the received signals in two phases gives:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_s \\ \boldsymbol{\Psi}\mathbf{h}_u \end{bmatrix}}_{\mathbf{a}} s + \underbrace{\begin{bmatrix} n_s \\ \boldsymbol{\Psi}\mathbf{n}_u + \mathbf{n}_d \end{bmatrix}}_{\mathbf{n}}$$

$$= \mathbf{a}s + \mathbf{n}, \tag{3.5}$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ with

$$\mathbf{R} = \mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \begin{bmatrix} \sigma_s^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix} \tag{3.6}$$

and

$$\mathbf{R}_2 = \sigma_u^2 \boldsymbol{\Psi}\boldsymbol{\Psi}^H + \sigma_d^2 \mathbf{I}_L. \tag{3.7}$$

It is noted that the objective of power allocation adopted in this paper is to maximize the SNR at the destination. As such the optimal combining operation at the destination is generally known as the maximal-ratio-combiner (MRC). However, the form of the receiver's filter and consequently the solution of the power allocation depend on the amount of CSI that the relays and destination have. The receiver's filters and formulations of the optimization problems are examined further in the following sections for the two CSI assumptions under consideration.

3.3 Power Allocation Under Assumption A

Under Assumption A, since all the instantaneous channels are known at the destination, the optimal receiver (filter) is

$$\mathbf{f} = \mathbf{R}^{-1}\mathbf{a}. \tag{3.8}$$

Let the output of the filter be represented as $\hat{s} = \mathbf{f}^H\mathbf{d} = \alpha \cdot s + \zeta$, where α and ζ are the effective gain and noise component. Then the *instantaneous* SNR, defined as $\gamma = \alpha^2 \mathbb{E}\{|s|^2\}/\text{var}\{\zeta\}$, can be computed as

$$\begin{aligned} \gamma &= P_S \mathbf{a}^H \mathbf{R}^{-1}\mathbf{a} \\ &= P_S \left[\frac{|h_s|^2}{\sigma_s^2} + \mathbf{h}_u^H \boldsymbol{\Psi}^H \mathbf{R}_2^{-1} \boldsymbol{\Psi}\mathbf{h}_u \right]. \end{aligned} \tag{3.9}$$

41

Since

$$\mathbf{R}_2^{-1} = \frac{1}{\sigma_d^2}\mathbf{I}_L - \frac{1}{\sigma_d^2}\boldsymbol{\Psi}\left(\frac{\sigma_d^2}{\sigma_u^2}\mathbf{I}_L + \boldsymbol{\Psi}^H\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^H, \tag{3.10}$$

the instantaneous SNR can be expressed as follows:

$$\gamma = P_S\left[\frac{|h_s|^2}{\sigma_s^2} + \frac{\|\mathbf{h}_u\|^2}{\sigma_u^2} - \frac{\sigma_d^2}{(\sigma_u^2)^2}\mathbf{h}_u^H\left(\boldsymbol{\Psi}^H\boldsymbol{\Psi} + \frac{\sigma_d^2}{\sigma_u^2}\mathbf{I}_L\right)^{-1}\mathbf{h}_u\right]. \tag{3.11}$$

However, since each relay does not know h_s and $\mathbf{h}_d$, it cannot compute the instantaneous SNR given in (3.11) for power allocation purpose. Instead, the relays strive to maximize the *average* value of the instantaneous SNR in (3.11), averaged over the distributions of h_s and $\mathbf{h}_d$. The average SNR can be computed as

$$\bar{\gamma}(A) = P_S\left[\frac{\Sigma^{(s)} - |\mu^{(s)}|^2}{\sigma_s^2} + \frac{\|\mathbf{h}_u\|^2}{\sigma_u^2} - \frac{\sigma_d^2}{(\sigma_u^2)^2}\mathbf{h}_u^H\mathbb{E}_{\mathbf{h}_d}\left\{\left(\boldsymbol{\Psi}^H\boldsymbol{\Psi} + \frac{\sigma_d^2}{\sigma_u^2}\mathbf{I}_L\right)^{-1}\right\}\mathbf{h}_u\right]. \tag{3.12}$$

Note that

$$\mathbb{E}_{\mathbf{h}_d}\left\{\left(\boldsymbol{\Psi}^H\boldsymbol{\Psi} + \frac{\sigma_d^2}{\sigma_u^2}\mathbf{I}_L\right)^{-1}\right\} = \mathbb{E}_{\mathbf{h}_d}\left\{\mathrm{diag}\left(\beta_1, \cdots, \beta_L\right)\right\}, \tag{3.13}$$

where $\beta_l = \left(\frac{|h_{dl}|^2|w_l|^2}{P_S|h_{ul}|^2+\sigma_u^2} + \frac{\sigma_d^2}{\sigma_u^2}\right)^{-1}$. Under the Rayleigh fading of downlink channels, the mean value of β_l is determined as follows:

$$\begin{aligned}
\mathbb{E}_{h_{dl}}\left\{\beta_l\right\} &= \int_0^{+\infty} \frac{\beta_l}{\Sigma_{ll}^{(d)}} \cdot \exp\left(-\frac{|h_{dl}|^2}{\Sigma_{ll}^{(d)}}\right) \mathrm{d}\,|h_{dl}|^2 \\
&= \frac{P_S\,|h_{ul}|^2 + \sigma_u^2}{|w_l|^2\,\Sigma_{ll}^{(d)}} \cdot \exp\left(\frac{\sigma_d^2\left(P_S\,|h_{ul}|^2 + \sigma_u^2\right)}{\sigma_u^2\,|w_l|^2\,\Sigma_{ll}^{(d)}}\right) \\
&\quad \times E_1\left(\frac{\sigma_d^2\left(P_S\,|h_{ul}|^2 + \sigma_u^2\right)}{\sigma_u^2\,|w_l|^2\,\Sigma_{ll}^{(d)}}\right),
\end{aligned} \tag{3.14}$$

where $E_1(x) = \int_x^{+\infty} \frac{e^{-t}}{t}\mathrm{d}t$, $x > 0$, is the exponential integral [8].

It is of interest to express the average SNR in terms of the power allocation variables $p_l = |w_l|^2$, $l = 1, \ldots, L$. To this end, let $\lambda = \sigma_d^2/\sigma_u^2$, $a_l = (P_S|h_{ul}|^2+\sigma_u^2)/\Sigma_{ll}^{(d)}$, $l = 1, \ldots, L$, and define

$$f_l\left(p_l\right) = \mathbb{E}_{h_{dl}}\left\{\beta_l\right\} = \frac{a_l}{p_l}\exp\left(\lambda\frac{a_l}{p_l}\right)E_1\left(\lambda\frac{a_l}{p_l},\right). \tag{3.15}$$

Then maximizing $\bar{\gamma}(A)$ in (3.12) is equivalent to minimizing $\sum_l f_l(p_l)|h_{ul}|^2$. The optimal power allocation problem can now be mathematically stated as follows:[3]

$$
\begin{aligned}
& \min_{p_1,\ldots,p_L} && \sum_{l=1}^{L} f_l\,(p_l)\,|h_{ul}|^2 \\
& \text{subject to} && p_l \geq 0,\ l = 1,\ldots,L \text{ and } \sum_{l=1}^{L} p_l \leq P_R.
\end{aligned}
\tag{3.16}
$$

The following lemma establishes the convexity of the objective function in (3.16).

Lemma 1. *With $\lambda > 0$ and $a_l > 0$, the function $f_l(p_l)$ is positive, decreasing, and convex in $p_l \geq 0$.*

Proof. It can be seen from (3.14) that $f_l(p_l)$ has the following form

$$
f_l(p_l) = \int_0^{+\infty} \chi_l(p_l)\eta(|h_{dl}|^2)\mathrm{d}|h_{dl}|^2,
\tag{3.17}
$$

where $\chi_l(p_l)$ is convex and decreasing in $p_l \geq 0$ and $\eta(|h_{dl}|^2)$ is a nonnegative function. Thus the decreasing monotonicity of $f_l(p_l)$ is obvious. Furthermore,

$$
\begin{aligned}
& f_l\,(tp_1 + (1-t)p_2) \\
&= \int_0^{+\infty} \chi_l\,(tp_1 + (1-t)p_2)\,\eta(|h_{dl}|^2)\mathrm{d}|h_{dl}|^2 \\
&\leq \int_0^{+\infty} [t\chi_l(p_1) + (1-t)\chi_l(p_2)]\,\eta(|h_{dl}|^2)\mathrm{d}|h_{dl}|^2 \\
&= tf_l(p_1) + (1-t)f_l(p_2),
\end{aligned}
\tag{3.18}
$$

with an arbitrary $t \in [0,1]$. Thus, the convexity of $f_l(p_l)$ in $p_l \geq 0$ is also obvious, which completes the proof. $\square$

Since the objective function is decreasing and convex in $p_1, p_2, \ldots, p_L$, while all the constraints are convex, according to the Karush-Kuhn-Tucker (KKT) condition for optimality of convex programming, the necessary and sufficient conditions for the

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[3]In this case, all the relays compute the optimal power allocation scheme and the destination does not have to send the power allocation information back to the relays.

optimal solution of (3.16) are:

$$
\begin{cases}
\dfrac{\partial \mathcal{L}(p_1, p_2, \ldots, p_L, \nu, \nu_1, \ldots, \nu_L)}{\partial p_l} = 0, \ l = 1, 2, \ldots, L; \\[2mm]
\nu \left(\sum_{l=1}^{L} p_l - P_R \right) = 0; \\[2mm]
\nu_l p_l = 0, \ l = 1, 2, \ldots, L,
\end{cases}
\tag{3.19}
$$

where the Lagrangian is

$$
\begin{aligned}
&\mathcal{L}(p_1, p_2, \ldots, p_L, \nu, \nu_1, \ldots, \nu_L) \\
&= \sum_{l=1}^{L} \frac{|h_{ul}|^2 a_l}{p_l} \exp\left(\lambda \frac{a_l}{p_l} \right) E_1\left(\lambda \frac{a_l}{p_l} \right) - \nu \left(P_R - \sum_{l=1}^{L} p_l \right) - \sum_{l=1}^{L} \nu_l p_l,
\end{aligned}
\tag{3.20}
$$

and $\nu \geq 0$, $\nu_l \geq 0$, $l = 1, 2 \ldots, L$ are the Lagrangian multipliers.

Making use of the fact that $dE_1(x)/dx = -E_0(x) = -\exp(-x)/x$, one can find

$$
\frac{\partial f_l(p_l)}{\partial p_l} = \frac{a_l}{p_l^2} \left[1 - \left(1 + \lambda \frac{a_l}{p_l} \right) \exp\left(\lambda \frac{a_l}{p_l} \right) E_1\left(\lambda \frac{a_l}{p_l} \right) \right].
\tag{3.21}
$$

Then it is straightforward to see that the optimal solution of (3.12) is $w_l = \sqrt{p_{l,\mathrm{opt}}}$ where $p_{l,\mathrm{opt}} = \max\{p_l, 0\}$, $l = 1, \ldots, L$, and p_l is the solution of the following nonlinear equation:

$$
|h_{ul}|^2 \frac{a_l}{p_l^2} \left[\left(1 + \lambda \frac{a_l}{p_l} \right) \exp\left(\lambda \frac{a_l}{p_l} \right) E_1\left(\lambda \frac{a_l}{p_l} \right) - 1 \right] = \nu,
\tag{3.22}
$$

and the optimal Lagrangian multiplier $\nu > 0$ is chosen such that $\sum_{l=1}^{L} p_{l,\mathrm{opt}} = P_R$.

It can be shown that the left-hand side of (3.22) is also decreasing and convex in $p_l \geq 0$ since all the component functions are positive, decreasing, and convex. Exploiting these properties, the optimal power allocation solution $\mathbf{p}_{\mathrm{opt}} = [p_{1,\mathrm{opt}}, \ldots, p_{L,\mathrm{opt}}]^T$ can be found using some efficient and fast computational algorithms. For the numerical results presented in Section V of this paper, a double iterative bisection procedure (IBP) shall be employed since it is very simple and well suited.[4] For completeness, the IBP algorithm is provided in detail in the Appendix (see [9] for a similar implementation).

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[4]Instead of IBP, other algorithms may also be employed. To compare the efficiency of different solvers is, however, beyond the scope of this paper.
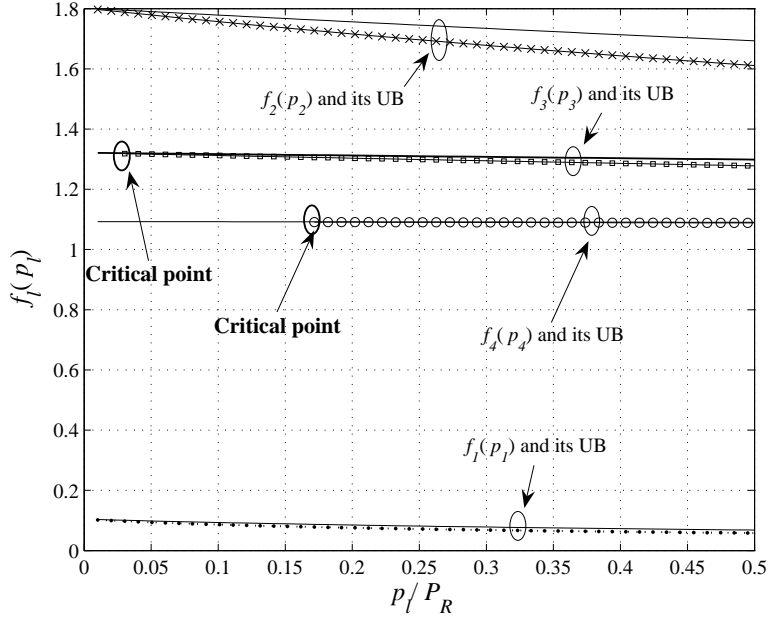
Figure 3.2 Objective functions $f_l(p_l)$, $l = 1, 2, 3, 4$, and their upper bounds (UB) with $P_S = P_R = 1$ dB and $\Sigma_{11}^{(u)} = -10$ dB, $\Sigma_{22}^{(u)} = 0$ dB, $\Sigma_{33}^{(u)} = \Sigma_{44}^{(u)} = \Sigma_{11}^{(d)} = \Sigma_{22}^{(d)} = 7$ dB, $\Sigma_{33}^{(d)} = 0$ dB, $\Sigma_{44}^{(d)} = -10$ dB.

Although the optimization problem in (3.16) can be efficiently solved by the IBP algorithm in most cases, numerical difficulty can arise. In particular, if a_l takes on a very large value while p_l is too small, $\exp(\lambda a_l/p_l)$ approaches ∞ extremely fast while $E_1(\lambda a_l/p_l)$ is almost zero. As a result, the values of $f_l(p_l)$ change vastly when crossing over this small value of p_l (these values of $f_l(p_l)$ may even be out of the computable range of computing softwares). Hereafter, we call such small value of p_l a "critical point", $p_{l,c}$. Consequently, there might be a critical point $(p_{l,c}, \nu_c)$ in the first derivative in (3.22), i.e.,

$$
\begin{cases}
p_l \approx p_{l,c}, & \nu \geq \nu_c \\
p_l \geq p_{l,c}, & \nu < \nu_c
\end{cases}, \tag{3.23}
$$

and (3.22) is thus in the so-called "ill-condition". Fig. 3.2 shows an example of critical points in $f_3(p_3)$ and $f_4(p_4)$. In general, one should find numerically the critical points first and then use (3.23) to approximate the curves given in (3.22) before solving the nonlinear equations and choosing the optimal Lagrange multiplier. This

approximation will prevent the iterative solution from a slow convergence.

To completely overcome the numerical issue due to the existence of critical points, one sub-optimal but reliable method is to minimize an upper bound of the objective function $\sum_{l=1}^{L} |h_{ul}|^2 f_l(p_l)$. Using the following inequality [10]:

$$E_1(x) < \exp(-x) \ln\left(1 + \frac{1}{x}\right), \tag{3.24}$$

the upper bound of $|h_{ul}|^2 f_l(p_l)$ can be established as

$$|h_{ul}|^2 f_l(p_l) < \frac{|h_{ul}|^2 a_l}{p_l} \ln\left(1 + \frac{p_l}{\lambda a_l}\right). \tag{3.25}$$

The following lemma states the convexity of the above upper bound.

Lemma 2. *With $\lambda > 0$ and $a_l > 0$, the function*

$$\chi_l(p_l) = \frac{a_l}{p_l} \ln\left(1 + \frac{p_l}{\lambda a_l}\right)$$

is positive, decreasing, and convex in $p_l \geq 0$.

Proof. It is obvious that $\chi_l(p_l)$ is positive for $p_l \geq 0$. Let $x = p_l/(\lambda a_l)$ and consider the function $\chi(x) = (1/x) \ln(1 + x)$. Since $x/(1 + x) < \ln(1 + x)$ for $x > 0$, the first derivative of $\chi(x)$ is

$$\frac{\mathrm{d}\chi(x)}{\mathrm{d}x} = -\frac{1}{x^2} \ln(1 + x) + \frac{1}{x(1 + x)} < 0, \quad \text{for} \quad x > 0, \tag{3.26}$$

and its second derivative can be expressed as

$$\frac{\mathrm{d}^2\chi(x)}{\mathrm{d}x^2} = \frac{2(1 + x)^2 \ln(1 + x) - x(3x + 2)}{x^3(1 + x)^2}. \tag{3.27}$$

Let $v(x) = 2(1 + x)^2 \ln(1 + x) - x(3x + 2)$. Since

$$\frac{\mathrm{d}v(x)}{\mathrm{d}x} = 4\left[(1 + x) \ln(1 + x) - x\right] > 0, \quad \text{for} \quad x > 0, \tag{3.28}$$

and $\lim_{x\to 0} v(x) = 0$, then $v(x) > 0$ and $\mathrm{d}^2\chi(x)/\mathrm{d}x^2 > 0$ for $x > 0$. It follows that $\chi(x)$ is decreasing and convex in $x > 0$. Therefore, $\chi_l(p_l)$ is positive, decreasing, and convex in $p_l \geq 0$. $\square$

Again, applying the KKT condition (3.19) for optimality of convex programming for the upper bound of the objective function given in (3.25), a sub-optimal power allocation scheme $\mathbf{p}_{\text{subopt}} = [p_{1,\text{subopt}}, \ldots, p_{L,\text{subopt}}]^T$ can be found as $p_{l,\text{subopt}} = \max\{p_l, 0\}$, $l = 1, \ldots, L$, where p_l's are the solutions of the following nonlinear equations:

$$\frac{|h_{ul}|^2 a_l}{p_l} \left[\frac{1}{p_l} \ln\left(1 + \frac{p_l}{\lambda a_l}\right) - \frac{1}{(p_l + \lambda a_l)}\right] = \mu, \qquad (3.29)$$

and the Lagrangian multiplier $\mu > 0$ is chosen such that $\sum_{l=1}^{L} p_{l,\text{subopt}} = P_R$. The set of above nonlinear equations can be solved efficiently and reliably by the IBP algorithm since there is no issue with critical points.

3.4   Power Allocation Under Assumption B

Under Assumption B, the relays cannot compute the optimal power allocation. Instead, the destination shall determine it and then send the information of the optimal power allocation back to the relays via some feedback channel. Furthermore, since the destination knows only h_s, $\mathbf{h}_d$, $(\boldsymbol{\mu}^{(u)}, \boldsymbol{\Sigma}^{(u)})$, it cannot perform the receiver given in (3.8) nor use the instantaneous SNR expression in (3.11) to optimize the power allocation. To overcome this difficulty, a modified signal processing at the relays shall be employed. Specifically, at each relay, instead of scaling the received signal with the gain in (3.2), each relay weights the received signal with

$$\bar{g}_l = \frac{h_{ul}^* w_l}{|h_{ul}| \sqrt{P_S |h_{ul}|^2 + \sigma_u^2}}, \quad l = 1, \ldots, L. \qquad (3.30)$$

In essence, the above amplification factor not only maintains the transmitted power of $|w_l|^2$ from the lth relay, but also corrects the random phase introduced by the uplink channel and prevents its destructive effect to propagate to the destination. Then the received signals at the destination in the second phase (over L time slots) can be expressed as

$$\bar{\mathbf{d}}_2 = \boldsymbol{\Psi}\bar{\mathbf{h}}_u \cdot s + \boldsymbol{\Psi}\bar{\mathbf{n}}_u + \mathbf{n}_d, \qquad (3.31)$$

where

$$\bar{\mathbf{h}}_u = [|h_{u1}|, \ldots, |h_{uL}|]^T$$

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and

$$\bar{\mathbf{n}}_u = \left[\frac{h_{u1}^*}{|h_{u1}|} n_{u1}, \ldots, \frac{h_{uL}^*}{|h_{uL}|} n_{uL} \right]^T.$$

Note that, with the assumption of uncorrelated Rayleigh fading uplink channels, $\bar{\mathbf{n}}_u$ is obtained by rotating the independent complex Gaussian random variables in $\mathbf{n}_u$ by independent uniform phases. It follows that $\bar{\mathbf{n}}_u$ is still a white Gaussian noise vector, i.e., $\bar{\mathbf{n}}_u \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_L)$.

The received signals at the destination over two transmission phases now become:

$$\bar{\mathbf{d}} = \begin{bmatrix} d_1 \\ \bar{\mathbf{d}}_2 \end{bmatrix} = \bar{\mathbf{a}} \cdot s + \bar{\mathbf{n}}, \tag{3.32}$$

where

$$\bar{\mathbf{a}} = \begin{bmatrix} h_s \\ \mathbf{W}\mathbf{H}_d\mathbf{F}_u\bar{\mathbf{h}}_u \end{bmatrix}, \quad \bar{\mathbf{n}} = \begin{bmatrix} n_s \\ \mathbf{W}\mathbf{H}_d\mathbf{F}_u\bar{\mathbf{n}}_u + \mathbf{n}_d \end{bmatrix},$$

with $\mathbf{H}_d$ defined as in (3.1), $\mathbf{W} = \mathrm{diag}(w_1, \ldots, w_L)$, and

$$\mathbf{F}_u = \mathrm{diag}\left(\frac{1}{\sqrt{P_S |h_{u1}|^2 + \sigma_u^2}}, \ldots, \frac{1}{\sqrt{P_S |h_{uL}|^2 + \sigma_u^2}} \right).$$

Based on (3.32) the optimal filter is simply given as[5]

$$\bar{\mathbf{f}} = \bar{\mathbf{R}}^{-1} \mathbb{E}_{\bar{\mathbf{h}}_u}\{\bar{\mathbf{a}}\}, \tag{3.33}$$

where $\bar{\mathbf{R}}$ is the covariance matrix of the noise vector $\bar{\mathbf{n}}$. It is computed as follows:

$$\begin{aligned} \bar{\mathbf{R}} &= \mathbb{E}\left\{ \bar{\mathbf{n}}\bar{\mathbf{n}}^H \right\} \\ &= \begin{bmatrix} \sigma_s^2 & \mathbf{0} \\ \mathbf{0} & \sigma_u^2 \mathbf{W}\mathbf{H}_d\mathbf{T}_u\mathbf{H}_d^H\mathbf{W}^H + \sigma_d^2 \mathbf{I}_L \end{bmatrix}, \end{aligned} \tag{3.34}$$

where

$$\begin{aligned} \mathbf{T}_u &= \frac{1}{\sigma_u^2}\mathbb{E}_{\bar{\mathbf{h}}_u}\left\{ \mathbf{F}_u\bar{\mathbf{n}}_u\bar{\mathbf{n}}_u^H\mathbf{F}_u^H \right\} = \mathbb{E}_{\bar{\mathbf{h}}_u}\left\{ \mathbf{F}_u\mathbf{F}_u^H \right\} \\ &= \mathrm{diag}\left(T_{u1}, \ldots, T_{uL} \right), \end{aligned} \tag{3.35}$$

---

[5]The proposed filter given in (3.33) is optimal in the sense that it maximizes the average SNR at its output, i.e., $\bar{\mathbf{f}} = \arg\max_{\mathbf{f}} \mathbb{E}_{\bar{\mathbf{h}}_u}\{\mathrm{SNR}\} = \arg\max_{\mathbf{f}} \frac{\mathbf{f}^H \mathbb{E}_{\bar{\mathbf{h}}_u}\{\bar{\mathbf{a}}\bar{\mathbf{a}}^H\}\mathbf{f}^H}{\mathbf{f}^H\bar{\mathbf{R}}^{-1}\mathbf{f}}$. The derivations in (3.34)-(3.41) show that $\mathbb{E}_{\mathbf{h}_u}\left\{ \bar{\mathbf{a}}\bar{\mathbf{a}}^H \right\} \approx \mathbb{E}_{\mathbf{h}_u}\{\bar{\mathbf{a}}\} \cdot \mathbb{E}_{\mathbf{h}_u}\{\bar{\mathbf{a}}^H\}$. Then, applying Proposition 1 in [1] to $\mathbb{E}_{\bar{\mathbf{h}}_u}\{\mathrm{SNR}\}$, one can find the optimal $\bar{\mathbf{f}}$.

and

$$T_{ul} = \mathbb{E}_{h_{ul}} \left\{ \frac{1}{P_S |h_{ul}|^2 + \sigma_u^2} \right\}$$

$$= \int_0^{+\infty} \frac{1}{P_S |h_{ul}|^2 + \sigma_u^2} \frac{e^{-\frac{|h_{ul}|^2}{\Sigma_{ll}^{(u)}}}}{\Sigma_{ll}^{(u)}} d |h_{ul}|^2$$

$$= \frac{e^{\frac{\sigma_u^2}{P_S \Sigma_{ll}^{(u)}}}}{P_S \Sigma_{ll}^{(u)}} E_1 \left(\frac{\sigma_u^2}{P_S \Sigma_{ll}^{(u)}} \right). \tag{3.36}$$

To complete the calculation of the optimal filter in (3.33), one needs to determine $\tilde{\mathbf{a}} = \mathbb{E}_{\mathbf{h}_u} \{\bar{\mathbf{a}}\}$. Under uncorrelated Rayleigh fading uplink channels, $\tilde{\mathbf{a}}$ is found as

$$\tilde{\mathbf{a}} = \mathbb{E}_{\bar{\mathbf{h}}_u} \{\bar{\mathbf{a}}\} = \begin{bmatrix} h_s \\ \mathbf{W} \mathbf{H}_d \mathbb{E}_{\bar{\mathbf{h}}_u} \{\mathbf{F}_u \bar{\mathbf{h}}_u\} \end{bmatrix} = \begin{bmatrix} h_s \\ \mathbf{W} \mathbf{H}_d \mathbf{q}_u \end{bmatrix},$$

where $\mathbf{q}_u = [q_{u1}, \ldots, q_{uL}]^T$, and

$$q_{ul} = \mathbb{E}_{h_{ul}} \left\{ \frac{|h_{ul}|}{\sqrt{P_S |h_{ul}|^2 + \sigma_u^2}} \right\}$$

$$= \int_0^{+\infty} \left(\frac{|h_{ul}|^2}{P_S |h_{ul}|^2 + \sigma_u^2} \right)^{\frac{1}{2}} \frac{e^{-\frac{|h_{ul}|^2}{\Sigma_{ll}^{(u)}}}}{\Sigma_{ll}^{(u)}} d |h_{ul}|^2$$

$$= \frac{1}{\sqrt{P_S}} \frac{\sigma_u^2 e^{\frac{\sigma_u^2}{2P_S \Sigma_{ll}^{(u)}}}}{2P_S \Sigma_{ll}^{(u)}} \left[K_1 \left(\frac{\sigma_u^2}{2P_S \Sigma_{ll}^{(u)}} \right) - K_0 \left(\frac{\sigma_u^2}{2P_S \Sigma_{ll}^{(u)}} \right) \right]. \tag{3.37}$$

Note that $K_\alpha(x)$ in (3.37) is the αth-order modified Bessel function of the second kind [8].

With the optimal filter in (3.33), the resulting average SNR at the filter's output can be shown to be:

$$\bar{\gamma}(B) = P_S \frac{\tilde{\mathbf{a}}^H \bar{\mathbf{R}}^{-1} \mathbb{E}_{\bar{\mathbf{h}}_u} \{\bar{\mathbf{a}} \bar{\mathbf{a}}^H\} \bar{\mathbf{R}}^{-1} \tilde{\mathbf{a}}}{\tilde{\mathbf{a}}^H \bar{\mathbf{R}}^{-1} \tilde{\mathbf{a}}}. \tag{3.38}$$

The calculation of the term $\mathbb{E}_{\bar{\mathbf{h}}_u} \{\bar{\mathbf{a}} \bar{\mathbf{a}}^H\}$ is as follows:

$$\mathbb{E}_{\bar{\mathbf{h}}_u} \{\bar{\mathbf{a}} \bar{\mathbf{a}}^H\} = \begin{bmatrix} |h_s|^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{W} \mathbf{H}_d \mathbf{Q} \mathbf{H}_d^H \mathbf{W}^H \end{bmatrix}, \tag{3.39}$$

where $\mathbf{Q} = \mathbb{E}_{\bar{\mathbf{h}}_u} \left\{ \mathbf{F}_u \bar{\mathbf{h}}_u \bar{\mathbf{h}}_u^H \mathbf{F}_u^H \right\}$. The (k, l)th element of $\mathbf{Q}$ is

$$[\mathbf{Q}]_{lk} = \begin{cases} Q_{ul}, & l = k \\ q_{ul} \cdot q_{uk}, & l \neq k \end{cases} , \quad l, k = 1, \ldots, L,$$

where

$$
\begin{aligned}
Q_{ul} &= \mathbb{E}_{h_{ul}} \left\{ \frac{|h_{ul}|^2}{P_S |h_{ul}|^2 + \sigma_u^2} \right\} \\
&= \int_0^{+\infty} \frac{|h_{ul}|^2}{P_S |h_{ul}|^2 + \sigma_u^2} \frac{e^{-\frac{|h_{ul}|^2}{\Sigma_{ll}^{(u)}}}}{\Sigma_{ll}^{(u)}} \mathrm{d} |h_{ul}|^2 \\
&= \frac{1}{P_S} \left[1 - \frac{\sigma_u^2}{P_S \Sigma_{ll}^{(u)}} e^{\frac{\sigma_u^2}{P_S \Sigma_{ll}^{(u)}}} E_1 \left(\frac{\sigma_u^2}{P_S \Sigma_{ll}^{(u)}} \right) \right]
\end{aligned} \tag{3.40}
$$

and q_{ul} is defined in (3.37).

Although being exact, the SNR expression in (3.38) depends on the power allocation matrix $\mathbf{W}$ in a very complicated manner. Hence, optimizing $\mathbf{W}$ to maximize (3.38) is very difficult, if not impossible. In what follows, we propose an approximation for $\bar{\gamma}(B)$ and use it to optimize the power allocation.

It follows from applying the Jensen's inequality to (3.37) and (3.40) that $q_{ul}^2 \leq Q_{ul}$. Furthermore, observe that when $P_S/\sigma_u^2 >> 1$, $\left(\frac{|h_{ul}|^2}{P_S |h_{ul}|^2 + \sigma_u^2} \right) \approx \frac{1}{P_S}$. Then $q_{ul} \approx \frac{1}{\sqrt{P_S}}$ while $Q_{ul} \approx \frac{1}{P_S}$. These results suggest that the approximation $q_{ul}^2 \approx Q_{ul}$ is asymptotically tight. To validate the accuracy of this approximation, numerical results are obtained with various values of the parameters in a broad range as the channel gains' variances vary from -10 dB to 20 dB. As shown in Fig. 3.3, at moderate and high SNR regions, say at $P_S/\sigma_u^2 \geq 4$ dB, $Q_{u1} \approx q_{u1}^2$ with the difference $\epsilon \leq 10^{-4}$ and at $P_S/\sigma_u^2 \geq 10$ dB the difference is $\epsilon \leq 4 \cdot 10^{-5}$. The same accuracy can be observed with $Q_{u2} \approx q_{u2}^2$ while the approximations $Q_{ul} \approx q_{ul}^2$ are much more accurate for Q_{u3} and Q_{u4}.

In summary, at moderate and high channel signal-to-noise ratio (CSNR), defined as $P_S/\sigma_u^2 = P_S/\sigma_d^2$, one can approximate $\mathbf{Q} \approx \mathbf{q}_u \mathbf{q}_u^H$ or $\mathbb{E}_{\mathbf{h}_u} \left\{ \bar{\mathbf{a}} \bar{\mathbf{a}}^H \right\} \approx \mathbb{E}_{\mathbf{h}_u} \left\{ \bar{\mathbf{a}} \right\} \cdot \mathbb{E}_{\mathbf{h}_u} \left\{ \bar{\mathbf{a}}^H \right\}$. With such an approximation, the average SNR in (3.38) can be approxi-
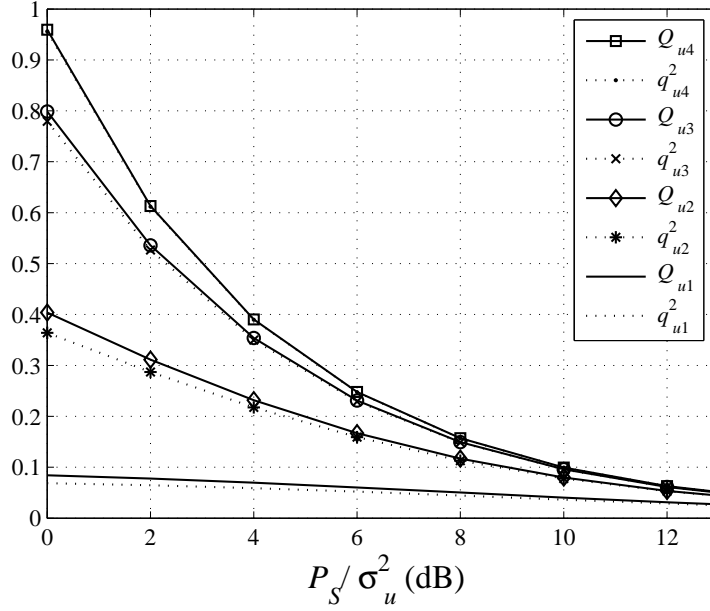
Figure 3.3 Accuracy of the approximation $Q_{ul} \approx q_{ul}^2$: Asymmetric model, $L = 4$, with $\Sigma_{11}^{(u)} = -10$ dB, $\Sigma_{22}^{(u)} = 0$ dB, $\Sigma_{33}^{(u)} = 10$ dB, $\Sigma_{44}^{(u)} = 20$ dB, $\Sigma_{11}^{(d)} = 20$ dB, $\Sigma_{22}^{(d)} = 10$ dB, $\Sigma_{33}^{(d)} = 0$ dB, $\Sigma_{44}^{(d)} = -10$ dB.

mated as:

$$\bar{\gamma}(B) \approx \tilde{\gamma}(B) = P_S \tilde{\mathbf{a}}^H \bar{\mathbf{R}}^{-1} \tilde{\mathbf{a}} =$$

$$P_S \left[\frac{|h_s|^2}{\sigma_s^2} + \frac{1}{\sigma_u^2} \mathbf{q}_u^H \mathbf{T}_u^{-H/2} \mathbf{T}_u^{-1/2} \mathbf{q}_u - \frac{\sigma_d^2}{(\sigma_u^2)^2} \mathbf{q}_u^H \mathbf{T}_u^{-H/2} \right.$$

$$\left. \times \left(\mathbf{T}_u^{H/2} \mathbf{H}_d^H \mathbf{W}^H \mathbf{W} \mathbf{H}_d \mathbf{T}_u^{1/2} + \frac{\sigma_d^2}{\sigma_u^2} \mathbf{I}_L \right)^{-1} \mathbf{T}_u^{-1/2} \mathbf{q}_u \right]. \qquad (3.41)$$

Now, the problem of designing the optimal power allocation $\mathbf{w} = [w_1, \ldots, w_L]^T$ in order to maximize the approximated average SNR, $\tilde{\gamma}(B)$, under the total relay power constraint, $\mathbf{w}^H \mathbf{w} = P_R$, can be equivalently expressed as follows:

$$\min_{w_1, \ldots, w_L} \quad \sum_{l=1}^{L} \frac{q_{ul}^2}{T_{ul} \left(T_{ul} |h_{dl}|^2 |w_l|^2 + \frac{\sigma_d^2}{\sigma_u^2} \right)},$$

$$\text{subject to} \quad \sum_{l=1}^{L} |w_l|^2 = P_R. \qquad (3.42)$$

It is obvious that the objective and the constraint functions are convex in $|w_l|^2 \in [0, P_R]$. By changing the variables $|w_l|^2 \rightarrow p_l$ and using the KKT condition (3.19)

for the convex optimization problem in (3.42), the optimal solution is found to be $w_{l,\text{opt}} = \sqrt{p_{l,\text{opt}}}$, $l = 1, \ldots, L$, where

$$p_{l,\text{opt}}(\alpha) = \frac{1}{T_{ul}|h_{dl}|^2} \max\left\{\alpha(q_{ul}|h_{dl}|) - \frac{\sigma_d^2}{\sigma_u^2}, 0\right\}, \tag{3.43}$$

and $\alpha > 0$ is chosen such that $\sum_{l=1}^{L} p_{l,\text{opt}}(\alpha) = P_R$.

At this point it is appropriate to comment on the proposed power allocations under Assumptions A and B. Firstly, in terms of feedback information required from the destination to the relays, feeding back the channel statistics (under Assumption A) requires less overhead as compared to feeding back the power allocation coefficients (under Assumption B) since the channel statistics vary slower than the power allocation coefficients. Secondly, in terms of the complexity of signal processing at relays, which is an important consideration in wireless relay networks, the complexity is less for power computation under Assumption B than under Assumption A due to the "centralized" and "distributed" natures of the two power allocation schemes. Lastly, but more importantly, in terms of the error performance, it is not clear which power allocation scheme will always be better. In fact this depends on the network configurations as demonstrated and discussed further in Section 3.5.

Before closing this section, it should be pointed out that our proposed power allocation schemes among the relays under both Assumptions A and B assume some fixed allocation of the total power between the source and L relays, i.e., both P_S and P_R are fixed. A more general problem would be to consider the allocation of the total power, $P_T = P_S + P_R$, among the source and L relays[6]. From the expressions of the average SNR in (3.12) and (3.41), it appears that such an optimization problem is intractable under the two partial CSI assumptions under consideration. A simpler alternative, albeit suboptimal, is to numerically vary the allocation of P_S and P_R

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[6]It is noted that, any power allocation involving the source needs to be carried out either at the relays or destination and the resulting source power quantity is fed back to the source. This is because it is assumed in this study that the source has no information about any channels in the network.

Table 3.1   Channel models used in the simulation where all links are uncorrelated Rayleigh fading.

Model	$\mathbf{\Sigma}^{(u)}$ (dB)	$\mathbf{\Sigma}^{(d)}$ (dB)	$\Sigma^{(s)}$ (dB)
I	$\mathrm{diag}(0, 0, 0, 0)$	$\mathrm{diag}(0, 0, 0, 0)$	0
II	$\mathrm{diag}(-10, -5, 5, 10)$	$\mathrm{diag}(-10, -5, 5, 10)$	-10
III	$\mathrm{diag}(20, 20, 20, 20)$	$\mathrm{diag}(0, 0, 0, 0)$	0
IV	$\mathrm{diag}(0, 0, 0, 0)$	$\mathrm{diag}(20, 20, 20, 20)$	0

while keeping $P_S + P_R$ at a constant and apply our proposed relay power allocation for each P_R. This suboptimal approach shall be used in the next section to investigate the impact of power allocation between the source and L relays.

3.5   Numerical Results

This section presents numerical results on the performance of the proposed wireless relay networks under the two partial CSI assumptions under consideration. Performance comparison to the design in [6] for the case of full CSI is also made. The results in Section 3.5.1 are obtained with equal-power allocation between the source and L relays, i.e., $P_S = P_R$, whereas the impact of allocating $P_S \neq P_R$ is examined in Section 3.5.2. Regarding power allocation among the L relays, the proposed scheme is obtained with two iterative bisection procedures, one inside the other, where the accuracy is set to be $\Delta \leq 10^{-6}$. For the case of symmetric channels, the original nonlinear equations in (3.22) are solved as there is no issue with critical points. On the other hand, for the case of asymmetric channels the upper bound of the objective function is minimized so that the numerical instability issue caused by critical points can be avoided.
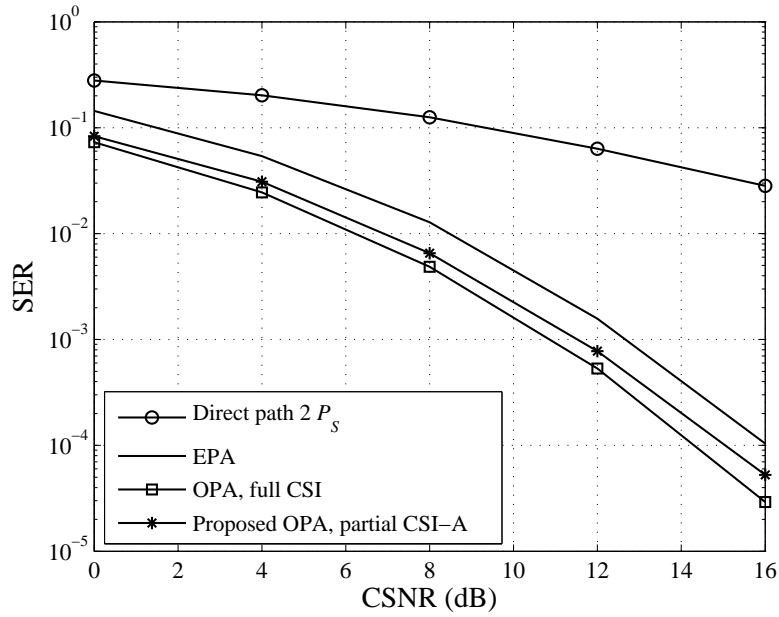
Figure 3.4 Performance comparison between the optimal power allocation (OPA) in [6] with full CSI and the proposed OPA under CSI Assumption A. $L = 4$, BPSK modulation, channel model II.
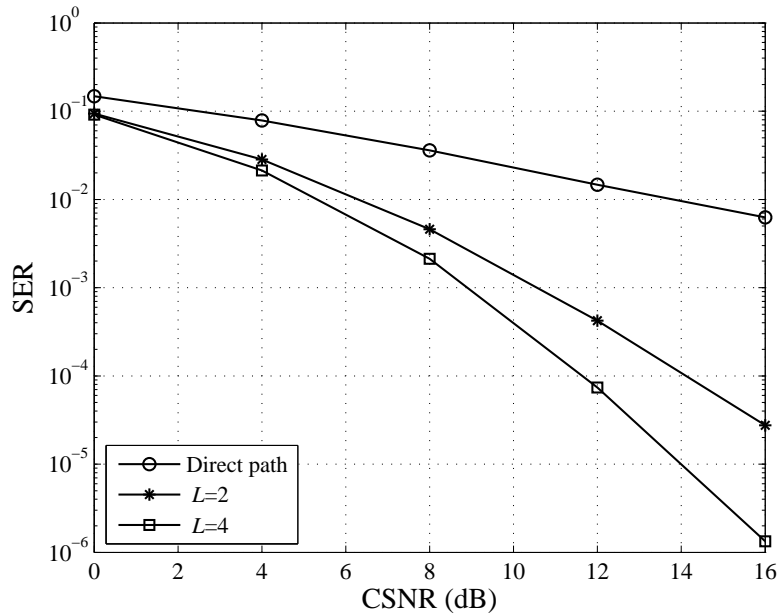
Figure 3.5 Performance of the proposed OPA under CSI Assumption A, with $L = 2$ and $L = 4$, BPSK modulation, channel model I.

3.5.1 Equal Power Allocation Between the Source and L Relays $(P_S = P_R)$

Consider a network with $L = 4$ relays. All the uplink and downlink channels are assumed to be uncorrelated Rayleigh fading, but with different variances (i.e., different average attenuation factors). They are referred to as asymmetric channels. In all simulations, binary phase-shift keying (BPSK) modulation is used at the source. With the normalization of unit noise power at the relays and destination, $\sigma_s^2 = \sigma_u^2 = \sigma_d^2 = 1$, the average CSNR is simply defined as $P_S/\sigma_s^2 = P_S/\sigma_u^2 = P_S/\sigma_d^2 = P_S$.

Under Assumption A, Fig. 3.4 compares the symbol-error-rate (SER) performances[7] of the wireless relay network that are achieved with (i) equal-power allocation (EPA) , (ii) the proposed power allocation and (iii) the optimal power allocation proposed in [6] under the full CSI assumption. The uplink and downlink Rayleigh fading channels in the network are modeled with $\mathbf{\Sigma}^{(u)} = \text{diag}(-10, -5, 5, 10)$ dB and $\mathbf{\Sigma}^{(d)} = \text{diag}(-10, -5, 5, 10)$ dB, respectively, while $\mathbf{\Sigma}^{(s)} = -10$ dB, i.e., channel model II in Table 3.1. Without the instantaneous information of the downlink channels, it can be seen that the performance loss is about 0.5 dB and 0.8 dB at the SER of 10^{-3} and 10^{-4}, respectively. The power saving by the proposed power allocation over the equal-power allocation is about 1 dB at high CSNR. Since the use of orthogonal transmission separates the signals received from the L relays, it can be verified from Figs. 3.4 and 3.5 that the diversity order is maintained with the proposed power allocation. This translates to a very significant performance improvement over the case of direct transmission between the source and destination as can be observed from these two figures. Note that, the results shown in Fig. 3.5 are obtained with the channel model I in Table 3.1.

Next, under Assumption B, since the destination does not know the instantaneous information of the uplink channels, the traditional MRC receiver, which is a *coherent* combiner, does not perform well. With the modification of the signal processing at

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[7]Since BPSK modulation is used, the symbol-error-rate is the same as the bit-error-rate.

the relays in (3.30), the receiver in (3.33) can detect the signal successfully. As a consequence, the performance improvement compared to the case of direct transmission is still very significant as illustrated in Fig. 3.6. Similar to Assumption A, the power saving offered by the proposed power allocation scheme over the equal-power allocation is about 1 dB at high CSNR. It is worth noting that the exact performance gains of the proposed schemes strongly depend on the fading statistics of the direct path, uplink and downlink channels.

The impact of CSI on the performance of the proposed method is illustrated in Fig. 3.7. In general, it can be seen that having more CSI information, i.e., under Assumption A as compared to Assumption B, leads to a better performance. However, it should be noted that under Assumption B, the optimal power allocation scheme is computed in a centralized manner at the destination, and hence it can make use of the information of the channels between the relays and the destination. In fact, this centralized computation together with the modified processing at the relays can lead to a better overall performance under Assumption B as compared to the distributed power allocation in Assumption A in some network configurations. This is demonstrated in Section 3.5.2 for the scenario that the average quality of the uplink channels is much better than that of the downlink channels (Fig. 3.9).

Also shown in Fig. 3.7 is the benchmark performance achieved by the optimal power allocation proposed in [6] under the *full* CSI assumption, i.e., the case that the relays and destination know all the instantaneous channels. In fact, it can be observed from Fig. 3.7 that the method proposed in [6] and our proposed method under Assumption A achieve the same diversity order, while the power loss experienced by our method due to partial CSI knowledge is less than 1 dB at the error performance level of 10^{-4}.
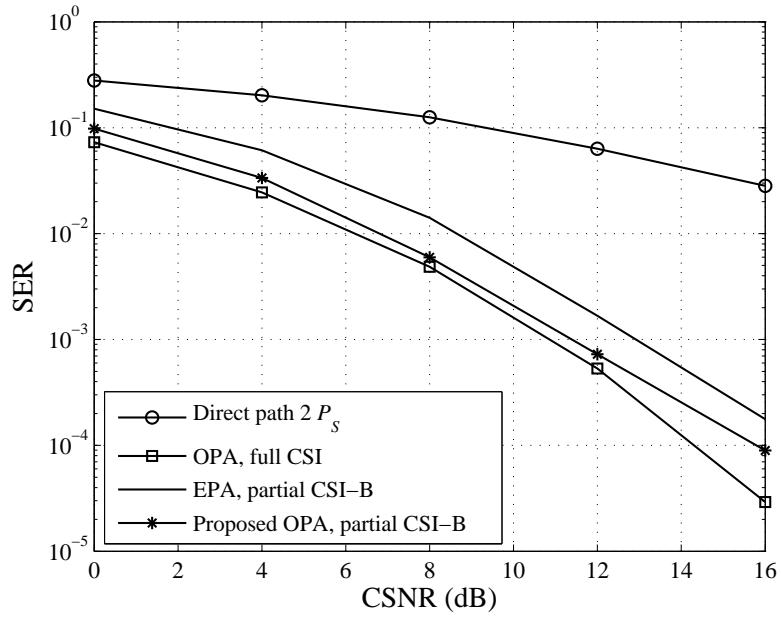
Figure 3.6 Performance comparison between the optimal power allocation (OPA) in [6] under the full CSI assumption and the proposed OPA under CSI Assumption B. $L = 4$, BPSK modulation, channel model II.
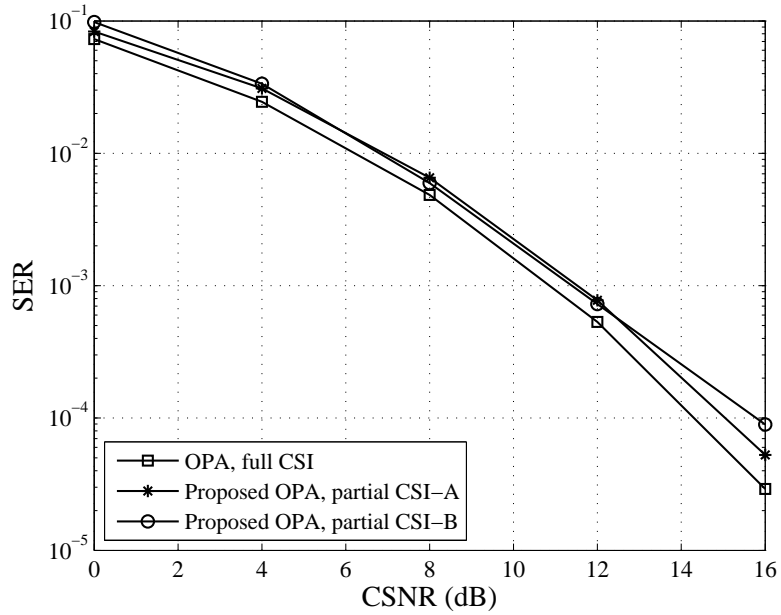
Figure 3.7 Impact of CSI on the performance of the proposed OPA. $L = 4$, BPSK modulation, channel model II.

3.5.2 Impact of Power Allocation Between the Source and L Relays

The impact of exercising different allocations between the source power, P_S, and relay power, P_R, on the SER performance is investigated in this section for both CSI Assumptions A and B. Figs. 3.8, 3.9 plot the SER when the ratio P_S/P_T is varied from 20% to 90% and the total power, $P_T = P_S + P_R$ is fixed. As before, the noise powers σ_s^2, σ_u^2 and σ_d^2 are normalized to unity.

Fig. 3.8 shows that for a symmetrical channel model (model I), the best SER performance is obtained if about 60% of the total power is allocated to the source. On the other hand, the optimal ratio is around 50% for an asymmetrical channel model (model II). Furthermore, when the uplink channels are in better conditions than the downlink channels, at least in the statistical sense, i.e., $\Sigma_{ll}^{(u)} >> \Sigma_{ll}^{(d)}$, it is reasonable to expect that more power is allocated to the relays in order to get better performances. This is clearly seen from Fig. 3.9 for channel model III in Table 3.1, where only 20% to 30% of the total power is assigned to the source. On the contrary, if the quality of the uplink channels is poor, as in model IV of Table 3.1, Fig. 3.9 tells that most of the total power should be assigned to the source. The optimal power assignment ratio P_S/P_T for channel model IV is about 80%.

Finally, an interesting observation from Fig. 3.9 is that, for the channel model III (where the average quality of the uplink channels is much better than that of the downlink channels), the error performance under Assumption B is better than that under Assumption A, even though there is less channel information under Assumption B as far as the whole network is concerned. As noted before, this is not surprising and can be explained by the effectiveness of the phase-matching processes at the relays and the fact that the centralized power computation under Assumption B can make use of the downlink channels between the relays and the destination.
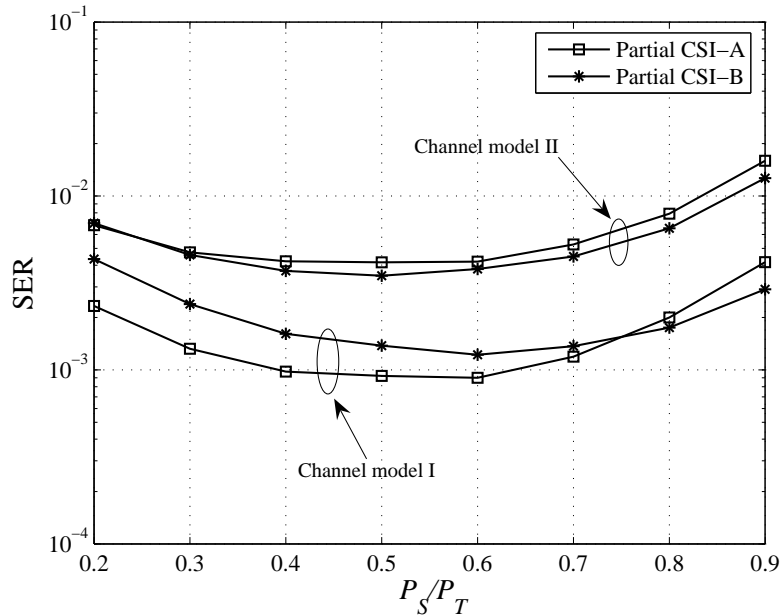
Figure 3.8 Average SER of the proposed model with different power allocation schemes between the source and L relays under CSI Assumptions A and B. $P_T = 12$ dB. $L = 4$, BPSK modulation, channel models I and II.

3.6 Conclusions

The problem of optimizing power allocation between the relays has been considered for wireless relay networks. The networks under consideration are the ones in which the source communicates with the relays and destination in the first phase and the relays forward signals to the destination in the second phase over orthogonal channels. Convex programming was used to obtain optimal and approximately optimal power allocation schemes to maximize the average signal-to-noise ratio at the destination under two *partial* CSI assumptions. Analysis and simulation results demonstrated the superiority of the proposed power allocation schemes over the equal-power allocation scheme. Comparison to a scheme previously obtained under the full CSI assumption illustrated performance losses experienced by the proposed schemes in a tradeoff for requiring less amounts of CSI. Finally, the impact of power allocation between the source and the relays was also investigated by computer simulation.
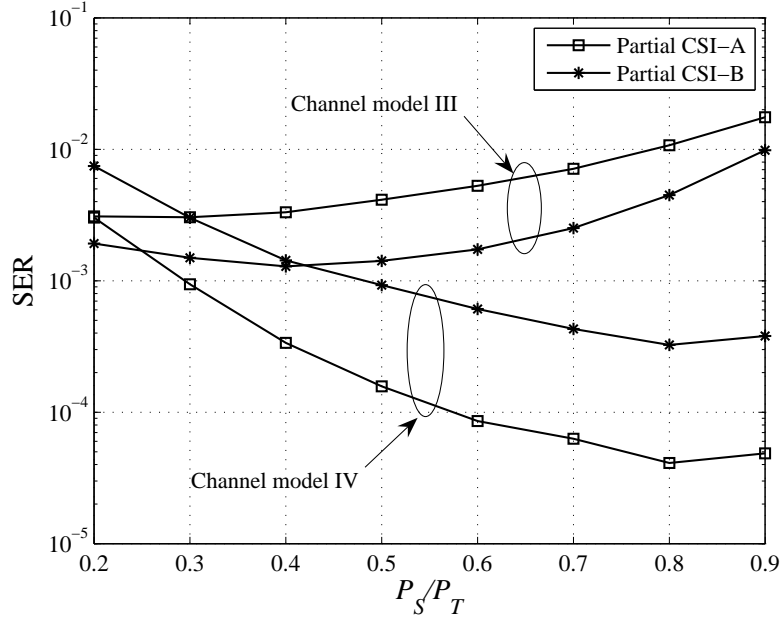
Figure 3.9 Average SER of the proposed model with different power allocation schemes between the source and L relays under CSI Assumptions A and B. $P_T = 8$ dB. $L = 4$, BPSK modulation, channel models III and IV.

3.A Double Iterative Bisection Procedure

3.A.1 Iterative Bisection Procedure (IBP)

For a monotonic (increasing or decreasing) function $h(t)$, the nonlinear equation

$$f(t) = h(t) - \nu = 0, \quad t \in [a, b]$$

can be solved using the following bisection method:

- If $f(a)f(b) > 0$, there is no solution in $[a, b]$.

- For $m = (a+b)/2$, reset $a = m$ if $f(a)f(m) > 0$ and reset $b = m$ if $f(a)f(m) < 0$. Repeat until $f(m) = 0$ or $|b - a| \leq \Delta$, where Δ specifies the required accuracy.

3.A.2 Double IBP

Exploiting the convexity and monotonicity of the first derivative of the objective function in (3.22), an effective procedure for locating the optimal solution of (3.16) is outlined with the following steps:

1. Compute ν_a and ν_b such that the solution of (3.16) belongs to $[\nu_a, \nu_b]$.

2. Find the critical point $(\nu_c, p_{l,c})$, if any, in each function of the first derivative of the objective function given in (3.22). Then use (3.23) to approximate the functions which have the critical point.

3. Apply IBP to locate the solution of (3.22) and of the approximate functions (found in Step 2) with ν_a and ν_b found in Step 1, i.e., $p_l(\nu_a)$ and $p_l(\nu_b)$, for $l = 1, \ldots, L$. If $p_l < 0$, then set $p_l = 0$.

4. Let $g(\nu) = \sum_{l=1}^{L} p_l(\nu) - P_R$. Calculate $g(\nu_a)$ and $g(\nu_b)$.

   - If $g(\nu_a)g(\nu_b) > 0$, there is no solution in $[\nu_a, \nu_b]$.

   - Otherwise, let $\nu_m = (\nu_a + \nu_b)/2$. Apply IBP to locate $p_l(\nu_m), l = 1, \ldots, L$. If $p_l < 0$, then set $p_l = 0$. Calculate $g(\nu_m)$. Reset $\nu_a = \nu_m$ if $g(\nu_a)g(\nu_m) > 0$ and reset $\nu_b = \nu_m$ if $g(\nu_a)g(\nu_m) < 0$. Repeat until $g(\nu_m) = 0$ or $|\nu_b - \nu_a| \leq \Delta$.

Note that for the optimization problem with the upper bound of the objective function, Step 2 is omitted since there is no critical point in the upper bound (3.25) nor in its first derivative (3.29).

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4. Beamforming in Non-Orthogonal Amplify-and-Forward Relay Networks

In the previous chapter, a wireless amplify-and-forward (AF) relay network model in which *orthogonal* transmissions are conducted between the relays and destination in the second phase has been considered. One major drawback of the orthogonal relaying scheme is that it requires more channel uses for each transmission (and therefore may not offer high bandwidth efficiency) compared to the *non-orthogonal* scheme. Based on the model examined in [1], the manuscript included in this chapter revisits the non-orthogonal AF relaying scheme with the second partial CSI assumptions considered in [1]. A number of interesting findings are discovered, especially when limited-rate feedback channels are taken into account.

Beamforming in Non-Orthogonal Amplify-and-Forward Relay Networks

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Abstract

This paper considers wireless amplify-and-forward relay networks in which the source communicates with the relays and destination in the first phase and the relays *simultaneously* forward signals to the destination in the second phase over uncorrelated Rayleigh fading channels. Examined is a scenario in which each relay only knows the perfect information of its source-relay channel while the destination knows the exact information of the relay-destination channels and the statistics of the source-relay channels. Based on a combiner developed at the destination, we propose an efficient beamforming scheme at the relays and develop its quantized version using Lloyd's algorithm to work with a limited-rate feedback channel. Simulation results show that the non-orthogonal relaying with the proposed beamforming scheme outperforms the orthogonal relaying with power allocation in terms of the ergodic capacity. In terms of the signal-to-noise ratio, the non-orthogonal scheme also becomes superior to the orthogonal scheme when the number of quantization regions increases.

Index terms

Wireless relay networks, non-orthogonal amplify-and-forward, beamforming, power allocation.

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4.1 Introduction

In recent years, designing transmission methods for wireless relay networks that can adapt to partial knowledge of channel state information (CSI) has gained a significant interest (see, e.g., [2–4]). This is because the trade-off between a potential performance improvement offered by a relay-assisted transmission and a large amount of feedback overhead has been well recognized to be an important issue. To reduce the amount of feedback information from the destination to the relays, a distributed suboptimal beamforming scheme is proposed in [5], whereas in [6, 7] relay selection scheme is recommended to be a good choice. Although the benefits in terms of implementation complexity and overhead information exchange have been demonstrated, the common limitation of the methods in [5–7] is that the instantaneous CSI of all involved channels in the system is required at the destination.

In our previous work [1], a wireless amplify-and-forward (AF) relay network model in which *orthogonal* transmissions are conducted between the relays and destination in the second phase has been considered. It is assumed that every relay only knows the instantaneous channel from the source to itself while the destination knows the instantaneous channels from the source and all the relays to itself, the first-order and second-order statistics of the channels from the source to all the relays. This practical assumption is also considered in [2] (referred to as Assumption III in [2]). When each relay is assigned an orthogonal channel, inter-relay interference is completely eliminated and the processing task of the destination becomes easier. One major drawback of the orthogonal relaying scheme is that it requires more channel uses for each transmission (and therefore may not offer high bandwidth efficiency) compared to the *non-orthogonal* scheme. However, developing an appropriate relaying scheme for the non-orthogonal case under such a partial CSI assumption has not been adequately investigated.

This paper extends the model examined in [1] to the non-orthogonal AF relaying scheme. Using the proposed signal processing approach, we cophase the signal received at each relay in order to prevent the destructive effects being propagated from

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the source-relay channels over the relay-destination channels. Under the assumption of uncorrelated Rayleigh fading channels, an *approximate* maximal-ratio combiner is employed at the destination, which allows us to derive a beamforming scheme for the relays based on an approximate averaged signal-to-noise ratio (SNR). Since the destination has to compute the optimal beamforming scheme and then sends this information back to all relays, vector quantization using Lloyd's algorithm [8] shall be implemented. With this scheme, the destination just broadcasts only some bits representing the index of the best beamforming vector in the codebook via a finite-rate feedback channel. A comparison in terms of the reliability (i.e., the SNR) and bandwidth efficiency (with the ergodic capacity [9]) between the orthogonal and non-orthogonal relaying schemes is carried out. The special case of relay selection is also examined and compared.

The rest of this paper is organized as follows. Section II summarizes the signal processing model proposed in [1] with some modifications for the *non-orthogonal* AF relaying scheme. Section III provides a beamforming scheme and its quantized version and discusses a relay selection scheme. A comparison of different relaying schemes in terms of the average SNR and ergodic capacity is presented in Section IV. Finally, some conclusions are given in Section V.

Notations: Italic, bold lower case and bold upper case letters denote scalars, vectors and matrices, respectively. The superscripts $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$ stand for transpose, Hermitian transpose, and statistical expectation operations, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a vector of complex Gaussian random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The notation $\mathbf{I}_L$ stands for an identity matrix of size $L \times L$ while $\mathrm{diag}(x_1, \ldots, x_L)$ is a diagonal matrix with the diagonal elements $x_1, \ldots, x_L$.

4.2   System Model

This section summarizes the signal model considered in Section IV of [1] and explains key differences when the non-orthogonal AF relaying scheme is employed.
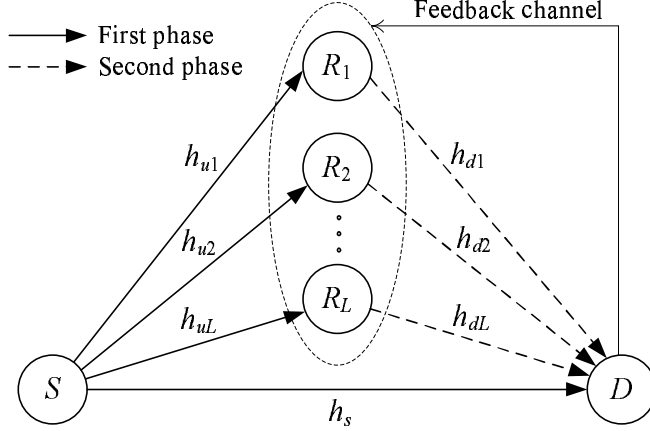
Figure 4.1 System model of the wireless relay network under consideration.

Illustrated in Fig. 4.1 is a wireless relay system in which the source terminal, S, communicates with the destination terminal, D, with the help of L relay terminals, $R_1, \ldots, R_L$. All terminals are equipped with one antenna and operate in a half-duplex mode. The transmission for every information symbol, s, happens in two phases. In the first phase, the source transmits the signal to the destination via the direct channel, $h_s \sim \mathcal{CN}(0, \Sigma^{(s)})$, and to the relays via the "uplink" (source-relay) channels, $\mathbf{h}_u = [h_{u1}, \cdots, h_{uL}]^T \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}^{(u)})$, where h_{ul} is the instantaneous channel coefficient between the source and the lth relay. In the second phase, the relays *simultaneously* forward their received signals to the destination via the "downlink" (relay-destination) channels $\mathbf{h}_d = [h_{d1}, \cdots, h_{dL}]^T \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}^{(d)})$. All channels are assumed to be uncorrelated Rayleigh fading (i.e., $\boldsymbol{\Sigma}^{(u)}$ and $\boldsymbol{\Sigma}^{(d)}$ are diagonal).

The signal received at the destination in this phase is

$$d_1 = h_s s + n_s, \tag{4.1}$$

where $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ represents additive white Gaussian noise (AWGN) with variance σ_s^2. Similarly, the signals received at the relays can be collectively written as

$$\mathbf{r} = [r_1, r_2, \cdots, r_L]^T = \mathbf{h}_u s + \mathbf{n}_u, \tag{4.2}$$

where $\mathbf{n}_u \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_L)$ accounts for AWGN with the same variance σ_u^2 at the relays.

Under the assumption that the lth relay only knows h_{ul} while the destination knows h_s, $\mathbf{h}_d$ and $\boldsymbol{\Sigma}^{(u)}$, appropriate signal processing operations need to be carried

out at both the relays and destination. First, since every relay cannot compute the beamforming scheme, the destination shall determine it and then sends the result back to the relays via some feedback channel[1]. Second, the random phase introduced by the uplink channel is corrected by setting the lth relay gain as

$$g_l = w_l e^{-j\theta_l} \left(P_S |h_{ul}|^2 + \sigma_u^2 \right)^{-1/2}, \qquad (4.3)$$

where $P_S = \mathbb{E}\{|s|^2\}$ is the average transmitted power of the source, θ_l is the phase of the uplink channel coefficient h_{ul}. With the above scaling factor, the transmitted power of the lth relay is limited to $|w_l|^2$. The received signal at the destination in the second phase can be expressed as[2]

$$
\begin{aligned}
d_2 &= \sum_{l=1}^{L} g_l r_l + n_d \\
&= \mathbf{w} \mathbf{H}_d \mathbf{F}_u \bar{\mathbf{h}}_u \cdot s + \mathbf{w} \mathbf{H}_d \mathbf{F}_u \bar{\mathbf{n}}_u + n_d,
\end{aligned}
\qquad (4.4)
$$

where $\mathbf{w} = [w_1, \ldots, w_L]$ is the beamforming vector used by the L relays, $\mathbf{H}_d = \text{diag}(h_{d1}, \cdots, h_{dL})$, $\mathbf{F}_u = \text{diag}\left((P_S|h_{u1}|^2 + \sigma_u^2)^{-1/2}, \ldots, (P_S|h_{uL}|^2 + \sigma_u^2)^{-1/2} \right)$, $\bar{\mathbf{h}}_u = [|h_{u1}|, \ldots, |h_{uL}|]^T$, $\bar{\mathbf{n}}_u = \left[n_{u1} e^{-j\theta_1}, \ldots, n_{uL} e^{-j\theta_L} \right]^T \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_L)$, $n_d \sim \mathcal{CN}(0, \sigma_d^2)$ is the AWGN experienced by the destination in the second phase.

The received signals at the destination in two phases can be represented as the following standard Gaussian vector form

$$\mathbf{d} = [d_1, d_2]^T = \bar{\mathbf{a}} \cdot s + \bar{\mathbf{n}}, \qquad (4.5)$$

where $\bar{\mathbf{a}} = \left[h_s, \mathbf{w} \mathbf{H}_d \mathbf{F}_u \bar{\mathbf{h}}_u \right]^T$, and $\bar{\mathbf{n}} = [n_s, \mathbf{w} \mathbf{H}_d \mathbf{F}_u \bar{\mathbf{n}}_u + n_d]^T$.

Lastly, since the instantaneous amplitudes of the uplinks $\mathbf{h}_u$ are not known at the destination, an *approximate* MRC filter that combines the signals received after two

---

[1]The feedback information may be in the form of analog feedback, i.e., the true beamforming vector, or in the form of finite-rate feedback, i.e., a quantized version of the true beamforming vector.

[2]In [1], the destination receives L orthogonal signals from L relays in the second phase. Therefore, a power allocation scheme at the relays is sufficient to help the destination constructively combines those signals. However, in the non-orthogonal case, power allocation alone is not the optimal scheme.

phases is given as [1]:

$$\mathbf{f} = \widetilde{\mathbf{R}}^{-1}\mathbb{E}_{\mathbf{h}_u}\left\{\bar{\mathbf{a}}\right\},\tag{4.6}$$

where $\widetilde{\mathbf{R}}$ is the covariance matrix of the noise vector $\bar{\mathbf{n}}$. It is computed as follows:

$$\widetilde{\mathbf{R}} = \mathbb{E}\left\{\bar{\mathbf{n}}\bar{\mathbf{n}}^H\right\} = \text{diag}\left(\sigma_s^2, \sigma_u^2\mathbf{w}\mathbf{H}_d\mathbf{T}_u\mathbf{H}_d^H\mathbf{w}^H + \sigma_d^2\right),\tag{4.7}$$

where

$$
\begin{aligned}
\mathbf{T}_u &= \frac{1}{\sigma_u^2}\mathbb{E}_{\mathbf{h}_u}\left\{\mathbf{F}_u\bar{\mathbf{n}}_u\bar{\mathbf{n}}_u^H\mathbf{F}_u^H\right\} \\
&= \mathbb{E}_{\mathbf{h}_u}\left\{\mathbf{F}_u\mathbf{F}_u^H\right\} = \text{diag}\left(T_{u1},\ldots,T_{uL}\right),
\end{aligned}\tag{4.8}
$$

and

$$T_{ul} = \mathbb{E}_{h_{ul}}\left\{F_{ul}^2\right\} = \frac{2\xi_l}{\sigma_u^2}\exp(2\xi_l)E_1(2\xi_l),\tag{4.9}$$

where $\xi_l = \sigma_u^2/(2P_S\Sigma_{ll}^{(u)})$ and $E_1(x) = \int_x^{+\infty}\frac{e^{-t}}{t}dt$, $x > 0$, is the exponential integral.

Under uncorrelated Rayleigh fading uplink channels, the term $\tilde{\mathbf{a}} = \mathbb{E}_{\mathbf{h}_u}\left\{\bar{\mathbf{a}}\right\}$ in (4.6) is found as

$$\tilde{\mathbf{a}} = \left[h_s, \mathbf{w}\mathbf{H}_d\mathbb{E}_{\mathbf{h}_u}\left\{\mathbf{F}_u\bar{\mathbf{h}}_u\right\}\right]^T = \left[h_s, \mathbf{w}\mathbf{H}_d\mathbf{q}_u\right]^T,\tag{4.10}$$

where $\mathbf{q}_u = \left[q_{u1},\ldots,q_{uL}\right]^T$, and

$$
\begin{aligned}
q_{ul} &= \mathbb{E}_{h_{ul}}\left\{F_{ul}|h_{ul}|\right\} \\
&= \frac{1}{\sqrt{P_S}}\xi_l\exp\left(\xi_l\right)\left[K_1\left(\xi_l\right) - K_0\left(\xi_l\right)\right],
\end{aligned}\tag{4.11}
$$

with ξ_l defined in (4.9) and $K_\alpha(x)$ is the αth-order modified Bessel function of the second kind.

Lemma 3. *With the approximate MRC filter given in (4.6), the estimate of the data symbol s at the filter's output is $\hat{s} = \mathbf{f}^H\mathbf{d}$ and the resulting average SNR can be shown to be:*

$$
\begin{aligned}
\overline{\text{SNR}}_{\text{non}} &\approx P_S\tilde{\mathbf{a}}^H\widetilde{\mathbf{R}}^{-1}\tilde{\mathbf{a}} \\
&= P_S\left[\frac{|h_s|^2}{\sigma_s^2} + \frac{\mathbf{w}\mathbf{H}_d\mathbf{q}_u\mathbf{q}_u^H\mathbf{H}_d^H\mathbf{w}^H}{\sigma_u^2\mathbf{w}\mathbf{H}_d\mathbf{T}_u\mathbf{H}_d^H\mathbf{w}^H + \sigma_d^2}\right].
\end{aligned}\tag{4.12}
$$

The proof can be carried out similarly as in Section IV of [1] and is omitted here for brevity. It is noted that, different from the orthogonal relaying scheme with power allocation in [1], the average SNR obtained in the non-orthogonal relaying scheme has a special form. This form is exploited in the next section to compute an efficient beamforming scheme.

4.3 Relay Beamforming Schemes

4.3.1 Proposed Relay Beamforming

Based on the average SNR expression given in (4.12), a general beamforming scheme can be numerically obtained under the total and individual power constraints at the relays via second order cone programming [10]. For the case that only the total transmit power at all the relays is limited, a closed-form beamforming scheme can be derived by making use of the following lemma (the proof follows directly by applying the Rayleigh-Ritz theorem [11]).

Lemma 4. *(from [2]) Let* $\mathbf{A}$ *be an* $n \times n$ *Hermitian matrix, and* $\mathbf{B}$ *be an* $n \times n$ *positive definite Hermitian. Furthermore, let* $\mathbf{B}$ *be decomposed as* $\mathbf{B} = \mathbf{L}\mathbf{L}^H$. *Then,*

$$\frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{B} \mathbf{x}} \leq \lambda_{\max} \quad \text{for all } \mathbf{x} \in \mathbb{C}^n, \tag{4.13}$$

where $\lambda_{\max}$ *is the largest eigenvalue of* $\mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^H)^{-1}$. *The equality holds if* $\mathbf{x} = c(\mathbf{L}^H)^{-1}\mathbf{u}_{\max}$, *where* c *is any non-zero constant and* $\mathbf{u}_{\max}$ *is the eigenvector of* $\mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^H)^{-1}$ *corresponding to* $\lambda_{\max}$.

With the total relay power constraint $\mathbf{w}\mathbf{w}^H = P_R$, the average SNR can be rewritten as

$$\overline{\text{SNR}}_{\text{non}} \approx P_S \left[\frac{|h_s|^2}{\sigma_s^2} + \frac{\mathbf{w}\mathbf{H}_d \mathbf{q}_u \mathbf{q}_u^H \mathbf{H}_d^H \mathbf{w}^H}{\mathbf{w}\left(\sigma_u^2 \mathbf{H}_d \mathbf{T}_u \mathbf{H}_d^H + \frac{\sigma_d^2}{P_R}\mathbf{I}_L\right)\mathbf{w}^H} \right]. \tag{4.14}$$

Since the second term of (4.14) is exactly the special case (with equality) consid-

ered in Lemma 4, one can readily find the following beamforming vector[3] to maximize the average signal-to-noise ratio $\overline{\text{SNR}}_{\text{non}}$:

$$\mathbf{w}_{\text{opt}} = c\tilde{\mathbf{w}} = c\mathbf{q}_u^H \mathbf{H}_d^H \mathbf{V}, \tag{4.15}$$

where $\mathbf{V} = \left(\sigma_u^2 \mathbf{H}_d \mathbf{T}_u \mathbf{H}_d^H + \frac{\sigma_d^2}{P_R}\mathbf{I}_L\right)^{-1}$ and $c = \sqrt{P_R/(\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H)}$. It follows that

$$\overline{\text{SNR}}_{\text{non}} \approx \frac{P_S}{\sigma_s^2}|h_s|^2 + \sum_{l=1}^{L} \frac{\frac{P_S}{\sigma_u^2}q_{ul}^2\frac{P_R}{\sigma_d^2}|h_{dl}|^2}{T_{ul}\frac{P_R}{\sigma_d^2}|h_{dl}|^2 + \frac{1}{\sigma_u^2}}. \tag{4.16}$$

With the average SNR given in (4.16), an interesting SNR comparison between the non-orthogonal and orthogonal relaying schemes is stated in the following corollary.

Corollary 1. *With the analog feedback, the average SNR achieved by the non-orthogonal relaying transmission with optimal beamforming scheme is always higher than the average SNR achieved by orthogonal relaying transmission with optimal power allocation.*

The proof of Corollary 1 is given in Appendix 4.A. It is worth mentioning that the above SNR comparison is also true for the case with perfect and full CSI assumption, in which the instantaneous SNR's of both schemes have the same form given as Eq. (10) in [2]. For the system model with a finite-rate feedback, further analysis and comparison are carried out in Section 4.4.

Given the beamforming scheme (4.15), the ergodic capacity of the transmission model under consideration can be calculated as

$$\mathcal{I} = \frac{1}{2}\mathbb{E}_{h_s,\mathbf{h}_u,\mathbf{h}_d}\left\{\log_2\left(1 + \frac{\mathcal{S}}{\mathcal{N}}\right)\right\} \tag{4.17}$$

where the received signal power is

$$\mathcal{S} = P_S\left|\frac{|h_s|^2}{\sigma_s^2} + \mathbf{q}_u^H \mathbf{H}_d^H \mathbf{V}\mathbf{H}_d \mathbf{F}_u \bar{\mathbf{h}}_u\right|^2 \tag{4.18}$$

---

[3]It is worth noting that a beamforming scheme usually includes two components: the phase compensation and the power allocation (see, e.g., [2, 10, 12]). The beamforming scheme (4.15) is also in this form.

and the averaged noise power is

$$\mathcal{N} = \frac{|h_s|^2}{\sigma_s^2} + \mathbf{q}_u^H \mathbf{H}_d^H \mathbf{V}^2 \left(\sigma_u^2 \mathbf{H}_d \mathbf{F}_u \mathbf{F}_u^H \mathbf{H}_d^H + \frac{\sigma_d^2}{P_R} \mathbf{I}_L \right) \mathbf{H}_d \mathbf{q}_u. \tag{4.19}$$

Obtaining a closed-form expression of the ergodic capacity defined in (4.17) appears to be very difficult. Nevertheless, a tight upper-bound of the ergodic capacity can be found as follows. Since the function $\log_2(1 + x)$ is concave in $x > 0$, applying Jensen's inequality to (4.17) yields

$$\begin{aligned}
\mathcal{I} &\leq \frac{1}{2} \log_2 \left(1 + \mathbb{E}_{h_s, \mathbf{h}_u, \mathbf{h}_d} \left\{ \frac{\mathcal{S}}{\mathcal{N}} \right\} \right) \\
&\approx \frac{1}{2} \log_2 \left(1 + \widehat{\mathrm{SNR}} \right),
\end{aligned} \tag{4.20}$$

where

$$\begin{aligned}
\widehat{\mathrm{SNR}} &= \mathbb{E}_{h_s, \mathbf{h}_d} \left\{ \overline{\mathrm{SNR}}_{\mathrm{non}} \right\} \\
&\approx \frac{P_S}{\sigma_s^2} \Sigma^{(s)} + \sum_{l=1}^{L} \frac{P_S q_{ul}^2}{\sigma_u^2 T_{ul}} \left[1 - \chi_l \exp(\chi_l) E_1(\chi_l) \right],
\end{aligned} \tag{4.21}$$

and $\chi_l = \sigma_d^2 / (\sigma_u^2 P_R T_{ul} \Sigma_{ll}^{(d)})$. Simulation results given in Section 4.4 will validate the tightness of the above upper bound.

4.3.2 Quantized Relay Beamforming

The beamforming scheme presented in (4.15) is computed at the destination and sent back to the relays via some feedback channel. For practical implementation we sketch in this subsection the vector quantization procedure based on Lloyd's algorithm to design the quantized relay beamforming codebook. This procedure can also be applied for the power allocation codebook design in the orthogonal relaying scheme (see Appendix 4.B). Let

$$\bar{\gamma}(\mathbf{w}|\mathbf{h}_d) = \frac{\mathbf{w} \mathbf{H}_d \mathbf{q}_u \mathbf{q}_u^H \mathbf{H}_d^H \mathbf{w}^H}{\mathbf{w} \left(\sigma_u^2 \mathbf{H}_d \mathbf{T}_u \mathbf{H}_d^H + \frac{\sigma_d^2}{P_R} \mathbf{I}_L \right) \mathbf{w}^H}. \tag{4.22}$$

Given a codebook of beamforming vectors

$$\mathcal{W} = \{ \mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_Z, \} \in \mathbb{C}^L, \tag{4.23}$$

where Z is the number of quantization regions and $\lceil \log_2 Z \rceil$, where $\lceil x \rceil$ is the smallest integer that is greater or equal to x, is the number of feedback bits, the average distortion with respect to the optimal beamforming vector can be defined as

$$\delta(\mathcal{W}) = \mathbb{E}_{\mathbf{h}_d} \left\{ \bar{\gamma}(\mathbf{w}_{\text{opt}}|\mathbf{h}_d) - \max_{1 \le z \le Z} \left\{ \bar{\gamma}(\mathbf{w}_z|\mathbf{h}_d) \right\} \right\}. \tag{4.24}$$

The codebook design using Lloyd's algorithm vector quantization can be summarized as follows [8]:

1. Randomly generate a codebook $\mathcal{W}^{(0)} = \{\mathbf{w}_1^{(0)}, \mathbf{w}_2^{(0)}, \ldots, \mathbf{w}_Z^{(0)}, \}$. Set $t = 1$.

2. Generate a set of N test channel realization vectors $\{\mathbf{h}_d^{(1)}, \mathbf{h}_d^{(2)}, \ldots, \mathbf{h}_d^{(N)}\}$. Divide this set into Z quantization regions with the zth region defined as

$$\mathcal{C}_z = \left\{ \mathbf{h}_d^{(n)} | \bar{\gamma}(\mathbf{w}_i^{(t-1)}|\mathbf{h}_d^{(n)}) \le \bar{\gamma}(\mathbf{w}_z^{(t-1)}|\mathbf{h}_d^{(n)}) \right\}, \tag{4.25}$$

for all $i \ne z, 1 \le n \le N$.

3. Construct a new codebook $\mathcal{W}^{(t)}$ with the zth codeword given as

$$\mathbf{w}_z^{(t)} = \arg\max_{\mathbf{w}} \mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \bar{\gamma}(\mathbf{w}|\mathbf{h}_d^{(n)}) \right\}, \tag{4.26}$$

where

$$\begin{aligned}
\mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \bar{\gamma}(\mathbf{w}|\mathbf{h}_d^{(n)}) \right\} &= \mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \frac{\mathbf{w}\mathbf{H}_d^{(n)}\mathbf{q}_u\mathbf{q}_u^H(\mathbf{H}_d^{(n)})^H\mathbf{w}^H}{\mathbf{w}\left(\sigma_u^2\mathbf{H}_d^{(n)}\mathbf{T}_u(\mathbf{H}_d^{(n)})^H + \frac{\sigma_d^2}{P_R}\mathbf{I}_L\right)\mathbf{w}^H} \right\} \\
&\approx \frac{\mathbf{w}\mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z}\left\{\mathbf{H}_d^{(n)}\mathbf{q}_u\mathbf{q}_u^H(\mathbf{H}_d^{(n)})^H\right\}\mathbf{w}^H}{\mathbf{w}\left(\sigma_u^2\mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z}\left\{\mathbf{H}_d^{(n)}\mathbf{T}_u(\mathbf{H}_d^{(n)})^H\right\} + \frac{\sigma_d^2}{P_R}\mathbf{I}_L\right)\mathbf{w}^H},
\end{aligned}$$

and $\mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \bar{\gamma}(\mathbf{w}|\mathbf{h}_d^{(n)}) \right\}$ is approximated by the first term of its Taylor series [2].[4]

---

[4]Note that if $|\mathcal{C}_z| = 0$ then $\mathbf{w}$ is randomly regenerated. If $|\mathcal{C}_z| = 1$, use the special case in Lemma 4 to find the closed-form solution, otherwise use the general solution given in Lemma 4. Also using a higher-order Taylor series approximation for $\mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \bar{\gamma}(\mathbf{w}|\mathbf{h}_d^{(n)}) \right\}$ might be more desirable, but appears to be complicated.

4. If $\delta(\mathcal{W}^{(t-1)}) - \delta(\mathcal{W}^{(t)}) > \epsilon$, set $t \leftarrow t + 1$ and go back to Step 2. Otherwise, set $\mathcal{W} = \mathcal{W}^{(t)}$ and STOP.

Depending on the initial codebook in Step 1 and the test set of channel vectors generated in Step 2, Lloyd's iterative procedure can result in local maxima. Therefore, to obtain a more reliable solution, a large test set is preferred. Although the off-line design of the codebook appears to be computationally intensive, the codeword selection process is really simple. Since the destination and all relays know the codebook, the destination only needs to broadcast the index of the optimal codeword to all the relays. This significantly reduces the number of feedback bits compared to the optimal beamforming scheme with *analog* feedback.

4.3.3 Relay Selection

Relay selection can be considered as a special scheme of both beamforming and power allocation. When full CSI is available at the destination, relay selection has been proven to be an efficient scheme with a simple feedback strategy [13]. In our scenario, the selection algorithm can only be implemented based on the instantaneous downlink channel coefficients and the statistics of the uplink channels. From (4.14), it can be inferred that the $l^\star$th relay is selected to forward the signal in the second phase if

$$l^\star = \arg\max_{l \in [1,L]} \frac{P_R |h_{dl}|^2 q_{ul}^2}{\sigma_u^2 P_R |h_{dl}|^2 T_{ul} + \sigma_d^2}, \tag{4.27}$$

and the feedback channel needs $\lceil \log_2 L \rceil$ bits to inform all the relays via a broadcast channel.

4.4 Numerical Results and Discussion

All the uplink and downlink channels are assumed to be under uncorrelated Rayleigh fading, but with different variances, namely $\Sigma_{11}^{(u)} = -10$ dB, $\Sigma_{22}^{(u)} = 0$ dB, $\Sigma_{33}^{(u)} = 10$ dB, $\Sigma_{44}^{(u)} = 20$ dB, $\Sigma_{11}^{(d)} = 20$ dB, $\Sigma_{22}^{(d)} = 10$ dB, $\Sigma_{33}^{(d)} = 0$ dB, $\Sigma_{44}^{(d)} = -10$ dB. Similarly, the direct channel is assumed to be $h_s \sim \mathcal{CN}(0, 1)$. Binary phase-shift
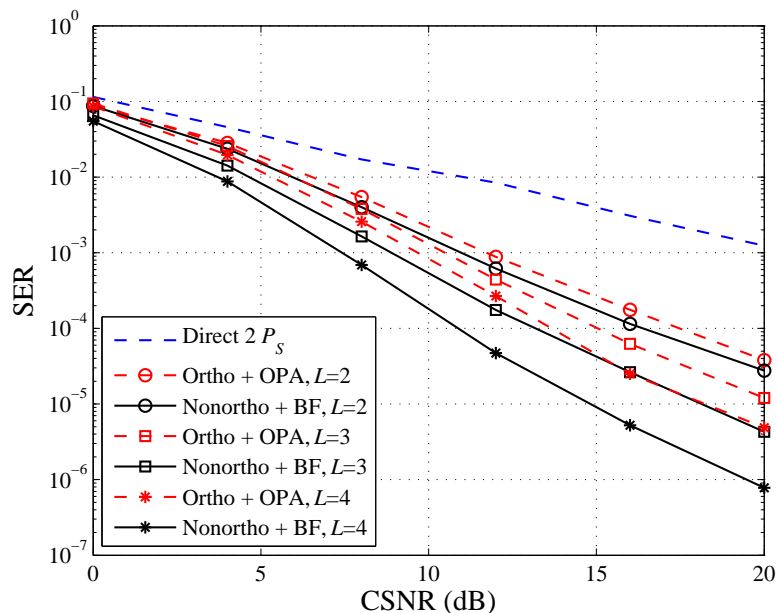
Figure 4.2 SER of the non-orthogonal relaying model with optimal beamforming scheme and the orthogonal relaying scheme with power allocation scheme in [1]. The number of relays $L = 2, 3, 4$.

keying (BPSK) modulation is used at the source. Assume $\sigma_s^2 = \sigma_u^2 = \sigma_d^2 = 1$, the average channel signal-to-noise ratio (CSNR) is simply defined as $P_S/\sigma_s^2 = P_S/\sigma_u^2 = P_S/\sigma_d^2 = P_S$. The total transmit power at the relays is chosen to be $P_R = P_S$.

Fig. 4.2 compares the symbol error rate (SER) performances achieved with the non-orthogonal relaying with optimal beamforming scheme and the orthogonal relaying with power allocation scheme in [1]. The number of relays are $L = 2, 3, 4$. It can be seen from the figure that the performance improvement (from the "*equivalent*" coding gain) with non-orthogonal relaying is significant, about 3 dB/relay at high SNR. However, since the destination does not know full CSI, the diversity order does not increase with the number of relays. This also agrees with the results of the orthogonal case under Assumption B in [1] and of the non-orthogonal case under Assumptions II and III in [2] where the diversity order is proved to be 2 only. As a reference, the SER obtained by direct transmission with a total transmit power of $P = P_S + P_R$ is also shown in Fig. 4.2. Even with such a large transmit power, the performance

of the direct transmission is significantly worse than that of the relay-assisted transmission. It should be pointed out that, in general, the performance improvement of the relay-assisted transmission strongly depends on the channel conditions among the source, relays and destination. The performance curves shown in Fig. 4.2 should be interpreted for the specific channel conditions under consideration.

Next, we compare the non-orthogonal relaying scheme under consideration with the orthogonal relaying scheme considered in [1] under the same CSI assumption. A broadcasting feedback channel accommodating 2 or 6 feedback bits is assumed. To adapt with these limited feedback rates, the quantized version of the beamforming vector (given in Section 4.3.2) and of the power allocation scheme used in [1] (given in Appendix 4.B) are employed. In the procedures, the number of test channel vectors is $N = 20,000$ and the termination criterion is chosen as $\epsilon = 10^{-4}\delta(\mathcal{W}^{(t-1)})$.

As predicted from Corollary 1, the simulation results in Fig. 4.3 confirm that when infinite-rate (i.e., analog) feedback is available, the beamforming scheme in the non-orthogonal case is superior in terms of the average SNR as compared to the power allocation scheme in the orthogonal case. This is due to the fact that the phases of both uplink and downlink channels can be perfectly matched at each relay, resulting in *a constructive superposition (basically addition of signal amplitudes) of all the co-phased signals at the destination.* Consequently, the energy of the combined signal is higher than that in the orthogonal case where *energies of different signals are accumulated, not their amplitudes.* When no feedback is available, it is expected that the non-orthogonal relaying scheme is inferior to the orthogonal scheme. For the case with only 2 feedback bits, performance of the orthogonal scheme is still better. This also reflects the sensitivity of the non-orthogonal relaying scheme to the quality of the phase estimates available at the relays. However, the gap is very small and can be practically removed and even reversed with more feedback bits (e.g., with 6 feedback bits or more in our simulation setup). It is also noted that under the assumed channel model, the performance of the power allocation scheme does not improve much as the number of feedback bits increases.
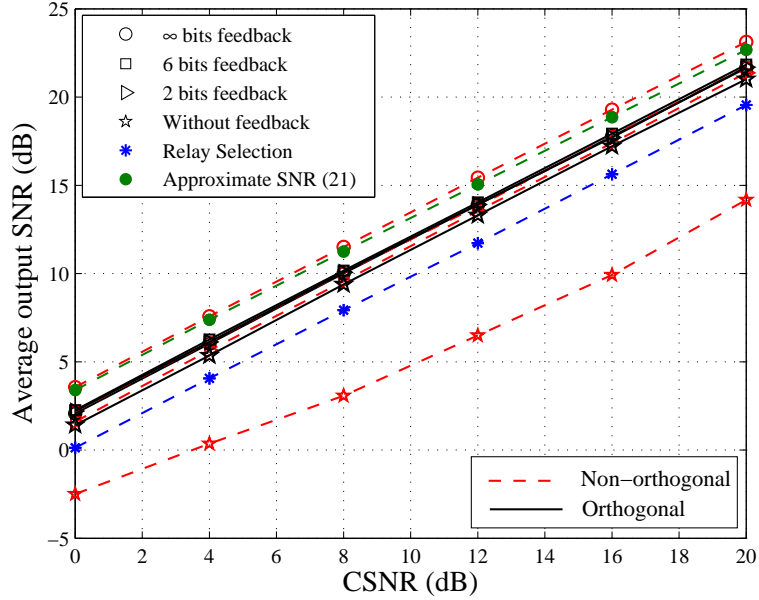
Figure 4.3   Average signal-to-noise ratios of the non-orthogonal relaying scheme with beamforming and the orthogonal relaying scheme with power allocation scheme in [1]. The number of relays $L = 4$.

Since the average SNRs of the two relaying schemes are not much different, their bandwidth efficiencies mostly depend on the number of channel uses (which is also the number of relays) required in the second phase. As shown in Fig. 4.4, the ergodic capacity of the non-orthogonal scheme is about 2.5 times larger than that of the orthogonal scheme at any SNR region (i.e., approximately $(L + 1)/2$ times for the cases with 2 or 6 feedback bits). Without feedback information, the non-orthogonal scheme performs a little bit worse at low and medium SNR regions. For a benchmark comparison, the capacity of the model considered in [2] under the assumption of full CSI available at the destination is also plotted. As expected, a decrease in capacity can be clearly observed when less CSI is available at both the transmitters and receiver.

Figs. 4.3 and 4.4 also plot curves (marked with filled circles) to validate the approximate $\widehat{SNR}$ given in (4.21) and the upper bound of the ergodic capacity given in (4.20), respectively. It is observed that both $\widehat{SNR}$ and the upper bound of the ergodic capacity are very close to the actual values. Other performance curves in Figs. 4.3
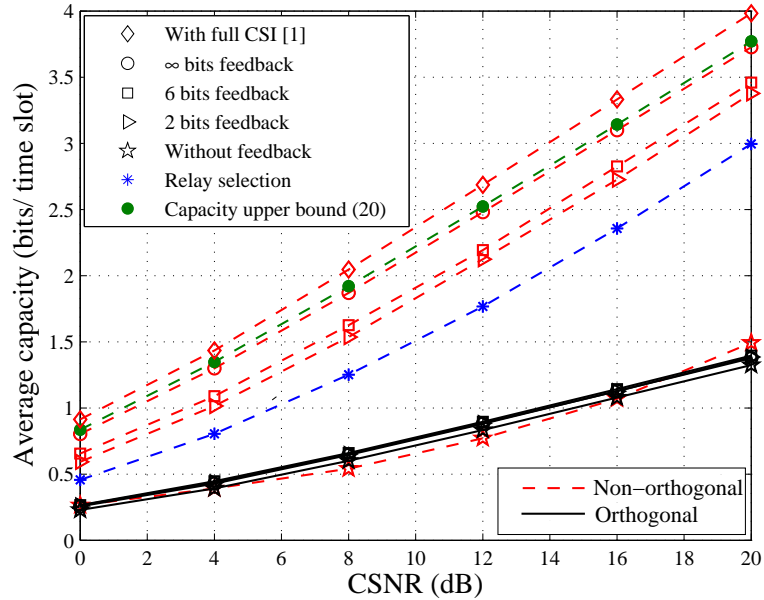
Figure 4.4 Ergodic capacity of the non-orthogonal relaying scheme with beamforming and the orthogonal relaying scheme with power allocation scheme in [1]. The number of relays $L = 4$.

and 4.4 also show that the relay selection scheme in (4.27) does not perform as well as the beamforming scheme with 2-bit feedback channel. However, it is superior to the power allocation scheme (even with analog feedback case). Thus, relay selection is a suitable solution when one needs to balance between the implementation complexity and performance gain.

4.5 Conclusions

Extended from our previously proposed orthogonal relaying model in [1], the problem of joint design of signal processing at the relays and destination has been considered for wireless *non-orthogonal* relay networks. With a highly accurate approximation of the signal-to-noise ratio, a closed-form beamforming scheme for the relays has been proposed. Although no diversity gain can be obtained as the number of relays increases, the error performance of the system can still be significantly improved. For a practical implementation with limited feedback from the destination,

the quantized beamforming scheme is also developed. Simulation results have shown that the proposed non-orthogonal relaying with beamforming scheme can outperform the orthogonal relaying with power allocation in terms of the signal-to-noise ratio, and consequently, the error performance as well as the bandwidth efficiency.

4.A   Proof of Corollary 1

Using the same notations in [1], let $\mathbf{W} = \mathrm{diag}(w_1, w_2, \ldots, w_L)$, $\tilde{\mathbf{a}} = [h_s, (\mathbf{W}\mathbf{H}_d\mathbf{q}_u)^T]^T$, and $\widetilde{\mathbf{R}} = \mathrm{diag}(\sigma_s^2, \sigma_u^2\mathbf{W}\mathbf{H}_d\mathbf{T}_u\mathbf{H}_d^H\mathbf{W}^H + \sigma_d^2\mathbf{I}_L)$. The average SNR for the orthogonal relaying transmission given in Eq. (41) in [1] can be rewritten as

$$
\begin{aligned}
\overline{\mathrm{SNR}}_{\mathrm{ortho}} &\approx P_S\tilde{\mathbf{a}}^H\widetilde{\mathbf{R}}^{-1}\tilde{\mathbf{a}} \\
&= P_S\left[\frac{|h_s|^2}{\sigma_s^2} + \mathbf{q}_u^H\mathbf{H}_d^H\mathbf{W}^H\left(\sigma_u^2\mathbf{W}\mathbf{H}_d\mathbf{T}_u\mathbf{H}_d^H\mathbf{W}^H + \sigma_d^2\mathbf{I}_L\right)^{-1}\mathbf{W}\mathbf{H}_d\mathbf{q}_u\right] \\
&= \frac{P_S}{\sigma_s^2}|h_s|^2 + \sum_{l=1}^{L}\frac{\frac{P_S}{\sigma_u^2}q_{ul}^2\frac{|w_l|^2}{\sigma_d^2}|h_{dl}|^2}{T_{ul}\frac{|w_l|^2}{\sigma_d^2}|h_{dl}|^2 + \frac{1}{\sigma_u^2}}.
\end{aligned}
\tag{4.28}
$$

For the non-orthogonal relaying transmission with our proposed beamforming scheme, the average SNR is given in (4.16). Both $\overline{\mathrm{SNR}}_{\mathrm{non}}$ given in (4.16) and $\overline{\mathrm{SNR}}_{\mathrm{ortho}}$ given in (4.28) have the same form, which are increasing functions of P_R and $|w_l|^2$, respectively. Since $\sum_{l=1}^{L}|w_l|^2 = P_R$, i.e., $|w_l|^2 \leq P_R \,\forall l$, it is obvious that substituting $|w_l|^2$ in (4.28) by P_R leads to $\overline{\mathrm{SNR}}_{\mathrm{ortho}} \leq \overline{\mathrm{SNR}}_{\mathrm{non}}$. The equality holds when $L = 1$.

4.B   Lloyd Algorithm for PA Codebook Design

With a power allocation codebook $\mathcal{P} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_Z\} \in \mathbb{R}_+^L$ where $\mathbf{p}_z = [p_{z,1}, p_{z,2}, \ldots, p_{z,L}]$, the procedure to generate the quantized version of the power allocation scheme proposed in [1] is similar to the one given in Section 4.3.2 except a

difference in Step 3 where a new codebook $\mathcal{P}^{(t)}$ with the zth codeword is found as

$$\mathbf{p}_z^{(t)} = \arg\min_{\mathbf{p}} \mathbb{E}_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \left\{ \bar{\eta}(\mathbf{p}|\mathbf{h}_d^{(n)}) \right\} \tag{4.29}$$

$$= \arg\min_{\mathbf{p}} \sum_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \sum_{l=1}^{L} \frac{q_{ul}^2/T_{ul}}{p_l T_{ul} |h_{dl}^{(n)}|^2 + \frac{\sigma_d^2}{\sigma_u^2}}$$

$$\approx \arg\min_{\mathbf{p}} \sum_{l=1}^{L} \frac{q_{ul}^2/T_{ul}}{p_l T_{ul} \frac{1}{|\mathcal{C}_z|} \sum_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} |h_{dl}^{(n)}|^2 + \frac{\sigma_d^2}{\sigma_u^2}},$$

where $|\mathcal{C}_z|$ is the cardinality of the set $\mathcal{C}_z$. Since each quantization region consists of *similar* channel vectors $\{\mathbf{h}_d^{(n)}\}$, another method to calculate the new codeword for each region is to find $\mathbf{p}_z^{(t)}$ for each channel vector $\mathbf{h}_d^{(n)}$, take the sample mean $\mathbf{p}_z^{(t)} = \frac{1}{|\mathcal{C}_z|} \sum_{\mathbf{h}_d^{(n)} \in \mathcal{C}_z} \mathbf{p}_z^{(t)}(\mathbf{h}_d^{(n)})$ and normalize the total power to P_R.

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5. Power Allocation in MMSE Relaying over Frequency-Selective Rayleigh Fading Channels

In Chapter 3 and Chapter 4, wireless relay networks with a single source are discussed. The manuscript included in this chapter considers a relay network with multiple sources and one relay. Since the transmissions from multiple sources to the relay and destination can be considered as the uplink channels in cellular networks, a new multiple access technique which is more suitable to the uplink transmissions is adopted. A framework for signal transmission is developed with two optimal power allocation (PA) schemes at the relay.

As mentioned in Section 2.2.4, it is of interest to maintain the fairness among all sources in the network with a limited transmit power at the relay. As such, the objectives of the PA schemes are to minimize the relay transmit power while maintaining the same signal-to-interference-plus-noise ratio (SINR) (or equivalently, the capacity) for all sources and to maximize the minimum SINR of the sources given a transmit power level at the relay. By visualizing the relationship between the SINR of the sources and the transmit power at the relay, several interesting observations are discussed, which motives the two proposed efficient algorithms.

Power Allocation in MMSE Relaying over Frequency-Selective Rayleigh Fading Channels

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Abstract

This paper develops an amplify-and-forward relaying scheme for multiuser wireless cooperative networks under frequency-selective block-fading. Single-carrier frequency division multiple-access with frequency-domain equalization technique is employed at both the relay and destination to combat the inter-block and inter-symbol interference caused by multipath propagation. With the assumption that the full channel state information (CSI) is available at the destination, the relay only knows the uplink channels while no CSI is available at the sources, two power allocation schemes are developed for the relay: (i) to minimize the total transmit power at the relay while maintaining the signal-to-interference-plus-noise ratio (SINR) for each user at the destination above a certain level, and (ii) to maximize the worst SINR among all the users subject to a constraint on total relay power. In the first problem, it is shown that SINR adaptation is needed not only to guarantee a feasible solution but also to significantly reduce the transmit power at the relay for certain channel conditions. In the second problem, a flexible multi-level water-filling scheme is developed, which can be easily modified to adapt to the different channel conditions as well as different quality-of-service provision strategies.

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Wireless relay networks, amplify-and-forward relaying, single-carrier FDMA, frequency-domain equalizer, power allocation, convex programming.

5.1 Introduction

To deal with frequency-selective fading experienced in high speed data transmission in broadband cooperative wireless networks, orthogonal frequency division multiplexing (OFDM) and its variant, orthogonal frequency division multiple-access (OFDMA), have been employed (see e.g., [1, 2]). Although OFDM/OFDMA offers many advantages in downlink transmission (i.e., from a base station to mobile stations), their drawbacks such as high peak-to-average power ratio (PAPR), sensitive to timing and frequency offsets and spectral null effect make it less attractive for uplink communications. Single-carrier frequency division multiple access (SC-FDMA) is an alternative that offers similar advantages as OFDMA, has been shown to be more suitable for uplink transmissions due to its low PAPR [3, 4]. In SC-FDMA block-based transmission, inter-block interference is also eliminated by applying cyclic prefix (CP) while inter-symbol interference within a block can be effectively mitigated using a simple one-tap frequency-domain equalizer.

Even when SC-FDMA is applied, due to the limitation on transmit power of mobile terminals, the terminals at the edge of a cell may still experience bad channel conditions. Under such a circumstance, relay-assisted transmission can be considered as a promising solution (see e.g., [5]). Studied in this paper is signal transmission over a multiuser cooperative network in which each user transmits its data to a destination (e.g., a base station) over a number of orthogonal subcarriers supported by one resource-constrained relay station working in amplify-and-forward (AF) mode. To combat the inter-block and inter-symbol interference caused by multipath propagation in frequency-selective block-fading channels, SC-FDMA with frequency-domain equalization (FDE) is employed at both the relay and destination. After receiving

the signals from all sources in the first phase, the relay employs a frequency-domain equalizer to compensate the distortion for the signals before forwarding them to the destination in the second phase of transmission.

An important issue in the multiuser cooperative network described above is how to effectively exploit the restricted resources provided by the relay. Together with bandwidth allocation techniques, power control has been widely studied with many transmission models. The general idea is how to jointly allocate the optimal power levels among some subcarriers and/or used by some transmitters in order to improve the quality of the received signals or to save the transmit power consumed by the whole network. In wireless relay network using OFDM/OFDMA technique, power allocation can be considered independently (see e.g., [6]) or jointly with bandwidth allocation [7, 8]. Although the exact solution varies and depends on each specific optimization problem, the optimal power allocation schemes usually follow a common structure of "water-filling" [9].

In [10], a heuristic proportional fairness scheduling in conventional uplink SC-FDMA systems has been proposed with an equal power allocation in order to maintain the low PAPR characteristics of the SC-FDMA. To the best of our knowledge, power allocation in a SC-FDMA cooperative wireless relay network as considered in this paper has not been studied yet. With the different signal processing performed in SC-FDMA, it is nontrivial to apply power allocation schemes proposed for general OFDMA to SC-FDMA systems. The major difference between OFDMA and SC-FDMA lies in how the subcarriers are related in the two systems. The inter-dependence of the subcarriers in SC-FDMA gives rise to the residual ISI and the harmonic mean form of the SINR, which makes the power allocation in SC-FDMA systems behave in a vastly different way as compared to the power allocation in OFDMA. This paper obtains the power allocation solutions for the following two problems: (i) minimizing the required transmit power at the relay subject to the constraint that the signal-to-interference-plus-noise ratio (SINR) of each user at the destination is above a predefined threshold, and (ii) maximizing the minimum SINR

among all the sources subject to a constraint on total relay power.

In the first problem, since all the sources transmit over orthogonal subcarriers, the optimal power allocation for each source is found to be a standard water-filling scheme. However, in contrast to the "*always-feasible*" solution when dealing with the power allocation in OFDMA, an adaptation process on the achievable SINR of the received signals at the destination is required to guarantee a feasible solution in SC-FDMA. In the second problem, based on an efficient algorithm proposed in [11] to solve a similar min-max problem, we develop a flexible multi-level water-filling scheme that can be easily modified to adapt to different channel conditions as well as different Quality-of-Service (QoS) in terms of the SINR requirement for each user. Since the proposed power allocations are carried out at the relay, the PAPR characteristics at the sources remain unchanged. Furthermore, as will be shown later in Section 5.5.4, the PAPR characteristics at the relay can actually be improved with our proposed power allocation schemes.

The remainder of the paper is organized as follows. Section II presents a transmission model for a multiuser wireless network assisted by a relay terminal using SC-FDMA. Section III develops a joint linear equalization in frequency domain at the destination. Two optimal power allocation schemes at the relay are formulated and solved in Section IV. Some numerical results that illustrate the effectiveness of the proposed schemes as well as the PAPR characteristics of the transmitted signal at the relay are presented and discussed in Section V. Finally, conclusions are drawn in Section VI.

Notations: Italic, bold lower-case and bold upper-case letters denote scalars, vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^{\mathcal{T}}$, $(\cdot)^\dagger$ and $(\cdot)^{\mathcal{H}}$ stand for complex conjugate, transpose, pseudo-inverse and Hermitian transpose, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a vector of complex Gaussian random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ while $\mathbf{I}_N$ stands for an $N \times N$ identity matrix. All signals are represented by their discrete-time equivalents in the complex baseband.

5.2 System Model
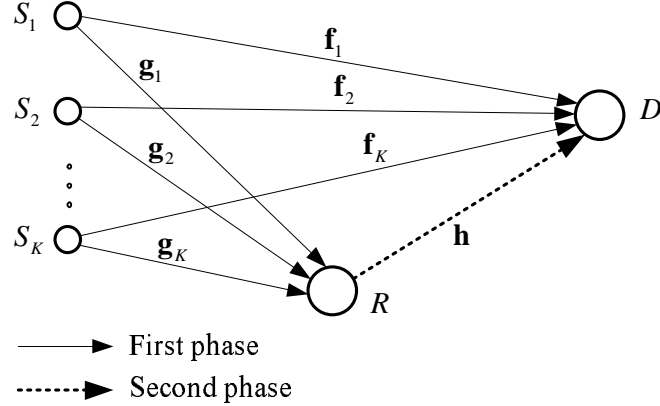
Figure 5.1 A multiuser cooperative network with K sources (users) and one relay.

Fig. 5.1 illustrates a wireless cooperative network where K source terminals (i.e., users), $S_1, \ldots, S_K$, communicate with the destination terminal, D, with the help of one relay terminal, R (e.g., the case of uplink transmission in cellular communications). All terminals are equipped with one antenna, which operates in a half-duplex mode. All the "direct" channels (sources-destination), "uplink" channels (sources-relay), and "downlink" channel (relay-destination) are assumed to be under independent frequency-selective Rayleigh block fading. Without loss of generality, transmission of one block is considered.

Fig. 5.2 outlines various signal processing operations at each source, relay and destination. They are described in detail in the following. With the AF relaying protocol, every transmission consists of two phases. In the first phase, the kth source transmits a block of Q data symbols at the rate of $1/T_s$, denoted by $\mathbf{d}_k = [d_{1,k}, \ldots, d_{Q,k}]^T$. Although different number of subcarriers can be assigned to each source, for simplicity, we assume that each source is assigned Q subcarriers and the total number of subcarriers available in the system is $N = KQ$. In this scenario, the system can handle K simultaneous transmissions (each with Q symbols in one block) without cochannel interference.

At each source, a Q-point discrete Fourier transform (DFT), represented by a $Q \times Q$ matrix $\mathcal{F}_Q$, is performed to produce a frequency-domain representation of the input
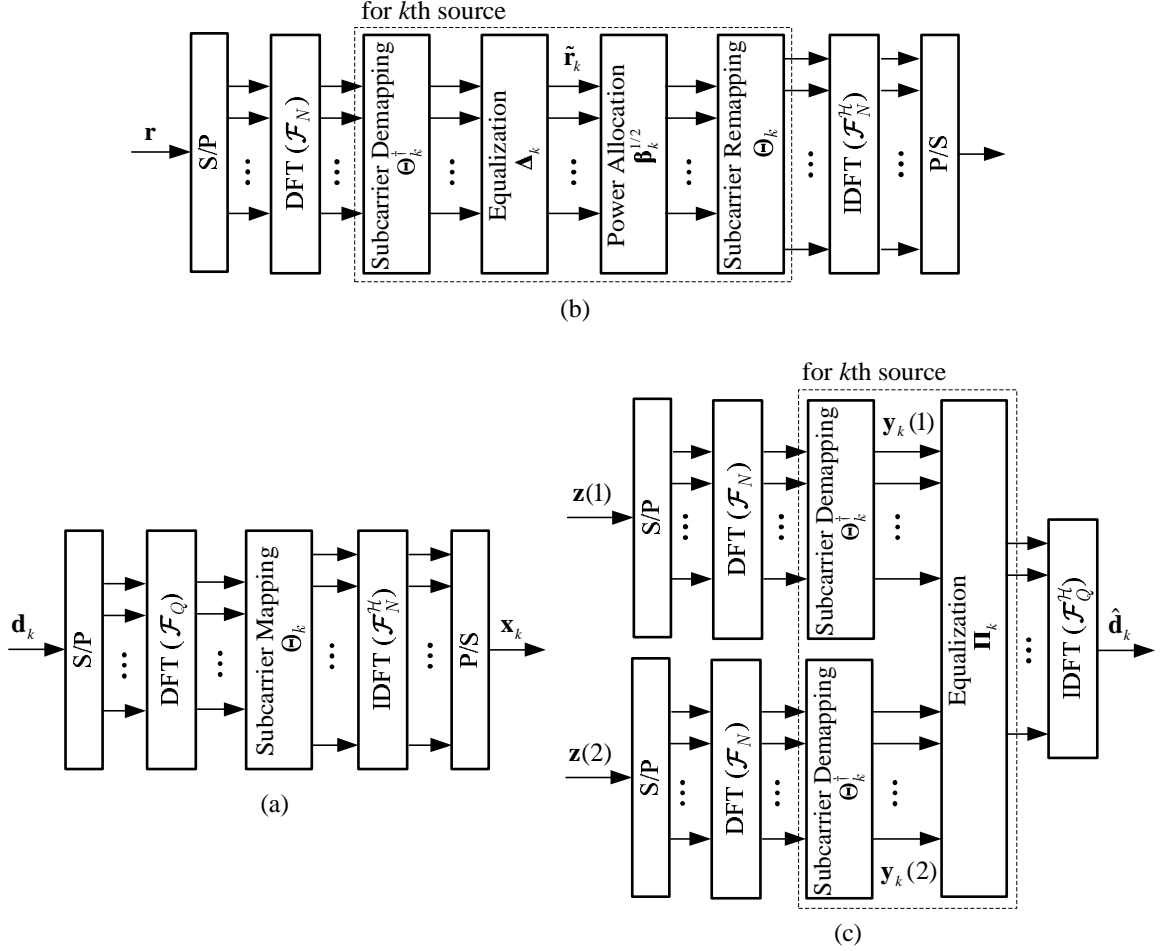
Figure 5.2 Various signal processing operations in the network: (a) at the transmitter of the kth source, (b) at the relay for the kth source signal, and (c) at the destination for the kth source signal.

symbols. The (p, q)th element of $\mathcal{F}_Q$ is defined as $[\mathcal{F}_Q]_{p,q} = (1/\sqrt{Q}) \exp(-j2\pi(p - 1)(q - 1)/Q)$ for $p, q = 1, \ldots, Q$. The assignment of Q DFT-precoded symbols to a subset Q out of N subcarriers is then carried out and can be described by an $N \times Q$ subcarrier allocation matrix $\boldsymbol{\Theta}_k$ (see [12] for some examples of this matrix). In this paper, the following two subcarrier allocation strategies (also referred to as mapping processes), known as localized FDMA (L-FDMA) and interleaved FDMA (I-FDMA), are considered:

$$[\boldsymbol{\Theta}_k]_{n,q} = \begin{cases} 1, & n = (k - 1)Q + q \\ 0, & \text{otherwise} \end{cases}, \quad \text{(L-FDMA)} \tag{5.1}$$

$$[\mathbf{\Theta}_k]_{n,q} = \begin{cases} 1, & n = (q-1)K + k \\ 0, & \text{otherwise} \end{cases}, \quad \text{(I-FDMA)} \qquad (5.2)$$

The result of the subcarrier mapping is a set of complex subcarrier amplitudes, where Q of the amplitudes are non-zero for each source. After that, an N-point inverse DFT (IDFT), denoted by $\mathcal{F}_N^{\mathcal{H}}$, transforms the subcarrier amplitudes to the complex time-domain signals $\mathbf{x}_k = [x_{1,k}, \ldots, x_{N,k}]^{\mathcal{T}}$, which is sampled at the rate of $1/T_c = K/T_s$. The signal components $x_{n,k}$'s are then transmitted sequentially. Performing similar derivations as in the traditional non-cooperative SC-FDMA (see e.g., [12]) it can be shown that a vector representation of the resultant sequence is as follows:

$$\mathbf{x}_k = \sqrt{P_{S,k}} \mathcal{F}_N^{\mathcal{H}} \mathbf{\Theta}_k \mathcal{F}_Q \mathbf{d}_k, \qquad (5.3)$$

where all symbols in one block are transmitted with the same power $P_{S,k}$ and we normalize $\mathbb{E}\{|d_{q,k}|^2\} = 1$, $q = 1, \ldots, Q$. Let $\mathbf{f}_k = [f_{0,k}, \ldots, f_{L_f-1,k}, 0, \ldots, 0]^{\mathcal{T}}$, $\mathbf{g}_k = [g_{0,k}, \ldots, g_{L_g-1,k}, 0, \ldots, 0]^{\mathcal{T}}$ and $\mathbf{h} = [h_0, \ldots, h_{L_h-1}, 0, \ldots, 0]^{\mathcal{T}}$ be $N \times 1$ vector representations of the time-dispersive "direct", "uplink", and "downlink" channels of the kth source, each with L_f, L_g and L_h non-zero coefficients sampled at the rate of $1/T_c$, respectively[1]. Assume $\max\{L_f, L_g, L_h\} \leq N$. Before being transmitted over the channels $\mathbf{f}_k$ and $\mathbf{g}_k$, a cyclic prefix (CP) is applied to $\mathbf{x}_k$.

With the insertion of CP, transmission over channel $\mathbf{f}_k$ and removal of the CP at the destination can be equivalently represented by the following $N \times N$ circulant channel matrix with the first column $\mathbf{f}_k$. Let $\mathbf{v}(1) = [v_1(1), \ldots, v_N(1)]^{\mathcal{T}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{v_1}^2 \mathbf{I}_N)$, whose elements are sampled at the rate of $1/T_c$, represents the effect of additive white Gaussian noise (AWGN) at the destination in the first phase. Assume perfect timing and frequency synchronization, the received signal at the destination can be

---

[1]All the derivations and treatments in this paper can also be applied to the case that the delay profiles of the channel fading experienced by different sources are different, i.e., $L_{f_k} \neq L_{f_j}$ and $L_{g_k} \neq L_{g_j}$ for $k \neq j$.

written as[2]

$$\mathbf{z}(1) = \sum_{k=1}^{K} \mathbf{F}_k \mathbf{x}_k + \mathbf{v}(1). \tag{5.4}$$

Similarly, at the relay, the received signal after CP removal is given by

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{G}_k \mathbf{x}_k + \mathbf{n}, \tag{5.5}$$

where $\mathbf{G}_k$ is a circular channel matrix corresponding to $\mathbf{g}_k$ and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$ represents AWGN at the relay.

At the relay, an N-point DFT is applied to the received signal $\mathbf{r}$. Next, the demapping process is carried out, which can be represented by the $Q \times N$ pseudo-inverse matrix $\mathbf{\Theta}_k^\dagger$. Then a frequency-domain equalizer (FDE), represented by a diagonal matrix $\mathbf{\Delta}_k$, compensates the distortion of the received signal. Two simple linear equalizers are the zero-forcing FDE (ZF-FDE) and the minimum mean-squared error FDE (MMSE-FDE). Due to its superior performance, the MMSE-FDE shall be adopted in our study. The output of the FDE corresponding to the signal from the kth source can be expressed by

$$
\begin{aligned}
\tilde{\mathbf{r}}_k &= \mathbf{\Delta}_k \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{r} \\
&= \mathbf{\Delta}_k \mathbf{\Theta}_k^\dagger \left(\sum_{j=1}^{K} \sqrt{P_{S,j}} \mathcal{F}_N \mathbf{G}_j \mathcal{F}_N^{\mathcal{H}} \mathbf{\Theta}_j \mathcal{F}_Q \mathbf{d}_j + \mathcal{F}_N \mathbf{n} \right) \\
&= \mathbf{\Delta}_k \mathbf{\Theta}_k^\dagger \left(\sum_{j=1}^{K} \sqrt{P_{S,j}} \mathbf{\Lambda}_j \mathbf{\Theta}_j \mathcal{F}_Q \mathbf{d}_j + \mathcal{F}_N \mathbf{n} \right) \\
&= \sqrt{P_{S,k}} \mathbf{\Delta}_k \tilde{\mathbf{\Lambda}}_k \mathcal{F}_Q \mathbf{d}_k + \mathbf{\Delta}_k \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{n},
\end{aligned} \tag{5.6}
$$

where $\mathbf{\Lambda}_k = \mathcal{F}_N \mathbf{G}_k \mathcal{F}_N^{\mathcal{H}}$ and

$$\mathbf{\Theta}_k^\dagger \mathbf{\Lambda}_j \mathbf{\Theta}_j = \begin{cases} \tilde{\mathbf{\Lambda}}_k = \mathrm{diag}(\tilde{\lambda}_{1,k}, \ldots, \tilde{\lambda}_{Q,k}), & j = k \\ \mathbf{0}, & \text{otherwise} \end{cases} \tag{5.7}$$

Note that $[\mathbf{\Lambda}_k]_{n,n} = \sum_{l=0}^{L_g-1} g_{l,k} \exp(-j2\pi(n-1)l/N)$, while $\tilde{\lambda}_{q,k}$ is chosen from $\{[\mathbf{\Lambda}_k]_{n,n}\}_{n=1}^N$ according to $\mathbf{\Theta}_k$ given in (5.1) or (5.2).

---

[2]Note that $\mathbf{F}_k$ is the $N \times N$ circulant channel matrix whose first column is $\mathbf{f}_k$.

The MMSE-FDE can be described by the $Q \times Q$ matrix $\boldsymbol{\Delta}_k = \text{diag}\,(\delta_{1,k}, \ldots, \delta_{Q,k})$, where each diagonal element is given by (see, e.g., [13])

$$\delta_{q,k} = \frac{P_{S,k}\tilde{\lambda}_{q,k}^*}{P_{S,k}|\tilde{\lambda}_{q,k}|^2 + \sigma_n^2}. \tag{5.8}$$

Let $\boldsymbol{\beta}_k = \text{diag}(\beta_{1,k}, \ldots, \beta_{Q,k})$ with the diagonal elements be the power gains the relay assigns to the Q symbols (corresponding to the Q subcarriers) of the kth source. As shown in Appendix 5.A, the elements of all $\boldsymbol{\beta}_k$'s should satisfy the following power constraint

$$\sum_{k=1}^{K}\sum_{q=1}^{Q} \frac{\beta_{q,k}P_{S,k}^2|\tilde{\lambda}_{q,k}|^2}{P_{S,k}|\tilde{\lambda}_{q,k}|^2 + \sigma_n^2} \leq P_R, \tag{5.9}$$

where P_R is the total transmit power of the relay.

After being equalized and allocated with different power levels, the signals are re-mapped to the assigned subcarriers, transformed into time domain using an IDFT and then forwarded to the destination. The received signal at the destination in the second phase can be written as

$$\mathbf{z}(2) = \mathbf{H}\mathcal{F}_N^{\mathcal{H}} \sum_{k=1}^{K} \boldsymbol{\Theta}_k \boldsymbol{\beta}_k^{1/2} \tilde{\mathbf{r}}_k + \mathbf{v}(2), \tag{5.10}$$

where $\mathbf{v}(2) \sim \mathcal{CN}(\mathbf{0}, \sigma_{v_2}^2 \mathbf{I}_N)$ represents the AWGN experienced at the destination in the second phase[3].

5.3 Joint MMSE Equalization at the Destination

The signals received at the destination in two phases can be jointly equalized in frequency domain before detection. To describe the MMSE-FDE at the destination, let $\mathbf{H}$ be the circulant channel matrix with the first column $\mathbf{h}$. Likewise, let $\boldsymbol{\Psi}_k = \mathcal{F}_N \mathbf{F}_k \mathcal{F}_N^{\mathcal{H}}$, $\boldsymbol{\Omega} = \mathcal{F}_N \mathbf{H} \mathcal{F}_N^{\mathcal{H}}$ and

$$\boldsymbol{\Theta}_k^\dagger \boldsymbol{\Psi}_j \boldsymbol{\Theta}_j = \begin{cases} \tilde{\boldsymbol{\Psi}}_k = \text{diag}(\tilde{\psi}_{1,k}, \ldots, \tilde{\psi}_{Q,k}), & j = k \\ \mathbf{0}, & \text{otherwise} \end{cases}, \tag{5.11}$$

---

[3]Here the mathematical model allows different noise powers at the destination in the first and second phases, i.e., $\sigma_{v_1}^2 \neq \sigma_{v_2}^2$. In practice, one however expects that $\sigma_{v_1}^2 = \sigma_{v_2}^2$.

$$\mathbf{\Theta}_k^\dagger \mathbf{\Omega} \mathbf{\Theta}_j = \begin{cases} \tilde{\mathbf{\Omega}}_k = \mathrm{diag}(\tilde{\omega}_{1,k}, \dots, \tilde{\omega}_{Q,k}), & j = k \\ \mathbf{0}, & \text{otherwise} \end{cases} . \qquad (5.12)$$

Then the signals received at the destination in two phases can be processed as follows:

$$
\begin{aligned}
\mathbf{y}_k(1) &= \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{z}(1) \\
&= \mathbf{\Theta}_k^\dagger \left(\sum_{j=1}^{K} \sqrt{P_{S,j}} \mathcal{F}_N \mathbf{F}_j \mathcal{F}_N^{\mathcal{H}} \mathbf{\Theta}_j \mathcal{F}_Q \mathbf{d}_j + \mathcal{F}_N \mathbf{v}(1) \right) \\
&= \mathbf{\Theta}_k^\dagger \left(\sum_{j=1}^{K} \sqrt{P_{S,j}} \mathbf{\Psi}_j \mathbf{\Theta}_j \mathcal{F}_Q \mathbf{d}_j + \mathcal{F}_N \mathbf{v}(1) \right) \\
&= \sqrt{P_{S,k}} \tilde{\mathbf{\Psi}}_k \mathcal{F}_Q \mathbf{d}_k + \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{v}(1), \qquad (5.13)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{y}_k(2) &= \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{z}(2) = \mathbf{\Theta}_k^\dagger \left(\sum_{j=1}^{K} \mathcal{F}_N \mathbf{H} \mathcal{F}_N^{\mathcal{H}} \mathbf{\Theta}_j \boldsymbol{\beta}_j^{1/2} \tilde{\mathbf{r}}_j + \mathcal{F}_N \mathbf{v}(2) \right) \\
&= \mathbf{\Theta}_k^\dagger \left[\sum_{j=1}^{K} \mathbf{\Omega}_j \mathbf{\Theta}_j \boldsymbol{\beta}_j^{1/2} \left(\sqrt{P_{S,j}} \boldsymbol{\Delta}_j \tilde{\mathbf{\Lambda}}_j \mathcal{F}_Q \mathbf{d}_j + \boldsymbol{\Delta}_j \mathbf{\Theta}_j^\dagger \mathcal{F}_N \mathbf{n} \right) + \mathcal{F}_N \mathbf{v}(2) \right] \\
&= \sqrt{P_{S,k}} \tilde{\mathbf{\Omega}}_k \boldsymbol{\beta}_k^{1/2} \boldsymbol{\Delta}_k \tilde{\mathbf{\Lambda}}_k \mathcal{F}_Q \mathbf{d}_k + \tilde{\mathbf{\Omega}}_k \boldsymbol{\beta}_k^{1/2} \boldsymbol{\Delta}_k \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{n} + \mathbf{\Theta}_k^\dagger \mathcal{F}_N \mathbf{v}(2). \quad (5.14)
\end{aligned}
$$

Let $\mathbf{M}_k(1) = \sqrt{P_{S,k}} \tilde{\mathbf{\Psi}}_k$, $\mathbf{M}_k(2) = \sqrt{P_{S,k}} \tilde{\mathbf{\Omega}}_k \boldsymbol{\beta}_k^{1/2} \boldsymbol{\Delta}_k \tilde{\mathbf{\Lambda}}_k$, and $\mathbf{M}_k = [\mathbf{M}_k(1), \mathbf{M}_k(2)]^{\mathcal{T}}$. The $Q \times 2Q$ joint FDE matrix $\mathbf{\Pi}_k$ can be found to be:[4]

$$\mathbf{\Pi}_k = \mathbf{M}_k^{\mathcal{H}} \left(\mathbf{M}_k \mathbf{M}_k^{\mathcal{H}} + \mathbf{R}_{nv} \right)^{-1}, \qquad (5.15)$$

where $\mathbf{R}_{nv} = \begin{bmatrix} \sigma_{v_1}^2 \mathbf{I}_Q & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{nv_2} \end{bmatrix}$ and $\mathbf{R}_{nv_2} = \sigma_n^2 \tilde{\mathbf{\Omega}}_k \boldsymbol{\beta}_k \boldsymbol{\Delta}_k \boldsymbol{\Delta}_k^{\mathcal{H}} \tilde{\mathbf{\Omega}}_k^{\mathcal{H}} + \sigma_{v_2}^2 \mathbf{I}_Q$. Furthermore, the estimates of the transmitted symbols of the kth user are $\hat{\mathbf{d}}_k = \mathcal{F}_Q^{\mathcal{H}} \mathbf{\Pi}_k \left[\mathbf{y}_k^{\mathcal{T}}(1) \mathbf{y}_k^{\mathcal{T}}(2) \right]^{\mathcal{T}}$.

Using the joint MMSE-FDE, the overall instantaneous SINR for the kth user can be computed as [14, 15]

$$\mathrm{SINR}_k = \frac{\eta_k}{1 - \eta_k}, \qquad (5.16)$$

---

[4]It should be noted that this linear equalizer can also be replaced by a decision-feedback equalizer (DFE). However, computation of the exact SINR, which is needed for calculating a power allocation scheme, appears intractable. Nevertheless, approximations or bounds on the performance of such a decision-feedback equalizer are available in the literature (e.g., see [21]) and can be used as the objective functions to find sub-optimal or heuristic power allocation schemes.

where η_k is the diagonal element of $\mathcal{F}_Q^{\mathcal{H}}\mathbf{\Pi}_k\mathbf{M}_k\mathcal{F}_Q$. Applying matrix inversion lemma $(\mathbf{X}_1 + \mathbf{X}_2\mathbf{X}_3\mathbf{X}_4)^{-1} = \mathbf{X}_1^{-1} - \mathbf{X}_1^{-1}\mathbf{X}_2(\mathbf{X}_3^{-1} + \mathbf{X}_4\mathbf{X}_1^{-1}\mathbf{X}_2)^{-1}\mathbf{X}_4\mathbf{X}_1^{-1}$, one has

$$
\begin{aligned}
\mathbf{\Pi}_k\mathbf{M}_k &= \mathbf{M}_k^{\mathcal{H}}\left(\mathbf{M}_k\mathbf{M}_k^{\mathcal{H}} + \mathbf{R}_{nv}\right)^{-1}\mathbf{M}_k \\
&= \mathbf{I}_Q - \left(\mathbf{I}_Q + \frac{1}{\sigma_{v_1}^2}\mathbf{M}_k^{\mathcal{H}}(1)\mathbf{M}_k(1)\right. \\
&\qquad\qquad \left. + \mathbf{M}_k^{\mathcal{H}}(2)\mathbf{R}_{nv_2}^{-1}\mathbf{M}_k(2)\right)^{-1}.
\end{aligned} \tag{5.17}
$$

Therefore η_k can be calculated as

$$
\eta_k = 1 - \frac{1}{Q}\sum_{q=1}^{Q}\frac{1}{\phi_{q,k}(\beta_{q,k})}, \tag{5.18}
$$

where

$$
\phi_{q,k}(\beta_{q,k}) = a_{q,k} + \frac{b_{q,k}\beta_{q,k}}{c_{q,k}\beta_{q,k} + \sigma_{v_2}^2}, \tag{5.19}
$$

and $a_{q,k} = 1 + \frac{P_{S,k}|\tilde{\psi}_{q,k}|^2}{\sigma_{v_1}^2}$, $b_{q,k} = P_{S,k}|\tilde{\omega}_{q,k}\delta_{q,k}\tilde{\lambda}_{q,k}|^2$, $c_{q,k} = |\tilde{\omega}_{q,k}\delta_{q,k}|^2\sigma_n^2$ are simply positive constants for a given set of $P_{S,k}$'s.

It is clear from (5.18) that η_k, and hence the instantaneous SINR_k, depends on power allocation factors $\beta_{q,k}$'s. It is therefore convenient to explicitly write these quantities as $\eta_k(\boldsymbol{\beta}_k)$ and $\text{SINR}_k(\boldsymbol{\beta}_k)$. More specifically, $\text{SINR}_k(\boldsymbol{\beta}_k)$ depends on the harmonic mean of $\phi_{q,k}(\beta_{q,k})$. Also observe that

$$
\begin{aligned}
\lim_{\beta_{q,k}\to\infty}\phi_{q,k}(\beta_{q,k}) &= a_{q,k} + \frac{b_{q,k}}{c_{q,k}} \\
&= 1 + \frac{P_{S,k}}{\sigma_{v_1}^2}|\tilde{\psi}_{q,k}|^2 + \frac{P_{S,k}}{\sigma_n^2}|\tilde{\lambda}_{q,k}|^2.
\end{aligned} \tag{5.20}
$$

The above implies that the performance of the system strongly depends on the gains of the "uplink" channels (especially when $|\tilde{\lambda}_{q,k}| >> |\tilde{\psi}_{q,k}|$). Moreover, up to a certain level, adding more relay power does not significantly improve the SINR of the received signal at the destination. This is a consequence of the fact that increasing relay power also increases the inter-symbol interference at the destination for each user. This observation is examined in more detail in the next section when the optimization of relay power allocation among users is addressed.

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Before closing this section, it is worth mentioning that with equal power allocation (EPA), each subcarrier is assigned the same power, i.e.,

$$\beta_{q,k} = \frac{P_R}{\sum\limits_{k=1}^{K}\sum\limits_{q=1}^{Q} \alpha_{q,k}}, \tag{5.21}$$

where

$$\alpha_{q,k} = \frac{P_{S,k}^2 |\tilde{\lambda}_{q,k}|^2}{P_{S,k}|\tilde{\lambda}_{q,k}|^2 + \sigma_n^2}. \tag{5.22}$$

Note that $\frac{1}{N}\sum\limits_{k=1}^{K}\sum\limits_{q=1}^{Q} \alpha_{q,k}$ is the power of each symbol in one block of the kth user at the output of the equalizer [14].

5.4 Power Allocation at the Relay

Under the assumption that full channel state information (CSI) is available at the destination, the relay can estimate the uplink channels perfectly while no CSI is available at the sources, this section solves the following two power allocation problems concerning the relay: (i) minimizing the sum of the total transmit power at the relay subject to the constraint that the SINR of each user at the destination is maintained at a predefined level, and (ii) maximizing the minimum of $\mathrm{SINR}_k(\boldsymbol{\beta}_k)$ subject to a constraint on total relay power. It is worth noting that the target performance adopted in the problems under consideration, SINR, is directly related to the user capacity[5]. Therefore, it is a sensible performance metric in both signal processing and communication theory viewpoints.

In both problems the power allocation solutions are computed at the destination and fed back to the relay. Another option could be to have the destination send the CSI of the direct paths and the downlink channels back to the relay so that the relay can compute the optimal power allocation. The relay then feeds forward this power allocation information to the destination in order to calculate the equalizer's

---

[5]The user capacity, C, has a monotonic relationship with the user's SINR, namely $C \sim \log_2(1 + \mathrm{SINR})$.

coefficients for the signals from all the sources. Compared to the latter option, it is obvious that our approach significantly reduces the overhead information exchanged between the relay and destination as well as the computational complexity of the relay. Also note that the destination can use a method proposed in [16] to estimate the CSI of the uplink channels.

5.4.1 Minimize the Relay Transmit Power

In this section, the total transmit power of the relay is minimized while the quality of the signal from each source received at the destination is maintained at a predefined level, denoted by $\overline{\text{SINR}}_k$. Since the SINR_k's are decoupled among all sources, the problem can be solved separately for each source. Without loss of generality, the problem is solved for the kth source in the following.

Let $\overline{\text{MSE}}_k = Q/(\overline{\text{SINR}}_k + 1)$ and $\boldsymbol{\beta}_k = \text{diag}(\beta_{1,k}, \ldots, \beta_{Q,k})$ as in (5.9). With $\alpha_{q,k}$ defined in (5.22), the total power the relay assigns to the kth user is $P_{R,k}(\boldsymbol{\beta}_k) = \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k}$. Then the power minimization problem can be formulated as follows:

$$\min_{\boldsymbol{\beta}_k} \quad P_{R,k}(\boldsymbol{\beta}_k) = \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} \tag{5.23}$$

$$\text{s.t.} \quad \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} \leq \overline{\text{MSE}}_k, \ 1 \leq k \leq K \tag{5.24}$$

$$\beta_{q,k} \geq 0, \quad 1 \leq q \leq Q. \tag{5.25}$$

Define

$$\text{MSE}_k(\boldsymbol{\beta}_k) = \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})}, \tag{5.26}$$

$$\chi_k = \sum_{q=1}^{Q} \frac{1}{a_{q,k}}, \tag{5.27}$$

$$\rho_k = \sum_{q=1}^{Q} \left(a_{q,k} + \frac{b_{q,k}}{c_{q,k}} \right)^{-1}. \tag{5.28}$$

Note that the parameters χ_k and ρ_k are exactly the $\text{MSE}_k(\boldsymbol{\beta}_k)$ when $\beta_{q,k} = 0, \forall q$ (the kth source is not supported by the relay) and when $\beta_{q,k} = +\infty, \forall q$ (the kth source is supported by the relay with an infinite power), respectively. With $\phi_{q,k}(\beta_{q,k})$ defined

in (5.19), it is clear that the problem is *only feasible* when $\overline{\text{MSE}}_k > \rho_k$. By examining the second derivative, one can easily show that the function $1/\phi_{q,k}(\beta_{q,k})$ is convex in $\beta_{q,k}$. Therefore, $\sum_{q=1}^{Q} 1/\phi_{q,k}(\beta_{q,k})$ is also convex in $\boldsymbol{\beta}_k$.

Since the objective function is linear and the constraint functions are all convex, problem (5.23)-(5.25) is a convex programming. The Lagrangian is:

$$\mathcal{L}_k(\boldsymbol{\beta}_k, \mu_k, \boldsymbol{\gamma}_k) = \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k}$$

$$+ \mu_k \left(\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} - \overline{\text{MSE}}_k \right) - \sum_{q=1}^{Q} \gamma_{q,k} \beta_{q,k}, \tag{5.29}$$

where μ_k and $\boldsymbol{\gamma}_k = [\gamma_{1,k}, \ldots, \gamma_{Q,k}]^T$ are the Lagrangian multipliers.

From the necessary and sufficient Karush–Kuhn–Tucker (KKT) optimality conditions [17], one has:

$$\alpha_{q,k} - \frac{\mu_k b_{q,k} \sigma_{v_2}^2}{\left[(a_{q,k} c_{q,k} + b_{q,k}) \beta_{q,k} + a_{q,k} \sigma_{v_2}^2 \right]^2} - \gamma_{q,k} = 0, \tag{5.30}$$

$$\mu_k \left(\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} - \overline{\text{MSE}}_k \right) = 0, \tag{5.31}$$

$$\gamma_{q,k} \beta_{q,k} = 0, \tag{5.32}$$

$$\mu_k \geq 0, \ \gamma_{q,k} \geq 0, \ \beta_{q,k} \geq 0. \tag{5.33}$$

Keeping in mind that $\chi_k > \rho_k, \forall k$, and the problem is only feasible when $\overline{\text{MSE}}_k > \rho_k$, we solve the above set of equations by considering the following two cases.

The first case is when $\overline{\text{MSE}}_k \geq \chi_k$. As noted before, since $a_{q,k}, b_{q,k}, c_{q,k}$ are positive for $1 \leq q \leq Q$, the expression in (5.19) implies that

$$\text{MSE}_k(\boldsymbol{\beta}_k) = \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} \leq \chi_k = \sum_{q=1}^{Q} \frac{1}{a_{q,k}} \leq \overline{\text{MSE}}_k. \tag{5.34}$$

The above means that the constraint on the SINR of the kth source is automatically met, regardless of the values of $\{\beta_{q,k}\}_{q=1}^{Q}$. Hence, to minimize the relay power, one sets $\beta_{q,k} = 0, \forall q$, i.e., the kth source does not need any support from the relay.

The second case is when $\rho_k < \overline{\text{MSE}}_k < \chi_k$, hence $\beta_{q,k} > 0$ for at least some q as the assistance from the relay is beneficial for the kth source. Then due to (5.32),

$\gamma_{q,k} = 0$. Since $\alpha_{q,k} > 0$ (see (5.22)), it follows from (5.30) that μ_k must be strictly positive and therefore at the optimality (5.31) forces the following equality:

$$\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} = \overline{\text{MSE}}_k. \tag{5.35}$$

Solving (5.30) gives the following optimal power allocation scheme

$$\beta_{q,k} = \left(\sqrt{\frac{\mu_k b_{q,k} \sigma_{v2}^2}{\alpha_{q,k}}} - a_{q,k} \sigma_{v2}^2 \right)^+ \frac{1}{a_{q,k} c_{q,k} + b_{q,k}}, \tag{5.36}$$

where $(x)^+ = \max\{0, x\}$ and μ_k can be found by solving (5.35), typically by some iterative technique. Here we also propose an efficient method to solve μ_k without tolerance as follows.

Define

$$\xi_{q,k} = \frac{\alpha_{q,k}(a_{q,k})^2 \sigma_{v2}^2}{b_{q,k}}, \tag{5.37}$$

and arrange $\xi_{q,k}$'s in the increasing order of subcarrier index q, i.e., $\xi_{q+1,k} \geq \xi_{q,k}$. Also arrange the sets $\{a_{q,k}\}, \{b_{q,k}\}, \{c_{q,k}\}$ and $\{\alpha_{q,k}\}$ in accordance with the increasing order of the set $\{\xi_{q,k}\}$ for each source. This means that $\beta_{q,k}$'s are in the decreasing order with respect to q.[6] Thus the indices of active subcarriers used at the relay to forward the signal of the kth source are in the range $1 \leq q \leq \hat{Q}_k \leq Q$, where $\hat{Q}_k$ is such that $\beta_{\hat{Q}_k,k} > 0$ and $\beta_{\hat{Q}_k+1,k} = 0$. Now, using (5.36) and (5.37) in (5.35) gives the following equation to solve for μ_k:

$$\vartheta_k(\mu_k) = \overline{\text{MSE}}_k - \sum_{q=1}^{\hat{Q}_k} \frac{c_{q,k}}{a_{q,k} c_{q,k} + b_{q,k}} - \sum_{q=\hat{Q}_k+1}^{Q} \frac{1}{a_{q,k}}$$
$$- \frac{1}{\mu_k^{1/2}} \sum_{q=1}^{\hat{Q}_k} \frac{\sqrt{\alpha_{q,k} b_{q,k} \sigma_{v2}^2}}{a_{q,k} c_{q,k} + b_{q,k}} = 0. \tag{5.38}$$

It should be noted that $\hat{Q}_k$ in (5.38) also depends on μ_k. Fortunately one can find an efficient algorithm to solve (5.38) by exploiting the monotonicity of $\vartheta_k(\mu_k)$ as stated in the following proposition.

---

[6]Note that from this point forward, q is the index after sorting and the proposed method deals with the sorted sets. All the sets shall be rearranged to the original order when the algorithm is completed.

Proposition 1. *Given a feasible set of the SINR constraints, i.e., $\overline{\mathrm{MSE}}_k > \rho_k$, $\forall k$, the function $\vartheta_k(\mu_k)$ is monotonically increasing in $\mu_k > 0$.*

Proof. See Appendix 5.B. □

In practice, the set of $\{\overline{\mathrm{MSE}}_k\}$ is usually imposed before the actual transmission process is carried out. To avoid infeasible cases, the values in the set of $\overline{\mathrm{MSE}}_k$ can be adjusted adaptively with the current channel conditions. Obviously, $\overline{\mathrm{MSE}}_k$ should be chosen such that $\overline{\mathrm{MSE}}_k > \rho_k$ and the transmit power required at the relay satisfies $\sum_k P_{R,k} \leq P_R$, where P_R is the maximum transmit power the relay can use.

From Proposition 1 and due to the water-filling structure of the optimal power allocation scheme in (5.36), one has the following corollary regarding the optimal solution of problem (5.23)-(5.25). This corollary will also be used in solving the min-max problem in Section 5.4.2.

Corollary 2. *Let $P_{R,k}^*(\overline{\mathrm{MSE}}_k)$ be the optimal value function of the optimization problem (5.23)-(5.25). Then the minimum transmit power required at the relay for each source, $P_{R,k}^*(\overline{\mathrm{MSE}}_k)$, and the total transmit power, $P_R^*(\overline{\mathrm{MSE}}_1, \ldots, \overline{\mathrm{MSE}}_k) = \sum_{k=1}^{K} P_{R,k}^*(\overline{\mathrm{MSE}}_k)$, are all strictly decreasing when $\overline{\mathrm{MSE}}_k > \rho_k$ increases for $k = 1, \ldots, K$.*

Based on the monotonic increase of $\vartheta_k(\mu_k)$ proved in Proposition 1, an efficient algorithm to compute the solution of (5.36) is outlined below:

Algorithm 1: Minimize the Relay Transmit Power

Input: K, Q, all CSI and $\{\overline{\text{MSE}}_k\}$.

Output: $\{\beta_{q,k}\}$ and $\{\mu_k\}$.

1. Adjust the values in the set $\{\overline{\text{MSE}}_k\}$ such that $\overline{\text{MSE}}_k > \rho_k, \forall k$, if needed.

2. Find all the sources that need assistance from the relay, i.e., the set $\mathcal{K} = \{k = 1, \ldots, K : \rho_k < \overline{\text{MSE}}_k < \chi_k\}$. For each $k \in \mathcal{K}$, rearrange the set $\{\xi_{q,k}\}$, computed as in (5.37), in increasing order with respect to q. Also rearrange all sets $\{a_{q,k}\}, \{b_{q,k}\}, \{c_{q,k}\}$ and $\{\alpha_{q,k}\}$ in accordance with the set $\{\xi_{q,k}\}$.

3. Using the fact that $\vartheta_k(\xi_{\hat{Q}_k,k}) \cdot \vartheta_k(\xi_{\hat{Q}_k+1,k}) < 0$, identify $\hat{Q}_k$ and then obtain μ_k from (5.38) in the range $(\xi_{\hat{Q}_k,k}, \xi_{\hat{Q}_k+1,k})$.

4. Compute $\beta_{q,k}$ using (5.36).

5. Rearrange $\{\beta_{q,k}\}$ in the reverse order as done in Step 2 and STOP.

5.4.2 Maximize the Minimum SINR_k

Let $\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K]$. The following max-min optimization problem can provide fairness among the sources in terms of the individual SINRs (and consequently the error performance):

$$\max_{\boldsymbol{\beta}} \quad \min_k \text{SINR}_k(\boldsymbol{\beta}_k) \tag{5.39}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{q=1}^{Q} \beta_{q,k}\alpha_{q,k} \leq P_R \tag{5.40}$$

$$\beta_{q,k} \geq 0, \ 1 \leq q \leq Q, \ 1 \leq k \leq K. \tag{5.41}$$

Since $\text{SINR}_k(\boldsymbol{\beta}_k) = \frac{Q}{\text{MSE}_k(\boldsymbol{\beta}_k)} - 1$, maximizing the minimum of $\text{SINR}_k(\boldsymbol{\beta}_k)$ is equiv-

alent to maximizing the minimum of $1/\mathrm{MSE}_k(\boldsymbol{\beta}_k)$. Furthermore, it is obvious that

$$
\begin{aligned}
\max_{\boldsymbol{\beta}} \min_{k=1,2,\ldots,K} \frac{1}{\mathrm{MSE}_k(\boldsymbol{\beta}_k)} &= \max_{\boldsymbol{\beta}} \frac{1}{\max\limits_{k=1,2,\ldots,K} \mathrm{MSE}_k(\boldsymbol{\beta}_k)} \\
&= \frac{1}{\min\limits_{\boldsymbol{\beta}} \max\limits_{k=1,2,\ldots,K} \mathrm{MSE}_k(\boldsymbol{\beta}_k)} \\
&\Leftrightarrow \min_{\boldsymbol{\beta}} \max_{k=1,2,\ldots,K} \mathrm{MSE}_k(\boldsymbol{\beta}_k).
\end{aligned} \tag{5.42}
$$

As $\mathrm{MSE}_k(\boldsymbol{\beta}_k)$ is convex, $\max_{k=1,2,\ldots,K} \mathrm{MSE}_k(\boldsymbol{\beta}_k)$ is also convex, and problem (5.39)-(5.41) can be equivalently rewritten as

$$
\min_{\boldsymbol{\beta}} \quad \max_{k} \left\{ \mathrm{MSE}_k(\boldsymbol{\beta}_k) \right\} \tag{5.43}
$$

$$
\text{s.t.} \quad \sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} \leq P_R \tag{5.44}
$$

$$
\beta_{q,k} \geq 0, \ 1 \leq q \leq Q, \ 1 \leq k \leq K. \tag{5.45}
$$

Recall that $\mathrm{MSE}_k(\boldsymbol{\beta}_k) = \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})}$. Thus by introducing a new variable $t = \max\{\mathrm{MSE}_k(\boldsymbol{\beta}_k)\}$, problem (5.43)-(5.45) can be transformed into

$$
\min_{\boldsymbol{\beta}, \, t} \quad t \tag{5.46}
$$

$$
\text{s.t.} \quad \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} \leq t, \ 1 \leq k \leq K \tag{5.47}
$$

$$
\sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} \leq P_R \tag{5.48}
$$

$$
\beta_{q,k} \geq 0, \ 1 \leq q \leq Q, \ 1 \leq k \leq K. \tag{5.49}
$$

Since the objective function is linear, constraints in (5.47) are convex and constraint (5.48) is linear, problem (5.46)-(5.49) is a convex programming. The Lagrangian is

$$
\mathcal{L}(\boldsymbol{\beta}, t, \boldsymbol{\mu}, \mu_0, \boldsymbol{\gamma}) = t + \sum_{k=1}^{K} \mu_k \left(\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} - t \right)
$$

$$
+ \mu_0 \left(\sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} - P_R \right) - \sum_{k=1}^{K} \sum_{q=1}^{Q} \gamma_{q,k} \beta_{q,k}, \tag{5.50}
$$

where $\boldsymbol{\mu} = \{\mu_k, k = 1, \ldots, K\}$, μ_0 and $\boldsymbol{\gamma} = \{\gamma_{q,k}, k = 1, \ldots, K, q = 1, \ldots, Q\}$ are the Lagrangian multipliers. The necessary and sufficient KKT optimality conditions on

the solution of problem (5.46)-(5.49) are [17]:

$$\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} \leq t_{\text{opt}}, \tag{5.51}$$

$$\sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} \leq P_R, \tag{5.52}$$

$$\mu_0 \geq 0, \ \mu_k \geq 0, \ \gamma_{q,k} \geq 0, \ \beta_{q,k} \geq 0, \tag{5.53}$$

$$\sum_{k=1}^{K} \mu_k = 1, \tag{5.54}$$

$$\mu_0 \alpha_{q,k} - \frac{\mu_k b_{q,k} \sigma_{v_2}^2}{\left[(a_{q,k} c_{q,k} + b_{q,k}) \beta_{q,k} + a_{q,k} \sigma_{v_2}^2 \right]^2} = \gamma_{q,k}, \tag{5.55}$$

$$\mu_k \left(\sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})} - t_{\text{opt}} \right) = 0, \tag{5.56}$$

$$\mu_0 \left(\sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} - P_R \right) = 0, \tag{5.57}$$

$$\gamma_{q,k} \beta_{q,k} = 0, \tag{5.58}$$

Now, let us examine the above conditions in more detail. First, from (5.55), if $\mu_0 = 0$, then $\mu_k = 0$ and $\gamma_{q,k} = 0$, $\forall q, k$. Therefore, μ_0 must be strictly positive and then, due to (5.57), at the optimality (5.52) holds with equality, i.e., $\sum\limits_{k=1}^{K} \sum\limits_{q=1}^{Q} \beta_{q,k} \alpha_{q,k} = P_R$.

Second, at the optimality, if $\mu_l = 0$ for some l, $1 \leq l \leq K$, it follows from (5.55) that $\gamma_{q,l} = \mu_0 \alpha_{q,l} > 0, \forall q$. Then by the complementary slackness conditions (5.58), $\beta_{q,l} = 0, \forall q$, which implies that the lth source is not supported by the relay. Substituting $\beta_{q,l} = 0$ into (5.51), one has $t_{\text{opt}} \geq \sum\limits_{q=1}^{Q} \frac{1}{\phi_{q,l}(0)} = \chi_l$.

Third, at the optimality, if $\mu_k > 0$ for some k, $1 \leq k \leq K$ then from (5.56) one has $t_{\text{opt}} = \sum\limits_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})}$. Furthermore, if $\beta_{q,k} = 0$ for some q, in order to satisfy the condition that $\gamma_{q,k} \geq 0$, it follows from (5.55) that

$$\frac{\mu_k}{\mu_0} \leq \frac{\alpha_{q,k} a_{q,k}^2 \sigma_{v_2}^2}{b_{q,k}} \overset{[\text{cf. (5.37)}]}{=} \xi_{q,k}. \tag{5.59}$$

Otherwise, if $\beta_{q,k} > 0$ for some q, then $\gamma_{q,k} = 0$. To reduce the number of parameters, define $\bar{\mu}_k = \mu_k/\mu_0$. Obviously μ_0 and $\{\mu_k\}$ can be found from the set $\{\bar{\mu}_k\}$ as
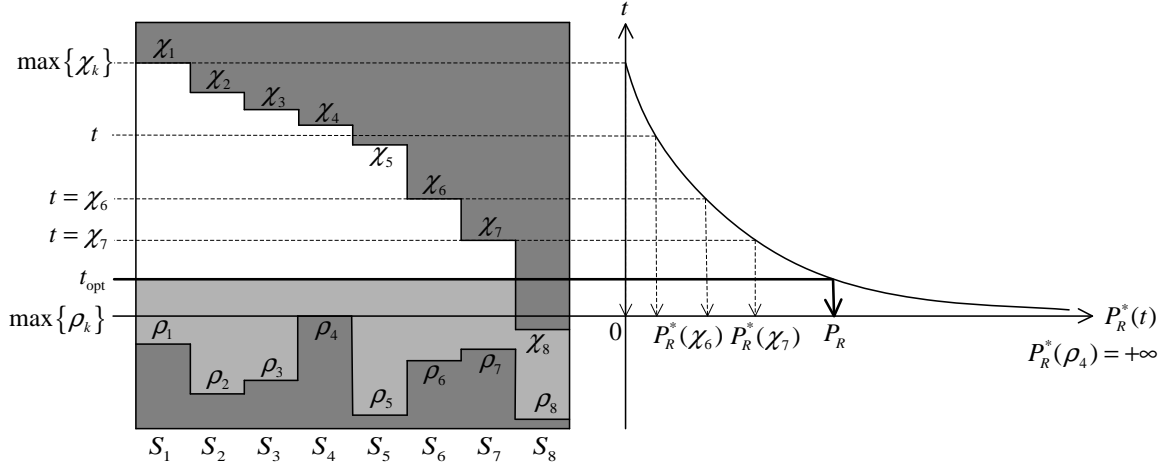
Figure 5.3 Power allocation in the min-max problem as a *water-discharging* scheme.

$\mu_0 = 1/\sum_{k=1}^{K} \bar{\mu}_k$ and $\mu_k = \mu_0 \bar{\mu}_k$. Based on the above observations one can eliminate $\gamma_{q,k}$ from (5.55) and (5.58) and solve for $\beta_{q,k}$ as follows:

$$
\beta_{q,k} = \begin{cases} \left(\sqrt{\frac{\bar{\mu}_k b_{q,k} \sigma_{v_2}^2}{\alpha_{q,k}}} - a_{q,k}\sigma_{v_2}^2 \right) \frac{1}{a_{q,k}c_{q,k}+b_{q,k}} & , \bar{\mu}_k > \xi_{q,k} \\ 0 & \text{otherwise} \end{cases} .
$$

Combining the above two cases of μ_k, the final optimal power allocation for problem (5.46)-(5.49) is given as follows

$$
\beta_{q,k} = \left(\sqrt{\frac{\bar{\mu}_k b_{q,k} \sigma_{v_2}^2}{\alpha_{q,k}}} - a_{q,k}\sigma_{v_2}^2 \right)^+ \frac{1}{a_{q,k}c_{q,k} + b_{q,k}}, \tag{5.60}
$$

where $\{\bar{\mu}_k\}$ are chosen to satisfy

$$
t_{\text{opt}} = \sum_{q=1}^{Q} \frac{1}{\phi_{q,k}(\beta_{q,k})}, \text{ for } \bar{\mu}_k > 0 \tag{5.61}
$$

$$
t_{\text{opt}} \geq \chi_k, \text{ for } \bar{\mu}_k = 0, \tag{5.62}
$$

$$
\sum_{k=1}^{K} \sum_{q=1}^{Q} \beta_{q,k}\alpha_{q,k} = P_R. \tag{5.63}
$$

Obviously the key step is to find $\bar{\mu}_k$, $k = 1, \ldots, K$. To this end, three important remarks regarding the optimal solution are given next.

Remark 1: Due to the fact that $\lim_{\beta_{q,k}\to+\infty} \phi_{q,k}(\beta_{q,k}) = a_{q,k}+b_{q,k}/c_{q,k}$, the optimal value of the objective function, t_{opt}, is restricted by $t_{\text{opt}} > \max_{1\leq k\leq K}\{\rho_k\}$ where ρ_k is defined in (5.28).

Remark 2: At the optimality, the set $\mathcal{K}$ which includes sources that need help from the relay[7] is nonempty (i.e., it must contain at least one source). All the sources in the set $\mathcal{K}$ have the same $\text{MSE}_k = t_{\text{opt}}$, $k \in \mathcal{K}$, i.e., they achieve the same SINR. All the other sources that do not need help from the relay (i.e., not in $\mathcal{K}$) achieve larger or equal SINRs compared to those in $\mathcal{K}$. Furthermore in the set $\min_{k\in\mathcal{K}}\{\chi_k\} > t_{\text{opt}}$ where χ_k is defined in (5.27). Any source $\tilde{k}$ that is not included in $\mathcal{K}$ has $\chi_{\tilde{k}} \leq t_{\text{opt}}$ and $\beta_{q,\tilde{k}} = 0$, $\forall q$. Obviously, from Remark 1 any source j that has $\chi_j \leq \max_{1\leq k\leq K}\{\rho_k\}$ is not included in $\mathcal{K}$ either.

Remark 3: Since only the sources in set $\mathcal{K}$ need help from the relay and hence participate in the power allocation process, identifying all sources in $\mathcal{K}$ and solving (5.60), (5.61), and (5.63) is sufficient to obtain the optimal power allocation for the network.

Based on these three remarks, we shall devise in the following an efficient algorithm to find $\{\bar{\mu}_k\}$, and hence $\{\beta_{q,k}\}$.

With χ_k defined in (5.27), arrange all the sources in decreasing order of χ_k, i.e., $\chi_k \geq \chi_{k+1}$. On the other hand, with $\xi_{q,k}$ defined in (5.37) arrange the subcarriers of each source in increasing order of $\xi_{q,k}$, i.e., $\xi_{q+1,k} \geq \xi_{q,k}$. It means that all the sets $\{a_{q,k}\},\{b_{q,k}\},\{c_{q,k}\}$ and $\{\alpha_{q,k}\}$ are arranged in accordance with the decreasing order of the sets $\{\chi_k\}$ for the index k and increasing order of the set $\{\xi_{q,k}\}$ for the index q. As a result, the set of sources participating in power allocation can then be defined as $\mathcal{K} = \{1,\ldots,\hat{K} \leq K\}$ such that $\beta_{q,\hat{K}} > 0$ for some q and $\beta_{q,\hat{K}+1} = 0$, $\forall q$. Likewise, for each active source $k \in \mathcal{K}$, the set of active subcarriers used at the relay to forward

---

[7]We use notation $\mathcal{K}$ in both Sections 5.4.1 and 5.4.2 to generally define the set of sources to be assigned some transmit power by the relay in the second phase of transmission (i.e., $P_{R,k} > 0$). In Section IV-A, $\mathcal{K} = \{k = 1,\ldots,K : \rho_k < \overline{\text{MSE}}_k < \chi_k\}$ while in Section IV-B, $\mathcal{K}$ is equivalently defined as $\mathcal{K} = \{k = 1,\ldots,K : \bar{\mu}_k > 0\}$.

the signal of the kth source is defined as $\{1, \ldots, \hat{Q}_k \leq Q\}$ such that $\beta_{\hat{Q}_k,k} > 0$ and $\beta_{\hat{Q}_k+1,k} = 0$.[8]

Next, for each source k in the active set $\mathcal{K}$, (5.61) can be rewritten as:

$$t_{\text{opt}} = \sum_{q=1}^{\hat{Q}_k} \frac{c_{q,k}}{a_{q,k}c_{q,k} + b_{q,k}} + \sum_{q=\hat{Q}_k+1}^{Q} \frac{1}{a_{q,k}}$$

$$+ \frac{1}{\sqrt{\bar{\mu}_k}} \sum_{q=1}^{\hat{Q}_k} \frac{\sqrt{\alpha_{q,k} b_{q,k} \sigma_{v2}^2}}{a_{q,k}c_{q,k} + b_{q,k}}. \tag{5.64}$$

Observe that the form of (5.64) is exactly the same as (5.38) if $\overline{\text{MSE}}_k$ and μ_k are replaced by t_{opt} and $\bar{\mu}_k$, respectively. The only but also important difference is that while $\overline{\text{MSE}}_k$ in (5.38) is known, the optimal value of t_{opt} also depends on $\hat{K}$ and needs to be found as well.

Following the same notation as in Section 5.4.1, let $P_R^*(t)$ be the minimum required relay power to achieve $\text{MSE}_k = t$ for all active sources. The problem can be interpreted as a *water-discharging* scheme from a cave and it is illustrated in Fig. 5.3. Here the cave is formed by K patches, where the top and bottom of the kth patch are χ_k and ρ_k, respectively. The variable t defines the water level. For each water level t, the relay power assigned to source k is determined by the relative distance from the top of the kth patch to the water level t, i.e., $\frac{\chi_k-t}{\chi_k-\rho_k}$ (in fact the relationship is not linear). Thus, the total relay power is related to the total amount of water that needs to be discharged from the cave (in all patches). Note that with source ordering based on χ_k's, one has $P_R^*(\chi_1) = 0$. The lower the water level t is the more and more water that needs to be discharged from the cave. Furthermore the water level can never reach the bottom of any patch since it would require to discharge an infinite volume of water. This is a consequence of the fact that $t_{\text{opt}} > \max_{1 \leq k \leq K}\{\rho_k\}$ and $P_R^*(\max_{1 \leq k \leq K}\{\rho_k\}) = \infty$.

To solve the problem, we first locate the range of t_{opt} over which the number of active sources $\hat{K}$ is fixed as follows. Let $\check{K}$ be the largest source index such that

---

[8]Similar to Section 5.4.1, from this point forward, k and q are the indices after sorting.

$\chi_{\check{K}} > \max_{1 \leq k \leq K}\{\rho_k\} > \chi_{\check{K}+1}$. Due to the monotonicity property of $P_R^*(t)$ and the ordering of $\chi_k \geq \chi_{k+1}$ one has $P_R^*(\chi_k) \leq P_R^*(\chi_{k+1}) < P_R^*(\max_{1 \leq k \leq K}\{\rho_k\}) = +\infty$, for $1 \leq k \leq \check{K} - 1$. Thus one concludes that t_{opt} lies in the range $[\chi_{j+1}, \chi_j)$ if $P_R^*(\chi_j) < P_R \leq P_R^*(\chi_{j+1})$ for some $j \in [1, \check{K}-1]$, or in the range $(\chi_{\check{K}}, \max_{1 \leq k \leq K}\{\rho_k\})$ if $P_R^*(\chi_{\check{K}}) < P_R$. For the scenario depicted in Fig. 5.3, $\check{K} = 7$. Since $P_R > P_R^*(\chi_7)$ one has $\hat{K} = \check{K} = 7$, i.e., only $S_8 \notin \mathcal{K}$. This means that t_{opt} must be in the range $(\max\{\rho_k\}, \chi_7)$. Other scenario could be $P_R^*(\chi_6) < P_R < P_R^*(\chi_7)$. For such a case, the size of $\mathcal{K}$ would be $\hat{K} = 6$ (i.e., $S_7, S_8 \notin \mathcal{K}$) and t_{opt} would be in the range (χ_7, χ_6).

Note that to find $P_R^*(\chi_k)$ for $1 \leq k \leq \check{K}$ one can implement Algorithm 1 proposed in Section 5.4.1. In general $\hat{K} \leq \check{K}$. Once the range of t_{opt} and the number of active sources have been found, the monotonicity of $P_R^*(t)$ again can be exploited to find t_{opt} and $\bar{\mu}_k$, $k \in \mathcal{K}$, using an efficient line search technique such as the bisection method [17].

In summary, to find the optimal t_{opt}, one can implement the following algorithm:

Algorithm 2: Maximize the Minimum SINR

Input: K, Q, all CSI and P_R.

Output: $\{\beta_{q,k}\}$ and $\{\bar{\mu}_k\}$.

1. Arrange all sources in an decreasing order of $\{\chi_k\}$, $1 \leq k \leq K$. Find $\check{K}$, the largest source index such that $\chi_{\check{K}} > \max_{1 \leq k \leq K}\{\rho_k\} > \chi_{\check{K}+1}$.

2. For each $k \in [1, \check{K}]$, rearrange the set $\{\xi_{q,k}\}$, given in (5.37), in increasing order with respect to q. Also, rearrange all sets $\{a_{q,k}\}, \{b_{q,k}\}, \{c_{q,k}\}$ and $\{\alpha_{q,k}\}$ in accordance with the sets $\{\xi_{q,k}\}$ and $\{\chi_k\}$.

3. Calculate $P_R^*(\chi_k)$, $k \in [1, \check{K}]$. This can be easily done using Algorithm 1 provided in Section IV-A (without Step 1) to solve the sum-power minimization. If $P_R \in (P_R^*(\chi_j), P_R^*(\chi_{j+1})]$, $j \in [1, \check{K} - 1]$, then $\hat{K} = j$ and $t_{\text{opt}} \in [\chi_{j+1}, \chi_j)$. If $P_R > P_R^*(\chi_{\check{K}})$, then $\hat{K} = \check{K}$ and $t_{\text{opt}} \in (\chi_{\check{K}}, \max_{1 \leq k \leq K}\{\rho_k\})$.
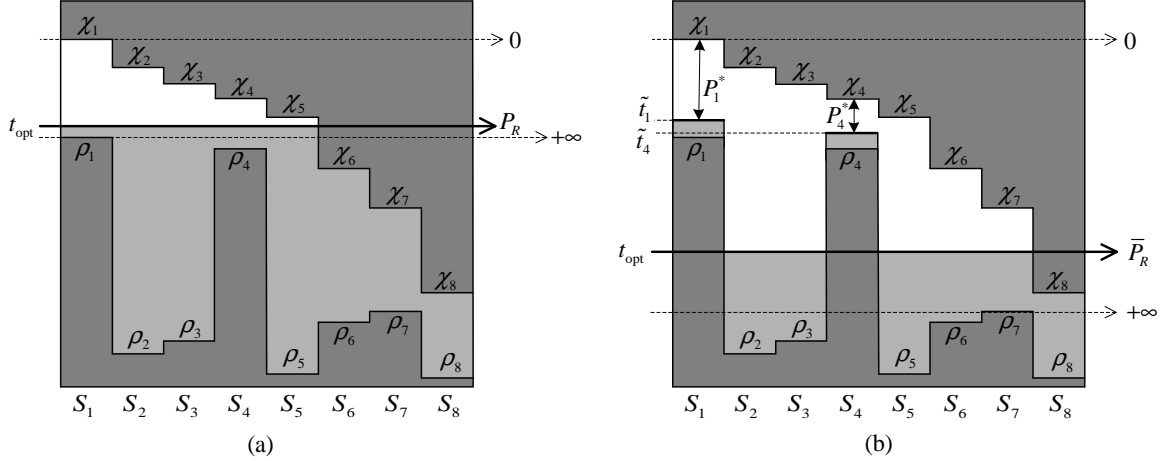
Figure 5.4 Power allocation with the modified min-max strategy which sets $\tilde{t}_k > \rho_k$ with corresponding amount of the relay transmit power P_k^* for $k = 1, 4$, and then continues reducing t_{opt} to a lower value with the remaining relay transmit power $\bar{P}_R = P_R - (P_1^* + P_4^*)$.

4. Using an efficient line search technique combined with Algorithm 1 to narrow the range of t such that $P_R^*(t) \to P_R$ iteratively, until satisfying some termination criterion[9].

At this point, it should be mentioned that the structure of the solution (5.60) is also known as a multi-level water-filling scheme where $\{\bar{\mu}_k^{1/2}\}$ are the water levels chosen to satisfy the constraints (5.61)–(5.63). These water levels indicate the power allocated on each subcarrier of each source. In fact reference [11] proposes an algorithm to find the optimal solution for a similar max-min problem. By testing the hypothesis with different upper and lower bounds of $\bar{\mu}_k$ to find the feasible set of t_{opt} and then using the bisection method to locate the optimal value of t_{opt}, all water levels $\bar{\mu}_k$'s can be found. Our problem is more complicated than the problem considered in [11] since in our case, together with $\bar{\mu}_k$ and $\hat{Q}_k$, the optimal value t_{opt} also depends on $\hat{K}$.

---

[9]The convergence is always guaranteed for Algorithm 2 due to the monotonic relationship between t and $P_R^*(t)$.

5.4.3 A Modified Max-Min Strategy

When one or a small number of sources experience severe channel conditions, $\frac{\chi_k - t_{\mathrm{opt}}}{\chi_k - \rho_k}$ may be very close to 1 and most (or all) of the available transmit power of the relay would be allocated to those sources. Consequently, the other sources will not get much (or any) help from the relay. Minimizing the weighted sum of MSE_k (e.g., see [11, 18]) or minimizing the basic level MSE with $\mathrm{MSE}_k = \tau_k \mathrm{MSE}$ (e.g., see [19] in which a basic data rate is maximized) can maintain a proportional fairness. Here, we adopt a heuristic method by adjusting $\tilde{t}_k = \tau_k t$ in order to maintain the performance of the "poor" sources at some acceptable level while the performance of other sources can be further improved.

As illustrated in Fig. 5.4, since $\frac{\chi_1 - t_{\mathrm{opt}}}{\chi_1 - \rho_1}$ and $\frac{\chi_4 - t_{\mathrm{opt}}}{\chi_4 - \rho_4}$ are quite large, we set $\tilde{t}_1$ and $\tilde{t}_4$ such that $\tilde{t}_1 > \rho_1$ and $\tilde{t}_4 > \rho_4$. These values of $\tilde{t}_1$ and $\tilde{t}_4$ require to allocate a certain relay powers, say P_1^* and P_4^*, for sources 1 and 4. We then allocate the remaining relay power, $\bar{P}_R$, for other sources in order to minimize the value of t for these sources. To this end, Algorithm 2 can be implemented with a modification that Step 1 of Algorithm 1 is included in each iteration in order to identify "poor" sources and obtain relay power allocations for them based on the sets $\{\rho_k\}$ and $\{\chi_k\}$, $k = 1, \ldots, \hat{K}$, and the *current* value of t_{opt}. Once the poor sources are identified and allocated powers accordingly, the power allocation for the remaining sources with a common t_{opt} is exactly the same as in the original problem. In general, some simple rule to select a set of reasonable values of $\tilde{t}_k$ to identify "poor" sources can be used. As an example, the rule implemented in Section 5.5.3 sets $\tilde{t}_k = 0.25\chi_k + 0.75\rho_k$ if $\frac{\chi_k - t_{\mathrm{opt}}}{\chi_k - \rho_k} > 0.75$ so that $\frac{\chi_k - \tilde{t}_k}{\chi_k - \rho_k} \leq 0.75, \forall k$. Note that other common ratios rather than 0.75 or different ratios for different users can also be applied.

Beside adapting to the channel conditions, Algorithm 2 can also be modified to support some strategies such as quality-of-service (QoS) based classification with different upper bounds of SINR for different classes of the sources (i.e., similar to the model with a basic rate proposed in [19]) or source admission control such that the relay only helps the "compensable" sources and refuses to support the other source(s)

which currently experience extremely deep fades (i.e., no relay transmit power is assigned to those "poor" sources). In other words, the three following options are considered by the relay: (i) do not help some sources, (ii) help all sources fully and fairly, or (iii) help some sources partially and the rest fully and fairly (i.e., proportional fairness).

5.4.4 Computational Complexity

In this subsection, a brief computational complexity analysis of the two proposed algorithms is presented. We follow the notation in [20] and use $\Theta(\cdot)$ to denote an asymptotically tight bound, $\mathcal{O}(\cdot)$ to denote an asymptotic upper bound.

Algorithm 1

The first step requires a running time of $\mathcal{O}(K)$. To find the set $\mathcal{K}$ requires $\mathcal{O}(|\mathcal{K}|)$ in running time while the computational cost to rearrange each set of $\{\xi_{q,k}\}$, $\{a_{q,k}\}$, $\{b_{q,k}\}$, $\{c_{q,k}\}$ and $\{\alpha_{q,k}\}$ is $\Theta(|\mathcal{K}|Q\log_2 Q)$. Step 3 in the worst-case runs in $\Theta(|\mathcal{K}|\log_2 Q)$ to find all $\hat{Q}_k$'s and $\mathcal{O}(|\mathcal{K}|)$ to find all μ_k's. Similarly, Step 4 runs in $\mathcal{O}(|\mathcal{K}|Q)$ while the cost of Step 5 is $\Theta(|\mathcal{K}|Q\log_2 Q)$. Therefore, Algorithm 1 approximately has a worst-case complexity of $\Theta(|\mathcal{K}|Q\log_2 Q)$.

Algorithm 2

Sorting $\{\chi_k\}$ requires $\Theta(K\log_2 K)$ in running time while finding $\check{K}$ needs $\mathcal{O}(K)$. The computational cost of Step 2 is $\max\{\Theta(\check{K}Q\lg Q), \Theta(\check{K}\log_2 \check{K})\}$. Since Algorithm 1 is used to find $P_R^*(\chi_k)$, Step 3 requires $\Theta(\check{K}Q\log_2 Q)$. Assuming the total number of iterations in Step 4 is M_{iter}, the cost of this step is $\Theta(M_{\text{iter}}\check{K}Q\log_2 Q)$. With a tolerance of ϵ, M_{iter} should satisfies $M_{\text{iter}} > \log_2\left(\frac{t_{\text{opt}}^{\text{upper}}-t_{\text{opt}}^{\text{lower}}}{\epsilon}\right)$, where $t_{\text{opt}}^{\text{upper}}$ and $t_{\text{opt}}^{\text{lower}}$ are the upper and lower bounds of the t_{opt} range found in Step 3. Putting all together, Algorithm 2 has approximately a complexity of $\max\{\Theta(M_{\text{iter}}\check{K}Q\log_2 Q), \Theta(\check{K}\log_2 \check{K})\}$.

5.5 Simulation Results

In this section, performance of the proposed power allocation schemes is studied via computer simulation. The total number of available subcarriers in the system (with a bandwidth $W = 5\text{MHz}$) is $N = 256$ and is assigned to $K = 16$ sources, each with a group of $Q = 16$ subcarriers. The sampling rate is 5×10^6 samples/sec. The cyclic prefix length is 16 samples (i.e., 3.2 μsec). Both I-FDMA and L-FDMA subcarrier mapping approaches are considered. Each source uses QPSK modulation to transmit signal at the same power level P_S. The total transmit power of the relay is set at $P_R = KP_S$. Rayleigh fading channels of order $L_f = 7$ are assumed for the direct paths $\mathbf{f}_k$, where the complex zero-mean Gaussian distributed taps have an exponential power profile of $\mathbb{E}\{|f_{lk}|^2\} = \exp(-l)/(\kappa_k \sum_{n=0}^{L_f-1} \exp(-n))$, $0 \leq l \leq L_f - 1$. Simulations in Section 5.5.1 are run with $\kappa_k = K, \forall k$, while other simulations are run with $\kappa_k = 2^{(k-9)}$ for $k > 9$ and $\kappa_k = 2$ for $k \leq 9$. For the Rayleigh "uplink" and "downlink" channels $\mathbf{g}_k, \mathbf{h}$, also assume $L_g = L_h = 7$, and the power profiles are $\mathbb{E}\{|g_{lk}|^2\} = \exp(-l)/(\sum_{n=0}^{L_g-1} \exp(-n))$, $0 \leq l \leq L_g - 1$, and $\mathbb{E}\{|h_l|^2\} = \exp(-l)/(\sum_{n=0}^{L_h-1} \exp(-n))$, $0 \leq l \leq L_h - 1$. The noise variances at the relay and destination are all normalized to unity, namely $\sigma_n^2 = \sigma_{v_1}^2 = \sigma_{v_2}^2 = 1$. Thus the input signal-to-noise ratio is simply defined as $\text{SNR} = P_S$.

5.5.1 Performance of Relay-Assisted Transmission

Fig. 5.5 illustrates the benefit of deploying a relay in SC-FDMA multiuser network by comparing the error performance obtained with and without the relay for the first user. When no relay is deployed, the transmit power at each source is increased to $2P_S$. On the other hand for the case of relay-assisted transmission the total relay power KP_S is equally divided among K sources and the power of each source is equally divided over all subcarriers, namely equal power allocation (EPA) (see (5.21)). Therefore the total transmit power per source is the same in both transmission methods. As can be seen from Fig. 5.5, since MMSE-FDE can effectively mitigate the inter-symbol interference, the relaying scheme significantly improves the
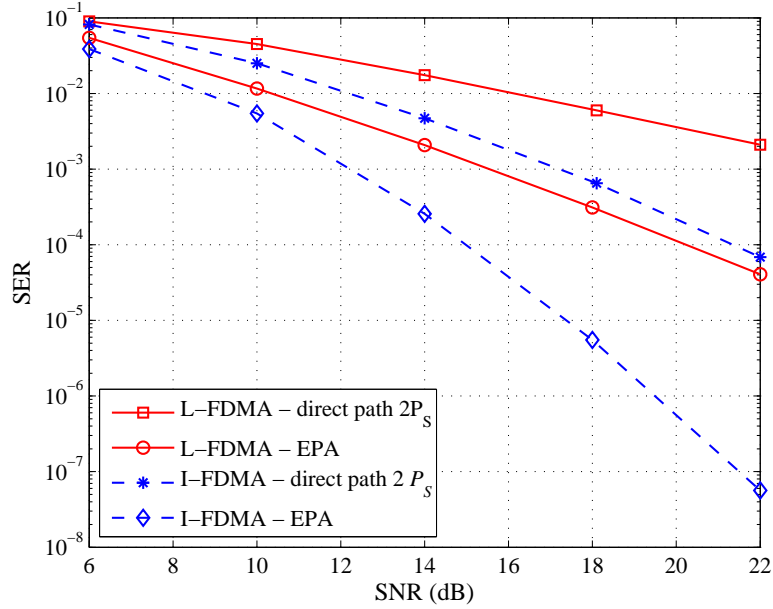
Figure 5.5   Performance comparison between transmissions without and with the help of relay for User 1 under equal power allocation (EPA).

error performance. In particular, at the SER level of 10^{-3} a reduction of about 5 dB in SNR can be achieved with the help of the relay in our simulation setup. Also observe that the I-FDMA mapping approach significantly outperforms the L-FDMA approach in both transmission methods. This is well expected and a consequence of the inherent frequency diversity of I-FDMA.[10]

5.5.2   Power Allocation for Relay Power Minimization

As discussed in Section 5.4.1, due to the special form of the SINR_k, the sum relay-power minimization problem does not always have a feasible solution. To compare the efficiency between the proposed optimal power allocation (OPA) scheme and the equal power allocation (EPA), Table 5.1 provides numerical results on the average relay power, P_R, required by the OPA and EPA schemes when the initial required

---

[10]It should be noted that, compared to the L-FDMA mapping approach which assigns a set of consecutive subcarriers to each source, the I-FDMA mapping approach, which exploits the frequency diversity by assigning a set of distributed subcarriers to each source, is more sensitive to carrier frequency offsets. As a consequence, the I-FDMA mapping needs a more efficient synchronization method.

SINRs are set as $\overline{\text{SINR}}_k = \overline{\text{SINR}}$, $\forall k$. As pointed out in Section 5.4.1, when the channel condition of the kth source is poor such that $\rho_k \geq \overline{\text{MSE}} = Q/(\overline{\text{SINR}} + 1)$, SINR_k can never meet the required value $\overline{\text{SINR}}$, no matter how large the transmit power available at the relay is. In this situation, we simply adjust $\overline{\text{MSE}}_k$ to $1.1\rho_k$. Note that this adjustment is adaptively carried out with each channel realization. In the simulation, the percentage of the sources that does not need help from the relay (denoted by % Higher), the percentage of the sources with the modified $\overline{\text{MSE}}_k$ (denoted by % Lower) and the minimum transmit power required at the relay to help the sources to achieve the adjusted set $\{\overline{\text{MSE}}_k\}$ for both schemes, EPA and OPA, are calculated and averaged over $10,000$ channel realizations.

Table 5.1   Comparison of Relay Powers by the OPA and EPA Schemes.

$\overline{\text{SINR}}$ (dB)	% Higher	% Lower	$\bar{P}_R^{(\text{EPA})}$ (dB)	$\bar{P}_R^{(\text{OPA})}$ (dB)
4	59.932	0.307	30.342	29.088
6	44.235	1.869	45.664	40.389
8	26.887	6.834	44.778	40.805
10	13.536	18.479	49.133	45.282
12	5.564	38.938	52.694	48.736

With the transmit power at each source set at $P_S = 14$ dB, Table 5.1 shows that the power saving offered by the proposed OPA scheme is very significant. In particular, the required transmit power at the relay using the OPA scheme is as low as $50\% - 70\%$ of the relay power in the EPA scheme. It should be noted that when the required SINR increases, one may need to increase the transmit power at each sources in order to reduce the percentage of the sources that have poor direct paths and uplink channels.
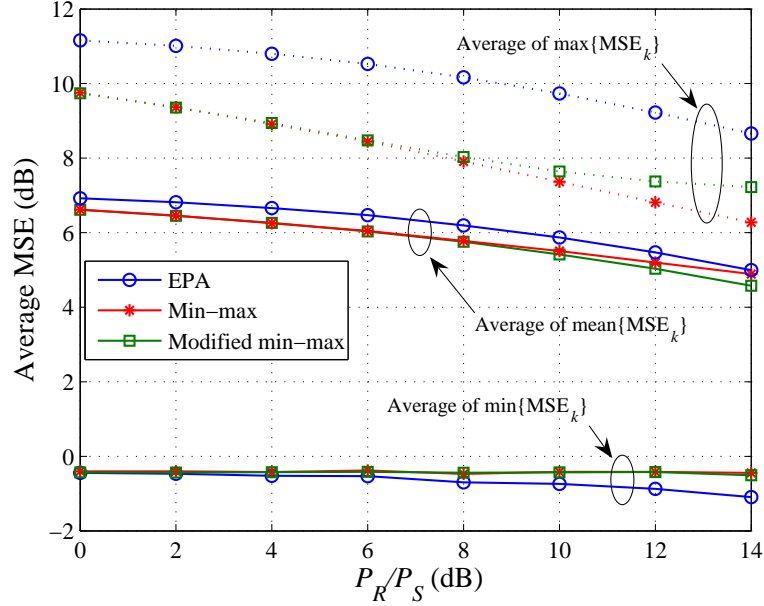
Figure 5.6 Averages of the maximum, mean and minimum of MSE_k obtained from the proposed power allocation schemes and the EPA with different relay transmit power P_R. The transmit power at each source is $P_S = 14$ dB and I-FDMA subcarrier mapping approach is used.

5.5.3 Power Allocation for Maximization of Minimum SINR

In order to compare the effectiveness of the proposed OPA based on the SINR max-min problem and an equal power allocation, averages of the maximum, mean and minimum MSEs are calculated and plotted in Fig. 5.6. It can be seen that the difference between the averaged maximum and minimum MSEs among the sources is reduced with the proposed OPA scheme, implying more fairness among the sources. However, due the strong dependence on the MSE of the "poor" source(s), the overall improvement in terms of maximum MSE is affected. As discussed in Section 5.4.2, the main reason is due to source(s) with a small difference $\chi_k - \rho_k$, which limits the range of the optimal water level t_{opt}.

To deal with the above mentioned scenario, the modified min-max solution is applied. Based on the instantaneous channel conditions, all the sources are classified into three groups, namely "poor", "fair" and "good". While the good sources do

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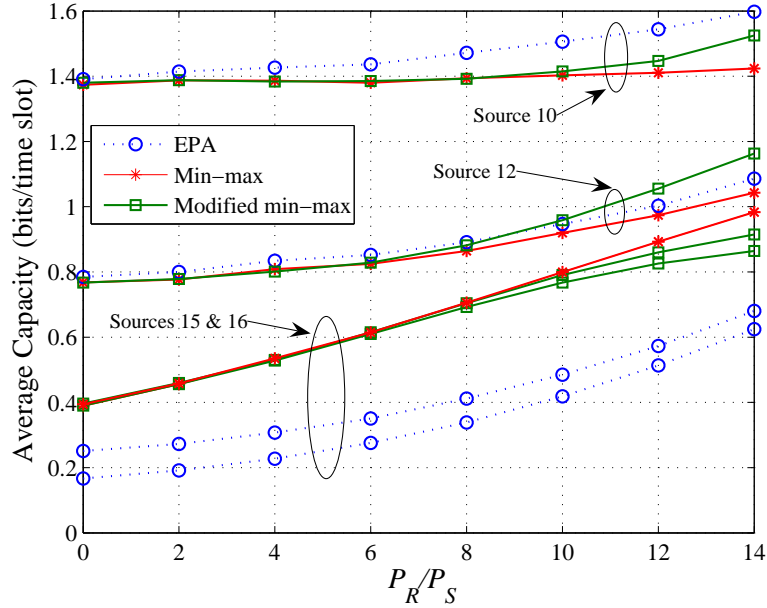
Figure 5.7 Averages of the capacity of each source obtained from the proposed power allocation schemes and the EPA with different relay transmit power P_R. The transmit power at each source is $P_S = 14$ dB and I-FDMA subcarrier mapping approach is used.

not need any support from the relay, the target MSE of the poor source j shall be set to some suitable value $\tilde{t}_j$. Also shown in Fig. 5.6 is the improvement achieved with the modified min-max algorithm when the MSEs of the poor sources are set as $\tilde{t}_k = 0.25\chi_k + 0.75\rho_k$ if $\frac{\chi_k - t_{\text{opt}}}{\chi_k - \rho_k} > 0.75$. With this modification, some poor sources will experience slightly worse performances. In contrast, the performance of the fair and good sources can be further improved, which leads to a reduction in the averaged mean value of the MSEs achieved by all sources. This behavior can be verified in Fig. 5.7 where the decrease in the average capacities of the poor sources, 15 and 16, is a trade-off for the increase in the capacities of the fair source 12 and the good source 10. Also observed from Fig. 5.7 that the capacity curves of the sources 15 and 16 when Algorithm 2 is applied overlap (the capacities of other sources are not included in the figure for a better clarity). As the transmit power at the relay increases, the modified strategy becomes more efficient.

Table 5.2   Percentages of poor, fair and good sources in the modified min-max power allocation.

P_R/P_S (dB)	Poor sources (%)	Fair sources (%)	Good sources (%)
0	0.004	19.324	80.672
4	0.271	24.256	75.473
8	2.855	30.068	67.077
12	13.496	35.123	50.391

Finally, shown in Table 5.2 are the percentages of the poor, fair and good sources with different transmit powers at the relay. Obviously, when we try to increase the optimal value of t, the percentage of the poor sources will increase. However, it is worth noting that even with a very high transmit power level available at the relay, the improvement gain for those poor sources is very limited once their MSE_k approaches ρ_k. With a small performance sacrifice experienced by the poor sources, the performance of other sources, including some good sources, in the network can be further improved. By choosing some appropriate criterion to classify the sources, the fairness for most of the sources in the network can be maintained by the modified min-max power allocation.

5.5.4   The PAPR Characteristics

Since the proposed power allocation schemes are carried out at the relay, the PAPR characteristics at each source is unchanged. On the other hand, as can be seen in Fig. 5.8, the PAPR characteristics at the relay with the proposed power allocation schemes is better than that with the EPA. This can be explained from the fact that both proposed schemes try to balance the SINR of the sources' signals received at the destination, and hence implicitly the power of the transmitted signals at the relay, according to the current fading conditions of all the channels. In contrast, if all the sources transmit signals with the same power as in the case of EPA, the variation in
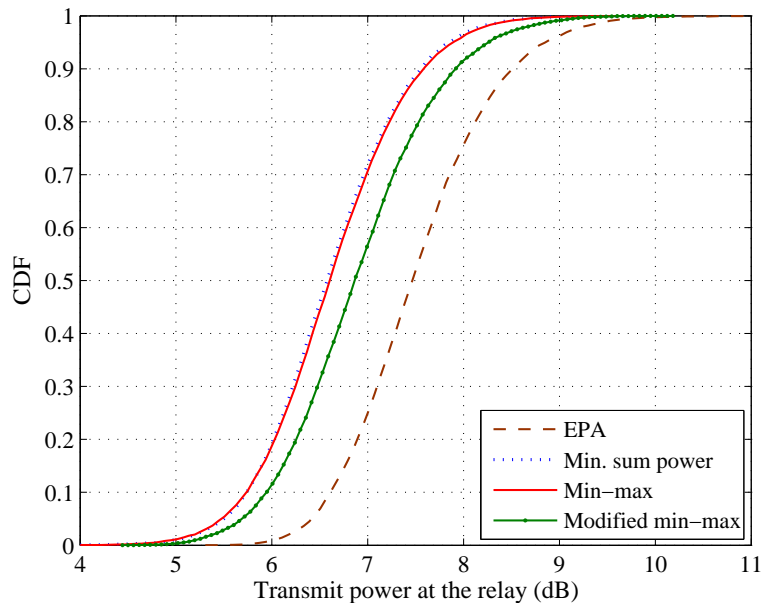
Figure 5.8   Peak-to-average-power-ratio (PAPR) characteristics of the proposed schemes.

channel conditions together with the noise at the relay lead to a higher PAPR of the transmitted signal.

5.6   Conclusion

In this paper, an amplify-and-forward relaying scheme for multiuser cooperative networks under frequency-selective block-fading channels was developed and studied. The single-carrier frequency division multiple access technique with frequency-domain equalization was shown to be able to mitigate the inter-block and inter-symbol interference effectively, which guarantees performance improvement offered by relay transmission. Two optimal power allocation schemes have been obtained to further improve the network performance. The first power allocation scheme offers a large saving of the total relay transmit power by optimally distributing the relay power among the subcarriers, while meeting the SINR constraint at the destination for each user. The second power allocation scheme and its modified version were shown to be able to maximize the worst-user performance under a constraint on the total relay

power. In particular, it was shown that the second power allocation scheme can be obtained efficiently by exploiting the optimal solution of the first power allocation problem. The second power allocation scheme can be flexibly modified to adapt with different channel conditions as well as different QoS provision strategies. Since all the power allocation schemes are carried out at the relay, the PAPR characteristics of each source is not affected. Interestingly, the PAPR characteristics of the transmitted signal at the relay can be improved by the proposed schemes.

5.A   Average Power of the Transmitted Signal at the Relay

It follows from (5.6) that

$$
\mathbb{E}\left\{\tilde{\mathbf{r}}_k\tilde{\mathbf{r}}_k^{\mathcal{H}}\right\} = P_{S,k}\boldsymbol{\Delta}_k\tilde{\boldsymbol{\Lambda}}_k\tilde{\boldsymbol{\Lambda}}_k^{\mathcal{H}}\boldsymbol{\Delta}_k^{\mathcal{H}} + \sigma_n^2\boldsymbol{\Delta}_k\boldsymbol{\Delta}_k^{\mathcal{H}} \tag{5.65}
$$

$$
= \text{diag}\left(|\delta_{1,k}|^2(P_{S,k}|\tilde{\lambda}_{1,k}|^2+\sigma_n^2),\ldots,\right.
$$

$$
\left.|\delta_{Q,k}|^2(P_{S,k}|\tilde{\lambda}_{Q,k}|^2+\sigma_n^2)\right)
$$

$$
= \text{diag}\left(\frac{P_{S,k}^2|\tilde{\lambda}_{1,k}|^2}{P_{S,k}|\tilde{\lambda}_{1,k}|^2+\sigma_n^2},\ldots,\frac{P_{S,k}^2|\tilde{\lambda}_{Q,k}|^2}{P_{S,k}|\tilde{\lambda}_{Q,k}|^2+\sigma_n^2}\right)
$$

With $\boldsymbol{\Theta}_k$ defined in (5.1) and (5.2), the covariance matrix of the transmitted signal at the relay can be calculated as

$$
\mathbb{E}\left\{\left(\mathcal{F}_N^{\mathcal{H}}\sum_{k=1}^{K}\boldsymbol{\Theta}_k\boldsymbol{\beta}_k^{1/2}\tilde{\mathbf{r}}_k\right)\left(\mathcal{F}_N^{\mathcal{H}}\sum_{k=1}^{K}\boldsymbol{\Theta}_k\boldsymbol{\beta}_k^{1/2}\tilde{\mathbf{r}}_k\right)^{\mathcal{H}}\right\}
$$

$$
= \mathcal{F}_N^{\mathcal{H}}\text{diag}\left(\boldsymbol{\beta}_1\mathbb{E}\left\{\tilde{\mathbf{r}}_1\tilde{\mathbf{r}}_1^{\mathcal{H}}\right\},\ldots,\boldsymbol{\beta}_K\mathbb{E}\left\{\tilde{\mathbf{r}}_K\tilde{\mathbf{r}}_K^{\mathcal{H}}\right\}\right)\mathcal{F}_N
$$

$$
\triangleq \mathcal{F}_N^{\mathcal{H}}\mathcal{D}\mathcal{F}_N, \tag{5.66}
$$

where $\mathcal{D}\triangleq\text{diag}(d_1,d_2,\ldots,d_N)$ and each d_i has the form of $\frac{\beta_{q,k}P_{S,k}^2|\tilde{\lambda}_{q,k}|^2}{P_{S,k}|\tilde{\lambda}_{q,k}|^2+\sigma_n^2}$ with $i = (k-1)Q+q$. Since all the diagonal elements of the covariance matrix $\mathcal{F}_N^{\mathcal{H}}\mathcal{D}\mathcal{F}_N$ are $\frac{1}{N}\sum_{i=1}^{N}d_i$, the transmitted power at the relay can be calculated as given in (5.9).

5.B   Proof of Proposition 1

To prove that $\vartheta_k(\mu_k)$ is monotonically increasing with $\mu_k > 0$, we need to prove that $\vartheta_k(\mu_k)$ is monotonically increasing with μ_k in each region $(\xi_{\hat{q}k}, \xi_{k,\hat{q}+1}]$ and $\vartheta_k(\mu_{k,1}) < \vartheta_k(\mu_{k,2})$ for $\mu_{k,1} \in (\xi_{\hat{q}-1,k}, \xi_{\hat{q}k}]$ and $\mu_{k,2} \in (\xi_{\hat{q}k}, \xi_{\hat{q}+1,k}]$, $\forall \hat{q} \in [2, Q-1]$.

Since all the subcarriers are arranged in increasing order of $\xi_{q,k}$ for each source (i.e., $\xi_{q+1,k} \geq \xi_{q,k}$), if $\mu_k \in (\xi_{\hat{q}k}, \xi_{\hat{q}+1,k}]$ then $\beta_{q,k} > 0$ for $1 \leq q \leq \hat{q}$. From (5.38), replacing $\hat{Q}_k$ by $\hat{q}$, it is clear that $\vartheta_k(\mu_k)$ monotonically increases with μ_k in each slot $(\xi_{\hat{q},k}, \xi_{\hat{q}+1,k}]$, $1 \leq \hat{q} \leq Q-1$.

For $\mu_{k,1} \in (\xi_{\hat{q}-1,k}, \xi_{\hat{q}k}]$ and $\mu_{k,2} \in (\xi_{\hat{q}k}, \xi_{\hat{q}+1,k}]$, from (5.38) one has:

$$\vartheta_k(\mu_{k,2}) - \vartheta_k(\mu_{k,1}) \tag{5.67}$$

$$= \frac{1}{a_{\hat{q},k}} - \frac{c_{\hat{q},k}}{a_{\hat{q}k}c_{\hat{q}k} + b_{\hat{q},k}} + \zeta_k - \frac{1}{\mu_{k,2}^{1/2}} \frac{\sqrt{\alpha_{\hat{q}k}b_{\hat{q}k}\sigma_{v_2}^2}}{a_{\hat{q}k}c_{\hat{q}k} + b_{\hat{q},k}}$$

$$> \frac{1}{a_{\hat{q},k}} - \frac{c_{\hat{q},k}}{a_{\hat{q}k}c_{\hat{q}k} + b_{\hat{q},k}} + \zeta_k - \frac{1}{\xi_{\hat{q},k}} \frac{\sqrt{\alpha_{\hat{q}k}b_{\hat{q}k}\sigma_{v_2}^2}}{a_{\hat{q}k}c_{\hat{q}k} + b_{\hat{q},k}} = \zeta_k,$$

where $\xi_{\hat{q}k}$ is given in (5.37) and $\zeta_k = \left(\frac{1}{\mu_{k,1}^{1/2}} - \frac{1}{\mu_{k,2}^{1/2}}\right) \sum_{q=1}^{\hat{q}-1} \frac{\sqrt{\alpha_{q,k}b_{q,k}\sigma_{v_2}^2}}{a_{q,k}c_{q,k}+b_{q,k}} > 0$. Thus $\vartheta_k(\mu_{k,1}) < \vartheta_k(\mu_{k,2})$ and the proof is completed.

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6. Decorrelate-and-Forward Relaying Scheme for Multiuser Wireless CDMA Networks

The manuscript included in this chapter is another contribution in developing an optimal power allocation scheme for relay networks with multiple sources. Different from the work presented in Chapter 5, the network model considered in this chapter consists of multiple relays. Adopting the code division multiple access (CDMA) technique, a multiuser relay-assisted transmission is transformed into multiple parallel single-source relaying transmissions where multiple access interference (MAI) is completely eliminated. A coordinated beamforming scheme at the relays combined with power allocation scheme at both the sources and relays is developed and solved by a successive convex method [1]. This method can be considered as a special case of alternative optimization [2].

Decorrelate-and-Forward Relaying Scheme for Multiuser Wireless CDMA Networks

Tung T. Pham, Ha H. Nguyen

Abstract

Decorrelate-and-forward relaying scheme for wireless synchronous CDMA networks where multiple sources communicate with a destination supported by multiple relays in flat Rayleigh fading channels is considered. By exploiting equicorrelated spreading sequences, a simple near-far resistant decorrelator together with a beamforming scheme at each relay and the maximal-ratio-combiner (MRC) at the destination are developed in order to achieve the full cooperative diversity for every source. Different from the case with a single source, the beamforming design in multiple-source networks needs to take into account the amounts of transmit power each relay spends to forward the signals from the sources. As such, we also propose a novel power allocation scheme to improve the fairness among the sources in terms of the instantaneous signal-to-noise ratio. Simulation results demonstrate the effectiveness of the proposed solution.

Index terms

Wireless relay networks, decorrelate-and-forward relaying, CDMA, MRC receiver, beamforming, power allocation, geometric programming.

6.1 Introduction

Relaying techniques for multi-source (or multiuser) cooperative networks have been recently considered and analyzed in a number of papers [3–6]. In particular,

power allocation for orthogonal amplify-and-forward (AF) relay networks with multiple source-destination pairs supported by multiple relays is considered in [3]. It is assumed in [3] that there is no direct transmission between a source and its destination and the transmission from each source to its destination is assisted by only one relay. Note that if the relay assignment is fixed at the connection setup phase, diversity is not fully exploited in the system. The simplicity of the network model considered in [3] leads to a simple power allocation problem in the form of standard geometric programming, which can be readily solved by some efficient numerical tools [7]. In contrast, relay selection schemes for orthogonal decode-and-forward (DF) cooperative networks with multiple source-destination pairs are proposed in [4], where each node has data to transmit to its own destination and also acts as a potential relay for other nodes. If all relays forward the information data for all sources, the number of orthogonal channels required is K for the first phase and $K(K-1)$ for the second phase, where K is the number of sources (which is also the number of source-destination pairs). Relay selection is therefore implemented in order to avoid the large bandwidth consumption required to support orthogonal channels. However, in order to maintain the full diversity order the relay selection schemes proposed in [4] require that each destination knows the set of all relays which successfully decode its source's information. Furthermore, the information on which relays should be chosen needs to be sent back from the destinations to all relays. It should also be pointed out that only equal transmit power at the relays is considered in [4].

In [5], a multiuser cooperative network with non-orthogonal transmission scheme using complex field network coding is proposed. Using the maximum likelihood multiuser detection, the full diversity order with a throughput as high as $1/2$ symbol per source per channel use is proved to be achieved at high signal-to-noise ratio (SNR) region. It is worth pointing out that in orthogonal transmission schemes, the throughput is only $1/(2K)$ symbol per source per channel use. However, when the number of users, K, increases (e.g., $K \geq 4$), although the diversity order is maintained, performance degradation is very severe. Furthermore, the fact that relay selection schemes

outperform parallel relaying schemes means that multiple-access interference (MAI) and inter-relay interference (IRI) might not be effectively eliminated when the number of users is large. Another AF relaying scheme with multiple source-destination pairs is considered in [6] with non-orthogonal transmissions from all the sources in the first phase and from all the relays in the second phase. With a well designed distributed beamforming scheme among all the relays, both MAI and IRI can be proportionally reduced with the number of relays. However the drawback is that perfect channel state information needs to be made available at all the relays.

Applying code-division multiple access (CDMA) technique in multiuser cooperative networks has also been studied in [8, 9]. Specifically, simulation results in [8] show that with the AF relaying scheme, spatial diversity may be achievable *only* when the MAI and IRI are efficiently mitigated. This conclusion in [8] is then proved for the scheme proposed in [9] where the minimum mean-squared error multiuser detector (MMSE-MUD) is applied at the relays and destination. By using the so called "relay-assisted decorrelator" (RAD), the signals forwarded from the relays are separated completely while the noise at the destination is kept uncorrelated. In [9], a beamforming technique is also implemented at the relays. However, the error performance is still limited since the beamforming vector for each user is calculated based on the statistical mean values of the sources' symbols decoded at the relays. Due to the complexity of multiuser detection, power allocation is not considered in [9].

In [10], a simple CDMA multiuser decorrelating scheme is introduced for non-cooperative transmission when equicorrelated spreading codes are employed. With this scheme, in addition to K spreading codes for K users, one spreading code is reserved for detecting the signals from all users. Therefore, the receiver consists of a bank of $K + 1$ matched filters. The detection of the transmitted signal from each user can be carried out by first subtracting the output of each matched filter to the output of the "reserved" matched filter so that the MAI is removed. This decorrelation approach is much preferred to the conventional linear multiuser detector [11] since it does not need a matrix inversion in its operation.

This paper develops a "decorrelate-and-forward" (DCF) relaying scheme for multi-source multi-relay cooperate wireless synchronous CDMA networks, which exploits the simple decorrelator [10] with a noise whitening process (e.g., see [9]) and the joint signal processing at the relays and destination proposed in [12]. The combined scheme can completely eliminate the MAI at the expense of a small reduction in bandwidth efficiency due to the need of having a reserved spreading code. This reduction is negligible when the number of user becomes large. As a result, the full diversity order can be achieved. Furthermore, we also propose a power allocation scheme in order to maximize the minimum signal-to-noise ratio (SNR) of the signals from all sources received at the destination under the total and/or individual transmit power constraints on the sources and relays. Assuming that perfect channel state information is available at the destination, all parameters of the beamforming scheme (used at the relays) and power allocation (used at the sources) are computed at the destination and then sent back to all sources and relays before the data transmissions are carried out. Simulation results show that the proposed scheme can successfully balance the instantaneous SNRs, and consequently the error performances, among all the users.

The rest of the paper is organized as follows. Section II describes the system model for a multiuser wireless CDMA network and the DCF relaying scheme. Section III proposes a power allocation scheme among the sources and/or relays in order to balance the instantaneous SNRs of the signals from all sources received at the destination under total and individual power constraints. Simulation results are presented and discussed in Section IV. Some concluding remarks are given in Section V.

Notations: Italic, bold lower-case and bold upper-case letters denote scalars, vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ stand for complex conjugate, transpose, and Hermitian transpose, respectively. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a vector of complex Gaussian random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, while $\text{diag}(x_1, x_2, \ldots, x_N)$ represents a diagonal matrix with $x_1, x_2, \ldots, x_N$ on its diagonal.

6.2 Multiuser Wireless CDMA Networks with Decorrelate-and-Forward Relaying Scheme
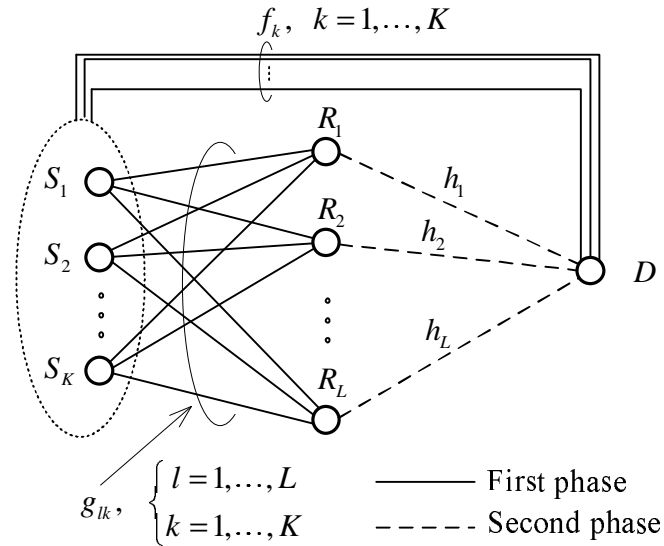
Figure 6.1 A multiuser cooperative network with K sources and L relays.

Figure 6.1 illustrates a synchronous wireless CDMA cooperative network where K source terminals (or users), $S_1, \ldots, S_K$, communicate with the destination terminal, D, with the help of L relay terminals, $R_1, \ldots, R_L$. All terminals are equipped with one antenna which operates in a half-duplex mode. All the "direct" channels (sources-destination), "uplink" channels (sources-relays), and "downlink" channels (relays-destination) are subject to independently Rayleigh distributed fading with different variances (i.e., the average attenuations of individual channels are different).

It is noted that synchronization in wireless cooperative networks is a challenging task. In general, if a central processing unit is available (e.g., can be implemented at the destination) and the energy consumption constraint is not so strict, network synchronization can be achieved in the same principle as in conventional cellular networks by exploiting a master-slave structure with the central unit broadcasting a training signal. Otherwise, mutual time and carrier synchronization methods (also referred to as distributed synchronization) could be applied [13, 14]. However, in this paper we assume perfect synchronization as also considered in [3, 8, 9, 15] and leave the synchronization issue as a potential problem for future works.

In the system model under consideration, each source is assigned a unique equicor-related signature waveform $c_k(t)$ of duration $0 \leq t \leq T_b$, where T_b is the symbol duration (which is also the bit duration if binary phase-shift keying (BPSK) modu-lation is used). All the signature waveforms are normalized to have unit energy, i.e., $\int_0^{T_b} c_k^2(t)\mathrm{d}t = 1, k = 1, \ldots, K$. Moreover, one spreading waveform, $c_0(t)$, is reserved for detecting the signals from all the sources. The constant cross-correlation between any two spreading waveforms is $\rho = \int_0^{T_b} c_k(t)c_i(t)\mathrm{d}t, \forall k \neq i$. Typically, each signature waveform, $c_k(t)$, is constructed from a spreading code $\mathbf{c}_k = [c_{1k}, \ldots, c_{Nk}]^T$ as follows:

$$c_k(t) = \sum_{n=1}^{N} c_{nk}p(t - (n - 1)T_c) \tag{6.1}$$

where $c_{nk} \in \{-1, +1\}$, $T_c = T_b/N$ is called the chip duration and $p(t)$ represents the chip waveform, which is limited to $[0, T_c]$ and has an energy of $1/N$. The length N of the spreading code is also known as the *processing factor*.

If random spreading codes are employed, then the MMSE-MUD can be used at the receiver sides to decorrelate the signals. However, by using equicorrelated spreading sequences, a much simpler receiver structure, which was originally proposed in [10] for *non-cooperative* CDMA transmission, can significantly reduce the overall complexity of the signal processing at the relays in the multi-user cooperative network.

With the DCF relaying protocol, every transmission consists of two phases. In the first phase, the kth source transmits symbol b_k through the channel f_k to the destination and through the channel g_{lk} to the lth relay. At the destination, the received signal can be written as follows:

$$y^{(1)}(t) = \sum_{k=1}^{K} \sqrt{P_{S,k}} f_k b_k c_k(t) + n^{(1)}(t), \quad t \in [0, T_b], \tag{6.2}$$

where $P_{S,k}$ is the transmitted power of the kth source and $n^{(1)}(t)$ represents a white Gaussian noise process at the destination in the first phase. The kth matched-filter's

output is

$$
\begin{aligned}
y_k^{(1)} &= \int_0^{T_b} y^{(1)}(t) c_k(t) \mathrm{d}t \\
&= \sqrt{P_{S,k}} f_k b_k + \rho \sum_{\substack{i=1 \\ i \neq k}}^{K} \sqrt{P_{S,i}} f_i b_i + \mathbf{c}_k^H \mathbf{n}^{(1)},
\end{aligned}
\tag{6.3}
$$

while the output of the reserved matched-filter is

$$
\begin{aligned}
y_0^{(1)} &= \int_0^{T_b} y^{(1)}(t) c_0(t) \mathrm{d}t \\
&= \rho \sum_{i=1}^{K} \sqrt{P_{S,i}} f_i b_i + \mathbf{c}_0^H \mathbf{n}^{(1)},
\end{aligned}
\tag{6.4}
$$

where $\mathbf{c}_k = [c_{1k}, \dots, c_{Nk}]^T$ is the spreading code corresponding to the kth user, $\mathbf{c}_0$ is the "reserved" spreading code and $\mathbf{n}^{(1)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ represents the additive white Gaussian noise vector.

Based on (6.3) and (6.4), the *soft estimate* of the information symbol b_k in the first phase can be calculated as

$$
\begin{aligned}
z_k^{(1)} &= y_k^{(1)} - y_0^{(1)} \\
&= (1 - \rho)\sqrt{P_{S,k}} f_k b_k + (\mathbf{c}_k^H - \mathbf{c}_0^H)\mathbf{n}^{(1)},
\end{aligned}
\tag{6.5}
$$

where the noise term is

$$
\eta_k^{(1)} = (\mathbf{c}_k^H - \mathbf{c}_0^H)\mathbf{n}^{(1)} \sim \mathcal{CN}(0, 2(1 - \rho)\sigma^2).
$$

It is noted that the result in (6.5) is similar to that obtained with the conventional decorrelator in a CDMA system, where the MAI is completely removed. The detector in (6.5) is therefore near-far resistant. The key difference is that the equicorrelation-based decorrelator does not require an inversion of the correlation matrix of the signature waveforms, which leads to a significant reduction in the complexity of the receivers, especially when the number of users is large. Compared to the *single-user* case, since the powers of the received signal and the noise in (6.5) are scaled by factors $(1 - \rho)^2$ and $2(1 - \rho)$, respectively, there is a reduction ratio of $\Delta_c =$

$2/(1-\rho)$ experienced by the SNR. It is noted that Δ_c can also be considered as a noise amplification factor.

Similarly, the signals produced by the equicorrelation-based decorrelator at the lth relay in the first phase can be represented by

$$r_{lk} = (1-\rho)\sqrt{P_{S,k}}g_{lk}b_k + \nu_{lk}, \quad k = 1, \ldots, K, \tag{6.6}$$

where g_{lk} is the "uplink" channel from the kth user to the lth relay, $\nu_{lk} \sim \mathcal{CN}(0, 2(1-\rho)\sigma^2)$ accounts for the white Gaussian noise experienced in this channel. In the second phase, each signal r_{lk} component is first normalized by α_{lk} and then processed by the "beamforming" coefficient by w_{lk}. To have a transmitted power of $|w_{lk}|^2$ for the information symbol of the kth user by the lth relay, the normalization factor α_{lk} is as follows:

$$\alpha_{lk} = \frac{1}{\sqrt{(1-\rho)^2 P_{S,k}|g_{lk}|^2 + 2(1-\rho)\sigma^2}}, \quad k = 1, \ldots, K, \tag{6.7}$$

After the normalization and beamforming operations, the K users' signal components could be re-spread and forwarded to the destination using the same set of spreading sequences $\{c_k(t)\}, k = 1, \ldots, K$ as in the first phase. However, with such a direct re-spreading at each relay, noise amplification will happen in the second phase of the transmission. In order to reduce noise amplification at the destination, a precoding operation shall be performed at each relay as described in the following.

Let $\mathbf{r}_l = [r_{l1}, \ldots, r_{lK}]^T$. Then it follows from (6.6) that $\mathbf{r}_l$ can be written as

$$\mathbf{r}_l = (1-\rho)\mathbf{P}_S\mathbf{G}_l\mathbf{b} + \boldsymbol{\nu}_l, \tag{6.8}$$

where $\mathbf{P}_S = \mathrm{diag}(\sqrt{P_{S,1}}, \ldots, \sqrt{P_{S,K}})$, $\mathbf{G}_l = \mathrm{diag}(g_{l1}, \ldots, g_{lK})$, $\mathbf{b} = [b_1, \ldots, b_K]^T$ and $\boldsymbol{\nu}_l = [\nu_{l1}, \ldots, \nu_{lK}]^T$. Define $\mathbf{W}_l = \mathrm{diag}(w_{l1}, \ldots, w_{lK})$ and $\boldsymbol{\alpha}_l = \mathrm{diag}(\alpha_{l1}, \ldots, \alpha_{lK})$. Then the signal components obtained by normalizing and beamforming $\mathbf{r}_l$, denoted by $\bar{\mathbf{r}}_l$, is $\bar{\mathbf{r}}_l = \mathbf{W}_l\boldsymbol{\alpha}_l\mathbf{r}_l$.

Now let $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]$ be the spreading code matrix and $\mathbf{R} = \mathbf{C}^H\mathbf{C}$ be the correlation matrix. Assume that $\mathbf{R}$ is non-singular (i.e., ρ should be in the range $[-1/(K-1), 1)$ [16]). The Cholesky decomposition of $\mathbf{R}$ is $\mathbf{R} = \mathbf{L}\mathbf{L}^H$ where $\mathbf{L}$ is

a $K \times K$ lower triangular matrix. Then, at the relays, instead of being re-spread by the spreading code matrix $\mathbf{C}$, $\bar{\mathbf{r}}_l$ is re-spread by the matrix $\mathbf{CL}^{-H}$. This is, in fact, a process of orthogonalization of the spreading codes since $\mathbf{L}^{-1}\mathbf{C}^H\mathbf{CL}^{-H} = \mathbf{I}$, which was also employed in [9] under the name of noise whitening. The signals to be forwarded to the destination, $\tilde{\mathbf{r}}_l = [\tilde{r}_{1l}, \ldots, \tilde{r}_{Nl}]^T$, can be written in the following form

$$\tilde{\mathbf{r}}_l = \mathbf{CL}^{-H}\bar{\mathbf{r}}_l = \mathbf{CL}^{-H}\mathbf{W}_l\boldsymbol{\alpha}_l\mathbf{r}_l. \tag{6.9}$$

With the above precoding operation, the despreading process carried out at the destination in the second phase gives

$$
\begin{aligned}
\mathbf{z}^{(2)} &= \mathbf{L}^{-1}\mathbf{C}^H\left(\sum_{l=1}^{L} h_l\tilde{\mathbf{r}}_l + \mathbf{n}^{(2)}\right) \\
&= (1-\rho)\sum_{l=1}^{L} h_l\mathbf{W}_l\boldsymbol{\alpha}_l\mathbf{P}_S\mathbf{G}_l\mathbf{b} + \sum_{l=1}^{L} h_l\mathbf{W}_l\boldsymbol{\alpha}_l\boldsymbol{\nu}_l + \mathbf{L}^{-1}\mathbf{C}^H\mathbf{n}^{(2)}, \tag{6.10}
\end{aligned}
$$

where h_l is the "downlink" channel from the lth relay to the destination and $\mathbf{n}^{(2)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ represents the additive white Gaussian noise vector. The kth element of $\mathbf{z}^{(2)}$ can be represented by

$$z_k^{(2)} = (1-\rho)\sum_{l=1}^{L} \sqrt{P_{S,k}}w_{lk}\alpha_{lk}h_lg_{lk}b_k + \eta_k^{(2)}, \tag{6.11}$$

where the noise term is

$$\eta_k^{(2)} \sim \mathcal{CN}\left(0, 2(1-\rho)\sigma^2\sum_{l=1}^{L}|h_l|^2|w_{lk}|^2(\alpha_{lk})^2 + \sigma^2\right).$$

Finally, the signal components $z_k^{(1)}$ and $z_k^{(2)}$ obtained in both phases of transmission are combined. Let $\mathbf{z}_k = [z_k^{(1)} z_k^{(2)}]^T$. Following Proposition 1 and Corollary 2 in [12] the maximal-ratio-combiner (MRC) filter $\mathbf{m}_k$ that maximizes the signal-to-noise ratio at its output, $\hat{b}_k = \mathbf{m}_k^H\mathbf{z}_k$, can be readily found to be

$$\mathbf{m}_k = \begin{bmatrix} 2(1-\rho)\sigma^2 & 0 \\ 0 & \Omega \end{bmatrix}^{-1} \begin{bmatrix} (1-\rho)f_k \\ (1-\rho)\mathbf{w}_k\mathbf{a}_k \end{bmatrix}, \tag{6.12}$$

where $\Omega = 2(1-\rho)\sigma^2\mathbf{w}_k\mathbf{A}_k\mathbf{A}_k^H\mathbf{w}_k^H + \sigma^2$, $\mathbf{w}_k = [w_{1k}, \ldots, w_{Lk}]$ is the vector of beamforming coefficients of the L relays for the kth user, $\mathbf{a}_k = [\alpha_{1k}g_{1k}h_1, \ldots, \alpha_{Lk}g_{Lk}h_L]^T$,

and $\mathbf{A}_k = \mathrm{diag}(\alpha_{1k}h_1, \ldots, \alpha_{Lk}h_L)$. Furthermore, the corresponding SNR at the output of the MRC for the kth user can be expressed as

$$\mathrm{SNR}_k = \frac{(1-\rho)P_{S,k}|f_k|^2}{2\sigma^2} + \frac{\frac{(1-\rho)P_{S,k}}{2\sigma^2}\mathbf{w}_k\mathbf{a}_k\mathbf{a}_k^H\mathbf{w}_k^H}{\mathbf{w}_k\left(\mathbf{A}_k\mathbf{A}_k^H + \frac{1}{2(1-\rho)P_{R,k}}\mathbf{I}\right)\mathbf{w}_k^H}, \qquad (6.13)$$

where $P_{R,k} = \|\mathbf{w}_k\|^2$ is the total transmitted power *all* relays assign for the kth source in the second phase.

Also from [12], the optimal "distributed" beamforming vector for the kth user that further maximizes SNR_k can be found by applying the Rayleigh-Ritz theorem [17] to the Rayleigh quotient form of the second term of (6.13). It is given as:

$$\mathbf{w}_k = \chi_k\mathbf{a}_k^H\left(\mathbf{A}_k\mathbf{A}_k^H + \frac{1}{(1-\rho)^2 P_{R,k}}\mathbf{I}\right)^{-1} = \chi_k\tilde{\mathbf{w}}_k, \qquad (6.14)$$

where $\tilde{\mathbf{w}}_k = [\tilde{w}_{1k}, \ldots, \tilde{w}_{Lk}]$ with

$$\tilde{w}_{lk} = \frac{(1-\rho)g_{lk}^*h_l^*\sqrt{P_{S,k}|g_{lk}|^2 + \frac{2}{1-\rho}\sigma^2}}{P_{R,k}|h_l|^2 + \frac{(1-\rho)}{2}P_{S,k}|g_{lk}|^2 + \sigma^2} \qquad (6.15)$$

and $\chi_k = \sqrt{P_{R,k}/\|\tilde{\mathbf{w}}_k\|^2}$. The beamforming vector given in Eq. (14) and Eq. (15) has a similar form to many filters or equalizers found in different signal processing problems, including the LMMSE chip-level equalizer (see e.g., [18]). These filters are usually designed to minimize some mean-squared error criterion or maximizes the signal-to-noise ratio under perfect channel state information.

It should be pointed out that, while the transmit powers of all relays in [12] are used for only one source, in our case they should be efficiently allocated for signals from all sources. The beamforming vector $\mathbf{w}_k$ itself includes the transmit power each relay assigns to the kth source. This fact leads to a complicated joint beamforming and power allocation problem. Moreover, up to this point we have only mentioned about the total relay transmit power assigned for each relay, $P_{R,k}$. The relationship between $P_{R,k}$ and the total transmit power of each relay will be discussed in Section 6.3 when the power allocation problem is formulated.

With the above optimal beamforming vectors, the instantaneous SNR in (6.13)

becomes

$$\text{SNR}_k = \frac{(1-\rho)P_{S,k}|f_k|^2}{2\sigma^2} + \sum_{l=1}^{L} \frac{\frac{(1-\rho)}{2\sigma^2}P_{S,k}|g_{lk}|^2 P_{R,k}|h_l|^2}{\frac{(1-\rho)}{2}P_{S,k}|g_{lk}|^2 + P_{R,k}|h_l|^2 + \sigma^2}. \qquad (6.16)$$

The form of the instantaneous SNR in (6.16) clearly shows that the full diversity order of $L+1$ can be obtained, provided that full channel state information (CSI) is available at the relays and destination (see, e.g., [19] or [12] for a proof). Also from (6.16), it can be seen that performance improvement by performing precoding at the relays depends on the instantaneous harmonic means of the "uplink" and "downlink" channel gains. In particular, if $|h_l| << |g_{lk}|$, the improvement is more pronounced.

Before closing this section, it should be mentioned that in order to compute the beamforming coefficients in a distributed fashion, i.e., at individual relays, information exchange among the relays is required. This is true even when the total transmitted power in the second phase is allocated equally among the users (i.e., $P_{R,k}$ is the same for all k). This is because although the lth relay can compute $\tilde{w}_{lk}$, $k = 1, \ldots, K$, in order to calculate χ_k and then w_{lk} it needs to know $\tilde{w}_{lk}$, $l = 1, \ldots, L$, which are calculated by other relays. A central processing unit is, therefore, necessary not only to keep the system overhead at an acceptable level but also to compute a more efficient power allocation scheme rather than the equal-power allocation scheme among the users in the second phase. Thus, it is reasonable to assume that the destination can estimate the direct path and all downlink channel coefficients while the relays estimate the uplink channel coefficients and forward them to the destination. The destination then computes all required parameters and send back the joint beamforming, $\{\mathbf{w}_l\}_{l=1}^{L}$, to the relays and power allocation scheme, $\{P_{S,k}\}_{k=1}^{K}$, to the sources. The destination can also estimate the "composite" channels (i.e., the products of the uplink and downlink channel coefficients of each relay) as proposed in [20, 21] and the downlink channel coefficients separately. In this case, each relay does not need to estimate its uplink channel and the destination will send back $\{\alpha_l \mathbf{w}_l\}_{l=1}^{L}$ instead of $\{\mathbf{w}_l\}_{l=1}^{L}$. It is also noted that all the aforementioned and following calculations are obtained under the assumption of perfect channel estimation. The next section considers the power

allocation problem in more detail.

6.3 Power Allocation

In order to further exploit all the instantaneous channel information, power control over the sources and/or relays can be performed to optimize some performance criteria of the network such as maximizing the minimum SNR_k, maximizing the total throughput, or minimizing the total power consumed.

Considered in this section is a power allocation scheme that maximizes the minimum SNR_k among all the sources under both the total power constraint and individual power constraints on the sources and relays. The problem is formulated as follows

$$\max_{P_{S,k}, P_{R,k}, \chi_k} \quad \min_k SNR_k \tag{6.17}$$

$$\text{s.t.} \quad \left(\sum_{k=1}^{K} P_{S,k} + \sum_{l=1}^{L} Q_l \right) \leq P_{\text{total}} \tag{6.18}$$

$$P_{S,k} \leq P_{S,k,\max}, \quad k = 1, \ldots, K, \tag{6.19}$$

$$Q_l \leq Q_{l,\max}, \quad l = 1, \ldots, L, \tag{6.20}$$

$$\chi_k^2 \sum_{l=1}^{L} |\tilde{w}_{lk}|^2 = P_{R,k} \tag{6.21}$$

where $Q_l = \sum_{k=1}^{K} \chi_k^2 |\tilde{w}_{lk}|^2$ is the lth relay's transmitted power, P_{total} is the maximum power the whole network is allowed to transmit, $P_{S,k,\max}$ and $Q_{l,\max}$ are the highest power levels the kth source and lth relay can use, respectively. If each source is assisted by only one relay and there are no direct connections from the sources to the destination, problem (6.17)–(6.21) becomes exactly the same as problem (1a)-(1d) in [3]. Moreover, the proposed scheme also provides a cooperative diversity (beyond what offered by power allocation) by exploiting the channel knowledge as in [3]. Similar problems with other objective functions can be readily extended, (e.g., see [22, 23]).

Since $P_{R,k}$ is one of the main variables of the optimization problem, the constraint (6.21) which comes from the normalization of the beamforming vector (6.14) needs to

be taken into consideration. Note that χ_k affects the total power allocated for each user in the second phase, $P_{R,k}$, thus, it also affects SNR_k. When the optimal set of $\{P_{S,k}, P_{R,k}\}_{k=1}^{K}$ are found, the beamforming schemes at all relays are then computed. In other words, the solution of the above problem is exactly a joint beamforming and power allocation scheme.

To solve problem (6.17)-(6.21) the *successive convex method* proposed in [1] might be applied. However, from (6.15) and (6.16), it can be seen that the approximations of (6.17), (6.18), (6.20) as posynomials and (6.21) as a monomial are very involved, even with a small number of relays (e.g., $L \geq 3$), and therefore, can cause a significant delay in computing the optimal power allocation scheme. In order to achieve a certain improvement level while simplifying the calculation, bounds can be used to formulate a solvable suboptimal problem as explained in the following.

Denote

$$\lambda_{lk} = \min\left\{\frac{(1-\rho)|g_{lk}|^2}{2\sigma^2}P_{S,k}, \frac{|h_l|^2}{\sigma^2}P_{R,k}\right\}. \tag{6.22}$$

At medium and high SNR regions, the following bounds are proved to be tight enough and widely used (e.g., see [24–26]):

$$\frac{1}{2}\lambda_{lk} \leq \frac{\frac{(1-\rho)}{2\sigma^2}P_{S,k}|g_{lk}|^2 P_{R,k}|h_l|^2}{\frac{(1-\rho)}{2}P_{S,k}|g_{lk}|^2 + P_{R,k}|h_l|^2 + \sigma^2} \leq \lambda_{lk}. \tag{6.23}$$

Applying (6.23), SNR_k given in (6.13) is lower bounded by

$$\text{SNR}_{k,\text{lower}} = \frac{(1-\rho)P_{S,k}|f_k|^2}{2\sigma^2} + \frac{1}{2}\sum_{l=1}^{L}\lambda_{lk}. \tag{6.24}$$

Using Lemma 1 in [1], $\text{SNR}_{k,\text{lower}}$ can be approximated as

$$\text{SNR}_{k,\text{lower}}^{*} = \frac{\left(\frac{(1-\rho)|f_k|^2}{2\sigma^2 \xi_1}P_{S,k}^{*}\right)^{\xi_1}}{2^{\left(\sum_{i=2}^{L+1}\xi_i\right)}}\prod_{i=2}^{L+1}\left(\frac{\lambda_{(i-1)k}}{\xi_i}\right)^{\xi_i}. \tag{6.25}$$

The above is basically the best local monomial approximation of $\text{SNR}_{k,\text{lower}}$ near a fixed set of $(P_{S,k}^{*}, P_{R,k}^{*})$ in the sense of the first-order Taylor approximation, where

$$\xi_1 = \frac{\frac{(1-\rho)|f_k|^2}{2\sigma^2}P_{S,k}^{*}}{\text{SNR}_{k,\text{lower}}(P_{S,k}^{*}, P_{R,k}^{*})},$$

$$\xi_i = \frac{1}{2}\frac{\lambda_{(i-1)k}(P_{S,k}^{*}, P_{R,k}^{*})}{\text{SNR}_{k,\text{lower}}(P_{S,k}^{*}, P_{R,k}^{*})}, \quad i = 2, \ldots, L+1.$$

Since $\sum_{l=1}^{L} Q_l = \sum_{l=1}^{L} \sum_{k=1}^{K} \chi_k^2 |\tilde{w}_{lk}|^2 = \sum_{k=1}^{K} P_{R,k}$, the constraint (6.18) can be replaced by

$$\sum_{k=1}^{K} (P_{S,k} + P_{R,k}) \leq P_{\text{total}}. \tag{6.26}$$

Using the bounds given in (6.23), one can approximately replace the constraint (6.20) by

$$Q_{l,\text{upper}} \leq Q_{l,\text{max}}, \tag{6.27}$$

since

$$
\begin{aligned}
Q_l &= \sum_{k=1}^{K} \chi_k^2 |\tilde{w}_{lk}|^2 \\
&= \sum_{k=1}^{K} \chi_k^2 \frac{(1-\rho)^2 |g_{lk}|^2 |h_l|^2 \left(P_{S,k} |g_{lk}|^2 + \frac{2}{1-\rho} \sigma^2 \right)}{\left[\frac{1-\rho}{2} P_{S,k} |g_{lk}|^2 + P_{R,k} |h_l|^2 + \sigma^2 \right]^2} \\
&\leq \sum_{k=1}^{K} \frac{4\sigma^4 \chi_k^2 \lambda_{lk}^2 \left(P_{S,k} |g_{lk}|^2 + \frac{2}{1-\rho} \sigma^2 \right)}{P_{S,k}^2 P_{R,k}^2 |g_{lk}|^2 |h_l|^2} \\
&= Q_{l,\text{upper}}. \tag{6.28}
\end{aligned}
$$

Note that $Q_{l,\text{upper}}$ is a posynomial in $P_{S,k}$, $P_{R,k}$, χ_k and λ_{lk}. The last and most challenge task is to handle the equality constraint (6.21) which clearly reflects the coupling relationship between the beamforming and power allocation scheme. If we use the lower bound in (6.23) to relax (6.21) by

$$\sum_{l=1}^{L} \frac{\left(P_{S,k} |g_{lk}|^2 + \frac{2\sigma^2}{1-\rho} \right) \sigma^4 \chi_k^2 \lambda_{lk}^2}{P_{S,k}^2 P_{R,k}^2 |g_{lk}|^2 |h_l|^2} \leq P_{R,k}, \tag{6.29}$$

then it is likely that χ_k may be set to some very small value in order to easily satisfy the approximated constraints (6.27)-(6.29). This is because χ_k has no *direct* effect on the objective function (6.17) in the sense that the formula of SNR_k given in (6.16) does not explicitly include the term χ_k.

To avoid this difficulty we propose a "heuristic" power allocation problem that can be formulated in the form of a combination of two sub-problems. The first one is to find the optimal power allocation scheme $(P_{S,k}, P_{R,k})$ with no individual constraint

on the transmitted power of each relay, while the second problem will adjust the set of $P_{R,k}$ in order to find the appropriate value of χ_k.

In order to transform the objective function (6.17) into a monomial, let $\gamma = \min_k \text{SNR}_k$. With the variables $\lambda_{lk}, k = 1, \ldots, K$, and $l = 1, \ldots, L$, defined in (6.22), the first sub-problem can be formulated as

$$\max_{P_{S,k}, P_{R,k}} \quad \gamma \tag{6.30}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} (P_{S,k} + P_{R,k}) \leq P_{\text{total}} \tag{6.31}$$

$$P_{S,k} \leq P_{S,k,\text{max}}, \tag{6.32}$$

$$\frac{\gamma}{\text{SNR}^*_{k,\text{lower}}} \leq 1, \tag{6.33}$$

$$\lambda_{lk} \leq \frac{(1-\rho)}{2\sigma^2} P_{S,k} |g_{lk}|^2, \tag{6.34}$$

$$\lambda_{lk} \leq \frac{P_{R,k} |h_l|^2}{\sigma^2}. \tag{6.35}$$

Denote $(P_{S,k}^\star, P_{R,k}^\star)$ the power allocation scheme obtained from solving problem (6.30)-(6.35). The second sub-problem, which can be considered as a tightening process [7], is in the form of an auxiliary geometric programming problem. It is written as follows:

$$\max_{\chi_k} \quad \prod_{k=1}^{K} \chi_k \tag{6.36}$$

$$\text{s.t.} \quad \chi_k^2 \sum_{l=1}^{L} |\tilde{w}_{lk}^\star|^2 \leq P_{R,k}^\star, \tag{6.37}$$

$$\sum_{k=1}^{K} \chi_k^2 |\tilde{w}_{lk}^\star|^2 \leq Q_{l,\text{max}}, \tag{6.38}$$

where $\tilde{w}_{lk}^\star$ is given in (6.14) and it is calculated based on $(P_{S,k}^\star, P_{R,k}^\star)$ obtained from solving problem (6.30)-(6.35). Note that the constraint (6.37) is still a relaxed version of the normalization of beamforming vector in (6.14). This inequality constraint will approach the equality by iteratively updating the optimal values $P_{R,k}^\star$ corresponding to $\chi_k^\star$ found in problem (6.36)-(6.38) as follows:

$$P_{R,k,\text{new}}^\star = (\chi_k^\star)^2 \sum_{l=1}^{L} |\tilde{w}_{lk}^\star(P_{R,k,\text{old}}^\star)|^2. \tag{6.39}$$

Due to the power constraint (6.38) on each relay, $P_{R,k}^\star$ found in problem (6.36)-(6.38) is less than or equal to $P_{R,k}^\star$ found in problem (6.30)-(6.35). Thus, instead of using the total transmitted power P_total allowed, the network uses a smaller amount, and a certain performance degradation is experienced, which depends on how strict the power constraints on the relays are.

If $\sum_{k=1}^K P_{R,k}^\star \leq \min_l \{Q_{l,\max}\}$, the normalization factor χ_k can be calculated directly based on $(P_{S,k}^\star, P_{R,k}^\star)$ obtained from solving problem (6.30)-(6.35) by normalizing the beamforming vectors given in (6.14). In particular, when $Q_{l,\max} \geq P_\text{total}$, which might be the case of fixed relays with a high power budget (or no individual power constraint applied on each relay), only the first sub-problem is needed. In this case, the first sub-problem can be solved efficiently using the successive convex method proposed in [1] while χ_k can also be calculated directly based on (6.14).

6.4   Simulation Results

In this section, the error performance of the equicorrelation-based decorrelating multiuser relaying scheme is studied. Performance of the scheme proposed in [9] is also simulated for comparison purposes. Finally, performance improvement achieved with the power allocation scheme proposed in Section 6.3 is illustrated.

6.4.1   Equal Power Allocation

All the direct channel path gains f_k, the "uplink" and "downlink" channel coefficients g_{lk}, h_l are assumed to be i.i.d. Rayleigh fading with variances $1/16, 1, 1$, respectively (i.e., a symmetric channel model). The power of AWGN at the relays and destination are normalized to be 1. The network consists of six sources (i.e., $K = 6$), each employs a shifted version of an m-sequence with spreading factor $N = 7$ and the cross-correlation factor $\rho = -1/7$. Note that Gold sequences or some Welch-bound sequences can be employed in this model since they are also equicor-

related spreading sequences.[1] Each source transmits with power $P_{S,k} = P_{\text{total}}/(2K)$ while L relays equally share the total transmitted power $P_{\text{total}}/2$ (i.e., each with power $Q_l = P_{\text{total}}/(2L)$). The symbol-error-rate (SER) is calculated based on the total power P_{total}.

Fig. 6.2 compares the performance of the proposed DCF relaying scheme and the scheme proposed in [9]. In this symmetric simulation set-up, the performance improvement by performing precoding in the proposed scheme is less than 1 dB, but the overall performance improvement compared to the MMSE RAD-MUD scheme in [9] is about 2.5 dB at the SER of 10^{-3}. When the transmitted powers of all sources and relays increase, due to the enhancement of multiple-access interference (MAI) and inter-relay interference (IRI), an error floor appears in the performance curve of the MMSE RAD-MUD scheme (at the SER $< 10^{-5}$). On the contrary, since the MAI and IRI are completely eliminated the diversity order of the DCF relaying scheme increases with the number of relays and can be verify to be $L + 1$ from Fig. 6.3.

6.4.2 Proposed Power Allocation

In order to evaluate the performance of the proposed power allocation, asymmetric independent Rayleigh fading channels with different variances of path gains shown in Table 6.1 is considered. The GP tool used in this simulation is GPPlab (see http://www.stanford.edu/ boyd/ggplab/). Since all the inequality constraints in problem (6.30)-(6.35) must be met with equality at optimality, with an identical constraint on the lower bound of the SNR of each user, the error performances of all users should be the same. As can be seen in Fig. 6.4, without the power constraints on the relays the fairness among all users is improved significantly by performing the

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[1]With one reversed spreading code, using binary equi-correlated sequences such as the shifted m-sequences chosen here is actually not as good as orthogonal spreading codes. Choosing non-binary Welch-bound equality (WBE) sequences [27] with a higher spectral efficiency would have been more meaningful. Besides, equi-correlated sequences are more robust against asynchronism than the orthogonal spreading codes. It should be noted, however, that the joint power allocation and beamforming scheme presented in this chapter can be applied to both cases of equi-correlated and orthogonal spreading sequences.
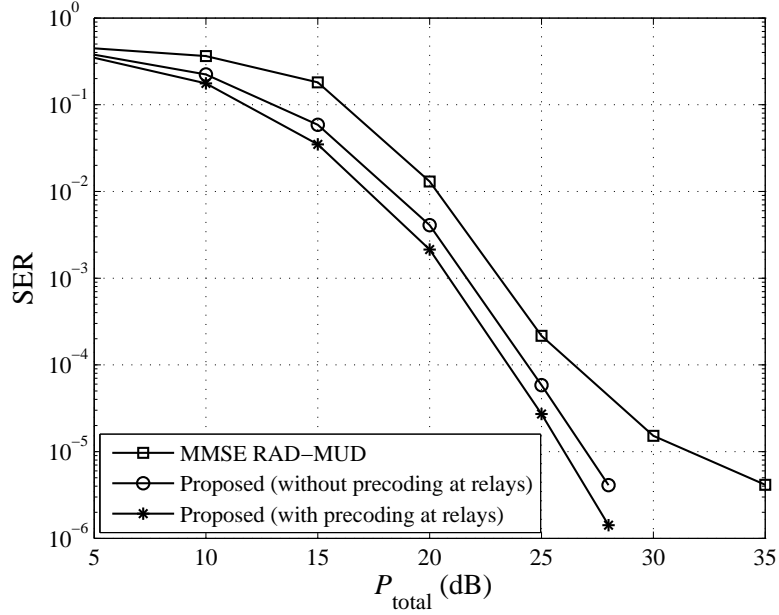
Figure 6.2 Performance comparison of the proposed scheme (with and without precoding at the relays), and the MMSE RAD-MUD proposed in [9]. Equicorrelated sequences with spreading factor $N = 7$. I.i.d. Rayleigh fading channels. Number of sources $K = 6$. Number of relays $L = 4$.

Table 6.1 Channel covariance matrices used in the simulation where all links are uncorrelated Rayleigh fading.

Covariance matrix	Value
$\boldsymbol{\Sigma}_{f_k}$	$\mathrm{diag}(0.10, 0.02, 0.01, 0.30, 0.08, 0.07)$
$\boldsymbol{\Sigma}_{g_{l1}}$	$\mathrm{diag}(1.4, 0.9, 0.5, 0.8)$
$\boldsymbol{\Sigma}_{g_{l2}}$	$\mathrm{diag}(0.4, 0.7, 0.5, 0.4)$
$\boldsymbol{\Sigma}_{g_{l3}}$	$\mathrm{diag}(0.8, 0.5, 0.6, 0.9)$
$\boldsymbol{\Sigma}_{g_{l4}}$	$\mathrm{diag}(1.7, 0.9, 2.0, 2.2)$
$\boldsymbol{\Sigma}_{g_{l5}}$	$\mathrm{diag}(1.4, 0.8, 1.5, 0.8)$
$\boldsymbol{\Sigma}_{g_{l6}}$	$\mathrm{diag}(1.1, 1.6, 1.2, 1.9)$
$\boldsymbol{\Sigma}_{h_l}$	$\mathrm{diag}(0.25, 0.75, 0.50, 0.75)$

proposed power allocation. The improvement decreases when the individual power constraints are also applied on the relays. In our simulation, the maximum transmit-
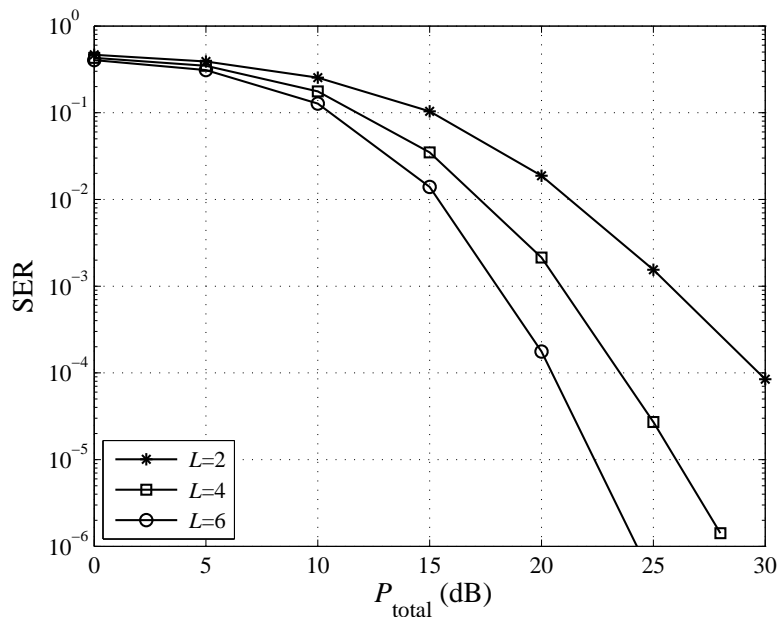
Figure 6.3 Performance of the proposed scheme with precoding at the relays. Equicorrelated sequences with spreading factor $N = 7$. I.i.d. Rayleigh fading channels. Number of sources $K = 6$. Number of relays $L = 2, 4, 6$.

ted power that each source and each relay in the network can use is P_{total}/K. The performance curves in Fig. 6.4 show that the proposed power allocation scheme works very well at moderate and high SNR regions, even with individual power constraints applied on all the sources and relays.

6.5   Conclusion

This paper has developed a DCF relaying scheme for multiuser wireless CDMA networks with equicorrelated spreading sequences. Since multiple-access interference (MAI) and inter-relay interference (IRI) are completely eliminated with DCF relaying, the full diversity order can be obtained for every user with a simple transceiver structure implemented at the relays and destination. A centralized max-min power allocation scheme based on geometric programming has been proposed under a fixed total and/or individual transmit powers of the sources and relays. Although being
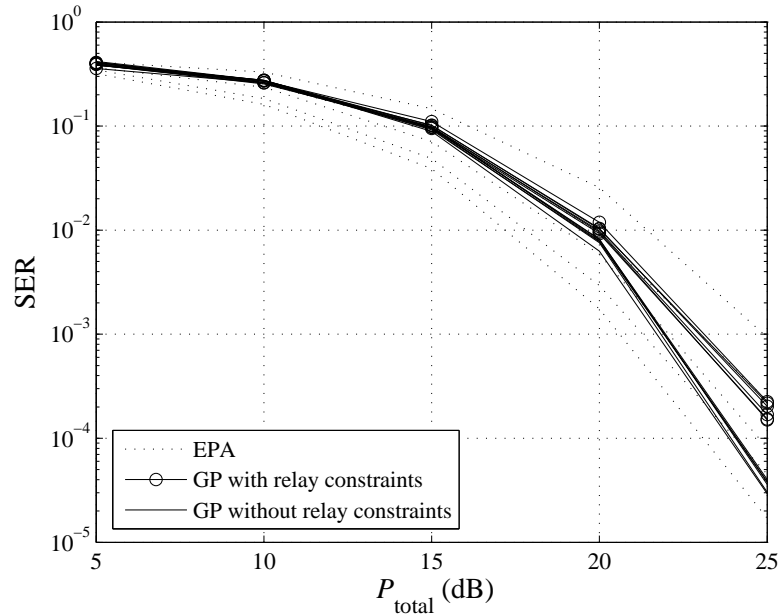
Figure 6.4 Performance of the proposed scheme with equal power allocation (EPA) and the max-min SNR based power allocation obtained by GP with and without individual relay constraints. Equicorrelated sequences with spreading factor $N = 7$. Asymmetric independent Rayleigh fading channels. Number of sources $K = 6$. Number of relays $L = 4$.

a heuristic solution, simulation results have shown that the proposed method can successfully provide the fairness among the users in terms of the signal-to-noise ratio regardless of their channel conditions.

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7. Relay Assignment for Max-Min Capacity in Cooperative Multiuser Wireless Networks

Tung T. Pham, Ha H. Nguyen, and Hoang D. Tuan, "Relay Assignment for Max-Min Capacity in Cooperative Multiuser Wireless Networks", submitted to *IEEE Transactions on Vehicular Technology*.

As indicated in Chapter 6, a combination of coordinated relay beamforming and power allocation schemes at both sources and relays may require a significant amount of overhead information and coordination. In this chapter, we consider a simple case with relay selection for each source and equal power allocation at all relays. Although being a sub-optimal scheme, the feedback requirement of the proposed scheme is significantly reduced. More importantly, it still performs much better than the conventional transmission model without relaying.

Since relay selection is basically a discrete variable problem with a continuous objective function, it can be cast as a mixed-integer optimization problem (MIP). We then use CPLEX software to find the benchmark solution. Next, a heuristic algorithm is proposed and its performance is compared against that of the benchmark solution under an IEEE 802.16j channel model. It is found that the performance of the heuristic algorithm is very close to the optimal solution but with a much lower computational complexity.

Relay Assignment for Max-Min Capacity in Cooperative Multiuser Wireless Networks

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Abstract

This paper is concerned with the problem of relay assignment in cooperative *multiuser* wireless networks. With the objective of maximizing the minimum capacity among all sources in the network, a mixed-integer linear programming (MILP) problem is formulated, which can be solved by some standard branch-and-bound algorithms. In order to reduce the computational complexity, a greedy solution in the form of a lexicographic bottleneck assignment algorithm is proposed. Simulation results obtained for the IEEE 802.16j uplink scenarios show that the greedy algorithm performs very close to the optimal solution, but at a much lower computational cost. The proposed greedy solution can also be tailored to provide further improvements on other network performance criteria.

Index terms

Wireless relay networks, amplify-and-forward relaying, relay selection, mixed-integer programming, greedy algorithm, lexicographic bottleneck assignment.

7.1 Introduction

In recent years, many relay-assisted wireless transmission models have been studied. For the model with a single source and multiple relays, references [1, 2] show

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that if full and perfect channel state information (CSI) is available at the relays and destination, then relay beamforming with maximal-ratio combining at the destination is the optimal scheme in maximizing the signal-to-noise ratio (SNR). In practice, at the expense of more bandwidth spent on channel estimation, the destination can obtain the CSI of all the associated links. However, high-resolution feedback information from the destination to all the relays can be very expensive, especially when multi-carrier modulation scheme is employed in the system. As a consequence, relay selection has emerged as a promising solution that can appropriately balance the overhead information exchange and performance improvement enjoyed with relaying [3].

For the model with multiple sources and multiple relays, it is commonly assumed that each source is assigned an orthogonal channel for its own transmission. Although it appears that multiple orthogonal transmissions from the sources can be treated as parallel independent transmissions, the inter-dependence among the relays in choosing the best subset of sources' signals to forward to the destination with a limited transmit power still makes signal processing/scheduling at relays a coupled and challenging problem. In some previous works [4,5], several approaches have been proposed to approximately solve the relay assignment problem. By omitting the direct paths from the sources to the destination in the transmission model, the resultant channel capacity for each source behaves favorably, which allows one to exploit some convex properties [6] in finding a sub-optimal solution. For the general case, which is especially applicable in cellular networks, the direct paths play a significant role and should not be ignored. In fact, transmission from any source to the destination can be successful in just one phase (i.e., the over the direct path from that source to the destination) or in two phases (in which all sources transmit their signals to the relays and destination in the first phase and the relays forward their received signals to the destination in the second phase). In terms of bandwidth efficiency, the advantage or disadvantage of a relay-assisted scheme over the conventional transmission method (i.e., without relays) essentially depends on the specific channel conditions. In other words, relay-assisted transmissions do not always offer better capacities. This fact

again emphasizes the importance of using an appropriate relay assignment scheme which solves both the power control problem at the physical layer as well as the scheduling issue at the medium access control layer.

In this paper, we develop an algorithm to find the optimal relay assignment scheme for a general cooperative wireless network model. When taking into account the direct transmission from all sources, the channel capacity functions become non-differentiable and non-convex. Solving the relay assignment problem therefore has to rely on mixed-integer optimization techniques. In order to find a sub-optimal solution with a lower computational cost, a greedy algorithm is proposed which performs very close to the optimal solution obtained by brute-force searching. Beside offering a promising solution with a much lower complexity, the greedy algorithm is also tailored to further improve other network performance criteria.

The rest of this paper is organized as follows. Section II presents the system model and formulates the relay assignment problem. Section III develops an algorithm to find the optimal relay assignment scheme and proposes a greedy solution. Numerical results are presented in Section IV. Finally, concluding remarks are given in Section V.

Notations: Italic, bold lower case and bold upper case letters denote scalars, vectors and matrices, respectively. The superscript $(\cdot)^T$ represents a matrix transpose operation. The notations $\mathbf{1}_K$ and $\mathbf{I}_L$ stand for a $K \times 1$ vector with all elements are equal to 1 and an identity matrix of size $L \times L$, respectively, while $|\mathcal{K}|$ is the cardinality of set $\mathcal{K}$.

7.2 System Model and Problem Formulation

Illustrated in Fig. 7.1 is a cooperative wireless network in which K source terminals, $S_1, \ldots, S_K$, transmit data to the destination terminal, D, with the help of L relay terminals, $R_1, \ldots, R_L$. All terminals are equipped with one antenna and operate in a half-duplex mode. Each source is assigned an orthogonal channel for its

own transmission to the destination while the relays may also use those orthogonal channels to assist the transmissions from all sources to the destination.

In this paper, we assume that the transmit power at each relay is equally assigned to the sources' signals it forwards. A joint optimization of relay assignment and power allocation in this case appears very complicated and deserves a separate treatment. Furthermore, implementing a power allocation scheme also requires more bandwidth for the information fed back to all the relays. An alternative is to perform power allocation for each relay once the relay assignment is done but this is beyond the scope of this paper.[1]
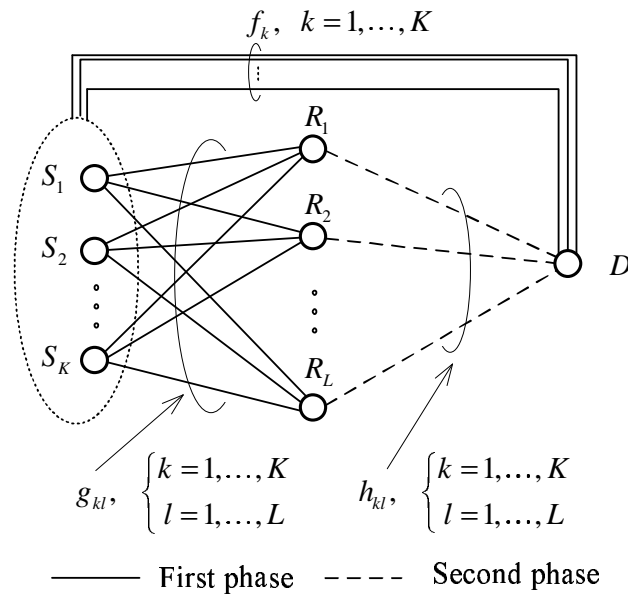
Figure 7.1 A multiuser cooperative network with K sources and L relays.

Although the relay assignment in this paper can be applied to any relaying protocols such as Amplify-and-Forward (AF) and Decode-and-Forward (DF), we adopt AF as an example to present the algorithm. In AF relaying model, the received signals at

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[1]The interested reader may refer to [7] for the power allocation problem in a model with a *single* relay.

the destination and the lth relay in the first phase can be represented, respectively, as

$$y_k^{(1)} = \sqrt{P\alpha_k}f_k s_k + n_k^{(1)}, \ k = 1, \ldots, K \tag{7.1}$$

$$z_{kl} = \sqrt{P\beta_{kl}}g_{kl}s_k + n_{kl}^{(r)}, \ \begin{matrix} k = 1, \ldots, K \\ l = 1, \ldots, L, \end{matrix} \tag{7.2}$$

where P is the transmitted signal power at every source, α_k, β_{kl} and f_k, g_{kl} are the path-loss (attenuation) parameters and the fading coefficient of the kth source – destination channel and of the kth source – lth relay channel, respectively. The variable s_k represents the transmitted symbol with unit power from the kth source, while $n_k^{(1)}$ and $n_{kl}^{(r)}$ denote the zero-mean additive white Gaussian noise (AWGN) with variance N_0 at the destination and lth relay, respectively.

By relay assignment we mean that each source's signal is forwarded by only one relay. This also implies that one relay can assist many sources and there can be relays that do not participate in the transmission process. In general, let the kth source be assisted by the ℓ_kth relay. Then the signals received at the destination in the second phase can be expressed as

$$\begin{aligned} y_k^{(2)} &= \sqrt{p_{k\ell_k}\delta_{k\ell_k}\gamma_{k\ell_k}}h_{k\ell_k}z_{k\ell_k} + n_{k\ell_k}^{(2)}, \ k = 1, \ldots, K \\ &= \sqrt{P\beta_{k\ell_k}p_{k\ell_k}\delta_{k\ell_k}\gamma_{k\ell_k}}h_{k\ell_k}g_{k\ell_k}s_k + \sqrt{p_{k\ell_k}\gamma_{k\ell_k}\delta_{k\ell_k}}h_{k\ell_k}n_{k\ell_k}^{(r)} + n_{k\ell_k}^{(2)}, \end{aligned} \tag{7.3}$$

where $\delta_{k\ell_k} = (P\beta_{k\ell_k}|g_{k\ell_k}|^2 + N_0)^{-1}$ and $p_{k\ell_k} = P_{\ell_k}/|\mathcal{K}_{\ell_k}|, \ \forall k \in \mathcal{K}_{\ell_k}$ are the normalization factor and the transmit power the ℓ_kth relay assigns to the signal of the kth source, respectively.[2] $\mathcal{K}_{\ell_k}$ denotes the set of sources the ℓ_kth relay forwards their signals, $\gamma_{k\ell_k}$ and $h_{k\ell_k}$ are the path-loss (attenuation) parameters and the fading coefficient of the ℓ_kth relay – destination channel for the kth source, respectively, and $n_{k\ell_k}^{(2)}$ represents the zero-mean additive white Gaussian noise with variance N_0 at the destination experienced by the signal from the kth source in the second phase.

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[2]Note that the total transmit power at the ℓ_kth relay, P_{ℓ_k}, is equally divided amongst the signals from those sources assisted by that relay.

Using the maximal-ratio combiner at the destination, the signal-to-noise ratio (SNR) for the signal of each source at the destination and its corresponding capacity can be expressed as [8]:

$$\text{SNR}_{k\ell_k}^{(\text{AF})} = \text{SNR}_k^{(\text{direct})} + \frac{\text{SNR}_{k\ell_k}^{(\text{up})}\text{SNR}_{k\ell_k}^{(\text{down})}}{\text{SNR}_{k\ell_k}^{(\text{up})} + \text{SNR}_{k\ell_k}^{(\text{down})} + 1}, \tag{7.4}$$

$$\mathcal{C}_{k\ell_k}^{(\text{AF})} = \frac{1}{2}\log_2\left(1 + \text{SNR}_{k\ell_k}^{(\text{AF})}\right), \tag{7.5}$$

where $\text{SNR}_k^{(\text{direct})} = \frac{P\alpha_k|f_k|^2}{N_0}$, $\text{SNR}_{k\ell_k}^{(\text{up})} = \frac{P\beta_{k\ell_k}|g_{k\ell_k}|^2}{N_0}$ and $\text{SNR}_{k\ell_k}^{(\text{down})} = \frac{P_{\ell_k}}{|\mathcal{K}_{\ell_k}|}\text{snr}_{k\ell_k}^{(\text{down})}$ where $\text{snr}_{k\ell_k}^{(\text{down})} = \frac{\delta_{k\ell_k}\gamma_{k\ell_k}|h_{k\ell_k}|^2}{N_0}$. Note that the pre-log factor of $\frac{1}{2}$ in the capacity expression reflects that signal transmission from a source to the destination is conducted in two phases. Similarly, for the direct transmission method, the channel capacity for each source can be computed as

$$\mathcal{C}_k^{(\text{direct})} = \log_2\left(1 + \text{SNR}_k^{(\text{direct})}\right). \tag{7.6}$$

From (7.5) and (7.6), it can be seen that when the direct channels from some sources are good enough, those sources' signals need not be forwarded by any relay in order to maximize their capacities.[3]

Various relaying strategies for different objectives in throughput maximization have been studied, which include maximizing the sum capacity of all sources or maximizing the minimum capacity among them [4,5,9]. In this paper, the objective of the relay assignment problem is to find the optimal sets of sources assisted by each of the L relays to maximize the minimum capacity among all sources. Different from [4,5], the direct paths are taken into account and no convex relaxation can be exploited to find an approximate solution. Compared to [9], we consider a more general case in which the number of relays is less than the number of sources. As a consequence, a standard perfect bipartite matching algorithm [10] (or the algorithm in [9]) cannot be applied.

---

[3]Of course, under such a scenario, the signals may still be forwarded if the relaying scheme offers a better signal quality and one is willing to sacrifice the bandwidth efficiency for an improvement in error performance.

To formulate the relay assignment problem of maximizing the minimum capacity, let $\{u_{kl}\}_{k=1,l=1}^{K,L+1}$ be the decision variables with $u_{kl} = 1$ if the kth source is assisted by the lth relay and 0 otherwise. With this definition, $u_{k(L+1)} = 1$ means no relay assists the kth source. A formulation for the optimization problem under consideration can be stated as

$$
\begin{aligned}
\max_{\{u_{kl}\}} \quad & \min_{k \in \{1,2,\dots K\}} \mathcal{C}_k \\
\text{s.t.} \quad & \sum_{l=1}^{L+1} u_{kl} = 1, \ \forall k
\end{aligned}
\tag{7.7}
$$

where $\mathcal{C}_k = \max\{\mathcal{C}_k^{(\text{direct})}, \mathcal{C}_{k\ell_k}^{(\text{AF})}\}$ and the constraints in (7.7) indicate that the signal from any source is forwarded by only one relay. Note that the term $\sum_{i=1}^{K} u_{il}$ in (7.8) is actually the cardinality of the set $\mathcal{K}_l$, i.e., $|\mathcal{K}_l|$. Furthermore, the capacity $\mathcal{C}_k$ can also be calculated as

$$
\begin{aligned}
\mathcal{C}_k \ = \ & \sum_{l=1}^{L} u_{kl} \cdot \frac{1}{2} \log_2 \left(1 + \text{SNR}_k^{(\text{direct})} + \frac{\text{SNR}_{kl}^{(\text{up})} P_l \text{snr}_{kl}^{(\text{down})}}{\left(\text{SNR}_{kl}^{(\text{up})} + 1\right) \sum_{i=1}^{K} u_{il} + P_l \text{snr}_{kl}^{(\text{down})}} \right) \\
& + u_{k(L+1)} \cdot \log_2 \left(1 + \text{SNR}_k^{(\text{direct})} \right)
\end{aligned}
\tag{7.8}
$$

Since each source's signal might be forwarded by one of L relays or not be forwarded by any relay (i.e., using direct transmission method), there are $L+1$ assignment possibilities for each source. As a result, for a network with K sources and L relays, the total number of possible assignment schemes is $(L+1)^K$. For an example of a small cooperative wireless network with $L = 3$ and $K = 16$, a brute-force search requires to check over more than 4×10^{10} possibilities to find the optimal solution, which is clearly impossible.

By introducing a new continuous variable, namely $\tau = \min_k\{\mathcal{C}_k\}$, problem (7.7) can be cast as the following mixed-integer *nonlinear* program (MINLP):

$$
\begin{aligned}
\max_{\tau, \{u_{kl}\}} \quad & \tau \\
\text{s.t.} \quad & \sum_{l=1}^{L+1} u_{kl} = 1, \ \forall k \\
& \tau \leq \mathcal{C}_k, \ \forall k \\
& \tau \geq 0, \ u_{kl} \in \{0,1\}, \ \forall k, l.
\end{aligned}
\tag{7.9}
$$

Since $\mathcal{C}_k$ given in (7.7) and (7.8) is non-convex and has a high degree of nonlinearity, solving such a non-convex MINLP problem is very challenging. The usual approach relies on successive approximations of the closely related continuous nonlinear programming and mixed-integer *linear* programming (MILP) problems and it does not guarantee whether the solution obtained in finite time duration is globally optimal [11]. This issue will be discussed further in Section 7.4. In the next section, we reformulate (7.7) as an MILP problem so that optimization software products such as CPLEX [12] or MINTO [13] can be readily used to find an optimal solution. More importantly, a greedy algorithm is also introduced and analyzed.

7.3 Proposed Relay Assignment Schemes

7.3.1 MILP-Based Solution

It is well-known that the complexity of an optimal solution for a general NP-hard mixed integer problem is not polynomial. Although finding efficient algorithms to solve MINLP problems has not achieved a significant progress, the field of MILP algorithms is now mature with many commercial softwares. In this subsection, a formulation in the form of MILP to the relay assignment problem is presented and discussed.

First, let matrix $\mathbf{A}_l$ define all possible assignment patterns for the lth relay when it forwards signal from *at least one source*. Specifically, let each element in $\mathbf{A}_l$ be 1 if the lth relay is assigned to a source and 0 otherwise. Then each matrix $\mathbf{A}_l$ has the size of $K \times (2^K - 1)$ with the rows corresponding to the sources' indices and the columns representing all possible assignment patterns. For example, with $K = 3$, a matrix $\mathbf{A}_l$ could be

$$\mathbf{A}_l = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{7.10}$$

For each assignment pattern on each relay (i.e., a column m in matrix $\mathbf{A}_l$), a $0-1$

decision variable x_{lm} is associated, which indicates whether the mth assignment pattern is applied on the lth relay ($x_{lm} = 1$) or not used ($x_{lm} = 0$). Corresponding to the sets of assignment patterns at all relays $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_{L+1}]$, the decision vector $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_{L+1}^T]^T$, where $\mathbf{x}_l = [x_{l1}, x_{l2}, \ldots, x_{l(2^K-1)}]^T$, is constituted. Note that matrix $\mathbf{A}_{L+1}$ represents all patterns of the direct transmission. In order to enforce that each source's signal is forwarded by only one relay or by direct transmission, the following set of equalities must hold [14]:

$$[\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_{L+1}]\mathbf{x} = \mathbf{1}_K. \tag{7.11}$$

It is worthwhile to point out that for each transmission several columns of $\mathbf{A}$, each column from a different sub-block $\mathbf{A}_l$, will be chosen by setting the value of the corresponding elements of $\mathbf{x}$ to be 1. By such a setting, a temporary network topology for the signals going from K sources to the destination is formed.

For each assignment pattern used by a relay, the minimum capacity among the sources supported by that relay can be calculated. Let $\mathcal{C}_{lm}$ be the minimum capacity among the sources in set $\mathcal{K}_l$ when the mth assignment pattern at the lth relay is selected. This capacity can be mathematically defined as

$$\mathcal{C}_{lm} = \min_{k \in \mathcal{K}_l(m)} \{\mathcal{C}_k\} \tag{7.12}$$

where $\mathcal{K}_l(m)$ is the set of sources the lth relay forwards their signals when the mth assignment pattern is chosen and $\mathcal{C}_k$ is defined in (7.7), which depends on the number of sources sharing the transmit power of the lth relay. Since each $\mathcal{K}_l(m)$ set has a fixed number of sources with a specific assignment pattern, the value of $\mathcal{C}_{lm}$ is therefore fixed, which allows a linear formulation of the problem. Specifically, the following change in the objective function of (7.7) can be made

$$\min_{k \in \{1,2,\ldots K\}} \mathcal{C}_k = \min_{\substack{l \in \{1,\ldots,L+1\} \\ m \in \{1,\ldots,2^K-1\}}} \{\mathcal{C}_{lm} : x_{lm} = 1\}. \tag{7.13}$$

Since only one pattern is applied at each relay, in each $\mathbf{x}_l$ only one element $x_{lm} = 1$ while all other elements $x_{l\hat{m}} = 0$ for $\hat{m} = 1, \ldots, 2^K - 1$, $\hat{m} \neq m$. If the lth relay is

not used, $x_{lm} = 0$, $\forall m$. Stated differently, to ensure that at most only one pattern is applied at each set $\mathcal{K}_l$, i.e., at most one column from each sub-block $\mathbf{A}_l$ is chosen, the elements in each decision vector $\mathbf{x}_l$ have to satisfy $\mathbf{1}_{(2^K-1)}^T \mathbf{x}_l \leq 1$.

By stacking the constraints on all $L+1$ sets, the following inequality constraint is formed:

$$\mathbf{Bx} \leq \mathbf{1}_{L+1}, \tag{7.14}$$

where $\mathbf{B} = \mathbf{I}_{L+1} \otimes \mathbf{1}_{(2^K-1)}^T$ and $\otimes$ represents the Kronecker product.

Combining (7.11), (7.13) and (7.14), problem (7.7) can be reformulated as a general bottleneck binary integer program in the following form [15]

$$
\begin{aligned}
\max_{\mathbf{x}} \quad & \min_{\substack{l \in \{1,\dots,L+1\} \\ m \in \{1,\dots,2^K-1\}}} \{\mathcal{C}_{lm} : x_{lm} = 1\} \\
\text{s.t.} \quad & \mathbf{Ax} = \mathbf{1}_K \\
& \mathbf{Bx} \leq \mathbf{1}_{L+1} \\
& x_{lm} \in \{0,1\}, \ \forall l, m
\end{aligned}
\tag{7.15}
$$

Let $v = \min\limits_{l \in \{1,\dots,L+1\}, \ m \in \{1,\dots,2^K-1\}} \{\mathcal{C}_{lm} : x_{lm} = 1\}$ be a new variable, then problem (7.15) can be transformed into

$$
\begin{aligned}
\max_{v,\mathbf{x}} \quad & v \\
\text{s.t.} \quad & \mathbf{Ax} = \mathbf{1}_K \\
& \mathbf{Bx} \leq \mathbf{1}_{L+1} \\
& v \leq \mathcal{C}_{lm} + \Gamma(1 - x_{lm}), \ \forall l, m \\
& x_{lm} \in \{0,1\}, \ \forall l, m,
\end{aligned}
\tag{7.16}
$$

where Γ is an arbitrary large number. Problem (7.16) can further be written in the standard form of a mixed-integer linear program:

$$
\begin{aligned}
\max_{\tilde{\mathbf{x}}} \quad & \mathbf{c}\tilde{\mathbf{x}} \\
\text{s.t.} \quad & \mathbf{A}_{\text{eq}}\tilde{\mathbf{x}} = \mathbf{1}_K \\
& \mathbf{A}_{\text{ineq}}\tilde{\mathbf{x}} \leq \mathbf{b}_{\text{ineq}},
\end{aligned}
\tag{7.17}
$$

where $\mathbf{c} = [1, \mathbf{0}_{1 \times (L+1)(2^K-1)}]$, $\tilde{\mathbf{x}} = [v, \mathbf{x}^T]^T$ consists of all binary variables except the first element v, $\mathbf{A}_{\text{eq}} = [\mathbf{0}_{K \times 1}, \mathbf{A}]$, $\mathbf{b}_{\text{ineq}} = \left[\mathbf{1}^T_{(L+2) \times 1}, (\Gamma + [\mathcal{C}_{11}, \mathcal{C}_{12}, \dots, \mathcal{C}_{(L+1)(2^K-1)}]) \right]^T$ and

$$\mathbf{A}_{\text{ineq}} = \begin{bmatrix} 1 & \mathbf{0}_{1 \times (L+1)(2^K-1)} \\ \mathbf{0}_{(L+1) \times 1} & \mathbf{B} \\ \mathbf{1}_{(L+1)(2^K-1) \times 1} & \Gamma \cdot \mathbf{I}_{(L+1)(2^K-1)} \end{bmatrix}. \tag{7.18}$$

Problem (7.17) can be solved by several optimization softwares using the branch-and-bound (BnB) technique and its variations [16]. However, for general problems, the computational complexity of BnB techniques is still exponential and in the worst case, its time complexity required to solve the problem is similar to that required by brute-force searching techniques. A comparison on the complexity of these techniques will be discussed in Section 7.4.

For the relay assignment problem under consideration, as mentioned at the end of Section 7.2, a typical number of sources K in a network could be as large as 50 to 100 or more, which means that MILP softwares become prohibitively complicated in a real-time network model. To overcome this complexity issue, specific properties of the relay assignment problem shall be exploited in the next subsection to construct an efficient algorithm whose performance can approach the optimal solution but with a much lower complexity.

7.3.2 Greedy Algorithm

This subsection develops a greedy algorithm based on the following two important observations:

1) In the original problem (7.7), if a source has $\mathcal{C}_k^{(\text{direct})} \geq \mathcal{C}_{k\ell_k}^{(\text{AF})}$ with $|\mathcal{K}_{\ell_k}| = 1$, it does not need a relay. Therefore, the size of the problem can be reduced by initially removing some "best" sources from the considering sets.

2) With the only objective to maximize the minimum capacity, the algorithm that solves problem (7.17) may result in one of many optimal solutions which identify one

or several worst source(s) whose capacities equal to the optimal objective function value. Those solutions differ in the relay assignment for the remaining sources. It means that the solution of (7.17) does not guarantee best possible capacities for all sources. In other words, the capacities of these sources can be further improved.

The objective of the new algorithm is to find a relay assignment meeting the constraints in (7.7) or, equivalently, in (7.17) that not only tries to maximize the minimum capacity but also improves the capacities of other sources. Such a relay assignment algorithm belongs to an integer programming class named *lexicographic* bottleneck assignment [17]. The main idea of the algorithm presented here is try to increase the minimum capacity by iteratively reassigning one source at a time from the relay that supports the source with the minimum capacity to another relay which gives the largest improvement. The pseudo-code for the algorithm is given in Algorithm 1 which consists of two steps: (1) the initial assignment step, and (2) source reallocation step.

The *initial assignment* step is perceived based on the first observation. It calculates all the possible capacities for each source with different supporting relay transmitting at its maximum power and assign the source to the set offering maximum capacity. After assigning all sources, the capacity for any source in any set which is assigned multiple sources is recalculated. Intuitively, this assignment is similar to assigning the sources to its "closest" relay (so as to have the highest possible overall SNR) and does not care about the transit power sharing issue at each relay. This step also helps to reduce the number of sources to be considered, i.e., the size of $\check{\mathcal{K}}$.

The second step of the algorithm consists of several iterations. At each iteration, a set $\mathcal{K}_{\ell_{k^\star}}$ which supports the *current* worst source(s) is selected. One source from this set will be moved to another set which corresponds to the $\ell_{\hat{k}^\star}$th relay. This move must provide the best improvement in the lowest capacity $\mathcal{C}_{\min}$. If no improvement is obtained, no move will be scheduled. In this case, all the sources in set $\mathcal{K}_{\ell_{k^\star}}$ will be removed from the considering set $\check{\mathcal{K}}$ and a new iteration starts. This is the key step that makes the performance of the greedy algorithm improve beyond the

Algorithm 1 Lexicographic bottleneck relay assignment

    Input: K, L, and all channel parameters.

    Output: The relay assignment scheme $\{\mathcal{K}_l\}_{l=1}^{L+1}$.

    Calculate $\{\mathcal{C}_k^{(\text{direct})}\}, \{\mathcal{C}_{kl}^{(\text{AF})}(1)\} \forall k, l$ % *Start of Initial assignment* %.

    for $k = 1 : K$ **do**

        $\mathcal{C}_k \leftarrow \max\{\mathcal{C}_K^{(\text{direct})}, \mathcal{C}_{kl}^{(\text{AF})}(1)\}$

    end for

    Find $\{\mathcal{K}_l\}_{l=1}^{L+1}$

    for $k \in \{\mathcal{K}_{\ell_k}\}_{\ell_k=1}^{L}$ **do**

        $\mathcal{C}_k \leftarrow \mathcal{C}_{k\ell_k}^{(\text{AF})}(|\mathcal{K}_{\ell_k}|)$

    end for

    Let $\check{\mathcal{K}} = \bigcup_{l=1}^{L} \mathcal{K}_l$ % *Start of Source Reallocation* %

    while $|\check{\mathcal{K}}| \geq 2$ **do**

        Find $[\mathcal{C}_{\min}, k^\star, \ell_{k^\star}] \leftarrow \min_k\{\mathcal{C}_k : k \in \check{\mathcal{K}}\}$

        for $\hat{k} \in \mathcal{K}_{\ell_{k^\star}}$ **do**

            $\hat{\mathcal{K}}_{\ell_{k^\star}}(\hat{k}) \leftarrow \mathcal{K}_{\ell_{k^\star}} \setminus \hat{k}$

            for $l = 1 : L + 1, l \neq \ell_{k^\star}$ **do**

                $\hat{\mathcal{K}}_l(\hat{k}) \leftarrow \mathcal{K}_l \cup \hat{k}$

            end for

            $\hat{\mathcal{C}}_k(\hat{k}) \leftarrow \mathcal{C}_{kl}^{(\text{AF})}(|\hat{\mathcal{K}}_l(\hat{k})|)$ or $\mathcal{C}_k^{(\text{direct})}$ if $\hat{k} \in \hat{\mathcal{K}}_{L+1}$

            $\hat{\mathcal{C}}_{\min}(\hat{k}) \leftarrow \min_k\{\hat{\mathcal{C}}_k(\hat{k}) : k \in \check{\mathcal{K}}\}$

        end for

        Find $[\mathcal{C}_{\min}^{(\text{offer})}, \hat{k}^\star, \ell_{\hat{k}^\star}] \leftarrow \max_{\hat{k}}\{\hat{\mathcal{C}}_{\min}(\hat{k}) : \hat{k} \in \mathcal{K}_{\ell_{k^\star}}\}$

        if $\mathcal{C}_{\min}^{(\text{offer})} > \mathcal{C}_{\min}$ **then**

            $\mathcal{K}_{\ell_{\hat{k}^\star}} \leftarrow \mathcal{K}_{\ell_{\hat{k}^\star}} \cup \hat{k}^\star$

            $\mathcal{K}_{\ell_{k^\star}} \leftarrow \mathcal{K}_{\ell_{k^\star}} \setminus \hat{k}^\star$

        else

            $\check{\mathcal{K}} \leftarrow \check{\mathcal{K}} \setminus \mathcal{K}_{\ell_{k^\star}}$

        end if

    end while

performance of the MILP solution. It is noted that if $|\check{\mathcal{K}}| = 1$, i.e., only one source is in the considering set, no further iteration is required since there is no more power sharing issue left to address.

From Algorithm 1, the main computational complexity comes from the *source reallocation* step. In the worst case (i.e., all sources currently are supported by only one out of L relays), the first iteration needs to compute all possible capacities for each source with K different transmitted powers at L different relays. It requires a complexity order of $\mathcal{O}(K^2L)$. However, all possible capacities for each source are computed one time at some iterations when needed and the following iterations only compare those values and select the best move. Furthermore, after each iteration, the number of sources and/or the number of relays to be considered also decreases, which significantly reduces the computation requirement for the next iterations. For the case when a significant number of sources located near the destination with good direct paths, the computational complexity of the relay assignment for the remaining sources is much lower than $\mathcal{O}(K^2L)$. Simulation results in the next section will support this statement.

7.4   Simulation Results

A wireless network with a circular cell of radius ρ is simulated. The destination D is located at the center of the cell while the locations of K sources are uniformly distributed within the cell. L relays are equally positioned on a ring of radius 0.7ρ. All channel fading coefficients are independently generated based on Rayleigh distribution with unit variance. All path-loss parameters $\{\alpha_k\}, \{\beta_{kl}\}$ and $\{\gamma_{kl}\}$ are calculated based on the COST-231/IEEE802.16j channel model with parameters given in Table 7.1.

Table 7.1   COST-231/IEEE802.16j Parameters

Parameter	Value
Antenna height and gain at D	30m 5dB
Antenna height and gain at R_l	25m 5dB
Antenna height and gain at S_k	1.5m -1dB
Transmit power at R_l (P_l)	24dBm
Transmit power at S_k (P)	20dBm
Frequency	2.0GHz
Channel bandwidth for each S_k	200kHz
Noise PSD	-174dBm/Hz
Rooftop height	24m
Building space	50m
Road orientation	90 degrees
Street width	12m

In the first part, a network scenario with $L = 3$ relays and cell radius $\rho = 3$km is simulated. Performances of the greedy solution and the solution of the MILP problem obtained with CPLEX software [18] are compared for different numbers of sources, ranging from 5 to 9. In (7.16), Γ is chosen to be $1.1 \max_{l,m}\{\mathcal{C}_{lm}\}$. It can be seen from Fig. 7.2 and Fig. 7.3 that the average minimum capacity achieved by the greedy solution is almost the same as that offered by the optimal solution (obtained by brute-force searching), as well as that of the MILP problem. As can be predicted, since the greedy algorithm also improves the capacities of other sources, it provides a higher average sum capacity compared to the MILP solution (see Fig. 7.3). More
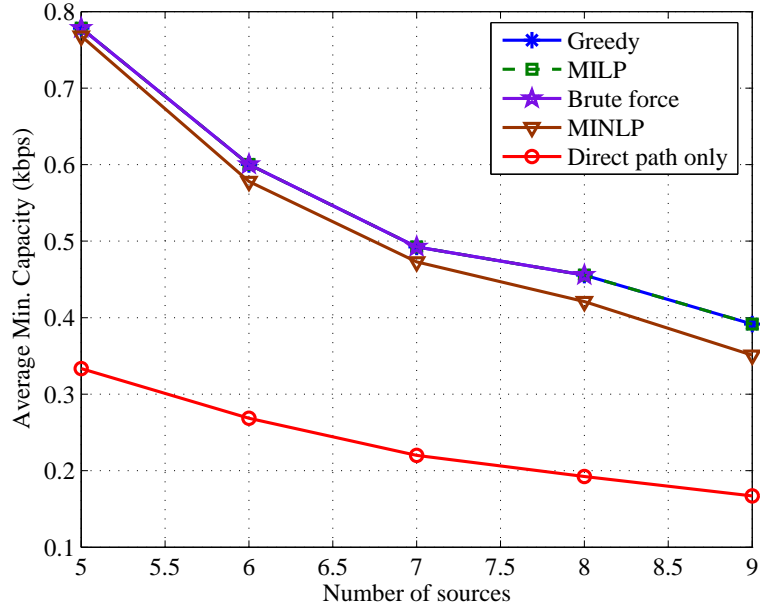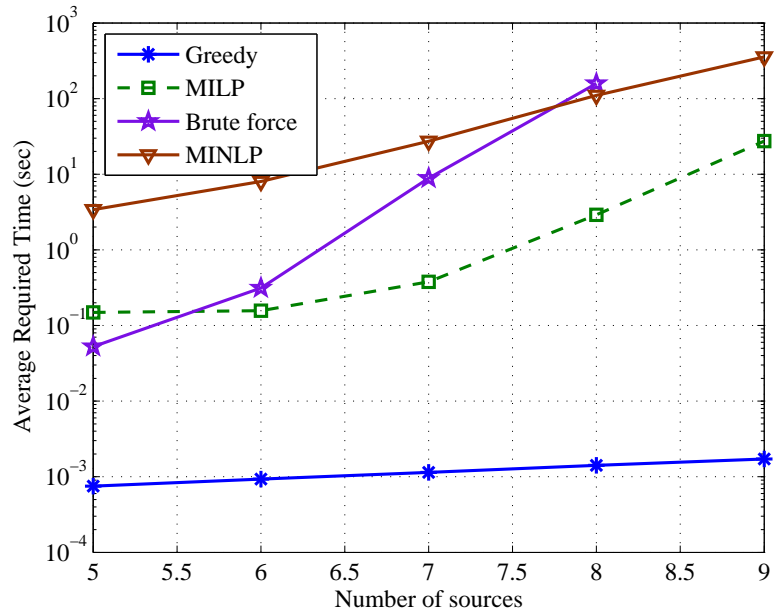
Figure 7.2 Average of minimum capacities obtained by the proposed greedy algorithm, the MILP and MINLP solutions. Cell radius $\rho = 3$km. Number of relays $L = 3$.

importantly, the average sum capacity obtained by the greedy algorithm approaches (practically overlaps) that of the optimal brute-force searching solution. It is also interesting to see from Fig. 7.3 that the sum capacity of the network without any relay is also higher than that of the MILP solution. This can be explained by the fact that in the MILP solution, a relay is still assigned to the sources whose capacities over the direct path are higher than the relay-assisted capacities. This results again emphasize that even with the reduction in path-loss offered by introducing two shorter transmission links (i.e., the source-relay and relay-destination links), the lengthened transmission period with one more phase (from the relay to destination) requires the relay-assisted transmission to be carefully designed in order to obtain any potential improvement in terms of bandwidth efficiency.
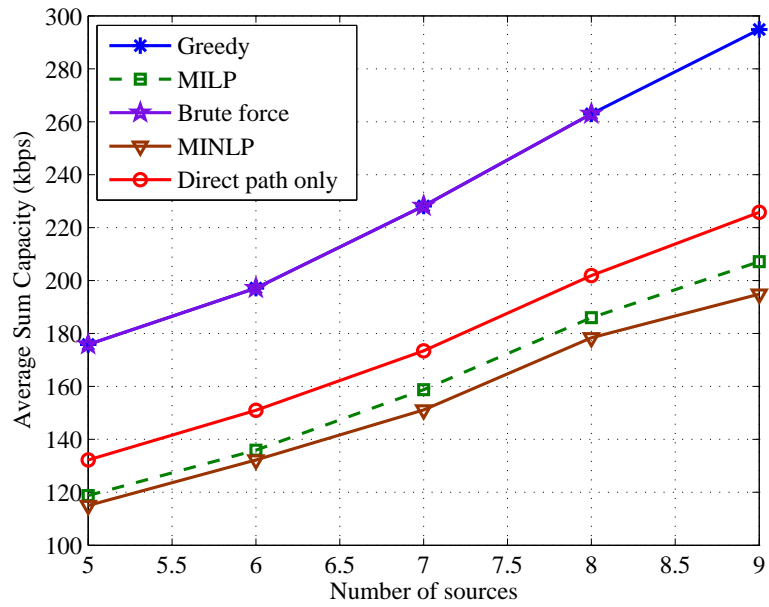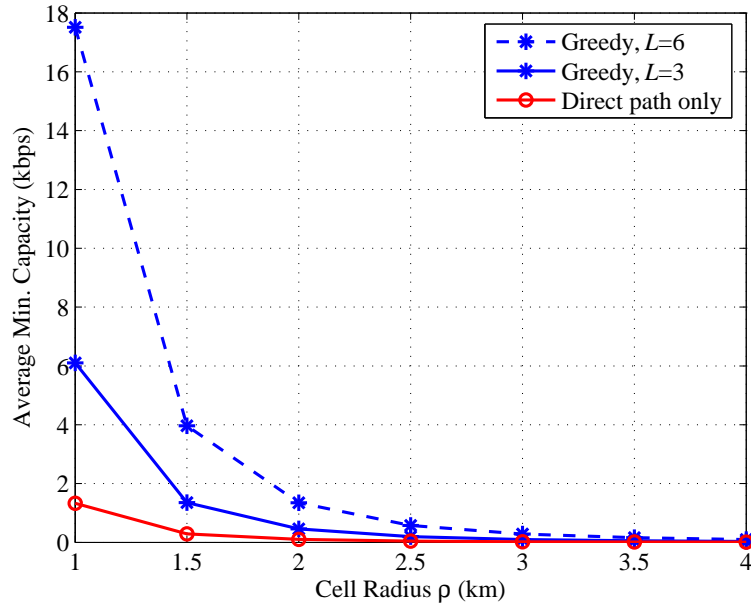
Figure 7.3 Average of sum capacities obtained by the proposed greedy algorithm, the MILP and MINLP solutions. Cell radius $\rho = 3$km. Number of relays $L = 3$.

**Figure 7.4** Average time required by the proposed greedy algorithm, the MILP and MINLP solutions. Cell radius $\rho = 3$km. Number of relays $L = 3$.

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The performance of the MINLP solution is also presented in Figures 7.2, 7.3 and 7.4. The solver *glcSolve* in [18] is used to solve problem (7.9). Since the convergence to the global optimal solution depends on the structure of the problem, a common way to obtain a good solution (which may approach the global optimal point) is to use iterations with the continually updated initial solution. Specifically, in our simulation, the number of iterations is $2^{(K-3)}$. It can be observed that with a much longer time required, both the average minimum and sum capacities obtained with the MINLP solution are still lower than those achieved by the MILP solution. The computational complexity of the MILP problem illustrated in Fig. 7.4 also shows that MILP softwares are too slow to apply for the relay assignment problem with a medium to larger number of sources. In such a scenario, the greedy algorithm is proven to be a practical choice. It is also noted that for the problem with a small number of sources, e.g., $K = 5$, the brute-force search runs faster than the MILP algorithm.

Figure 7.5 Average of minimum capacities obtained by the proposed greedy algorithm. Number of sources $K = 64$. Number of relays $L = 3, 6$.

Finally, a network model with $K = 64$ sources is simulated. The radius of the cell varies from 1km to 4km. The average of the min capacity and the sum capacity of the network achieved by the proposed greedy algorithm with $L = 3$ and $L = 6$ relays are shown in Fig. 7.5 and Fig. 7.6, respectively. When cell radius increases, both min capacity and sum capacity of the network decrease exponentially. However, the relay-assisted transmission method always performs better than the direct transmission method. These figures also illustrate that installing more relays in the cell offers a higher capacity for all sources.
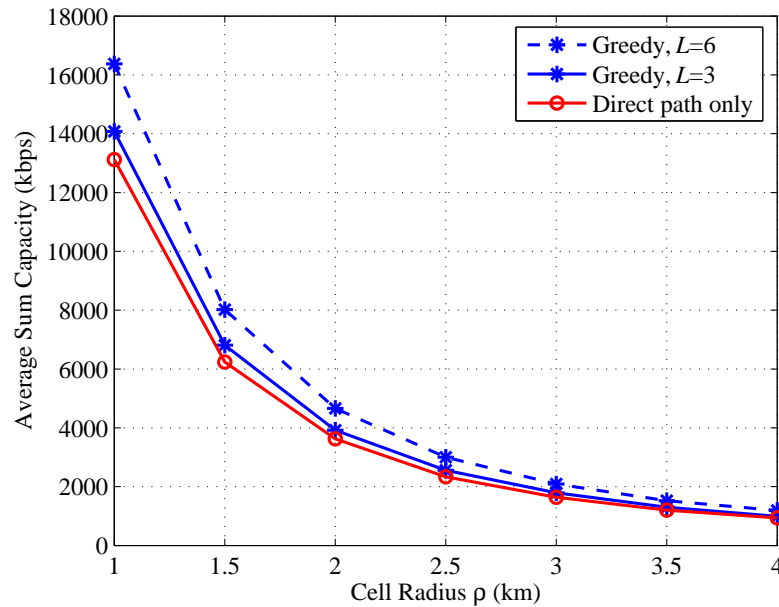
Figure 7.6 Average of sum capacities obtained by the proposed greedy algorithm. Number of sources $K = 64$. Number of relays $L = 3, 6$.

7.5 Conclusions

In this paper, the relay assignment problem in cooperative multiuser wireless networks has been addressed. The problem is formulated as a general bottleneck binary integer optimization problem and can be solved by several mixed-integer programming softwares. Based on two important observations on the considered network

model, we proposed a greedy algorithm in the form of lexicographical bottleneck assignment problem. The proposed greedy algorithm not only yields a solution that performs closely to the optimal solution with much a lower complexity, but it can also provide further improvements on other performance criteria of the network.

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8. Summary and Suggestions for Further Study

8.1 Summary

This thesis focussed mainly on developing power allocation schemes for wireless amplify-and-forward (AF) relay networks with different network topologies and channel state information assumptions. Specifically, the main contributions of this thesis are summarized as follows:

- Signal processing and power allocation issues in orthogonal AF relay networks have been developed under two partial CSI assumptions. Potential gains over conventional transmission without relaying techniques have been investigated in Chapter 3. Simulation results with different scenarios revealed that the performance improvement offered by relaying techniques strongly depends on various factors, such as the CSI availability at the relays and/or destination, and the specific conditions of the associated communication channels.

- As a further step of investigating the relaying networks under partial CSI assumptions, the thesis also developed a non-orthogonal relay transmission model with a modified beamforming scheme and limited rate feedback channels (Chapter 4). An interesting comparison between the orthogonal and non-orthogonal relay networks has been carried out to give a different viewpoint of the trade-offs involved in assessing these two network models.

- Moving from a relay network with a single source to a relay network with multiple sources, the role of power allocation changes significantly. We are interested in the fairness among multiple sources in terms of the error rate

and capacity. Research works that exploit this kind of resource to balance the quality-of-service (QoS) experienced by all sources result in two novel power allocation schemes for networks with a single relay (Chapter 5) and multiple relays (Chapter 6), respectively.

- In studying relay networks with multiple sources and multiple relays, a method to achieve a substantial performance improvement with a limited CSI requirement was investigated with relay selection and equal power allocation scheme (Chapter 7). Considered in the context of IEEE 802.16j WiMAX network model, the proposed algorithm has shown to be efficient in terms of capacity fairness and overall system throughput.

8.2   Suggestions for Further Studies

Currently, research efforts on different aspects of relaying techniques are still needed in order to put the cooperative/relaying ideas into real world applications. In particular, a relay-enabled mode to WiMAX has been developed in the IEEE 802.16j standard. Several cooperative relay features have also been proposed in LTE Advanced standards [1].

While conducting our research works, several issues arose that should be interesting for further studies. These issues are elaborated next.

- The first issue that exists in any closed-loop technique is the rate of channel variations. Specifically, in fast fading channels, updating power allocation scheme at a high rate implies a large increase in the amount of feedback information. In such situations, the objective of a power allocation scheme should be based on a long-term parameter such as ergodic capacity or the stability of some buffers equipped at the transmitters. An example of this is recently considered in [2–5].

- Another issue that could be present in fast fading channels is the quality of channel estimation process [6, 7]. Channel estimation errors and delays can deteriorate the potential performance improvement. A comparison between

coherent transmission methods (which require channel estimation) and non-coherent transmission methods (in which channel estimation is not needed) in relay networks under such imperfect channel conditions should give useful information.

- In relay networks with multiple sources, to avoid communication burden of the feedback/feedforward information, distributed signal processing schemes with limited information exchange are preferred. Current research directions in distributed signal processing for relay networks still focus on reservation-based multiple-access protocols such as OFDMA, TDMA, etc., which are most suitable to the centralized systems with regular traffic. In contrast, research works on distributed algorithms in relay network using contention-based protocols such as Aloha or CSMA have not been paid much attention. An exception is the study of routing schemes in CDMA/TDMA-based sensor networks [8].

- Finally, a complete study of wireless relay networks which takes into account both uplink and downlink channels has not been carried out yet. Such a study will give a better understanding of the advantages and disadvantages of relaying techniques when applied in cellular systems.

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