A Framework for Analyzing and Designing Scale Invariant Remote Sensing Algorithms

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***Abstract*— The land surface exhibits heterogeneity across a range of spatial scales. Remote sensors provide integrated information at the pixel scale, however, there is important spatial variability at scales smaller than the scale of the sensor. On the other hand, large scale models that use remotely sensed data do not require them at the same spatial resolution at which remote sensors are required to operate. In this paper, a framework for testing aggregation-disaggregation properties of remote sensing algorithms is presented. The proposed framework provides a systematic approach for parameterizing the land surface heterogeneity effects. For the estimation of the pixel scale response, the lumped response should be modified by the variance and covariance terms. This representation of land surface heterogeneity could lead to substantial savings in remote sensing data storage and management. Using simulated land and vegetation scenarios, we have successfully parameterized subpixel scale heterogeneity effects for the estimation of vegetation index, by modeling the variances and covariance terms with the pixel scale values.**

I. Introduction

The biosphere-atmosphere transfer, ecosystem process, and biogeochemistry models all require land surface parameters. Currently, the most feasible way to obtain these parameters over large areas is through satellite remote sensing. Land surface parameters including surface temperature, leaf area index, normalized difference vegetation index, fraction of photosynthetically active radiation, and soil moisture are needed at a variety of spatial scales, ranging from meters to hundreds of kilometers, for different process models. Processing global data sets at scales as fine as tens of meters is prohibitive with the existing computational capability. For example, over 130 million 1-km pixels would be required to cover the land surface across the globe, and consequently, use of remote sensing measurements at 1 km or finer resolution presents a significant challenge [6].

There are at least two widely debated issues in the use of satellite multispectral data for the characterization and representation of land surface and ecosystem parameters. First, land surface parameters exhibit important spatial and temporal variability at scales smaller than the scale of measurement. For instance, in the semiarid region of the Southwest United States vegetation density varies at scales ($\sim$meters) that are much smaller than the pixel scale of current multispectral sensors (tens of meters to kilometers). Several studies have provided substantial information on the spatial variability of vegetation and other terrain attributes at scales smaller than 100 m [20], [21], [25]. Pixel scale measurements provide integrated reflectance information, as a result, a *disaggregation* methodology is needed to decipher subpixel landscape characteristics.

Second, large-scale models that use remotely sensed land surface parameters do not require them at the same spatial resolution at which the remote sensors are required to operate. For example, current generation general circulation models use input levels of hundreds of kilometers, and cannot use land surface parameters below that spatial resolution. One outstanding research question critical to the integration of satellite derived data into global models is how adequately the inherent spatial heterogeneity is represented at scales commensurate with current generation global models [10]. To address this question, an *aggregation* methodology is needed that can bridge the scale gap between the scale of satellite measurements and large scale model resolution by taking into account the role of spatial heterogeneity in landscape parameters and responses.

The characterization of small-scale land surface heterogeneity in modeling the land-atmosphere system and in designing remote sensing algorithms have become a central focus of many recent studies [2], [26]. There is a considerable debate, however, over the influence of subgrid scale land surface heterogeneity on the grid scale response and over how to parameterize the effects of spatial heterogeneity in remote sensing algorithms. Bonan *et al.* [2] found that the heterogeneity of land surface is important to the grid level sensible and latent heat fluxes, whereas Wood and Lakshmi [26] found that latent heat flux is not particularly sensitive to the land surface heterogeneity.

An increasingly popular approach attempts to account for the effect of land surface heterogeneity on landscape responses by using the so called scale invariant algorithms or parameterizations. A scale invariant algorithm uses pixel level mean values of parameters, but promises to account for subpixel level heterogeneity (disaggregation). It also promises to provide a solution for the aggregation problem. This notion of deriving scale invariant land surface parameterizations and remote sensing algorithms is very appealing. For the retrieval of remotely sensed land surface parameters, if scale invariant algorithms could be designed, one would be able

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to use coarser resolution radiance data to infer landscape properties that need finer resolution radiance data. This would result in substantial savings in terms of computational and data storage requirements. Recently, Hall *et al.* [5] developed the conditions for scale invariance and have shown that mathematical structures relating radiation and surface fluxes to remote sensing parameters are nearly scale invariant for FIFE [First ISLSCP (International Satellite Land Surface Climatology Project) Field Experiment] study area. Their approach has primarily focused on a particular form of surface parameterizations, especially a product function like reflected component of shortwave radiation, and incorporated the effect of heterogeneity in landscape structure. For the chosen form of the parameterization, effects of heterogeneity was accounted for only through the covariance between parameters. Sellers *et al.* [19] have explored the relationship between surface conductance and spectral vegetation indices using the FIFE data set and have shown that near-linear relationship between these two parameters lead to scale invariance for the studied cases. Wood and Lakshmi [26] showed that the normalized difference vegetation index (NDVI) is scale invariant at the FIFE experiment site. The relative homogeneity of the FIFE site compared to a GCM (General Circulation Model) grid block, however, could prevent the generalization of this conclusion. In fact, recent results regarding scale invariance over heterogeneous land surfaces are rather mixed. For example, Pielke *et al.* [14] suggest that heterogeneity in land surface characteristics could produce significant differences in surface fluxes across heterogeneous patches. Consequently, they argue that land surface responses cannot be considered scale invariant. Mahrt and Sun [11] have reviewed several approaches to represent spatially averaged surface fluxes over heterogeneous surfaces. Based on the analysis of three different field experiment data, they argue that the effective exchange coefficients for spatially averaged surface fluxes are approximately independent of averaging scale, except for the case of weak flow over highly contrasted heterogeneous surfaces.

In order to address some of these questions regarding the effects of landscape heterogeneity on the performance of remote sensing algorithms, we will focus on the following objectives in this paper.

- Develop a framework to test the scaling properties (aggregation-disaggregation) of remote sensing algorithms.
- Demonstrate the utility of the above framework for the parameterization of land surface heterogeneity effects for the disaggregation problem.

Our primary focus in this paper will be on the disaggregation problem. Specifically, we will use the NDVI as an example and derive the variances and covariances of subpixel level radiances, using only pixel level information. The plan of the paper is as follows: in the next section we define a few terms that have specific meaning in this paper. In the following section we formulate a general framework to study the aggregation-disaggregation properties of remote sensing algorithms. Subsequently, we elaborate the procedure developed in Section III for two remote sensing algorithms. In Section IV, we include a proposed parameterization of NDVI that accounts for subpixel scale heterogeneity by using only pixel scale information, and concluding remarks are given in Section V.

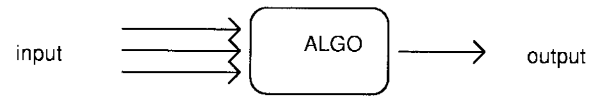


Fig. 1. Remote sensing algorithm takes a set of input parameters (e.g., Radiances) and produces an output (e.g., NDVI).

II. Definitions

A few terms used in this paper have specific meaning, we define those here. A heterogeneous land surface at the coarser resolution may be viewed as a collection of land surface elements at the finer resolution. The land surface at the coarser resolution is defined as pixel and at the finer resolution as subpixel here.

Remote sensing algorithm $ALGO$ takes a set of parameters as inputs and produces an output (Fig. 1). For example, the normalized deviation vegetation index is an algorithm. This algorithm takes two channels of radiances as input and produces the vegetation index as output.

In a lumped algorithm, the system is spatially averaged. On the other hand, the distributed algorithm considers land surface radiance properties taking place at various points in space and characterizes the variables as a function of spatial dimensions.

Here, a lumped algorithm takes pixel scale radiances as input and produces an output at the pixel scale. The output from a lumped algorithm is lumped output at the pixel scale. By a lumped algorithm, we mean that the domain is spatially homogeneous with regard to its inputs, parameters, and outputs (Fig. 2, right part).

Distributed algorithm calculates the pixel scale response by first dividing the pixel into a number of subpixels which can be assumed to be homogeneous. Then the response of each subpixel is aggregated by a suitable kernel (e.g., areal weighted average) to get the pixel scale output. Thus, a distributed algorithm accounts for spatial variability of inputs, parameters, and outputs within a pixel. The estimation of pixel scale response from subpixels through a suitable kernel is referred as aggregation (Fig. 2, left part).

Scale invariant algorithm means that the empirical relationship developed from point observation can be used for larger areas, e.g., at the pixel scale. A scale invariant algorithm will produce the pixel scale response if we use average parameters over the pixel as input. Quasiscale-invariant algorithm means that the resulting error from using an algorithm to estimate the pixel scale response would be small.

III. Analysis of Scale Invariance: A Framework

If a remote sensing algorithm is valid for a point scale or fine resolution, can we use this algorithm at a coarse resolution? Under what conditions, is the algorithm scale invariant? In order to answer these questions, we begin our discussion for

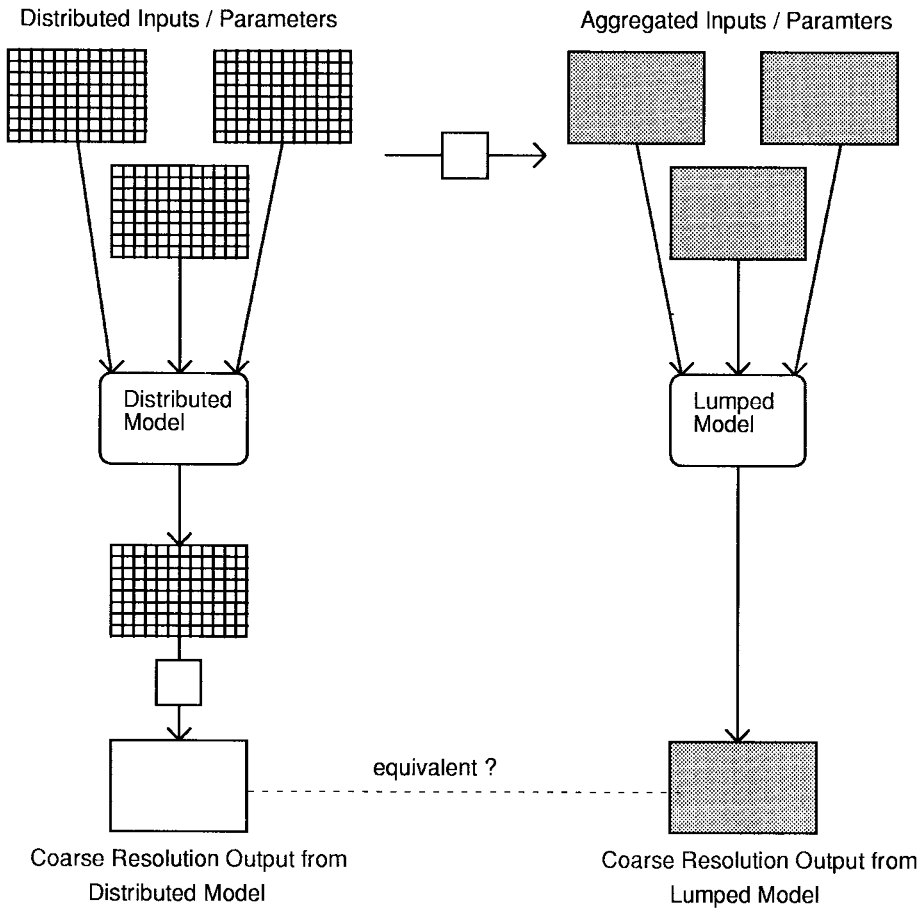


Fig. 2. Scheme for aggregation and scaling in land surface modeling and remote sensing.

an algorithm at the fine resolution. Then, we will discuss two requirements for scaling to be valid.

Let $f(P_1, P_2, \cdots, P_n)$ be the joint density function of parameter set $\{P_1, P_2, \cdots, P_n\}$ over a pixel (for example, 120 m square pixel), then the fractional area that has the same set of parameters $\{P_1, P_2, \cdots, P_n\}$ in the range of $\{\Delta P_1, \Delta P_2, \cdots, \Delta P_n\}$ will be

$$f(P_1, P_2, \cdots, P_n) \cdot \Delta P_1 \Delta P_2 \cdots \Delta P_n \tag{1}$$

where $\Delta P_1, \Delta P_2, \cdots$, and ΔP_n are interval of parameters. We assume here that each parameter is discretized into Q equal intervals. This is equivalent to dividing a pixel into Q patches with fractional area corresponding to the frequency of each interval as defined by (1).

Let the output from the algorithm at a point q be G_q having a parameter set at the point q $\{p_1, p_2, \cdots, p_n\}_q$, for a given remote sensing algorithm

$$G_q = ALGO(\{P_1, P_2, \cdots, P_n\}_q). \tag{2}$$

Then the pixel scale output will be

$$\overline{G}_D = \sum_{q=1}^{Q} ALGO(\{P_1, P_2, \cdots, P_n\}_q) \cdot f(\{P_1, P_2, \cdots, P_n\}_q)\, \Delta P_1\, \Delta P_2 \cdots \Delta P_n. \tag{3}$$

The subscript "D" indicates that the pixel scale output is aggregated from the output of a distributed model. Taking limit to the above equation with respect to Q (this is equivalent to dividing the pixel into finer elements), we have

$$\overline{G}_D = \int_{P_1} \int_{P_2} \cdots \int_{P_n} ALGO(P_1, P_2, \cdots, P_n) \cdot f(P_1, P_2, \cdots, P_n) \cdot dP_1 dP_2 \cdots dP_n. \tag{4}$$

This is a generalized formulation for the estimation of pixel scale quantities by taking subpixel heterogeneity into account. Interactions among remote sensing inputs and parameters within a pixel call for a joint stochastic distribution for these inputs and parameters. The lack of knowledge of the joint parameter density function of a remote sensing algorithm, however, makes practical use of (4) difficult. Nonetheless, (4) may be used to elucidate some properties of a scale invariant algorithm.

Let the density function for ith parameter be $f_i(P_i)$, then the pixel scale value of this parameter, which is the pixel average of this parameter, will be

$$\overline{P}_i = \int_{P_i} P_i f_i(P_i)\, dP_i. \tag{5}$$

Let $\{\overline{P}_1, \overline{P}_2, \cdots, \overline{P_n}\}$ be the pixel scale parameter set. If the point algorithm is valid at the pixel scale, we can use the algorithm to get the lumped output over a pixel as

$$\overline{G}_L = ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n). \tag{6}$$

If the algorithm is scale invariant, (4) and (6) will lead to same pixel scale output. In Hu and Islam [8], we have discussed at least two conditions under which the scale invariant assumption is appropriate. We briefly review the conditions here. First, *If the parameters are homogeneous over the pixel scale, then the algorithm is scale invariant.* Second, *If the algorithm can be described as a linear combination of inputs and parameters, then the algorithm is scale invariant.* Under these two conditions, we can show (4) and (6) are equivalent. Thus for homogeneous land surface or linear algorithms, remote sensing algorithms can be scaled up or down without any error.

For the naturally occurring heterogeneous land surfaces, either of these two conditions would be difficult to satisfy. Thus, theoretically, there may not be any simple scale invariant algorithms. However, if the error is very small between the aggregated output from a distributed algorithm and the output from a lumped algorithm, a quasiscale-invariant relationship could still be feasible. Such a quasiscale-invariant algorithm could be very useful for remote sensing application since it would greatly reduce the computational and storage requirement. Let us divide a pixel into m equal size subpixels and $\{P_1, P_2, \cdots, P_n\}_k$ be the parameter set for an algorithm at the subpixel k, and $\{\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n\}$ be the average parameter set for that algorithm over the pixel. Now using the theory of small perturbation, and neglecting third and higher order terms, we have

$$\begin{aligned} &ALGO(\{P_1, P_2, \cdots, P_n\}_k) \\ &\quad \approx ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n) \\ &\qquad + \left[(P_1 - \overline{P}_1)_k \frac{\partial}{\partial P_1} + (P_2 - \overline{P}_2)_k \frac{\partial}{\partial P_2} + \cdots \right. \\ &\qquad \left. + (P_n - \overline{P}_n)_k \frac{\partial}{\partial P_n}\right] ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n) \\ &\qquad + \frac{1}{2} \cdot \left[(P_1 - \overline{P}_1)_k \frac{\partial}{\partial P_1} + (P_2 - \overline{P}_2)_k \frac{\partial}{\partial P_2} + \cdots \right. \\ &\qquad \left. + (P_n - \overline{P}_n)_k \frac{\partial}{\partial P_n}\right]^2 ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n). \quad (7) \end{aligned}$$

Similar linearization approach using Taylor series has been used before in hydrologic and ecological literature [1], [3]. Most of these studies, however, considered one parameter at a time and have described the effects of heterogeneity in terms of variance of parameters. Using (7), we get the aggregated output over the pixel from a distributed algorithm as

$$\begin{aligned} \overline{G}_D &= \frac{1}{m} \sum_{k=1}^{m} ALGO(\{P_1, P_2, \cdots, P_n\}_k) \\ &\approx ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n) \\ &\quad + \frac{1}{m} \sum_{k=1}^{m} \frac{1}{2} \cdot \left[(P_1 - \overline{P}_1)_k \frac{\partial}{\partial P_1} \right. \\ &\quad \left. + (P_2 - \overline{P}_2)_k \frac{\partial}{\partial P_2} + \cdots (P_n - \overline{P}_n)_k \frac{\partial}{\partial P_n}\right]^2 \\ &\quad \cdot ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n). \quad (8) \end{aligned}$$

Note that $\overline{G}_D$ in (8) accounts for the effects of the surface heterogeneity. To get the lumped output over the pixel, $\overline{G}_L$, usually pixel level average parameters are used:

$$\overline{G}_L = ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n). \quad (9)$$

The difference between aggregated response from a distributed algorithm and lumped response can be estimated by

$$\begin{aligned} \overline{G}_D - \overline{G}_L &\approx \frac{1}{m} \sum_{k=1}^{m} \frac{1}{2} \cdot \left[(P_1 - \overline{P}_1)_k \frac{\partial}{\partial P_1} \right. \\ &\quad \left. + (P_2 - \overline{P}_2)_k \frac{\partial}{\partial P_2} + \cdots (P_n - \overline{P}_n)_k \frac{\partial}{\partial P_n}\right]^2 \\ &\quad \cdot ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n). \quad (10) \end{aligned}$$

Expanding (10), we have following equation which includes variance of parameters, covariance between parameters, and derivative terms that account for the degree of nonlinearity:

$$\begin{aligned} \overline{G}_D - \overline{G}_L &\approx \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2}{\partial P_i^2} ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n) \\ &\quad \cdot \frac{1}{m} \sum_{k=1}^{m} (P_i - \overline{P}_i)_k^2 + \frac{1}{2} \sum_{\substack{i=1 \\ l=1 \\ i \neq l}}^{n} \\ &\quad \cdot \frac{\partial^2}{\partial P_i \partial P_l} ALGO(\overline{P}_1, \overline{P}_2, \cdots, \overline{P}_n) \\ &\quad \cdot \frac{1}{m} \sum_{k=1}^{m} (P_i - \overline{P}_i)_k (P_l - \overline{P}_l)_k. \quad (11) \end{aligned}$$

The conditions described before are still valid for (11). If the parameters are homogeneous, then $P_i - \overline{P}_i$ is zero; for linear $ALGO$, the derivatives of $ALGO$ are zero, thus the scale invariance holds. If the algorithm is weakly nonlinear or the parameters are mildly inhomogeneous over the pixel, a quasiscale-invariant relationship may still be found because under such conditions, the right hand side of (11) will be small. In (11), the correlation between the remote sensing inputs and parameters also plays an important role in the nonlinear algorithms.

If the algorithm is not scale invariant, (11) would still provide a method to parameterize the pixel scale output from an algorithm. In order to estimate the pixel scale output, the lumped output should be modified by the algorithm parameter heterogeneity. For a given algorithm, the derivatives of an algorithm to its parameters are known. Thus, if we can parameterize the variances and covariances of parameters by their corresponding pixel scale mean values, (11) can be used to estimate the pixel scale response by taking into account the parameter heterogeneity. Furthermore, (11) can also be used to design scale invariant remote sensing algorithms. For naturally occurring land surfaces, homogeneity assumption would almost never be appropriate. Thus, in designing scale invariant remote sensing algorithms, one should look for linear combination of inputs and parameters.

IV. Applications

Equation (11) provides a systematic and generic framework for examining scale invariant properties of different algorithms. Here, we demonstrate the utility and effectiveness of (11) to determine scale invariant characteristics of two commonly used remote sensing algorithms: NDVI and TM-derived sensible and latent heat fluxes. We also use this framework to explain some findings from recent field experiments.

A. Normalized Difference Vegetation Index (NDVI)

The NDVI is usually calculated by using channel 1 (red) and channel 2 (infrared) data sensed by Advanced Very High Resolution Radiometer (AVHRR) or Thematic Mapper (TM):

$$\mathrm{NDVI} = \frac{R_2 - R_1}{R_2 + R_1} \tag{12}$$

where R_1 represents channel 1, and R_2 represents channel 2 reflectance or radiance. The reflectance or radiance R_1 is often referred to as the red and R_2 the near-infrared band. The NDVI is perhaps one of the most widely used vegetation index and the utility of NDVI for satellite assessment and monitoring of the global vegetation cover has been demonstrated over a decade [9]. The NDVI data have been related to physical properties of vegetation [16], [18], [23] and seasonal integrals of the NDVI have been correlated with the accumulation of above ground biomass [4], [24]. The NDVI data have been used in the studies of ecosystem structure and function, land cover classification, and carbon and biogeochemistry cycle of the earth [17], [22].

Substituting (12) into (11), we have the difference between aggregated normalized difference vegetation index $\overline{\mathrm{NDVI}}_D$, and a lumped estimate of the normalized difference vegetation index $\overline{\mathrm{NDVI}}_L$ as

$$\begin{aligned}\overline{\mathrm{NDVI}}_D - \overline{\mathrm{NDVI}}_L &= \frac{2\overline{R}_2}{(\overline{R}_2+\overline{R}_1)^3}\frac{1}{m}\sum_{k=1}^{m}(R_1-\overline{R}_1)_k^2 - \frac{2\overline{R}_1}{(\overline{R}_2+\overline{R}_1)^3} \\ &\quad \cdot \frac{1}{m}\sum_{k=1}^{m}(R_2-\overline{R}_2)_k^2 + \frac{2(\overline{R}_2-\overline{R}_1)}{(\overline{R}_2+\overline{R}_1)^3} \\ &\quad \cdot \frac{1}{m}\sum_{k=1}^{m}(R_1-\overline{R}_1)_k\cdot(R_2-\overline{R}_2)_k. \end{aligned} \tag{13}$$

We recall that $\overline{\mathrm{NDVI}}_D$ is estimated from a distributed NDVI algorithm while $\overline{\mathrm{NDVI}}_L$ is calculated from a lumped NDVI algorithm. The relative difference between these two estimates may be found as

$$\begin{aligned}\frac{\overline{\mathrm{NDVI}}_D - \overline{\mathrm{NDVI}}_L}{\overline{\mathrm{NDVI}}_L} &= \frac{2\overline{R}_2}{(\overline{R}_2+\overline{R}_1)^2(\overline{R}_2-\overline{R}_1)}\frac{1}{m}\sum_{k=1}^{m}(R_1-\overline{R}_1)_k^2 \\ &\quad - \frac{2\overline{R}_1}{(\overline{R}_2+\overline{R}_1)^2(\overline{R}_2-\overline{R}_1)}\frac{1}{m}\sum_{k=1}^{m}(R_2-\overline{R}_2)_k^2 \\ &\quad + \frac{2}{(\overline{R}_2+\overline{R}_1)^2}\frac{1}{m}\sum_{k=1}^{m}(R_1-\overline{R}_1)_k\cdot(R_2-\overline{R}_2)_k. \end{aligned} \tag{14}$$

From (13) and (14), we see that the relative error between the lumped and distributed NDVI algorithms depends on the variances of the red and near-infrared band radiances from the surface and the covariance between these two radiances. If the distribution of red and near-infrared band radiances is highly heterogeneous, then a simple scaling relationship for NDVI algorithm might not be found because in such cases the variances of the red or near-infrared band radiance would be large. Thus, for NDVI algorithm, we need to determine on what scale, the red and near-infrared bands are nearly homogeneous in order to use the scaling algorithm of NDVI. A hypothetical example is provided in Section IV-D to illustrate the effect of surface heterogeneity on the NDVI algorithm.

B. TM-Derived Sensible and Latent Heat Fluxes

Sensible and latent heat fluxes can be estimated using the TM thermal band (10.45–12.5 μm, with a resolution of 120 m) aboard Landsat following a procedure suggested by Holwill and Stewart [7]. The relationship between surface radiometric temperature and emittance is given for the Landsat thermal channel by Markham and Barker [12] as

$$T_g = \frac{K_2}{\ln\left(\dfrac{K_1}{R_g+1}\right)} \tag{15}$$

where R_g is surface emittance in $(\mathrm{Wm^{-2}Sr^{-1}}\,\mu\mathrm{m^{-1}})$, K_1 and K_2 are constant coefficients. At the FIFE site, $K_1 = 607.76\ \mathrm{Wm^{-2}Sr^{-1}}\,\mu\mathrm{m^{-1}}$ and $K_2 = 1260.56$ K after atmospheric calibration for August 15, 1987 [26].

In order to estimate the sensible heat flux, the exchange coefficient should be determined. Holwill and Stewart [7] presented a procedure to estimate the exchange coefficient by combining the surface station data with TM satellite thermal data. A TM-derived surface temperature is estimated for each location of surface flux stations. The TM surface temperature estimates and the station sensible heat flux measurements can be combined to provide an exchange coefficient at each station from the TM data in the following form:

$$g_{st} = \frac{H_{st}}{\rho C_p (T_g - T_a)} \tag{16}$$

where H_{st} is the observed station sensible heat and subscript "st" is for station. T_a is the observed station air temperature, and T_g the TM-derived surface temperature. ρ is the air density and C_p is the specific heat of air at constant pressure.

The exchange coefficients at all flux stations, g_{st}, are interpolated across the field experiment site through geostatistical kriging. We indicate this exchange coefficient field as g_r. Similarly, T_a is also interpolated across the site. Using exchange coefficient g_r and air temperature T_a fields and the TM-derived surface temperature T_g (all at a 120-m resolution), the sensible heat flux can be estimated over the domain by inverting (16). That is

$$H_r = \rho C_p g_r \left[\frac{K_2}{\ln\left(\dfrac{K_1}{R_g}+1\right)} - T_a\right] \tag{17}$$

where H_r is the sensible heat flux field derived from TM data. Subscript "r" is for remote sensing. Substituting (17) into (11), we have the difference of the aggregated sensible heat flux from a distributed sensible heat flux algorithm and the lumped sensible heat flux from a lumped sensible heat flux algorithm as

$$\overline{H}_{rd} - \overline{H}_r = \frac{1}{2}\rho C_p \overline{g}_r \left(\frac{\overline{T}_g}{\overline{D}}\right)^2 (2\overline{T}_g - K_2 - 2\overline{R}_g)\frac{1}{m} \cdot \sum_{k=1}^{m}(R_g - \overline{R}_g)_k^2 + \rho C_p \frac{\overline{T}_g^2}{\overline{D}}\frac{1}{m} \cdot \sum_{k=1}^{m}(g_r - \overline{g}_r)_k \cdot (R_g - \overline{R}_g)_k - \rho C_p \frac{1}{m} \cdot \sum_{k=1}^{m}(g_r - \overline{g}_r)_k \cdot (T_a - \overline{T}_a)_k \tag{18}$$

where

$$\overline{T}_g = \frac{K_2}{\ln\left(\dfrac{K_1}{\overline{R}_s + 1}\right)}$$
$$\overline{D} = K_2\overline{R}_g + \overline{R}_g^2.$$

The first term on the right hand side of (18) is due to the nonlinearity of sensible heat flux to the surface emittance, the second term in (18) is due to the correlation between the exchange coefficient and the surface emittance, and the last term in (18) is due to the correlation between the exchange coefficient and the air temperature at the observation level. A higher air temperature will lead to a more stable atmosphere, and more stable the atmosphere is, smaller will be the exchange coefficient. Thus, the exchange coefficient and air temperature are negatively correlated. The first two terms in (18) are essentially due to the correlation between exchange coefficient and surface temperature. Because the exchange coefficient and surface temperature are positively correlated, the right hand side of (18) is positive. Thus the heterogeneity of surface emittance, exchange coefficient, and air temperature at observation level would introduce errors in scaling up of sensible heat flux in the remote sensing algorithm.

The TM-derived latent heat flux can be estimated by assuming that the sum of the averaged latent and sensible heat fluxes for the station data and the TM-derived fluxes would be equal. Based on analyzes of the remote sensing algorithm for the sensible heat flux, we can infer that the heterogeneity of land surface would also introduce errors in scaling up of latent heat flux in the remote sensing algorithm.

C. An Illustration for Comparing Lumped, Distributed, and Proposed NDVI Algorithms

Here, we compare and contrast three approaches to estimate NDVI over a heterogeneous pixel consisting of a mixture of totally vegetated area (v) with fraction area f, and nonvegetated area (g) with fractional area $1 - f$. Following Price [15], we assume that the different surfaces are completely resolved by the sensor, and use $R_{1v} = 0.12$, $R_{2v} = 0.48$, $R_{1g} = 0.18$, and $R_{2g} = 0.20$. The lumped normalized difference vegetation index at the pixel scale $\overline{\text{NDVI}}_L$ is calculated by using (12) with pixel level average radiances and $\overline{R}_1$ and $\overline{R}_2$. The aggregated normalized difference vegetation index over the pixel scale from the distributed algorithm $\overline{\text{NDVI}}_D$is calculated by first applying (12) to get normalized vegetation index from each subpixel, then averaging over the pixel. Our proposed framework estimates the normalized difference vegetation index over the pixel by adding a correction term to the lumped estimate of the normalized difference vegetation index. That is

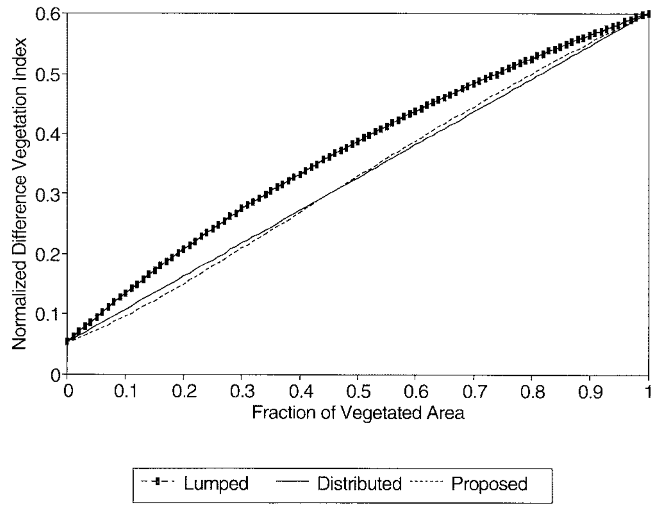


Fig. 3. Comparison of NDVI estimates from three approaches for a hypothetical example of heterogeneous land surface conditions.

$$\overline{\text{NDVI}}_P = \overline{\text{NDVI}}_L + \text{ correction term.} \tag{19}$$

The correction term accounts for the effect of surface heterogeneity. From (13), it can be expressed as

$$\text{correction term} = \frac{2\overline{R}_2}{(\overline{R}_2 + \overline{R}_1)^3}\frac{1}{m}\sum_{k=1}^{m}(R_1 - \overline{R}_1)_k^2 - \frac{2\overline{R}_1}{(\overline{R}_2 + \overline{R}_1)^3} \cdot \frac{1}{m}\sum_{k=1}^{m}(R_2 - \overline{R}_2)_k^2 + \frac{2(\overline{R}_2 - \overline{R}_1)}{(\overline{R}_2 + \overline{R}_1)^3} \cdot \frac{1}{m}\sum_{k=1}^{m}(R_1 - \overline{R}_1)_k \cdot (R_2 - \overline{R}_2)_k. \tag{20}$$

Fig. 3 compares and contrasts NDVI from these three approaches. Lumped algorithm always overestimates the NDVI when the surface heterogeneity exists. NDVI from our proposed algorithm is quite close to the distributed algorithm. This demonstrates that our proposed framework for including the effects of surface heterogeneity performs well and higher order heterogeneity effects may be neglected.

D. A Proposed Parameterization to Account for the Subpixel Heterogeneity

In the above example, we take advantage of the assumption that subpixel radiances are available. In reality, subpixel level radiances would be rarely available. Consequently, for the practical use of our proposed framework, variances and covariance of R_1 and R_2 must be known in advance. In the absence of a probability distribution function (or second order statistical moments), a parameterization of the variances and covariance of R_1 and R_2 by the pixel scale radiances is desirable.

For the example of Section IV-C, we find that the relationship between the normalized standard deviation of radiance (NS): for R_1 or R_2 and the normalized mean radiance over the pixel (NA) is a half ellipse (Fig. 4). That is

$$NS = \sqrt{1 - 4(NA - 0.5)^2} \tag{21}$$

where

$$NA = \frac{\overline{R} - \overline{R}_{\min}}{\overline{R}_{\max} - \overline{R}_{\min}} \tag{22a}$$

$$NS = \frac{S - S_{\min}}{S_{\max} - S_{\min}} \tag{22b}$$

where S is for the standard deviation of reflectance or radiance. The bar on the symbol is for the pixel level values or average, and subscripts $\min$ and $\max$ are for the minimum and maximum possible values over pixels. The above relationship can be derived analytically by using the definitions of average radiance and variance of radiance over a pixel. For example, average radiance for channel 2 radiance can be expressed as:

$$\overline{R}_2 = R_{2v} f + R_{2g}(1 - f). \tag{23}$$

Since R_{2g} is the lowest radiance and R_{2v} is the highest radiance for channel 2, from (23), we have

$$\overline{R}_{2\ \min} = R_{2g}$$

and

$$\overline{R}_{2\ \max} = R_{2v}. \tag{24}$$

Then the normalized pixel average radiance for channel 2 is

$$NA_2 = \frac{\overline{R}_2 - \overline{R}_{2\ \min}}{\overline{R}_{2\ \max} - \overline{R}_{2\ \min}} = f. \tag{25}$$

The variance of channel 2 radiance can be calculated as follows:

$$\begin{aligned} V_2 &= \frac{1}{m} \sum_{k=1}^{m} (R_2 - \overline{R}_2)_k^2 \\ &= f(R_{2v} - \overline{R}_2)^2 + (1 - f)(R_{2g} - \overline{R}_2)^2. \end{aligned} \tag{26}$$

Using (23)–(25), one finds that the variance can be rewritten as

$$\begin{aligned} V_2 = (\overline{R}_{2\ \max} - \overline{R}_{2\ \min})^2 \frac{\overline{R}_2 - \overline{R}_{2\ \min}}{\overline{R}_{2\ \max} - \overline{R}_{2\ \min}} \\ \cdot \left(1 - \frac{\overline{R}_2 - \overline{R}_{2\ \min}}{\overline{R}_{2\ \max} - \overline{R}_{2\ \min}}\right). \end{aligned} \tag{27}$$

The variance reaches its maximum when the normalized pixel average radiance is 0.5. Thus, the maximum variance is

$$V_{2\ \max} = \tfrac{1}{4} (\overline{R}_{2\ \max} - \overline{R}_{2\ \min})^2. \tag{28}$$

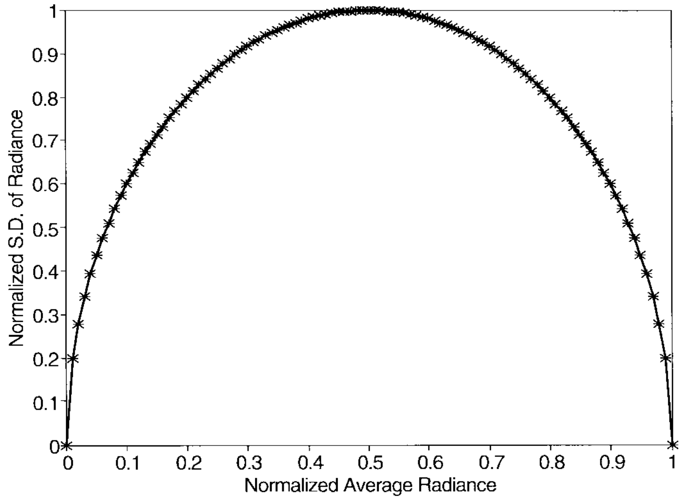


Fig. 4. Relationship between normalized standard deviation and the normalized average radiance.

From (25), (27) and (28), the relationship between standard deviation of radiance and the normalized pixel average radiance is expressed as

$$S_2 = S_{2\ \max} \sqrt{1 - 4(NA_2 - 0.5)^2}. \tag{29}$$

Because the minimum standard deviation of radiance is zero, we can rewrite(29) as

$$NS_2 = \sqrt{1 - 4(NA_2 - 0.5)^2} \tag{30}$$

which is (21) for the channel 2 radiance.

The covariance between the radiances of the channels 1 and 2 can be written as

$$\begin{aligned} \text{Cov}\ (R_1, R_2) &= \frac{1}{m} \sum_{k=1}^{m} (R_1 - \overline{R}_1)_k \cdot (R_2 - \overline{R}_2)_k \\ &= f(R_{1v} - \overline{R}_1)(R_{2v} - \overline{R}_2) + (1 - f) \\ &\quad \cdot (R_{1g} - \overline{R}_1)(R_{2g} - \overline{R}_2). \end{aligned} \tag{31}$$

By using (22a), (23), and (28), one can rewrite (31) as

$$\text{Cov}\ (R_1, R_2) = -4 S_{1\ \max} S_{2\ \max} (1 - NA_1)(1 - NA_2). \tag{32}$$

Thus, the covariance between the radiances of the channels 1 and 2 is only related to the pixel averages through the normalized average radiances.

Combining (19), (20), (29), and (32), one has the following expression for our proposed algorithm for estimating pixel NDVI while accounting for the surface heterogeneity:

$$\begin{aligned} \overline{\text{NDVI}}_P = \overline{\text{NDVI}}_L &+ \frac{2\overline{R}_2}{(\overline{R}_2 + \overline{R}_1)^3} S_{1\ \max}^2 [1 - 4(NA_1 - 0.5)^2] \\ &- \frac{2\overline{R}_1}{(\overline{R}_2 + \overline{R}_1)^3} S_{2\ \max}^2 [1 - 4(NA_2 - 0.5)^2] \\ &+ \frac{2(\overline{R}_2 - \overline{R}_1)}{(\overline{R}_2 + \overline{R}_1)^3} (-4) S_{1\ \max} S_{2\ \max} \\ &\cdot (1 - NA_1)(1 - NA_2). \end{aligned} \tag{33}$$

Since this formula is derived directly from the definitions of variances and covariance, the plot of $\overline{\mathrm{NDVI}}_P$ versus vegetation fraction should be exactly the same as in Fig. 3. However, the beauty of (33) is that we only need the information at the pixel scale for estimating the NDVI that includes subpixel scale heterogeneity effects. This will substantially reduce the computation and storage requirements. The above relationships between the average, standard deviation, and covariance of radiances need to be validated from the observed radiance and NDVI data. For our illustration, we have used the most commonly used NDVI algorithms. Many studies, however, have found the NDVI to be sensitive to soil, atmospheric conditions, sun-view geometry, and the presence of dead material [13]. We hope future studies would seek to utilize our approach to analyze improved version of the NDVI by correcting for soil and atmospheric sources of variances.

V. Concluding Remarks

A method has been developed to test aggregation-disaggregation properties of remote sensing algorithms. This analytical framework permits identification of two conditions under which upscaling or downscaling of remote sensing algorithms can be performed without incurring significant errors. These two conditions are existence of homogeneous land surface or linear representation of remote sensing algorithms. For commonly encountered land surface and remote sensing algorithms, these two conditions would often be violated.

To account for effects of heterogeneity and nonlinearity, we propose a new representation by parameterizing the variance and covariance terms with pixel scale mean values of parameters. This representation could lead to substantial savings in remote sensing data processing and management. We have demonstrated the utility of this framework by analyzing remote sensing algorithms for the estimation of sensible heat flux and vegetation index. Our preliminary analysis suggests that TM-derived sensible heat flux and normalized difference vegetation index cannot be scaled up or down without introducing error. Here, we have only used two commonly used remote sensing algorithms to demonstrate the utility and effectiveness of our proposed approach. We hope future studies would utilize the proposed framework to test aggregation-disaggregation properties of other remote sensing algorithms.

To parameterize the effects of subpixel scale heterogeneity in the estimation of NDVI, we have developed an explicit functional relationship between pixel scale mean radiance and correction terms involving variance and covariance of radiances. Although this framework can provide a reasonably accurate estimation of NDVI over heterogeneous surfaces, we must emphasize here that to keep the analytical framework tractable, we have neglected third and higher order correction terms. Before extending the framework to include higher order correction terms, future studies should seek to validate the proposed functional relationship between pixel scale values and correction terms by using remotely sensed data. To test whether we can infer subpixel scale land surface properties by using only pixel scale information, we need remote sensing measurement from at least two sensors, one with a very fine resolution and the other with a coarser resolution, for the same region. We are currently analyzing data sets from several sensors and hope to present our findings in the future.

References

[1] L. E. Band, D. L. Peterson, S. W. Running, J. Coughlan, R. Lammers, J. Dungan, and R. Nemani, “Forest ecosystem processes at the watershed scale: Basis for distributed simulation,” *Ecological Modeling,* vol. 56, no. 171–196, 1991.

[2] G. B. Bonan, D. Pollard, and S. L. Thompson, “Influence of subgrid-scale heterogeneity in leaf area index, stomatal resistance, and soil moisture on grid-scale land-atmosphere interactions,” *J. Climate,* vol. 6, no. 1882–1897, 1993.

[3] E. Bresler and G. Dagan, “Variability of yield of an irrigated crop and its causes: Statement of the problem and methodology,” *Water Resource Res.,* vol. 24, no. 3, pp. 318–387, 1988.

[4] S. N. Goward and D. G. Dye, “Evaluating North American primary productivity with satellite observations,” *Adv. Space Res.,* vol. 7, pp. 165–174, 1987.

[5] F. G. Hall, K. F. Huemmrich, S. J. Goetz, P. J. Sellers, and J. E. Nickeson, “Satellite remote sensing of surface energy balance: Success, failures, and unresolved issues in FIFE,” *J. Geophys. Res.,* vol. 97, no. D17, pp. 19061–19089, 1992.

[6] F. G. Hall, J. R. Townshend, and E. T. Engman, “Status of remote sensing algorithms for estimating land surface parameters, *Remote Sens. Environ.,* vol. 51, pp. 138–156, 1995.

[7] C. J. Holwill and J. B. Stewart, “Spatial variability of evaporation derived from aircraft and groundbased data,” *J. Geophys. Res.,* vol. 97, no. D17, pp. 18673–18680, 1992.

[8] Z. Hu and S. Islam, “Analysis of the influence of surface heterogeneity on the remote sensing algorithms,” in *AGU Spring Meet., EOS Trans.,* 1995, vol. 76, no. 17, p. 116.

[9] A. R. Huete, “An error and sensitivity analysis of the atmospheric- and soil-correcting variants of the NDVI for the MODIS-EOS,” *IEEE Trans. Geosci. Remote Sensing,* vol. 32, no. 4, pp. 897–905, 1994.

[10] C. O. Justice, J. R. G. Townshend, and V. L. Kalb, “Representation of vegetation by continental data sets derived from NOAA-AVHRR data,” *Int. J. Remote Sens.,* vol. 12, pp. 999–1021, 1991.

[11] L. Mahrt and J. Sun, “Dependence of surface exchange coefficients on averaging scale and grid size,” *Q. J. R. Meteorol. Soc.,* vol. 121, pp. 1835–1852, 1995.

[12] B. L. Markham and J. L. Barker, “Landsat MSS & TM post-calibration dynamic ranges, exoatmospheric reflectones, and at satellite temperatures,” *EOSAT Landsat Tech. Note,* vol. 1, pp. 3–8, 1986.

[13] R. B. Myneni, G. Asrar, D. Tanre, and B. J. Choudhry, “Remote sensing of solar radiation absorbed and reflected by vegetated land surfaces,” *IEEE Trans. Geosci. Remote Sensing,* vol. 30, pp. 302–314, 1992.

[14] R. Pielke, G. A. Dalu, J. S. Snook, T. J. Lee, and T. G. F. Kittel, “Nonlinear influence of mesoscale land use on weather and climate,” *J. Climate,* vol. 4, pp. 1053–1069, 1991.

[15] J. C. Price, “Using spatial context in satellite data to infer regional scale evapotranspiration,” *IEEE Trans. Geosci. Remote Sensing,* vol. 28, no. 5, pp. 940–948, 1990.

[16] S. D. Prince, C. O. Justice, “Coarse resolution remote sensing of the Sahelian environment,” *Int. J. Remote Sens.,* vol. 12, pp. 1133–1421, 1991.

[17] S. W. Running, “Estimating terrestrial primary productivity by combining remote sensing and ecosystem simulation,” *Ecological Studies, Vol. 79, Remote Sensing of Biosphere,* H. Mooney and R. Hobbs., Eds. New York: Springer-Verlag, 1990, pp. 65–86.

[18] P. J. Sellers, “Canopy reflectance, photosynthesis and transpiration,” *Int. J. Remote Sens.,* vol. 6, pp. 1335–1372, 1985.

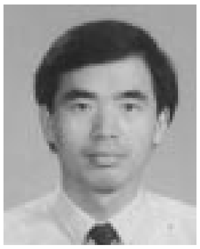
[19] P. J. Sellers, M. Heiser, and F. G. Hall, “Relations between surface conductance and spectral vegetation indices and intermediate (100 m^2 to 15 km^2) length scales,” *J. Geophys. Res.,* vol. 97, no. D17, pp. 19033–19059, 1992.

[20] J. R. G. Townshend, “The spatial resolving power of Earth resources satellites: A review,” *Progr. Phys. Geog.,* vol. 5, pp. 32–55, 1981.

[21] J. R. G. Townshend and C. O. Justice, “Selecting the spatial resolution of satellite sensors for global monitoring of land transformations,” *Int. J. Remote Sens.,* vol. 9, pp. 187–236, 1988.

[22] J. R. G. Townshend, C. O. Justice, W. Li, C. Gurney, J. McManus, “Global land cover classification by remote sensing: Present capabilities and future capabilities,” *Remote Sens. Environ.,* vol. 35, pp. 243–255, 1991.

[23] C. J. Tucker, "Red and photographic infrared linear combinations monitoring vegetation," *Remote Sens. Environ.*, vol. 8, pp. 127–150, 1979.
[24] C. J. Tucker, B. N. Holben, J. H. Elgin, and J. E. McMurtry, "Remote sensing of total dry matter accumulation in winter wheat," *Remote Sens. Environ.*, vol. 11, pp. 171–189, 1981.
[25] C. E. Woodcock and A. H. Strahler, "The factor of scale in remote sensing," *Remote Sens. Environ.*, vol. 21, pp. 311–332, 1987.
[26] E. F. Wood and E. Lakshmi, "Scaling water and energy fluxes in climate systems: Three land-atmospheric modeling experiments," *J. Climate*, vol. 6, pp. 839–857, 1993.



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