Fuzzy Logic Controller for Bidirectional Garaging of a Differential Drive Mobile Robot *

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Abstract

This paper presents a procedure for the design of a fuzzy logic controller for the garaging of a wheeled mobile robot, with non-holonomic constraints and discrete control signal levels. The design procedure is based on the virtual fuzzy magnet principle, which implies the definition of fictive points in the garage surroundings, and which, in an appropriate manner, by robot attraction, provides an efficient garaging process. The proposed fuzzy logic controller has four input variables: two represent the robot's distancing from fictitious fuzzy magnets, and two are angles that define the orientation of the vehicle. The output variables are related to voltages sent to motors in charge of propelling the left and right wheel of the robot. The efficiency and shortfalls of the proposed algorithm are analyzed by means of both detailed simulations and multiple re-runs of a real experiment. Special attention is devoted to the analysis of different initial robot configurations and the effect of an error in the estimation of the current position of the robot on garaging efficiency.

Keywords: bidirectional garaging, fuzzy controller, fictitious fuzzy magnets, non-holonomic mobile robot, real-time control

1 Introduction

The process of designing algorithms for the control of Wheeled Mobile Robot(WMR) is a big challenge, as it brings along all the problems that can be encountered in control theory. In the first place, there are non-linear and non-stationary phenomena characteristic of these vehicles, and then there are the complexity of real time control, reliable communication with appurtenant sensors, and the extraction of precise information from measurements provided by these sensors. These problems contain a high level of uncertainty regarding the surroundings of autonomous robots, and an efficient control algorithm is expected to have a reaction highly correlated to the status of those surroundings [1]. Finally, each of the algorithms endeavors, more or less, to model and formulate human behavior in the process of solving these problems. In line with the described problems and challenges arising from the algorithms for control of

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autonomous robot systems, literature offers various approaches to their solution. There are approaches which do not target imitation of human behavior but instead choose implementation of simple or well-known control forms. These are time-varying controllers, usually characterized by low convergence speed [2, 3], or time-invariant controllers with discontinuities which provide exponential convergence speed [4, 5]. The approaches to multi-level controllers use, at a lower level, the traditional proportional, integral and differential laws of control, whereas the higher, intelligent level, most often deals with an imitation of human behavior, in order to avoid constraints imposed by the surroundings [6, 7], or with adequate selection of the desired robot trajectory [8].

From a mobile robot control perspective, robot garaging can be solved as a stabilization problem or as a problem of following an *a priori* generated garaging trajectory, on the one hand, or as a vision problem, on the other hand. Robot control methods based on vision do not require the identification of the position or localization of the robot within the work area [9], contrary to other approaches which require accurate localization of the controlled object, whether by posture stabilization control methods [10, 11, 12], trajectory stabilization [13] or path following methods [14, 15], i.e. path tracking methods [16, 17].

Garaging by following a trajectory introduces another problem, that of generating the garaging trajectory itself. The characterization and/or generation of mobile robot trajectories, along with different constraints and/or optimizations, are described in [18, 19, 20, 21, 22].

In presenting the idea behind fuzzy logic, Zadeh [23] illustrates its application in car parking. In [24], Zadeh sets forth four rationales in cases where it is prudent to apply a fuzzy-logic-based methodology. To illustrate the case where a rationale is not needed, he mentions parking problems and states: "In this case, there is a tolerance for imprecision that can be exploited to achieve tractability, robustness, low solution cost, and better rapport with imprecision is an issue of central importance in computing with words".

In the literature, the fuzzy method is generally used to address WMR parking/garaging problems of the following three types of vehicles: car-like mobile robots, WMRs with one or more trailers, and differential drive mobile robots.

Reference [25] reports on an experimental study of the Fuzzy Logic Controller (FLC) for garaging car models. The fuzzy control design is based on the fuzzy conclusion of the Takagi – Sugeno (T-S) type [26], used to model drivers' garaging experiences. In [27], the hardware for car model garaging was similar to that used in [25] and a fuzzy controller was designed for fourteen oral instructions. The fuzzy point stabilization control of a car model, with two FLCs (the first for the steering angle and the second for the acceleration or deceleration force) is discussed in [12]. Although simulation results are presented, the constraints of the proposed system are not addressed.

In [28], FLCs were designed for the tasks of parallel parking and backward garaging. The number of fuzzy rules was reduced in the originally designed heuristic FLC, based on pre-defined criteria, and the FLC was then tuned applying an evolutionary strategy. In [29, 30], parking control using visual sensors by means of neuron networks and an FLC are discussed. The control strategy has been verified by an autonomous mobile robot experiment. Reference [31] proposes an intelligent garage parking system based on a fuzzy target and reinforcement learning. A navigation methodology based on a sub-optimal reference trajectory, which is independent of the vehicle model is treated in [32]. The approach is along which the vehicle is guided with minimal error by means of an FLC and testing involved parallel parking in reverse.

A hierarchical fuzzy drive control system for differential-drive WMR control is presented in [33]. Positioning with respect to the surroundings was based on data obtained from a TV camera. The core of the system is a fuzzy drive expert system, which uses these data and two sets of rules to control the speed and orientation of the WMR. Experiments conducted with straight and circuitous trajectories, using a real WMR, show that the results are comparable to human driving skills under these conditions. In [34], the fuzzy drive expert system was enhanced using predictive fuzzy control with a forecast learning function. WMR experiments with a straight trajectory and two obstacles confirm the advantages of forecast learning fuzzy control relative to predictive fuzzy control and basic fuzzy control.

Reference [35] discusses a fuzzy control system used to navigate a mobile robot with five and ten trailers, in reverse, which is presented by a T-S fuzzy model. A stability criterion derived from the Lyapunov approach and linear matrix inequalities guarantees stability of the fuzzy controller for a simplified non-linear model. Simulations are used to illustrate the efficiency of the system and they corroborate a good match of the results obtained from fuzzy models and original models. A discrete fuzzy controller with a unit transport delay is analyzed in [36], and computer simulations of a backward – driven WMR with a single trailer confirm its advantage over models which do not address a transport delay.

This paper proposes a new methodology for differential-drive mobile robot garaging based on fuzzy logic and a new fictitious fuzzy magnet concept. The proposed approach can be applied in cases where the level of the control variable is discrete and where the number of quantization levels is relatively small. Compared to other algorithms which address this type of problem, the proposed algorithm is very simple, it does not rely on a WMR model, and it has twelve fuzzy rules whose parameters have a clear physical meaning. The proposed FLC is of the T-S type, it is generated manually (in a manner similar to human driving skills), and it relies on two fictitious fuzzy magnets, one of which is located in front of the parking garage and the other in the center of the parking garage. The fictitious fuzzy magnets contain fuzzy control rules which allow for bidirectional single-stage garaging, which is characteristic of only a few solutions available in the literature [18, 21].

This paper addresses in detail the sensitivity of the system to noise and systematic errors of sensors used in the estimation of the WMR position, and the sensitivity to any change in the parameters of the FLC membership function. A large number of computer simulations and repeated real experiments were used to systematically test the characteristics and constraints of the proposed fictitious fuzzy magnet concept. The second section of this paper describes the posed problem. The third section is dedicated to the FLC design methodology. It defines the fictitious fuzzy magnet concept, the controller structure, the input and output variables, the corresponding membership functions, and the fuzzy rules. The fourth section deals with simulation and experimental results of the application of a designed FLC. Garaging results are presented for a large number of different initial positions of the robot, accompanied by an estimate of final positioning error. The results obtained indicate the high efficiency of the proposed method and the possibility of its application in a large number of different situations. The concluding section of the paper contains a summary of the main features of the proposed algorithm and of its applicability, constraints, and possible areas of improvement so that it can be used to address a broader range of problems.

2 Garaging Problem

The garaging/parking problem implies that a mobile robot is guided from an initial configuration $(x_{r(0)}, y_{r(0)}, \psi_{r(0)})$ to a desired one (x_G, y_G, ψ_G), such that it does not collide with the garage. Figure 1 shows the garage and robot parameters; the position of the garage is defined by the coordinates of its center – C_m, while its orientation is defined by the angle of the axis of symmetry of the garage – S_g relative to the y axis, identified as β in the figure. The width of the garage is designated by W_g and the length by L_g, and it is understood that garage dimensions enable the garaging of the observed object. The implication is that point (x_G, y_G) is inside the garage and is set to coincide with the center of the garage C_m, such that $C_m(x_G, y_G)$. In the case of bidirectional garaging, the targeted configuration is not uniquely defined because the objective is for the longitudinal axis of symmetry of the robot to coincide with that of the garage, such that $\delta = 0$ or $\delta = \pi$, meaning that there are two solutions for the angle $\psi_G : \psi_{G_1} = \beta,\ \psi_{G_2} = \beta + \pi$. For reasons of efficiency, the choice between these two possibilities should provide the shortest travel distance of the mobile robot. The controller proposed in this paper does not require *a priori* setting of the angle ψ_G, because it has been designed in such a way that the mobile robot initiates the garaging process from the end closer to the garage.

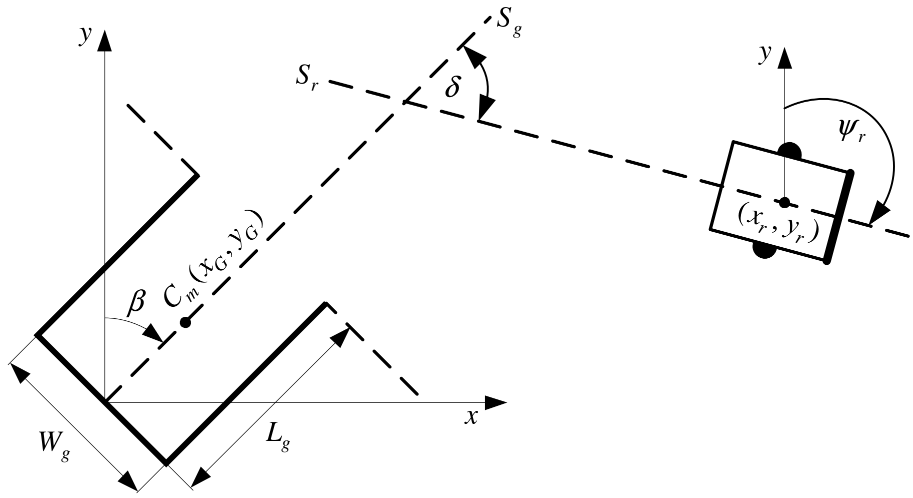

Figure 1: Garage and robot parameters.

The bidirectional garaging problem is similar to the stabilization problem, also known as the "parking problem", formulated in [37]: "the robot must reach a desired configuration $x_G,\ y_G,\ \psi_G$ starting from a given initial configuration $(x_{r(0)}, y_{r(0)}, \psi_{r(0)})$". The differences are that there are two solutions for the desired configuration in bidirectional garaging problems and that the stabilization problem need not necessarily involve constrains imposed by the presence of the garage.

Although the bidirectional garaging problem is addressed in this paper as a stabilization problem, the proposed solution can also be used as a garaging trajectory generator because it includes non-holonomic constraints of the mobile robot.

3 Fuzzy Controller Desing

3.1 Fictitious fuzzy magnets concept

A given point $A(x_A, y_A)$, is assumed to be in the Cartesian coordinate system and to represent the position of the fictitious fuzzy magnet FM. The fictitious fuzzy magnet FM is defined as a arranged pair comprised of its position A and an added sub-set of fuzzy rules, $FRsS$ (Fuzzy Rules subSet):

$$FM = (A, FRsS) \tag{1}$$

The input variables of the $FRsS$ are the WMR distance and orientation relative to point A, while its output variables are the speeds of the left and right wheels of the WMR. Point (x_r, y_r) is the center of the mobile robot (Figure 2). The angle between the longitudinal axis of symmetry of the robot S_r and the segment which connects points (x_r, y_r) and A is denoted by α. If

$(x_r(k), y_r(k))$ denote the coordinates of the WMR center at the sample time k, then the distance of the robot from the position of the fictitious fuzzy magnet $d_A(k)$ is:

$$d_A(k) = \sqrt{(x_A - x_r(k))^2 + (y_A - y_r(k))^2} \qquad (2)$$

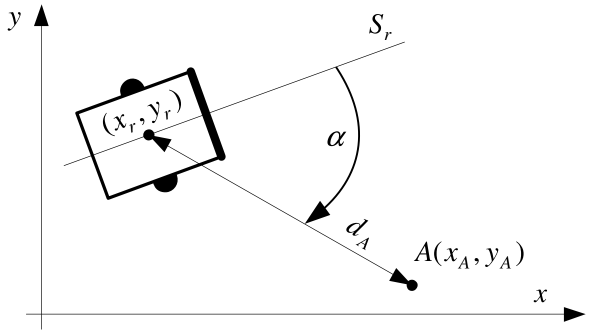

Figure 2: Mutual spatial positions of fictitious fuzzy magnet and vehicle.

The speed of the left and right wheels of the WMR is denoted by $v_L(k)$ and $v_R(k)$, respectively. The $FRsS$ of r control rules is defined as follows for the discrete T-S fuzzy system:

$$\begin{array}{ll} \textit{Control Rule } i: & \text{If } d_A(k) \text{ is } M_{i1} \text{ and } \alpha(k) \text{ is } M_{i2} \\ & \text{then } v_L(k) = C_{i1} \text{ and } v_R(k) = C_{i2} \end{array} \quad i = 1,\ 2, \ldots, r \qquad (3)$$

where $d_A(k)$ and $\alpha(k)$ are premise variables. The membership function which corresponds to the i^{th} control rule and the j^{th} premise variable is denoted by M_{ij}, and C_{ij} are constants. System outputs are $v_L(k)$ and $v_R(k)$, obtained from:

$$\begin{bmatrix} v_L(k) \\ v_R(k) \end{bmatrix} = \frac{\sum_{i=1}^{r} M_{i1}(d_A(k)) \cdot M_{i2}(\alpha(k)) \cdot [C_{i1}\ C_{i2}]^T}{\sum_{i=1}^{r} M_{i1}(d_A(k)) \cdot M_{i2}(\alpha(k))} \qquad (4)$$

where $M_{ij}(x_j(k))$ is the degree of membership of $x_j(k)$ in M_{ij}.
From a human control skill perspective, the garaging of a vehicle is comprised of at least two stages: approach to the garage and entry into the garage. The adjustment of the vehicle's position inside the garage might be the third stage, but the need for this stage depends on the success of the second stage (i.e., driver skills). Similarly, the FLC proposed in this paper uses two selected points, called fictitious fuzzy magnets: one immediately in front of the garage and the other inside the garage. However, the algorithm is not executed in stages. Instead, the first point is used to approach the garage and while the vehicle is entering the garage, it serves as a reference point for proper orientation, similar to human driving skills. The second point is both a target and a reference point. The entire algorithm is executed in a single stage. Other than human steering skills, the results shown in [21] for the minimum wheel travel distance were used to define the rules and membership functions.
The fictitious fuzzy magnets, created according to (1), are denoted by FM_{F_m} and FM_{C_m}. Their positions are identified by C_m and F_m (Figure 3). The fuzzy rule subsets $FRsS_{F_m}$ and $FRsS_{C_m}$ will be determined at a later stage. The point F_m lies on the garage axis of symmetry S_g at a distance d_F from the front line. Let us imagine that these points are fictitious fuzzy magnets with attraction regions around them, and if the mobile object/vehicle finds itself in that region, the attraction force will act on it. The activity of these fictitious fuzzy magnets will be neutralized in point C_m, thus finishing the garaging process.

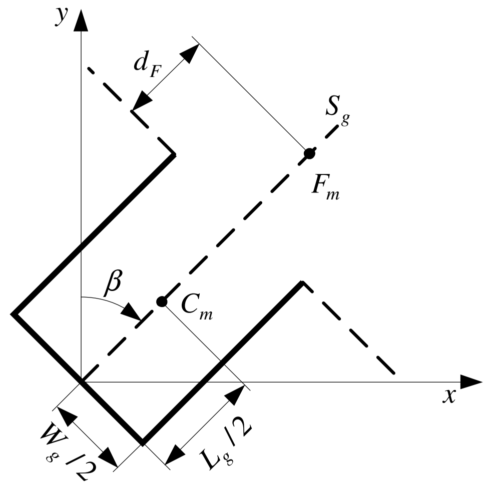

Figure 3: Locations of fictitious fuzzy magnets C_m and F_m.

Implementation of this concept implies the definition of interaction between fictitious fuzzy magnets and the vehicle. Figure 4 presents parameters which define mutual spatial positions of fictitious magnets and the vehicle. The distance from the vehicle to the fictitious fuzzy magnet C_m is marked as d_{C_m}, whereas the distance to the fictitious magnet F_m is marked as d_{F_m}.

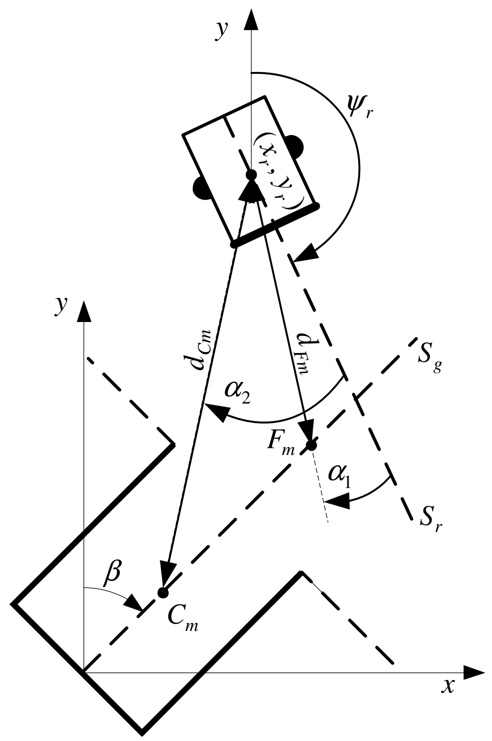

Figure 4: Mutual spatial positions of fictitious fuzzy magnets and vehicle.

The key step in the design of such an approach is the observation of these two distances as fuzzy sets which need to be attributed appropriate membership functions in an unusual way (Figure 5). Each of these variables is attributed only one linguistic variable, and these do not cover the complete set of possible values. However, such a concept fully complies with the idea of fictitious fuzzy magnets. The membership function $'near'\left(\mu_{near}\left(d_{C_m}\right)\right)$, which pertains to the distance from the vehicle to the fictitious fuzzy magnet C_m, enables the action of this fictitious magnet only in its immediate vicinity, whereas the membership function $'far'(\mu_{far}(d_{F_m}))$ does not enable the action of the fictitious magnet F_m in the immediate vicinity of its location. In this way, the vehicle is at all times in the attraction region of at least one fictitious magnet; a detailed procedure for the selection of parameters and fuzzy rules will be provided in the next section.

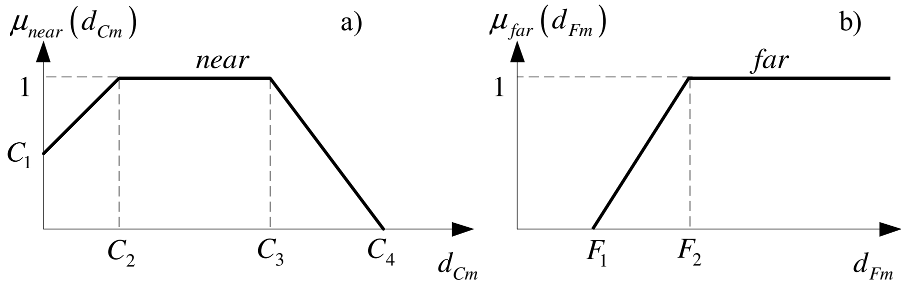

Figure 5: Definition of membership functions for distances: a)– d_{Cm} b) – d_{Fm}.

Distances from fictitious fuzzy magnets to the vehicle are not sufficient input variables for a proper vehicle control system. It is also necessary to know the vehicle orientation to the fictitious fuzzy magnets. The simplest way to introduce this orientation is by angles α_1 and α_2 presented in Figure 4. If angle α_1 is close to zero, then the vehicle's front end is oriented toward the fictitious fuzzy magnet F_m, whereas for the value of angle α_1 close to angle π, the vehicle's rear end is oriented toward F_m. This fact is important, as will be shown further in this text, from the aspect that the objective of vehicle control will be to reduce the angles α_1 and α_2 to a zero or π level, depending on initial positioning conditions.
Linguistic variables α_1 and α_2 are commonly called *Direction*, and are described by the same membership functions which indicate the vehicle orientation to the fictitious fuzzy magnets. The linguistic variable *Direction* is defined through following membership functions:

$$Direction\{Front, FrontLeft, FrontRight, Back, BackLeft, BackRight\} \tag{5}$$

or abbreviated as:

$$D\{F, FL, FR, B, BL, BR\} \tag{6}$$

as presented in Figure 6.

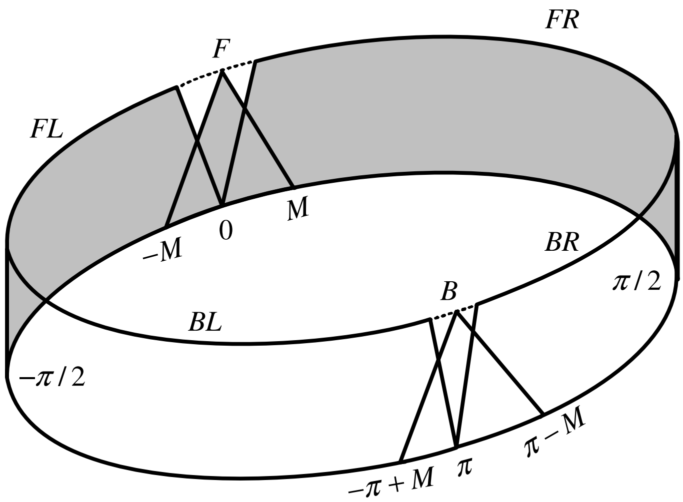

Figure 6: Illustration of membership functions for variable *D*.

In order to enable bidirectional garaging, the variable *Direction* is strictly divided into two groups – the angles related to orientation at the front $\{F, FL, FR\}$, which are grey in Figure 6, and the angles related to orientation at the rear $\{B, BL, BR\}$. The method proposed in this paper analyzes mobile objects with an equal ability of maneuvering by front and rear pace; therefore, the objective of defined membership functions and fuzzy rules is to provide identical garaging performance in both cases. For this reason, the membership function B (*Back*) is divided into two sub-functions B' and B'', in the following manner:

$$B = S(B', B'') \tag{7}$$

where S represents the operator of $S-norm$, which corresponds to the fact that the union of sets B' and B'' makes set B. The B' and B'' sets being disjunctive, the calculation of $S-norm$ is not important [38]. Consequently the linguistic variable *Direction* becomes attributed to seven membership functions $D\{F, FL, FR, B', B'', BL, BR\}$, as shown in Figure 7.

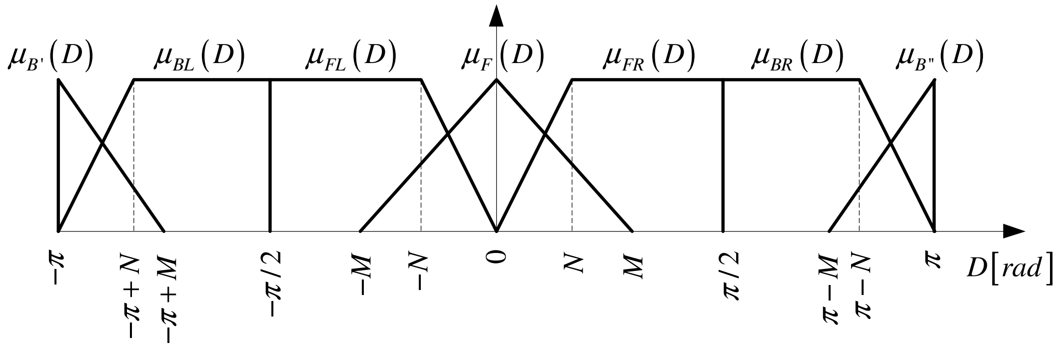

Figure 7: Membership functions of the linguistic variable *Direction*.

A conflict occurs when the orientation angles exactly equal $\pi/2$ or $-\pi/2$. Assuming that membership functions BL and BR, or FL and FR, are completely symmetrical, the controller will not produce any control command because this case is, from the aspect of a decision on vehicle garaging by front or rear pace, totally undefined. Although the probability of the occurrence of such a situation equals zero, it is overcome by the introduction of insignificant asymmetry by a slight overlapping of membership function pairs (FL, BL) and (FR, BR).

3.2 Output variables

The definition of output variables of the FLC requires a clear definition of the object of control. The algorithm described in this paper is applied to the control of a Hemisson robot. This is a mobile vehicle of symmetrical shape, with two wheels and a differential drive. Wheels are independently excited, and their speeds may be defined independently from each other. Guidance is controlled by the difference in wheel speeds. Each of these speeds can be set as one of the integer values in the interval $[-9, 9]$, (Figure 8), where a negative value means a change in the direction of wheel rotation. The dependence between the command and wheel speed is not quite linear causing some additional problems in the process of vehicle control.

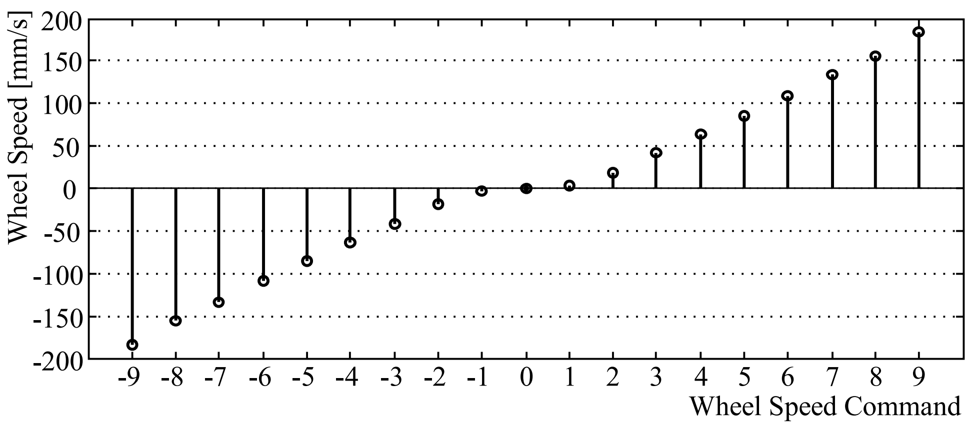

Figure 8: Wheel speed of the Hemisson robot as a function of wheel speed command.

Specifically, for the subject case, the selected output variables of the FLC are the speed commands of the left and right wheels: VL and VR, respectively. Accordingly, this discrete set of integer values from the interval $[-9,9]$ is selected as the domain of membership functions attributed to the output variables.

3.3 Fuzzy rules and fuzzy rule adjustment

Fuzzy rules are defined based on the following principles:

- As the distance from the vehicle to the garage grows, the total speed of the vehicle should increase;
- As the distance from the vehicle to the garage shrinks, the total speed of the vehicle should drop;
- The difference between the wheel speeds causes turning, which must depend on the robot's orientation toward the garage;
- In the case of good orientation, the robot speed may be maximal.

The set of rules is comprised of two subsets: $FRsS_{F_m}$ and $FRsS_{C_m}$, which correspond to the fictitious fuzzy magnets FM_{F_m} and FM_{C_m}, respectively. The first six rules in Table 1 are $FRsS_{F_m}$ and the last six rules are $FRsS_{C_m}$. The rules are shown in the form of (3), but instead of wheel speeds v_L and v_R, speed commands VL and VR, whose inter-dependency is shown in Figure 8, are used for a more straightforward presentation.

Table 1: FLC I – fuzzy rules base.

Rule	d_{F_m}	α_1	d_{C_m}	α_2	VL	VR
1	far	B			-9	-9
2	far	BL			-3	-9
3	far	FL			3	9
4	far	F			9	9
5	far	FR			9	3
6	far	BR			-9	-3
7			$near$	B	-9	-9
8			$near$	BL	0	-3
9			$near$	FL	0	3
10			$near$	F	9	9
11			$near$	FR	3	0
12			$near$	BR	-3	0

For $T-norm$, the minimum method was selected, for $S-norm$ the maximum method was selected. Membership functions of input variables d_{F_m} and d_{C_m} independently activate particular rules, an action which results in a considerable reduction in the number of rules. The rule which produces zero commands on both wheels does not exist; this might lead to the wrong conclusion that the vehicle never stops and does not take a final position. Since the points C_m and F_m are on the garage axis of symmetry, when the vehicle finds itself near its final position the difference between orientation angles α_1 and α_2 is close to $\pm\pi$. In this case, at least two rules generating opposite commands are activated, and their influence becomes annulled. However, with regard to the endless set of possible combinations of input variables, such a selection of fuzzy rules does not guarantee that the vehicle will stop at the desired position. It is therefore, necessary to pay close attention to the selection of parameters d_{F_m}, M, N, C_i, $i=1,2,3,4$ and F_i, $i=1,2$.

With regard to the large number of parameters, in the first iteration the parameter d_{F_m} should be adopted, observing the following limitation:

$$d_{F_m} > \sqrt{(W_r/2)^2 + (L_r/2)^2} \tag{8}$$

where W_r represents the robot width, and L_r the robot length, with the recommendation $d_{F_m} \approx W_r$.
The selection of parameters M and N, in their geometrical sense as presented in Figure 7, influences the nature of the maneuver (curve and speed of turning) performed by the vehicle during garaging, especially at the beginning of the garaging process when the robot is at some distance from the garage. When the values of M and N are low, the vehicle rotates with a very small curve diameter; as these values increase, the arches circumscribed by the vehicle also increase. During the selection of these values, a compromise must be made to maintain the maneuvering capabilities of the vehicle, as well as the constraints imposed by geometry, namely, the ratio between the vehicle dimensions and the width of the garage. Generally, these parameters are adjusted in such a way that the vehicle circumscribes larger arches when distant from the garage, whereas more vivid maneuvers are needed in the vicinity of the garage.
As such, in the second iteration, parameters M and N related to input α_1 are adjusted, followed by those related to the input α_2, with the following constraints:

$$\pi/2 > M \geq N > 0 \tag{9}$$

Coefficients C_3, C_4, F_1 and F_2 enable gradual activation of fictitious magnet C_m and the deactivation of fictitious magnet F_m, as the robot approaches its final garaging point. The selection of these parameters has a critical impact on the performance of the entire fuzzy controller, and the following constraints must be strictly observed:

$$C_4 > C_3, F_2 > F_1 \geq 0 \tag{10}$$

$$F_1 > 0 \Rightarrow C_4 > L_g/2 + d_{F_m} + F_1 \tag{11}$$

$$F_1 = 0 \Rightarrow C_4 > L_g/2 + d_{F_m} - F_2 \tag{12}$$

The constraints specified ensure that the vehicle stays within the attraction region of at least one fictitious magnet at all times. The dominant impact on vehicle garaging is obtained by overlapping fictitious fuzzy magnet attraction regions, and proper vehicle garaging is achieved through the adjustment of the region in which both fictitious fuzzy magnets are active.
The selection of the above parameters might enable vehicle stopping near point C_m. The simultaneous adjustment of the garaging path and stopping at the target position, due to a large number of free parameters, requires a compromise which may have considerable impact on the quality of the controller. The introduction of parameters C_1 and C_2 enables unharnessed adjustment of vehicle stopping. The selection of parameters C_1 and C_2, enabled by theoretical exact stopping of vehicles at point C_m, will cause oscillations in movement around point C_m due to the dynamics of the vehicle, a factor which was neglected in the process of controller design. Accordingly, the selected desired final point of garaging is in the immediate vicinity of the fictitious magnet C_m location. Coefficients C_1 and C_2 take into account the neglected dynamics of the vehicle, and their adjustment is performed experimentally, observing the following constraints:

$$d_{C_m} < L_g/2 \Rightarrow d_{F_m} \leq L_g/2 + d_F \tag{13}$$

The above relation ensures that at least two rule – generating opposed commands are always activated around point C_m. If the variable d_{F_m}, should take on a value larger than $L_g/2 + d_F$,

which means that the vehicle has practically surpassed the target point, angles α_1 and α_2 will become nearly equal, which results in the activation of rules that generate commands of the same sign, which in turn causes oscillations around point C_m.
The experimentally – determined values of these parameters, for the observed example of the Hemisson robot, are presented in Table 2.

Coeff.	Value	Coeff.	Value
$M(\alpha 1)$	$2\pi/9$	C_1	$5/13$
$N(\alpha 1)$	$\pi/12$	C_2	3.2 cm
$M(\alpha 2)$	$5\pi/18$	C_3	16 cm
$N(\alpha 2)$	$\pi/18$	C_4	40.0 cm
F_1	4.0 cm	d_F	12.0 cm
F_2	44.5 cm		

Table 2: FLC I - coefficient values.

4 Analysis of Simulation and Experimental Results

In order to test the applicability and efficiency of the above-described method for different adjustments of the fuzzy controllers and in cases where an error is introduced in setting the angles, a test set of N initial conditions was created. This set of initial configurations $(x_{i,(t=0)}, y_{i,(t=0)}$, and $\psi_{i,(t=0)})$, $i = 1 : N$, was formed such that the robot is placed at equidistant points on the x and y axes at distances of 1cm, while the angle $\psi_{i,(t=0)}$ was a uniform-distribution random variable in the interval $[-\pi 2/, \pi/2]$. The test configuration set was divided into two sub-sets: the first satisfies the condition that the robot in its initial configuration is at an adequate distance from the garage and contains N_d elements, while the other includes initial configurations near the garage and contains $N_c = N - N_d$ elements.
Garaging performance was measured by the distance of the robot from the target position d_{C_m} (Fig. 4), and the angle δ, which showed that the robot and garage axes of symmetry did not coincide:

$$\delta = |\psi_G - \psi| \tag{14}$$

An approximate dynamic model of the Hemisson robot was used for the simulations. The inertia of the kinematic model given in [8] was described by the presence of a lowpass Butterworth filter, with cutoff frequency at 1.225Hz. The sampling period of the FLC was $T_s = 0.2$s. The size of the garage was 16cm×20cm, and the size of the robot was 10cm×12cm. FLC I was designed to conform to the constraints defined in the previous section, with the objective of ensuring successful garaging. The results of simulations with FLC I, for the set of initial conditions N_d (Tables 1 and 2), are shown in Fig. 9.
In all N_d cases, the garaging process was completed with no collision occurring between the robot and the garage. In view of the non-linear setting of the speeds and the long sampling period, the conclusion can be drawn that the results of garaging were satisfactory. The average deviation in N_d cases was $\overline{d}_{C_m} = 1.07$cm, while the average angle error was $\overline{\delta} = 3.18°$.
FLC I was modified based on the results obtained by [21], relating to minimum wheel travel. The rules were modified such that when the robot is some distance away, it rotates around its wheel roughly in the direction of point F_m. Both the parameters and the membership function were altered and the new system was named FLC II. The differences between FLC I and FLC II are shown in Tables 3 and 4.
The results of FLC II simulations with the set of initial conditions N_d are shown in Fig. 10.

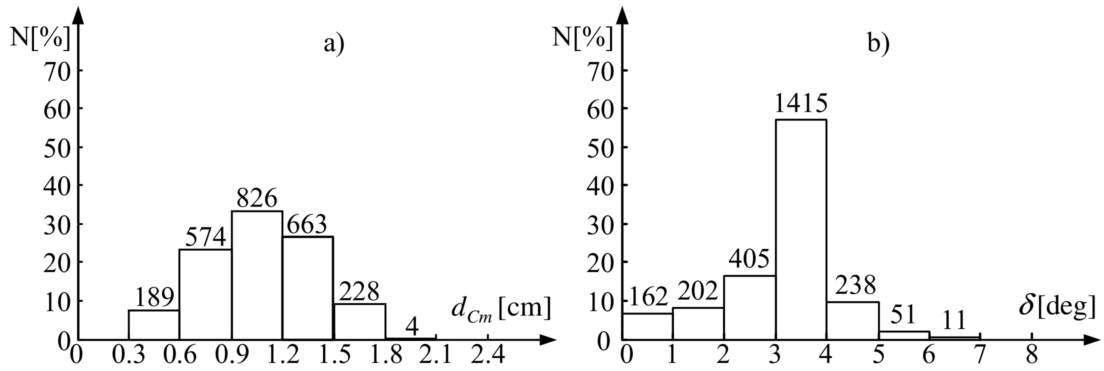

Figure 9: FLC I in N_d simulations: a) histograms of robot distances from final position, b) deviations from final orientation.

Table 3: FLC II - Modified fuzzy rules.

Rule	d_{F_m}	α_1	d_{C_m}	α_2	VL	VR
2	far	BL			0	−9
3	far	FL			0	9
5	far	FR			9	0
6	far	BR			−9	0

Figures 9 and 10 suggest that FLC II exhibits better final orientation characteristics ($\overline{\delta} = 1.37°$ on average), while the error in the distance of the robot from its final position is greater than that obtained with FLC I ($\overline{d}_{C_m} = 1.1$cm on average).

4.1 Robustness of the system

Since the system was designed for use with a real vehicle, its efficiency needed to be examined under less-than-ideal conditions. To operate the system, the positions and orientations of the robot for each selection period must be determined, whereby a robot position and orientation need not necessarily coincide with the real position and orientation. Since inputs into the system consist of two distances and two angles whose accuracy depends directly on the accuracy of the determination of the position of the robot during the garaging process, the effect of inaccuracies of the vehicle coordinates on the efficiency of the garaging process were analyzed. The set of experiments was repeated for FLC I and FLC II with N_d initial conditions, and the vehicle coordinate determination error was modeled by noise with uniform distribution within the range $[-1\text{cm}, 1\text{cm}]$. Figure 11 shows histograms of distance and orientation deviations from the targeted configuration, under the conditions of simulated sensor noise, for FLC I. It was found that the system retained its functionality but that the deviations were greater than those seen in the experiment illustrated in Fig. 9.

Table 4: FLC II - coefficient values.

Coeff.	Value	Coeff.	Value
$M(\alpha 1)$	$7\pi/36$	C_1	$20/63$
$N(\alpha 1)$	$\pi/12$	C_2	4.3 cm
$M(\alpha 2)$	$\pi/4$	C_3	16 cm
$N(\alpha 2)$	$\pi/18$	C_4	40.0 cm
F_1	4.0 cm	d_F	11.0 cm
F_2	44.5 cm		

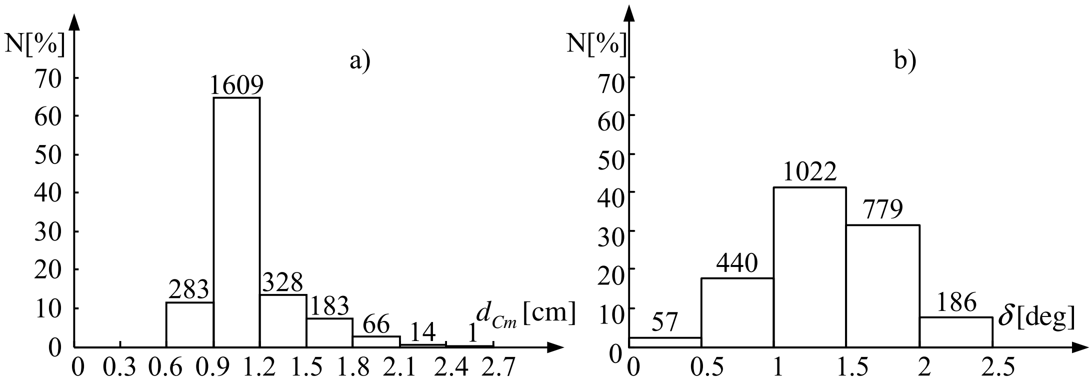

Figure 10: FLC II in N_d simulations: a) histograms of robot distances from final position, b) deviations from final orientation.

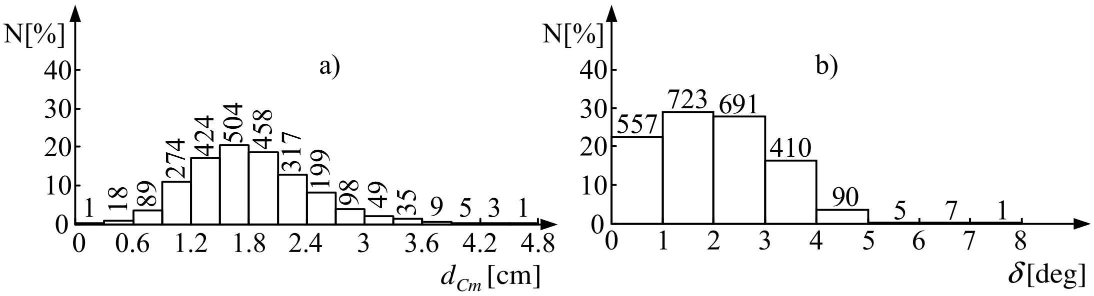

Figure 11: FLC I with simulated sensor noise in N_d simulations: a) histograms of robot distances from final position, b) deviations from final orientation.

The average distance deviation for N_d cases was $\overline{d}_{Cm} = 1.82$cm, while the average angle error was $\overline{\delta} = 2.08°$. Figure 12 shows the results of repeated experiments with FLC II.
Table 5 shows extreme and mean values obtained from the garaging experiments. The subscript N denotes that the simulations were conducted under simulated sensor noise conditions. The sensor noise mostly affected the maximum distances from the target position $d_{Cm\,\max}$; the mean values of angle error $\overline{\delta}$ were unexpectedly lower under simulated sensor noise conditions.

	$\overline{\delta}$ [°]	$\delta_{\max}$ [°]	$\overline{d}_{Cm}$ [cm]	$d_{Cm\,\max}$ [cm]
FLC I	3.18	6.83	1.07	1.88
FLC$_N$ I	2.01	7.17	1.82	4.59
FLC II	1.37	2.35	1.10	2.41
FLC$_N$ II	1.19	9.52	1.93	4.80

Table 5: Extreme and mean values obtained from garaging experiments.

4.2 Limitations of the proposed FLC

Simulations were conducted for the sub-set N_c of initial conditions, under which the robot was not at a sufficient distance from the garage and was placed at the points of an equidistant grid, at 0.25cm distances, where the initial orientation of the robot $\psi_{i,(t=0)}$ was a uniform-distribution random variable in the interval $[-\pi/2, \pi/2]$. Figure 13 shows the regions of initial conditions where the probability of collision of the robot with the garage is greater than zero. Full lines

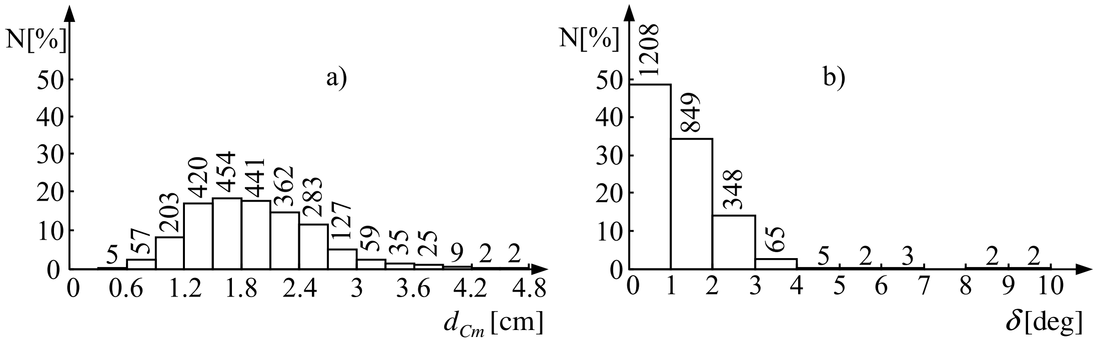

Figure 12: FLC II with simulated sensor noise: a) histograms of robot distances from final position, b) histograms of deviations from final orientation.

identify the limits of the region under ideal conditions, while dotted lines denote simulated sensor noise conditions.

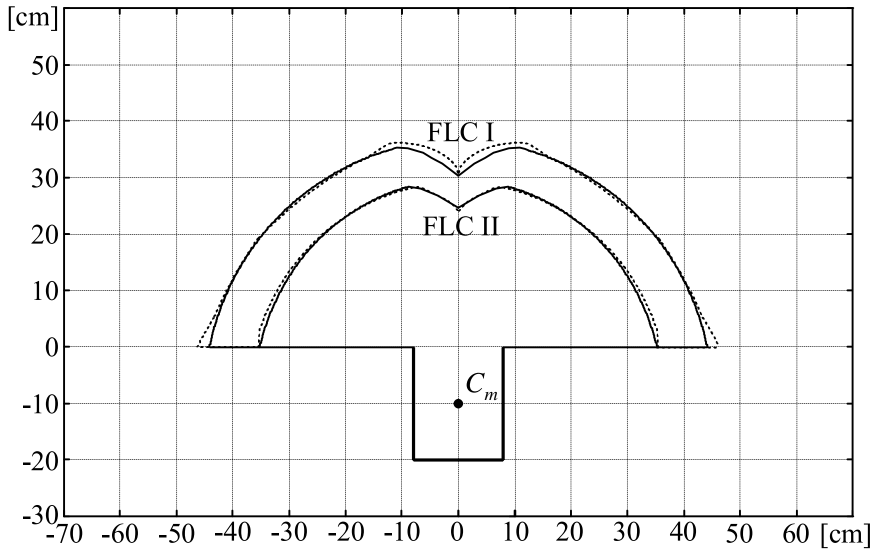

Figure 13: Limits of the system – region of initial configurations with probable collision of the robot with the garage.

It follows that FLC II has a smaller restricted area and, in view of the results shown in Table 5, its overall characteristics are better than those of FLC I. Figure 13 shows that sensor noise has little effect on the restricted area of initial conditions, which is an indicator of the robustness of the system to sensor noise.

4.3 Experiment with a real robot

An analogous experiment was performed with a real mobile vehicle (mobile Hemisson robot) and a real garage whose dimensions are 16cm×20cm. A block diagram of the garaging experiment is shown in Fig. 14. A personal computer with a Bluetooth interface (d) and web camera (f) – resolution 640×480 pixels, were used during the experiment. The initial conditions were determined prior to the initiation of the garaging process, namely (a): the position and orientation of the garage q_G, and the position and orientation of the robot $q_{r(0)}$. Based on the position of the robot and the position and orientation of the garage, in block (b), input variables d_{C_M}, d_{F_m}, α_1 and α_2 were calculated for the FLC (c). FLC outputs were wheel speed commands for the Hemisson robot (VL and VD), which were issued to the robot via the Bluetooth interface.

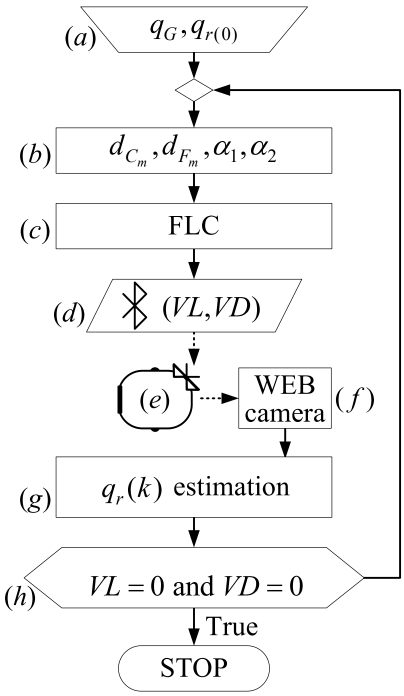

Figure 14: Block diagram of real – time experiment.

The web camera performed a successive acquisition of frames in real time, with a 200 milliseconds repetition time, which dictated the sampling period for the entire fuzzy controller. In block (g), web camera frames were used to determine the position of the robot and compute its orientation. During the garaging process, wheel speed commands were different from zero. When both commands became equal to zero, garaging was completed and block (h) stopped the execution of the algorithm.

Figures 15a and 15b show twelve typical trajectories obtained during the process of garaging of a Hemisson robot in a real experiment. Figure 15a contains illustrative trajectories where there is a significant deviation of real trajectories (dotted line) from trajectories based on a FLC II controller simulation (full line). These differences are mainly the result of robot wheel eccentricity and the ultimate resolution of the camera used to assess the robot position and orientation. Figure 15b shows trajectories obtained during the course of garaging from "difficult" initial positions (initial angle between the axis of symmetry of the robot and the axis of symmetry of the garage near $\pi/2$). These trajectories were obtained equally under front- and back-drive conditions (the front of the robot is identified by a thicker line). All of the illustrated trajectories are indicative of good performance of the proposed garaging algorithm.

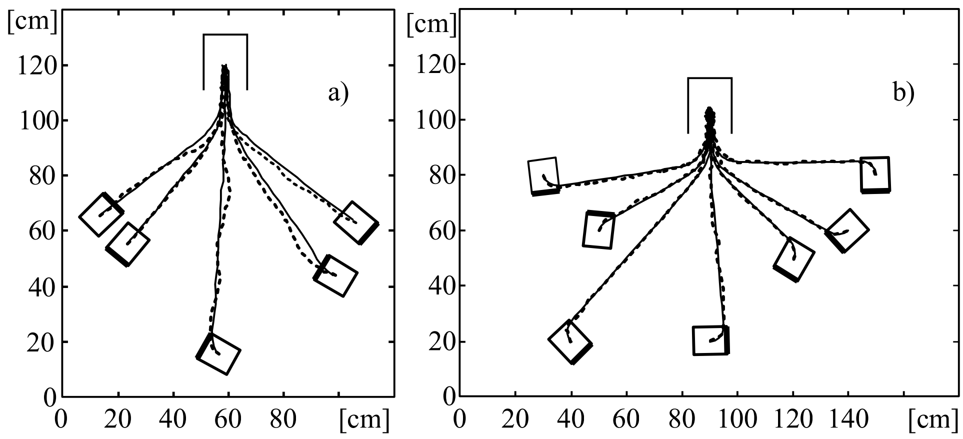

Figure 15: Real – time garaging experiment results.

5 Conclusion

This paper proposes and analyses a new WMR garaging algorithm founded upon on a basic FLC. Compared to algorithms which address similar problems, the proposed system uses a new concept referred to as "fictitious fuzzy magnets". This concept allows for navigation to the target in a single maneuver, without changing the direction of WMR travel. The symmetry of the differential-drive WMR is utilized fully, such that the algorithm provides a bidirectional solution to the WMR garaging problem. The robot is automatically parked from the end of the robot which is closer to the garage entrance. The algorithm can be applied when the control variable is of the discrete type and where there are relatively few quantization levels. A detailed analysis of simulation and experimental results illustrates the efficiency of the proposed algorithm and its robustness in the case of a random or systematic WMR position estimation error, as well as its limitations (or shortfalls). The most significant shortfall of the proposed algorithm is that it does not provide a solution which will ensure that regardless of initial conditions the garage parking process is completed with no collision with the garage. The geometrical position of the initial conditions which lead to a collision of the robot with the garage is a compact, finite area which is discussed in the paper. Some of the constraints mentioned in this paper could be overcome in further research aimed at improving the proposed algorithm through a higher level of FLC complexity. Additionally, if a larger number of fictitious fuzzy magnets are introduced, the concept could be used to perform more intricate garage parking tasks. Re-configuration of controller outputs would render the proposed algorithm applicable for garage parking of a broader class of car-like mobile robots.

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