

# Transactions Letters

## Asymptotically Tight Bounds on the Capacity and Outage Probability for QAM Transmissions over Rayleigh-Faded Data Channels with CSI

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**Abstract**—In this contribution, novel quickly computable analytical upper and lower bounds are presented on the symmetric capacity for flat-faded Rayleigh channels with finite-size quadrature amplitude modulation constellations when perfect channel-state information at the receiving site is available; the proposed bounds are asymptotically tight both for high and low signal-to-noise ratios. Furthermore, an easily computable expression is also provided for a reasonably tight evaluation of the resulting outage probability.

**Index Terms**—Capacity bounds, outage probability, packet transmission, Rayleigh channel.

### I. MOTIVATIONS OF THE WORK AND SYSTEM MODELING

**A**N EFFECTIVE means to improve the performance of wireless data systems impaired by fading phenomena consists in resorting to channel coding combined with bandwidth-efficient modulation formats [1, Ch. 4]. So, since the ultimate throughput supported by a coded channel is dictated by the corresponding capacity, starting from the basic results of [4], this last parameter has been largely investigated in literature for faded links. In this regard, several results about capacity and cutoff rate of faded channels can be found, for example, in [1, Ch. 4], [10], [12], and [13]; furthermore, capacity of fading channels has been analyzed, in general, in [3], [7], and [8], whereas the class of faded links with a “block-type” memory has been considered in [2], [5], [11], and [13].

In this contribution, we focus on the capacity and outage probability of systems for data transmissions with finite-size quadrature amplitude modulation (QAM) constellations over flat-faded Rayleigh channels. More in detail, we assume that the baud-rate sampled baseband complex sequence  $\{y(i) \in \mathbb{C}\}$  received at the output of a flat-faded link impaired by additive white Gaussian noise (AWGN) is modeled as

$$y(i) = g(i)x(i) + w(i) \quad (1)$$

where, under the usual assumption of coherent demodulation,  $\{g(i)\}$  is a nonnegative generally time-correlated Rayleigh-distributed gain sequence independent of  $\{x(i)\}$  and  $\{w(i)\}$ ,

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while this last is a zero-mean complex AWGN sequence with variance  $N_o/2$  per component. The transmitted  $q$ -ary complex data stream  $\{x(i) \in A_x \equiv \{\alpha_1, \dots, \alpha_q\} \subset \mathbb{R}^1\}$  is a memoryless zero-mean sequence with equidistributed symbols with variance  $\sigma_x^2$ . Now, assuming  $\{g(i)\}$  ergodic<sup>1</sup> and also maximum-likelihood soft decoding with perfect channel-state information, the symmetric capacity  $C^{*2}$  of the channel in (1) can be obtained via an application of the chain rule of [1, eqs. (4.6.11)–(4.6.14)], [2, eqs. (2.5)–(2.9)] as reported below<sup>3</sup>

$$\begin{aligned} C^{*} &\equiv \lim_{N \rightarrow \infty} \frac{1}{N} I(x^N; Y^N, G^N) \\ &\stackrel{(a)}{=} \lim_{N \rightarrow \infty} \frac{1}{N} I(X^N; Y^N | G^N) \\ &\stackrel{(b)}{=} \lim_{N \rightarrow \infty} \frac{1}{N} \left[ \sum_{i=1}^N I(Y(i); X(i) | G(i)) \right] \\ &\stackrel{(c)}{=} I(Y^1; X^1 | G^1) \end{aligned} \quad (2)$$

where  $I(\cdot; \cdot, \cdot)$  stands for the average mutual information functional (thereafter given in nats/channel symbol, unless otherwise stated) and  $X^N, Y^N$ , and  $G^N$  are subsequences of  $N$  elements picked out from  $\{x(i)\}$ ,  $\{y(i)\}$ , and  $\{g(i)\}$ , respectively. Therefore, as in [1, eqs. (4.6.14), (4.7.13)], [2, eq. (2.9)] the symmetric capacity  $C^{*}$  can be directly computed via the following expectation:

$$\begin{aligned} C^{*} &\equiv E_g \{ \tilde{C}^{*}(g) \} \\ &\equiv \frac{2\sigma_x^2}{N_o \bar{\gamma}_c} \int_{g=0}^{\infty} g \exp\left(-\frac{g^2 \sigma_x^2}{N_o \bar{\gamma}_c}\right) \tilde{C}^{*}(g) dg \end{aligned} \quad (3)$$

<sup>1</sup>Under the ergodic assumption on  $\{g(i)\}$ , the channel in (1) is “information stable” so that the limiting expression in (2) for the capacity  $C^{*}$  holds [6, Sec. I].

<sup>2</sup>According to a current taxonomy [1, p. 350], we have qualified as “symmetric” the capacity in (2) which represents the average mutual information of the channel (1) for discrete equidistributed input symbols. As it is known [2, Sec. II-C], [4], the actual capacity  $C_G$  of the channel is achieved for continuous Gaussian-distributed coding alphabet and is given by the formula in (16) of Section IV where  $Ei(x)$  is the usual exponential integral function [9, p. 933].

<sup>3</sup>As in [1, eqs. (4.6.12), (4.6.13)], the equality (a) in (2) follows from the independence of  $G^N$  from  $X^N$ , which makes the mutual information  $I(X^N; G^N)$  vanish. Furthermore, similarly to [2, eq. (2.7)], the memoryless assumption on the noise  $\{w(i)\}$  guarantees that the equality (b) of (2) holds while (c) is a consequence of the stationarity of the channel.

where  $\bar{\gamma}_c$  is the average signal-to-noise ratio (SNR) per channel symbol and [1, p. 352, eq. (4.5.18)]<sup>4</sup>

$$\tilde{C}^*(g) \equiv \log q - q^{-1} \sum_{j=1}^q \int_{y \in \mathbb{C}} (\pi N_o)^{-1} \exp(-\|y - g\alpha_j\|^2 / N_o) \cdot \log \left[ 1 + \sum_{\substack{s=1 \\ s \neq j}}^q \exp((\|y - g\alpha_j\|^2 - \|y - g\alpha_s\|^2) / N_o) \right] dy \quad (4)$$

is the symmetric capacity of the channel (1) *conditioned* on the determination  $g$  of the channel gain.

Now, as also pointed out in [1, Sec. 4.6.3], for each value of  $\bar{\gamma}_c$ , the evaluation of the capacity formulas in (3), (4) is cumbersome and, in principle, it should require numerical integrations over the *overall* complex plane  $\mathbb{C}$ , followed by a final numerical integration on  $\mathbb{R}^+$ . Furthermore, the behavior of the outage probability of the system *does not clearly stand out* from (3) so that a direct utilization of the latter for system design purposes does not appear attractive. The analytical relationship presented in the following two sections allow us to bypass these drawbacks.

## II. THE PROPOSED UPPER AND LOWER BOUNDS ON THE SYMMETRIC CAPACITY

In the Appendix, it is shown that the conditional capacity  $\tilde{C}^*(g)$  in (4) can be upper and lower bounded as

$$LB(g) \leq \tilde{C}^*(g) \leq UB(g) \quad (5)$$

where the following relationships take place:

$$LB(g) \equiv \log q - \log \left[ 1 + \frac{1}{q} \sum_{j=1}^q \sum_{\substack{s=1 \\ s \neq j}}^q \exp \left( -\frac{g^2 \|\alpha_j - \alpha_s\|^2}{4N_o} \right) \right] \quad (6)$$

$$UB(g) \equiv \log q - \frac{1}{q} \sum_{j=1}^q \log \left[ 1 + (q-1) \exp \left( -\frac{g^2}{N_o(q-1)} \left( \sum_{\substack{s=1 \\ s \neq j}}^q \|\alpha_j - \alpha_s\|^2 \right) \right) \right] \quad (7)$$

So, by inserting the bounds (6), (7) into (3), it can be proved (see the Appendix) that the following chain of upper and lower bounds holds for  $C^*$  in (3):

$$LB_2 \leq LB_1 \leq C^* \leq UB_1 \leq UB_2 \quad (8)$$

with the positions below reported

$$LB_1 \equiv \log q - \log \left[ 1 + \frac{1}{q} \sum_{j=1}^q \sum_{\substack{s=1 \\ s \neq j}}^q \left( 1 + \|\alpha_j - \alpha_s\|^2 \left( \frac{\bar{\gamma}_c}{4\sigma_X^2} \right) \right)^{-1} \right] \quad (9)$$

<sup>4</sup>The relationship in (4) is the formula for the symmetric capacity of an unfaded AWGN channel with  $q$ -ary QAM input symbols. Equation (4) is directly obtained from [1, p. 352, eq. (4.5.18)] by exploiting the Gaussianity of the noise  $\{w(i)\}$ .

$$LB_2 \equiv \log q - \log \left[ 1 + (q-1) \left( 1 + d_{\min}^2 \left( \frac{\bar{\gamma}_c}{4\sigma_X^2} \right) \right)^{-1} \right] \quad (10)$$

$$UB_1 \equiv \frac{q-1}{q} \sum_{j=1}^q (h_j^2 + 1)^{-1} {}_2F_1(1, h_j^2 + 1; h_j^2 + 2; 1 - q) \quad (11)$$

$$UB_2 \equiv (q-1)(h^2 + 1)^{-1} {}_2F_1(1, h^2 + 1; h^2 + 2; 1 - q). \quad (12)$$

In the above expressions, the parameters  $\{h_j^2, 1 \leq j \leq q\}$  and  $h^2$  in (11) and (12) are defined as

$$h_j^2 \equiv \left( \frac{(q-1)\sigma_X^2}{\bar{\gamma}_c} \right) \left( \sum_{\substack{s=1 \\ s \neq j}}^q \|\alpha_j - \alpha_s\|^2 \right)^{-1}, \quad 1 \leq j \leq q$$

$$h^2 \equiv \frac{\sigma_X^2}{\bar{\gamma}_c d_{\max}^2}$$

with  $d_{\max}^2$  and  $d_{\min}^2$  denoting the maximum- and minimum-squared Euclidean distances between two distinct constellation points.

*Remark 1:* All the bounds in (8) are *asymptotically exact both for high and low SNR's*; in fact, they approach zero and  $\log q$  for  $\bar{\gamma}_c \rightarrow 0$  and  $\bar{\gamma}_c \rightarrow \infty$ , respectively.  $\diamond$

*Remark 2:* The computation of the lower bounds in (9) and (10) is direct. The evaluation of the functions in (11) and (12) can be quickly accomplished via standard numerical routines typically based on power-series expansions [9, pp. 1065–1071]. Furthermore, for  $UB_1$  in (11), the following simple asymptotic expression holds (see the Appendix):

$$UB_1 \cong \log q - \frac{(q-1)}{q} \sum_{j=1}^q \left[ 1 + \left( \frac{\bar{\gamma}_c}{(q-1)\sigma_X^2} \right) \cdot \left( \sum_{\substack{s=1 \\ s \neq j}}^q \|\alpha_j - \alpha_s\|^2 \right) \right]^{-1}, \quad \text{for } \bar{\gamma}_c \rightarrow \infty \quad (13)$$

which gives rise to a direct evaluation of the bound for medium to large SNR's (see Section IV).  $\diamond$

## III. ON THE EVALUATION OF THE OUTAGE PROBABILITY OF THE SYSTEM FOR DATA TRANSMISSIONS VIA MULTIPLE INTERLEAVED PACKETS

In actual time-division multiple-access systems that operate on fading environments, to obtain a diversity gain the overall  $NK$ -long codeword, which encodes a user message, is generally split in  $K$  packets of  $N$  symbols, and these packets are then transmitted sufficiently spaced in time [2, p. 361, and references therein]. Thus, by resorting to the common “block-fading channel model” of [2, Sec. II-A], [5] and [11], as in the case of the pan-European GSM standard, we can assume that the fading phenomena are constant over each packet [2, eq. (2.2)], and a deep interleaving is also used so as to guarantee independent fadings from one packet to the other [1, Sec. 4.7.2], [2, Sec. II], [5]. So, by indicating as  $\underline{g}^K \equiv [g_1, \dots, g_K]^T$ , the  $K$ -variate vector, which collects the realizations of the channel gains in (1) over the  $K$  trans-

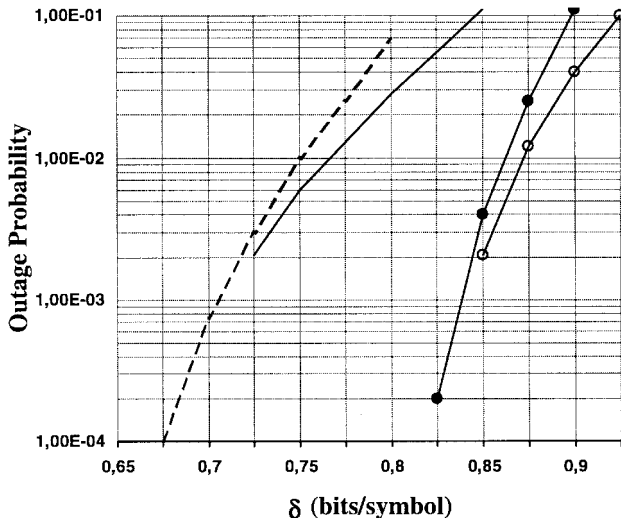


Fig. 1.  $P_{\text{OUT}}$  (in a  $\log_{10}$  scale) versus the outage parameter  $\delta$  for BPSK constellations with  $K = 4$  packets. The actual values of  $P_{\text{OUT}}$  (obtained via Monte Carlo simulations) are marked as  $\text{---}$  and  $\ominus$  for  $\bar{\gamma}_c = 15$  dB and  $\bar{\gamma}_c = 20$  dB, respectively. The corresponding values assumed by the relationship in (14) are plotted as  $\cdots$  and  $\bullet$  for  $\bar{\gamma}_c = 15$  dB and  $\bar{\gamma}_c = 20$  dB, respectively.

mitted packets, the resulting conditional mutual information [2, eq. (2.11)]  $I(\underline{g}^K) \equiv (1/K)\sum_{m=1}^K \tilde{C}^*(g_m)$  is a random variable whose distribution  $P_{\text{OUT}}(\delta; K) \equiv P(I(\underline{g}^K) \leq \delta)$  is commonly referred as the outage probability of the system [2, Sec. II-B], [11]. Now, due to the integral form of (4), an exact analytical computation of  $P_{\text{OUT}}(\delta; K)$  is a hard task to be accomplished;<sup>5</sup> so, on the basis of the Chernoff-bound reported in (A.5) of the Appendix, we have developed the following expression for an approximate evaluation of  $P_{\text{OUT}}(\delta; K)$ :

$$P_{\text{OUT}}(\delta; K) \approx \ll \exp\{-K\lambda[LB_1 - \delta]\}, \quad 0 \leq \delta \leq LB_1 \quad (14)$$

where for medium to large SNR's the Chernoff-type parameter  $\lambda$  in (14) is given by the optimized expression reported below

$$\lambda = \log(LB_1/\delta)/(\log q - LB_1), \quad 0 \leq \delta < LB_1. \quad (15)$$

For  $\delta < LB_1$ , the expression we propose in (14) for the approximate evaluation of the outage probability is asymptotically *tight*, and for large  $K$ , it certainly approaches the actual value of  $P_{\text{OUT}}(\delta; K)$ . Furthermore, the numerical results reported in Figs. 1 and 2 support the conclusion that the presented analytical formula is able to give rise to reliable evaluations of  $P_{\text{OUT}}(\delta; K)$  even for values of  $K$  of practical interest that, for example, range from four to eight for the half-rate and full-rate GSM standards [2].

<sup>5</sup>It is part of a quite common (but misleading) folklore the opinion that outage probability is related to the ergodic behavior of the channel; on the contrary, capacity-versus-outage is recognized to be a meaningful performance index when the ergodic assumption falls short, as for the case of block-fading channels with limited  $K$  [13, p. 2631 and references therein]. In the interesting contribution [11], Chernoff-type bounds on the outage probability are presented for continuous Gaussian-shaped coding alphabets; unfortunately, due to the integral form of (4), the developments of [11] do not seem applicable in our case.

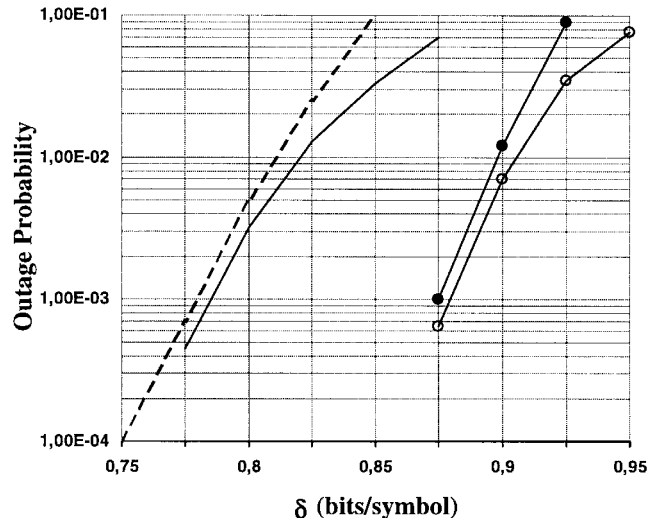


Fig. 2. The same as in Fig. 1 for  $K = 8$ .

*Remark:* The expression at the right-hand-side (RHS) of (14) is obtained by introducing the approximate relationship (A.8) in the Chernoff-bound (A.7), so that it should be properly regarded as an approximation on  $P_{\text{OUT}}(\delta; K)$ .<sup>6</sup> The satisfactory behavior exhibited by this approximation at least for medium to high SNR's and values of the outage probability ranging from  $10^{-3}$  to  $10^{-1}$  is partially due to the exchange of the expectation with the exponential function introduced in (A.8); in fact, in doing so, the weakness of the Chernoff-bound tends to be partially compensated. Furthermore, it can be also checked out that the dependence on  $\delta$  of the (optimized) Chernoff-like parameter in (15) significantly improves the reliability of the RHS of (14), especially in the limit cases of  $\delta \rightarrow 0$  and  $\delta \rightarrow LB_1$ <sup>7</sup> (see the last part of the Appendix for some additional comments on these subjects).  $\diamond$

#### IV. NUMERICAL EXAMPLES AND CONCLUSIVE REMARKS

The tightness of the presented bounds on  $C^*$  has been tested for several QAM constellations and the numerical results obtained for 4PSK, 8PSK, and 16 QAM modulation formats are drawn in Figs. 3–5. For comparison purposes on the same figures, even the actual capacity  $C_G$  of the channel (1) is reported; this last is obtained for continuous Gaussian-distributed coding alphabet and is given by the usual formula [2, eq. (2.20)], [4]

$$C_G \equiv -\exp(\bar{\gamma}_c^{-1})Ei(-\bar{\gamma}_c^{-1}). \quad (16)$$

The above  $C_G$  can be viewed as the ultimate throughput supported by the channel in (1) when suitably shaped large-size QAM constellations are used [1, p. 350 and references therein].

Now, an examination of Figs. 3–5 shows, indeed, that the bounds  $LB_1$  and  $UB_1$  of (9) and (11) *closely approach* the symmetric capacity  $C^*$  for SNR's below 5 dB and over 15

<sup>6</sup>For this reason, we have adopted in (14) the symbol “ $\approx \ll$ ” in place of the standard “ $\leq$ ”.

<sup>7</sup>In fact, the Chernoff-like parameter in (15) approaches infinite and zero for  $\delta \rightarrow 0$  and  $\delta \rightarrow LB_1$ , respectively.

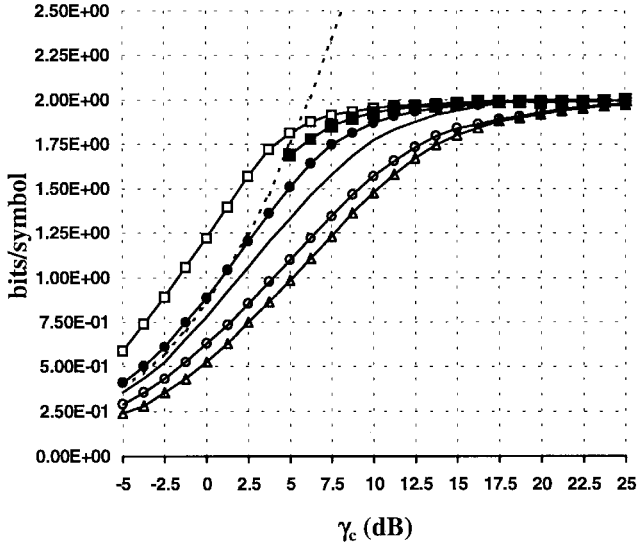


Fig. 3. Behavior of  $LB_1$  ( $-\circ-$ ),  $LB_2$  ( $-\triangle-$ ),  $UB_1$  ( $-\bullet-$ ),  $UB_2$  ( $-\square-$ ),  $C_G$  ( $\cdots$ ), and the asymptotic expression of (13) ( $-\blacksquare-$ ) for a 4PSK constellation. The actual capacity  $C^*$  of (3) is marked by a continuous line ( $-\text{---}$ ).

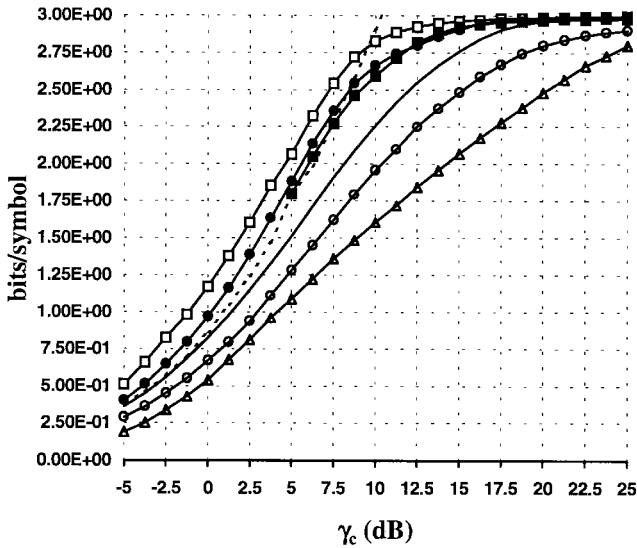


Fig. 4. The same as in Fig. 3 for an 8PSK constellation.

dB. In addition, the reported curves confirm that the simple asymptotic expression for  $UB_1$  in (13) gives rise to tight evaluations of  $C^*$  for SNR's over 10–12 dB. The figures also show that the simpler bounds  $LB_2$  and  $UB_2$  of (10) and (12) differ from the corresponding  $LB_1$  and  $UB_1$  within 1–1.5 dB, and they allow reasonably tight evaluations of  $C^*$  for  $\bar{\gamma}_c$  below 3–5 dB and over 15–17 dB.

Some results about the cases of data channels affected by Rice and Nakagami fading are reported in [14].

APPENDIX  
DERIVATION OF THE MAIN RESULTS

The bound  $UB(g)$  in (7) can be obtained by rewriting the conditional capacity  $C^*(g)$  in (4) as  $C^*(g) = \log q - H(X|Y, g)$  and then recognizing that for the conditional en-

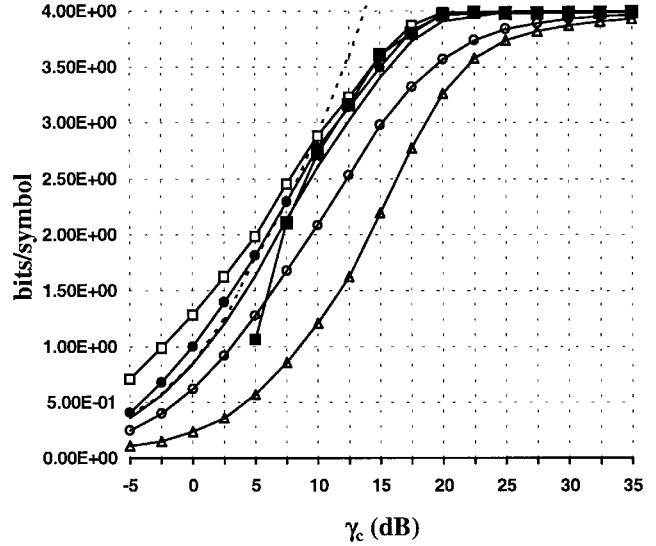


Fig. 5. The same as in Fig. 3 for a 16QAM constellation.

trophy  $H(X|Y, g)$  the following chain of lower bounds holds:

$$\begin{aligned}
 & H(X|Y, g) \\
 & \equiv \frac{1}{q} \sum_{j=1}^q E_{Y/X=\alpha_j} \left\{ \log \left[ 1 + \sum_{\substack{s=1 \\ s \neq j}}^q \exp \left( \frac{\|y - g\alpha_j\|^2 - \|y - g\alpha_s\|^2}{N_o} \right) \right] \right\} \\
 & \stackrel{(a)}{\geq} \frac{1}{q} \sum_{j=1}^q E_{Y/X=\alpha_j} \left\{ \log \left[ 1 + (q-1) \exp \left( \frac{1}{N_o(q-1)} \sum_{\substack{s=1 \\ s \neq j}}^q (\|y - g\alpha_j\|^2 - \|y - g\alpha_s\|^2) \right) \right] \right\} \\
 & \stackrel{(b)}{\geq} \frac{1}{q} \sum_{j=1}^q \log \left\{ 1 + (q-1) \exp \left( \frac{1}{N_o(q-1)} \sum_{\substack{s=1 \\ s \neq j}}^q E_{Y/X=\alpha_j} \{ \|y - g\alpha_j\|^2 - \|y - g\alpha_s\|^2 \} \right) \right\} \\
 & \equiv \frac{1}{q} \sum_{j=1}^q \log \left\{ 1 + (q-1) \exp \left( -\frac{g^2}{N_o(q-1)} \sum_{\substack{s=1 \\ s \neq j}}^q \|\alpha_j - \alpha_s\|^2 \right) \right\} \tag{A.1}
 \end{aligned}$$

where (a) in (A.1) arises from an exploitation of the arithmetic-geometric inequality [9, p. 1126], and (b) is obtained via the Jensen's inequality applied to the U-convex function  $f(x) \equiv \log(1 + e^x)$ . Then,  $UB_1$  in (11) is obtained by averaging  $UB(g)$  over the chi-square pdf of  $g^2$  and, thus, resorting to [9, p. 334, eq. (3.194.5)] for computing the resulting integral. Furthermore, since

$$UB(g) \leq \log q - \log \left\{ 1 + (q-1) \exp \left( -\frac{d_{\max}^2}{N_o} g^2 \right) \right\} \tag{A.2}$$

by averaging the RHS of (A.2) with respect to the pdf of  $g^2$ , we obtain  $UB_2$  in (12). In addition, we note that for vanishing  $N_o$ , the bound in (7) can be well approximated as

$$UB(g) \cong \log q - \left( \frac{q-1}{q} \right) \sum_{j=1}^q \exp \left( - \frac{g^2}{N_o(q-1)} \sum_{\substack{s=1 \\ s \neq j}}^q \|\alpha_j - \alpha_s\|^2 \right), \quad (A.3)$$

$N_o \rightarrow 0$

therefore, by averaging again the RHS of (A.3) over the pdf of  $g^2$ , we obtain the asymptotic expression in (13).

The lower bound  $LB(g)$  in (6) on  $\tilde{C}^*(g)$  is the cutoff rate of the AWGN channel in (1) conditioned on  $g$ ; it can be directly obtained from [1, eq. (4.3.37)] by exploiting the assumed equidistribution of the input symbols. Now, an application of the Jensen's inequality to the  $\cap$ -convex function  $f(x) \equiv \log(1+x)$  leads to the following lower bound:

$$E_g \{ LB(g) \} \geq \log q - \log \left\{ 1 + \frac{1}{q} \sum_{j=1}^q \sum_{\substack{s=1 \\ s \neq j}}^q E_g \left[ \exp \left( - \frac{\|\alpha_j - \alpha_s\|^2}{4N_o} g^2 \right) \right] \right\} \quad (A.4)$$

so that  $LB_1$  in (9) is obtained by carrying out the averages in (A.4). Finally,  $LB_2$  in (10) directly arises from  $LB_1$  by noting that  $d_{\min}^2 \leq \|\alpha_j - \alpha_s\|^2 \forall j \neq s$ .

As far as the outage probability is concerned, an application of the Chernoff-bound leads to the following exponential expressions for dominating  $P_{\text{OUT}}(\cdot; \cdot)$ :

$$\begin{aligned} P_{\text{OUT}}(\delta; K) &\leq \min_{\lambda \geq 0} [E\{\exp(-\lambda(\tilde{C}^*(g) - \delta))\}]^K \quad (A.5) \\ &\equiv \min_{\lambda \geq 0} \left( \exp(\lambda \delta K) \right. \\ &\quad \cdot \left. \left[ \int_{g=0}^{\infty} \exp(-\lambda \tilde{C}^*(g)) p(g) dg \right]^K \right) \quad (A.6) \\ &\equiv \min_{\lambda \geq 0} \{ \exp(\lambda \delta K) [E\{\exp(-\lambda \tilde{C}^*(g))\}]^K \} \quad (A.7) \end{aligned}$$

where  $p(g)$  in (A.6) indicates the Rayleigh-type pdf of the channel gain in (1). Unfortunately, due to the integral form of (4), the expectation in the above bound resists analytical evaluation, so that the computation of the latter requires expensive multiple nested numerical integrations over unbounded domains [see (4), (A.6)]. In the following, we attempt to bypass this drawback by resorting to a suitable *approximation*. Toward this end, we simply note that both  $E\{\exp(-\lambda \tilde{C}^*(g))\}$

and  $\exp(-\lambda E\{\tilde{C}^*(g)\})$  are below the unity; so, at least for large  $K$  the following *approximate* relationship can be assumed to hold:

$$[E\{\exp(-\lambda \tilde{C}^*(g))\}]^K \cong [\exp(-\lambda E\{\tilde{C}^*(g)\})]^K. \quad (A.8)$$

Therefore, from (A.7) and (A.8) the relationship (14) directly arises. Although the latter should be properly regarded as an approximation on  $P_{\text{OUT}}(\cdot; \cdot)$ ; nevertheless, the reported performance plots support its actual effectiveness, at least for medium to high SNR's and values of the outage probability ranging from  $10^{-3}$ – $10^{-1}$ . To this regard, we also note that the relationship (A.8) *exactly* holds for any finite value of  $K$  when the pdf of  $\tilde{C}^*(g)$  collapses in a Dirac-delta. This condition can be assumed approximately met for sufficiently large SNR's (that is, for SNR's bigger than about 13–14 dB); in fact, in this case the pdf of  $\tilde{C}^*(g)$  tends to become a tall spike of small width and, thus, approaches a Dirac-delta stood at  $\log q$  (see (4) and the remark of Section III).

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