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Research Article

Multiway Physical-Layer Network Coding via Uniquely Decodable Codes

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We focus on a multiway relay channel (MWRC) network where two or more users simultaneously exchange information with each other through the help of a relay node. We propose for the first time to apply ternary uniquely decodable (UD) code sets that we have developed to allow each user to uniquely recover the information bits from the noisy channel environment. One of the key features of the proposed scheme is that it utilizes a very simple decoding algorithm, which requires only a few logical comparisons. Simulation results in terms of bit error rate (BER) demonstrate that the performance of the proposed decoder is almost as good as the maximum-likelihood (ML) decoder. In addition to that through simulations, we show that the proposed scheme can significantly improve the sum-rate capacity, which in turn can potentially improve overall throughput, as it needs only two time slots (TSs) to exchange information compared to the conventional methods.

1. Introduction

Network coding (NC) has attracted a lot of attention in the research community due to its capability to enhance the throughput of lossless wireline networks in a multicast environment [1]. In a two-way bidirectional relay channel, NC requires three time slots (TSs) for information exchange while the conventional scheme requires four. Therefore, NC can potentially improve the channel throughput by 4/3 times in bidirectional channels.

A lot of work has been done to investigate some of the NC issues in wireless environments [2–5]. The conventional NC data forwarding schemes based on decode-and-forward (DF) relaying are not able to fully utilize the wireless channel. More recently, denoise-and-forward (DNF) relaying adopting the physical-layer network coding (PNC) proposed by Zhang et al. [6] can potentially provide further throughput improvements of the wireless channel. The PNC has several attractive features; in particular, it is relatively simple to implement compared to the conventional NC method as mentioned in [7]. Unlike the conventional two-way relay channel (TWRC), which requires four time slots, PNC requires only two time slots; hence, it can potentially double the throughput. A large volume of research on PNC, mainly focusing on the TWRC, has been reported in the literature, which outperform the conventional NC technique in terms of throughput and achievable data rates. In [7, 8], the authors present a PNC scheme based on frequency-shift keying (FSK).

More recently, a code-division multiple-access (CDMA) based analog network coding (ANC) scheme that utilizes amplify-and-forward (AF) relaying has been introduced in [9, 10] for multipath and asynchronous underwater acoustic sensor networks (UW-ASNs). The authors developed an adaptive RAKE receiver that equalizes the received signal and then jointly estimates the two multipath faded channels. The relay node cancels the interference before decoding the information of interest. The simulation and experiment results demonstrate that their proposed scheme can significantly improve the channel utilization by up to 50% for unidirectional and 100% for bidirectional networks compared to the conventional DS-CDMA scheme.

One of the main constraints of PNC and ANC techniques is the limitation on the number of users that can simultaneously transmit to the relay node. The concept of a multiway relay channel (MWRC), which was introduced in [11], is a generalization of a TWRC, where $K > 2$ users simultaneously aim to achieve full information exchange with the help of a single relay node. A major application of MWRC is in satellite communication in which multiple users around the world simultaneously exchange data through a satellite. Several works studied different multiple access schemes to achieve MWRC. For example, in [12, 13] authors present a transmission method that uses binary phase-shift keying (BPSK), where multiuser detection (MUD) is achieved at the expense of an increase of channel use by identifying "minority" nodes. In [14], MUD relies on iterative multiuser detection of users at the receiver. Nonorthogonal multiple access schemes can be considered to be potential candidate for MWRC.

A well-known example of nonorthogonal multiple access is CDMA. The existence of uniquely decodable (UD) codes for overloaded synchronous CDMA where the number of multiplexed signals K is greater than the signature length L over the antipodal alphabet $\{\pm 1\}$ with linear MUD in noiseless channels is reported in [15]. However, in a noisy channel the high computation cost involved in MUD, which may increase in an exponential order with the number of users, has limited use in practical applications.

An interesting technique referred to as interleave-division multiple-access (IDMA) was introduced in [16]. This technique is attractive because of its low-complexity receiver design. A similar technique referred to as sparse code multiple access (SCMA) that possesses a low-complexity receiver is also presented in [17]. Authors in [18] utilize the UD code set over the ternary alphabet $\{\pm 1, 0\}$, which as mentioned before has linear MUD in noiseless environment only. In addition to that the authors present a decision decoding scheme at the relay for the $K = 3$ user case only, with $8 = 2^3$ decision regions in a noisy channel.

In this paper, we focus on a MWRC network, where $K \geq 2$ users exchange information with each other simultaneously via the help of a relay node. For MWRC networks utilizing PNC, it is proved in [19] that $2(K-1)$ TSs are required. Hence, we propose for the first time to apply ternary UD code sets, which will allow each user to uniquely recover the information bits from a noisy channel. One of the attractive features of the proposed scheme is that it utilizes a very simple decoding algorithm, which requires only a few logical comparisons. Simulation results in terms of bit error rate (BER) demonstrate that the performance of the proposed decoder is very close to the maximum-likelihood (ML) decoder. Moreover, through simulations, we show that the proposed scheme can significantly improve the sum-rate capacity, which in turn can potentially improve overall throughput, as it needs only two TSs to exchange information compared to the conventional methods.

The rest of the paper is organized as follows. In Section 2, we present the system model. In Section 3, we discuss the construction of the uniquely decodable code sets followed by decoding algorithm in Section 4. The complexity analysis is performed in Section 5. After illustrating simulation results in Section 6, a few conclusions are drawn in Section 7.

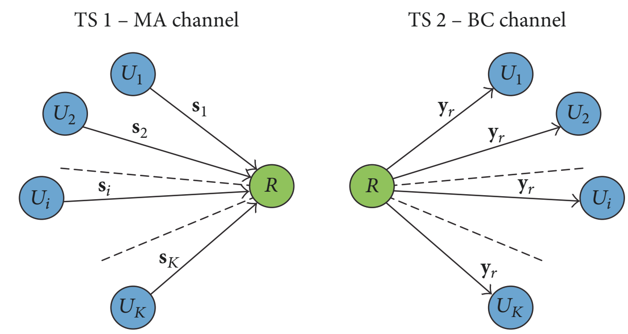

Figure 1: PNC with AF criteria.

The following notations are used in this paper. All boldface lower case letters indicate column vectors and upper case letters indicate matrices, $(\,)^T$ denotes transpose operation, $\mathbb{C}$ denotes the set of all complex numbers, * denotes conjugate, sgn denotes the sign function, $|\cdot|$ denotes complex amplitude, and $\lfloor\cdot\rfloor$, $\lceil\cdot\rceil$ are the floor and ceiling functions, respectively.

2. System Model

We consider a MWRC network with $K \geq 2$ users where each user intends to transmit binary data to all the other user nodes; that is, full data exchange is desired. Direct communication is assumed infeasible, and thus, users can only exchange information through the help of a relay node, as shown in Figure 1. We assume that perfect synchronization is available during the whole transmission, and all channel state information (CSI) is known at all the user nodes and the relay node [20]. Unlike the conventional PNC technique in which DNF is utilized, in this paper, we explore PNC with the AF criteria. Notice that the proposed scheme is still coined PNC and not ANC because of synchronization assumption and the fact that the relay node can potentially detect all of the users' information even without the knowledge of users' previous binary data. The communication takes place in two phases, that is, multiple access (MA) and broadcast (BC) phases. In each TS1, that is, the MA phase, each user encodes the data sequence and then simultaneously transmits their signals to the relay through an additive white Gaussian noise (AWGN) channels. The received signal at the relay node can be expressed as

$$\mathbf{y}_r = \sum_{k=1}^{K} \alpha_k h_k \mathbf{c}_k d_k + \mathbf{n} = \sum_{k=1}^{K} h_k \mathbf{s}_k + \mathbf{n}, \quad (1)$$

where $\mathbf{c}_k \in \{\pm 1, 0\}^{L\times 1}$ is the ternary spreading code assigned to the user k from UD code set $\mathbf{C}_{L\times K}$, h_k stands for the complex channel gain between the k-th user and the relay node, $d_k \in \{\pm 1\}$ is BPSK modulated data bits, and the channel noise $\mathbf{n}$ is assumed to be AWGN. Since each user has perfect CSI at the transmitter it is reasonable to utilize transmit diversity scheme, where it precodes the signal by $\alpha_k = h_k^*/|h_k|^2$ before the transmission to mitigate the channel effects [20]. For each user, the encoder combines its data sequence d_k into $\mathbf{s}_k = \alpha_k \mathbf{c}_k d_k$, which is then transmitted through the MA channel, as shown in Figure 2.

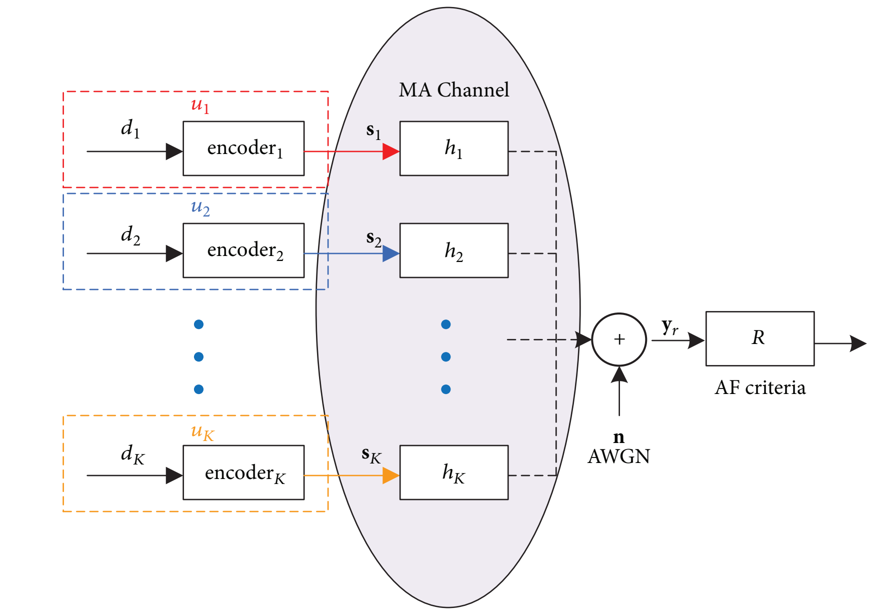

Figure 2: MA phase (TS1).

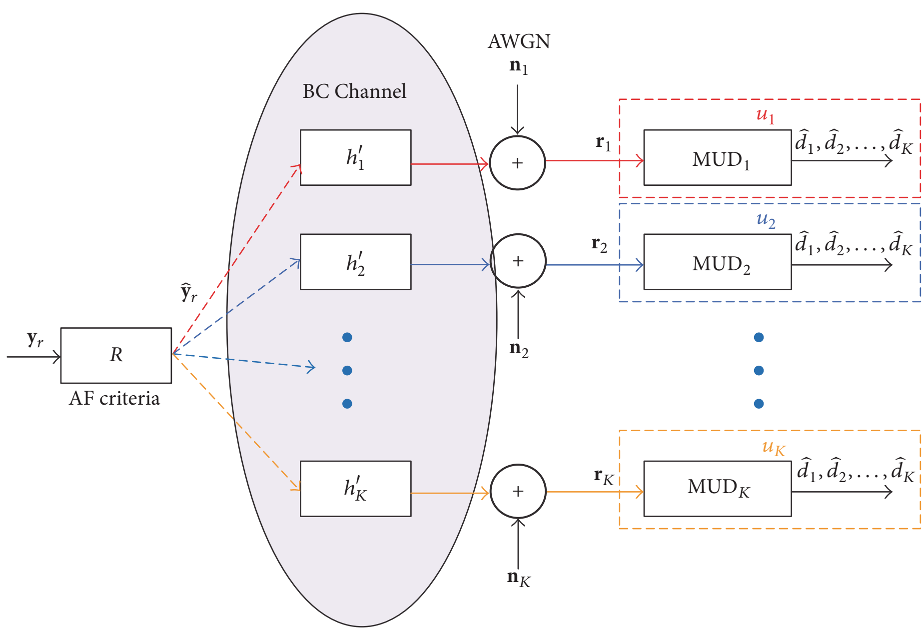

Figure 3: BC phase (TS2).

The BC phase is shown in Figure 3. In TS2, the relay node may apply the AF criteria and amplify the received vector $\mathbf{y}_r$ to obtain $\hat{\mathbf{y}}_r$ depending on the channel conditions. The relay node broadcasts the combined sum-signals to all the user nodes. Each end user's received signal is given by

$$\mathbf{r}_k = h'_k \hat{\mathbf{y}}_r + \mathbf{n}_k, \tag{2}$$

where h'_k is the complex channel gain between the relay node and the k-th user and $\mathbf{n}_k$ is AWGN. Note that to get rid of the channel effects each user can multiply the received signal by $\alpha'_k = h'^{*}_k / |h'_k|^2$ before applying the MUD. In the following section, we will present the construction of the proposed UD code sets $\mathbf{C}_{L\times K}$ and their linear MUD decoder.

The beauty behind the proposed scheme is that any user node can uniquely decode every user's binary data from the received sum-signal without using any table, as discussed in [18], or previous decoded data, which is the case for the ANC. Compared to the conventional PNC, sum data rate is much greater than 1. At the relay, we apply AF criteria instead of performing DNF.

3. Iterative Construction

In the proposed MWRC system, we utilize the UD code set that allows K-users to exchange information with the help of the relay node in only 2 TSs. These codes along with the proposed linear MUD decoder make a perfect candidate for MWRC systems.

We recall that a ternary code set $\mathbf{C} \in \{0, \pm 1\}^{L\times K}$ is uniquely decodable over signals $\mathbf{x} \in \{\pm 1\}^{K\times 1}$ or $\mathbf{x} \in \{0, 1\}^{K\times 1}$, if and only if, for any $\mathbf{x}_1 \neq \mathbf{x}_2$, $\mathbf{C}\mathbf{x}_1 \neq \mathbf{C}\mathbf{x}_2$ or, equivalently, $\mathbf{C}(\mathbf{x}_1 - \mathbf{x}_2) \neq \mathbf{0}_{L\times 1}$ [15]. We can rewrite the unique decodability necessary and sufficient condition as $\mathrm{Null}(\mathbf{C}) \cap \{0, \pm 2\}^{K\times 1} = \{0\}^{K\times 1}$ or in an equivalent manner as

$$\mathrm{Null}(\mathbf{C}) \cap \{0, \pm 1\}^{K\times 1} = \{0\}^{K\times 1}. \tag{3}$$

Let $f_t(L)$ represent the maximum number of columns (signals) that matrix can have for a given L and still be uniquely decodable. For the ternary code matrix with codes of length $L = 2$, $f_t(2)$ is simple and can be found by looking at the total number of possible columns $3^2 = 9$. Excluding the $[0, 0]^T$ column, half of the remaining is the negative of the other half, which makes it a total of 4 distinct columns that can be chosen to be $[0, 1]^T$, $[1, 0]^T$, $[1, -1]^T$, and $[1, 1]^T$. We conclude that no possible distinct combinations of these 4 columns satisfy uniquely decodability criteria (3). Out of all the possible combinations there are only few matrices with number of columns 3 that satisfy (3); therefore, $f_t(2) = 3$. Every possible ternary matrix of dimension 2×3 that has uniquely decodable property can be reduced to

$$\mathbf{C}^1_{2\times 3} = \begin{bmatrix} +1 & +1 & +1 \\ +1 & 0 & -1 \end{bmatrix}, \tag{4}$$

by applying operations such as multiplying columns by negative one, permuting rows, and columns. For the case of $L = 3$ and $L = 4$ it can be shown with an exhaustive search that $f_t(3) = 5$ and $f_t(4) = 8$, respectively.

In the preparation of general construction of matrices having $L = 2^i$, where $i \geq 2$, we carefully choose our seed matrix $\mathbf{C}^2_{4\times 8}$ from distinct uniquely decodable matrices, which are found by exhaustive search,

$$\mathbf{C}^2_{4\times 8} = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & +1 & +1 & +1 & 0 & -1 & -1 & -1 \\ +1 & +1 & 0 & -1 & 0 & +1 & 0 & -1 \\ +1 & 0 & 0 & -1 & 0 & -1 & 0 & +1 \end{bmatrix}. \tag{5}$$

Next, we are ready to propose a general $L_i \times K_i$ code set design when $L_i = 2^i$ with $K_i = 2^{i+1} + 2^{i-2} - 1$, $i = 3, 4, \ldots$. Starting from $\mathbf{C}^2_{4\times 8}$ the following recursive relation defines a sequence of matrices. The ith recursive matrix $\mathbf{C}^i_{L_i\times K_i}$ is formed as follows:

$$\mathbf{C}^i_{L_i\times K_i} = \begin{bmatrix} +1 & \cdots & +1 & +1 & +1 & \cdots & +1 \\ +1 & \cdots & +1 & 0 & -1 & \cdots & -1 \\ & & & 0 & & & \\ & \widehat{\mathbf{C}}^{i-1} & & 0 & & \mathbf{0} & \\ & & & \vdots & & & \\ & \mathbf{0} & & 0 & & \widehat{\mathbf{C}}^{i-1} & \\ & & & 0 & & & \end{bmatrix}, \tag{6}$$

where $L_i = 2L_{i-1}$, $K_i = 2K_{i-1} + 1$, and $\widehat{\mathbf{C}}^{i-1}$ is derived by eliminating the first row of $\mathbf{C}^{i-1}_{L_{i-1}\times K_{i-1}}$. The above code sequences $\mathbf{C}^i_{L_i\times K_i}$ preserve the uniquely decodability property, given that $\widehat{\mathbf{C}}^{i-1}$ is a UD matrix.

4. The Proposed Fast Decoder

In this section, we present our proposed fast decoder algorithm (FDA). In the system with signature matrix $\mathbf{C} \in \{\pm 1, 0\}^{L\times K}$, where the columns are the user spreading codes. At k's user, the received vector $\boldsymbol{r}_k$ after multiplication by α'_k is expressed by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{c}_k x_k + \mathbf{n} = \mathbf{C}\mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{c}_j \in \{\pm 1, 0\}^{L\times 1}$ are signatures for $1 \leq j \leq K$, $\mathbf{x} \in \{\pm 1\}^{K\times 1}$ is user data, and $\mathbf{n}$ is AWGN noise. The objective of the receiver is the following: given the received vector $\mathbf{y}$ and $\mathbf{C}$ recover the user data $\hat{\mathbf{x}}$ such that the mean square error $E\{\|\mathbf{x} - \hat{\mathbf{x}}\|^2\}$ is minimized. It is known that obtaining the ML solution is generally NP-hard [21].

For our detection problem, where the overloaded signature matrix has a UD structure, can be solved efficiently if there is a function that maps $\mathbf{y} \longmapsto \hat{\mathbf{y}} \in \Lambda$, where Λ is a $\mathbb{Z}$-module with rank L. It is equivalent to finding the closest point in a lattice Λ, such that

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}'\in\Lambda} \left\| \mathbf{y} - \mathbf{y}' \right\|^2. \tag{8}$$

Gaining the knowledge of $\hat{\mathbf{y}}$, one of the points in Λ generated by $\mathbf{C}$, we can obtain $\hat{\mathbf{x}}$ uniquely, since $\mathbf{C}$ satisfies the uniquely decodability criteria (3). However, there is no known polynomial algorithm that can obtain $\hat{\mathbf{y}}$ from $\mathbf{y}$.

We first present the general form of the proposed FDA for the $\mathbf{C}^i_{L_i\times K_i}$, $i \geq 2$ case, where the vector 1 is defined as $\mathbf{1} \in 1^{K\times 1}$ and the quantizer $Q : \mathbb{R} \longmapsto \mathcal{N}$, $z_1 = Q(y, -K, K)$ is a mapping of $y \in \mathbb{R}$ to the constellation of $\{\pm K, \pm(K-2), \ldots\}$. The output of the quantizer z_1 shows the number of -1s in $\hat{\mathbf{x}}$. Furthermore, let m_1, m_2, m_3, m_{11}, k_1, k_2, and k_3 represent the number of -1's at (1, 2), 3, 4, 1, 6, 7, and 8 locations of $\hat{\mathbf{x}}$, respectively. Note that when $z_1 = K$ or $z_1 = -K$

Input: $\mathbf{y}$
(1) $z_1 \longleftarrow Q(y_1, -K, K)$
(2) **if** $z_1 = |K|, \hat{\mathbf{x}} \longleftarrow \operatorname{sgn}(z_1)\mathbf{1}$
(3) **else**
(4) $n \longleftarrow (K - z_1)/2$
(5) $z_2 \longleftarrow Q(y_2, -(K - |z_1|), K - |z_1|)$
(6) $n_l \longleftarrow (2n - z_2)/4, n_r \longleftarrow n - n_l$
(7) $n_l \longleftarrow \lfloor n_l \rfloor, n_r \longleftarrow \lfloor n_r \rfloor$
(8) **if** $K = 8, \hat{\mathbf{x}} \longleftarrow subDecoder(\mathbf{y}, n_l, n_r)$
(9) **else**
(10) $\hat{\mathbf{y}}_l \longleftarrow [(2^i + 2^{i-3} - 1 - 2n_l), y_3, \ldots, y_{2^{i-1}+1}]^T$
(11) $\hat{\mathbf{y}}_r \longleftarrow [(2^i + 2^{i-3} - 1 - 2n_r), y_{2^{i-1}+2}, \ldots, y_{2^i}]^T$
(12) $\hat{\mathbf{x}}_l \longleftarrow decoder(\hat{\mathbf{y}}_l), \hat{\mathbf{x}}_r \longleftarrow decoder(\hat{\mathbf{y}}_r)$
(13) $x_m \longleftarrow z_l - (\hat{\mathbf{x}}_l^T\mathbf{1} + \hat{\mathbf{x}}_r^T\mathbf{1}), \hat{\mathbf{x}} \longleftarrow [\hat{\mathbf{x}}_l^T, x_m, \hat{\mathbf{x}}_r^T]^T$
Output: $\hat{\mathbf{x}}$

Algorithm 1: Fast decoder algorithm (FDA).

Input: $\mathbf{y}, n, n_l, n_r$
(1) **If** $n_l = 0, [m_1, m_2, m_3, m_{11}] \longleftarrow [0, 0, 0, 0], S_l \longleftarrow 1$
(2) **elseIf** $n_l = 4, [m_1, m_2, m_3, m_{11}] \longleftarrow [2, 1, 1, 1], S_l \longleftarrow 1$
(3) **If** $n_r = 0, [k_1, k_2, k_3] \longleftarrow [0, 0, 0], S_r \longleftarrow 1$
(4) **elseIf** $n_r = 3, [k_1, k_2, k_3] \longleftarrow [1, 1, 1], S_r \longleftarrow 1$
(5) **If** $S_l = 1$ AND $S_r = 0$,
(6) $[k_1, k_2, k_3] \longleftarrow rightDecoder(\mathbf{y}, m_1, m_2, m_3, m_{11})$
(7) **If** $S_l = 0$ AND $S_r = 1$,
(8) $[m_1, m_2, m_3, m_{11}] \longleftarrow leftDecoder(\mathbf{y}, k_1, k_2, k_3)$
(9) **else**, $S_l = 0$ AND $S_r = 0$
(10) $[m_1, m_2, m_3, m_{11}, k_1, k_2, k_3] \longleftarrow lrDecoder(\mathbf{y})$
(11) $[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4] \longleftarrow -2[m_{11}, (m_1 - m_{11}), m_3, m_2] + 1$
(12) $\hat{x}_5 \longleftarrow -2(n - n_l - n_r) + 1$
(13) $[\hat{x}_6, \hat{x}_7, \hat{x}_8] \longleftarrow -2[k_1, k_2, k_3] + 1$
Output: $\hat{\mathbf{x}}$

Algorithm 2: subDecoder algorithm.

Input: $\mathbf{y}, n_r, m_1, m_2$
(1) $y_{3m} \longleftarrow (y_3 - 1)/2 - m_2 + m_1$
(2) $z_{3m} \longleftarrow Q(y_2, -1, +1)$
(3) $k_2 \longleftarrow \lfloor (z_{3m} + n_r)/2 \rfloor$
(4) $k_3 \longleftarrow z_{3m} + n_r - 2k_2$
(5) $k_1 \longleftarrow n_r - k_2 - k_3$
Output: $[k_1, k_2, k_3]$

Algorithm 3: rightDecoder algorithm.

Input: $\mathbf{y}, n_l, k_1, k_2$
(1) $y_{3k} \longleftarrow (y_3 - 1)/2$
(2) $z_{3k} \longleftarrow Q(y_2, -k_1 + k_2 + \delta_{\min}, -k_1 + k_2 + \delta_{\max})$
(3) $m_2 \longleftarrow \lfloor (z_{3k} - k_2 + k_1 + n_l)/2 \rfloor$
(4) $m_3 \longleftarrow z_{3k} - k_2 + k_1 + n_l - 2m_2$
(5) $m_1 \longleftarrow n_l - m_2 - m_3$
(6) **If** $m_1 = 2, m_{11} \longleftarrow 1$
(7) **elseIf** $m_1 = 0, m_{11} \longleftarrow 0$
(8) **elseIf** $y_4/2 - k_1 - m_2 + k_2 \geq -0.5, m_{11} \longleftarrow 0$
(9) **else**, $m_{11} \longleftarrow 1$
Output: $[m_1, m_2, m_3, m_{11}]$

Algorithm 4: leftDecoder algorithm.

Input: $\mathbf{y}, n_l, n_r$
(1) $y_{3n} \longleftarrow (y_3 - 1)/2, d_3 \longleftarrow e^{10}$
(2) $z_{3n} \longleftarrow Q(y_2, -\delta_{\min} - 1, \delta_{\max} + 1)$
(3) **for** $\delta_3 \in \{-1 + \gamma_{\min}, \ldots, -1 + \gamma_{\max}\}$
(4) $m_2' \longleftarrow \lfloor (z_{3n} - \delta_3 + n_l)/2 \rfloor$
(5) $m_3' \longleftarrow z_{3n} - \delta_3 + n_l - 2m_2'$
(6) $m_1' \longleftarrow n_l - m_2' - m_3', k_2' \longleftarrow \lfloor (\delta_3 + n_r)/2 \rfloor$
(7) $k_3' \longleftarrow n_r + \delta_3 - 2k_2', k_1' \longleftarrow n_r - k_2' - k_3'$
(8) **If** $m_1' = 2, m_{11}' \longleftarrow 1$
(9) **elseIf** $m_1' = 0, m_{11}' \longleftarrow 0$
(10) **elseIf** $y_4/2 - k_1' - m_2' + k_2' \geq -0.5, m_{11}' \longleftarrow 0$
(11) **else** $m_{11}' \longleftarrow 1$
(12) **if** $d_3' \longleftarrow |y_4/2 + m_{11}' - m_2' - k_1' + k_2'| < d_3$
(13) $[m_1, m_2, m_3, m_{11}] \longleftarrow [m_1', m_2', m_3', m_{11}']$
(14) $[k_1, k_2, k_3] \longleftarrow [k_1', k_2', k_3']$
(15) $d_3 \longleftarrow d_3'$
Output: $[m_1, m_2, m_3, m_{11}, k_1, k_2, k_3]$

Algorithm 5: lrDecoder algorithm.

only one comparison is required. The algorithm proceeds by computing n, n_l, and n_r, which denote the number of -1's in $\hat{\mathbf{x}}$, $[\hat{x}_1, \ldots, \hat{x}_{(K-1)/2}]$, and $[\hat{x}_{(K-1)/2+1}, \ldots, \hat{x}_K]$, respectively.

For the case of $\mathbf{C}^1_{2\times 3}$ the decoding is trivial and will not be covered in this article, instead we start with the nonsymmetric case of $\mathbf{C}^2_{4\times 8}$. The FDA, shown in Algorithm 1, calls the *subDecoder* (Algorithm 2) at line (8) with $[y_1, \ldots, y_4]^T$, n_l, and n_r parameters. This algorithm will proceed in four different paths depending on n_l and n_r. If n_l is 0 or 4 then the *leftDecoder* (Algorithm 4) will never be called and will assign $[m_1, m_2, m_3, m_{11}] = [0, 0, 0, 0]$ or $[m_1, m_2, m_3, m_{11}] = [2, 1, 1, 1]$, respectively. Similarly, if n_r is 0 or 3 then the *rightDecoder* (Algorithm 3) will never be called and will assign $[k_1, k_2, k_3] = [0, 0, 0]$ or $[k_1, k_2, k_3] = [1, 1, 1]$, respectively. Therefore, the trivial case is when both the *leftDecoder* and the *rightDecoder* are not required; other scenarios are that the *rightDecoder* is called when the *leftDecoder* is not required, the *leftDecoder* is called when the *rightDecoder* is not required, and the last case is when left and right decoder, *lrDecoder*, is called (Algorithm 5).

The *rightDecoder* and the *leftDecoder* decoders are straightforward, having the knowledge of $(\mathbf{y}, n_r, m_1, m_2)$ the *rightDecoder* computes (k_1, k_2, k_3), and similarly, having the knowledge of $(\mathbf{y}, n_l, k_1, k_2)$, the *leftDecoder* computes (m_1, m_2, m_3, m_{11}) (Algorithms 3 and 4).

Note that the parameters in the *leftDecoder* and the *lrDecoder* are computed as such $\delta_{\min} = -\text{rnd}(3(n_l + 1)/5)$, $\delta_{\max} = \text{mod}(\text{rnd}(3n_l/5), 2)$, $\gamma_{\min} = (\operatorname{sgn}(\eta - 1/10) + 1)\eta/2$, and $\gamma_{\max} = \lambda(\zeta - 3)/2 - 1$, where $\eta = \zeta + \delta_{\min} - \delta_{\max} - 1$,

Table 1: Complexity of the proposed ternary codes.

Decoder	Complexity	(4 × 8)	(8 × 17)	(16 × 35)
Proposed	Comparisons	5.86	17.98	50.24
ML	Comparisons	2^8	2^{17}	2^{35}

$\lambda = \text{sgn}(31/10 - \zeta) + 1$ and ζ is the index of the constellation returned by $Q(\cdot)$ function (Algorithms 4 and 5).

Having all the required information now the *subDecoder* assigns $[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_6, \hat{x}_7, \hat{x}_8] = -2[m_{11}, (m_1 - m_{11}), m_3, m_2, k_1, k_2, k_3] + 1$ and $\hat{x}_5 = -2(n - n_l - n_r) + 1$. Now we completed the case when $K = 8$; the rest of the algorithm in FDA proceeds by applying the general *decoder* algorithm with the inputs of $\hat{\mathbf{y}}_l$ and $\hat{\mathbf{y}}_r$ to obtain $\hat{\mathbf{x}}_l$ and $\hat{\mathbf{x}}_r$, respectively, to find the middle element $x_m = z_l - (\hat{\mathbf{x}}_l^T \mathbf{1} + \hat{\mathbf{x}}_r^T \mathbf{1})$. The decoded data is $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_l^T, x_m, \hat{\mathbf{x}}_r^T]^T$. In the following section, we study the complexity of the proposed fast decoder analytically.

5. Complexity Analysis

The proposed decoder, discussed in Section 4, deciphers all the users' data at the receiver side in a recursive manner. In this section, we demonstrate the computational complexity analytically. It is important to state that the proposed FDA neither requires any multiplications nor additions; instead, only a few comparisons are performed in the $Q(\cdot)$ function. First, we will look at the average number of comparisons required for the $\mathbf{C}^2_{4\times 8}$ case, whose decoding algorithm is presented in the *subDecoder* algorithm. Since, our proposed $\mathbf{C}^2_{4\times 8}$ matrix is nonsymmetric, we will analyze the complexity of decoding all the 2^8 possible input vectors. By closely analyzing FDA algorithm the comparison required for $n = 0, 1, 2, 3, 4, 5, 6, 7, 8$ is $1, 25, 144, 289, 488, 369, 155, 28, 1$, respectively, and there are $\binom{8}{n}$ of input vectors per n. There are a total of 1500 comparisons; hence, the average computational complexity is $T_2 = 1500/256 = 5.86$. The recursive structure of our proposed matrices for $i \geq 3$ possesses symmetries that enable us to present the general case. In order to express the relationship for T_i, where $i \geq 3$, we will first introduce a few definitions. Let us define

$$G_i = \sum_{j=0}^{2^i + 2^{(i-3)} - 1} \binom{2^{(i+1)} + 2^{(i-2)} - 1}{j} (j+1), \tag{9}$$

$$\begin{aligned} H_i = \sum_{j=1}^{2^i + 2^{(i-3)} - 1} \Bigg\{ & \binom{2^i + 2^{(i-3)} - 1}{\lceil \frac{j-1}{2} \rceil}^2 (j+1) \\ & + 2 \sum_{k=0}^{\lfloor (j-1)/2 \rfloor} \binom{2^{(i-3)} - 1}{k} \binom{2^i + 2^{(i-3)} - 1}{j-k} (2k+1) \\ & + 2 \sum_{k=0}^{\lfloor (j-2)/2 \rfloor} \binom{2^i + 2^{(i-3)} - 1}{k} \binom{2^i + 2^{(i-3)} - 1}{j-k-1} (2k+2) \Bigg\}, \end{aligned} \tag{10}$$

$$\begin{aligned} U_i = 4\left(2^{2^i - 1} - 2\right) + 2 \sum_{j=2}^{2^i + 2^{(i-3)} - 1} \Bigg\{ & \binom{2^i + 2^{(i-3)} - 1}{\lceil \frac{j-1}{2} \rceil}^2 \\ & + 2 \sum_{k=1}^{\lfloor (j-1)/2 \rfloor} \binom{2^i + 2^{(i-3)} - 1}{k} \binom{2^i + 2^{(i-3)} - 1}{j-k} \\ & + 2 \sum_{k=1}^{\lfloor (j-2)/2 \rfloor} \binom{2^i + 2^{(i-3)} - 1}{k} \binom{2^i + 2^{(i-3)} - 1}{j-k-1} \Bigg\}, \end{aligned} \tag{11}$$

where G_i is the number of comparisons that are required in the first call of the $Q(\cdot)$ function. If the input vector contains j number of -1's, in $Q(\cdot)$ function it needs $(j+1)$ comparisons, as shown in (9). Note that, due to symmetry, we do not consider all the input vectors $\mathbf{x} \in \{\pm 1\}^K$, instead, only half of them, that is, $2^i + 2^{(i-3)} - 1$. The H_i is related to the number of comparisons required in the second call of the $Q(\cdot)$ function, while the last term U_i shows how many times left and/or right subdecoders are called. The general relation for $i \geq 3$ can be expressed as

$$T_i = \frac{1}{2^{2^{(i+1)} + 2^{(i-2)} - 2}} \left[G_i + H_i + U_i \times \widehat{T}_{i-1} \right], \tag{12}$$

where

$$\widehat{T}_{i-1} = \frac{1}{2^{2^i + 2^{(i-3)} - 2} - 1} \left[2^{2^i + 2^{(i-3)} - 2} T_{i-1} - G_{i-1} \right], \tag{13}$$

is the altered version of the T_{i-1} by excluding the number of comparisons in the first call of the $Q(\cdot)$ calculations.

In Table 1, we show the complexity results for (4×8), (8×17), (16×35) using the proposed FDA and ML algorithms. As we can see, the complexity of ML decoder increases exponentially, while the proposed decoder has fairly small complexity even for a relatively large matrix size (16×35).

6. Simulation Results

In this section, we evaluate the performance of the MWRC network by employing our proposed ternary uniquely decodable codes at the physical layer. All the simulations at the physical layer of the proposed scheme are performed in Matlab. We consider wireless transmission between $K = 8$ and $K = 17$ users. In the MA phase, each user k spreads its data $d_k \in \{\pm 1\}$, using BPSK modulation and the proposed ternary code $\mathbf{c}_k$, and then transmits to the relay node through a slow Rayleigh fading channel. The relay node in BC phase transmits the combined sum-signals to all the user nodes.

We assume that the transmission is synchronous and all CSI is known at all the user as well as relay nodes. Each user gets rid of the complex channel effects and then

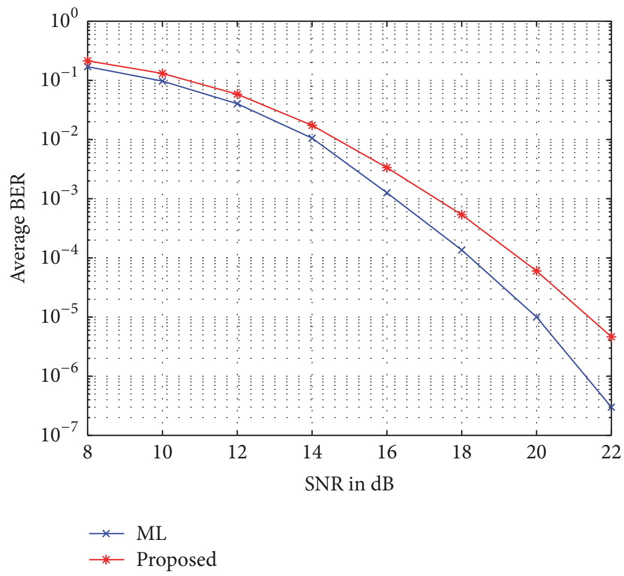

FIGURE 4: Average BER versus SNR for the UD codes, $\mathbf{C}^2_{4\times8}$.

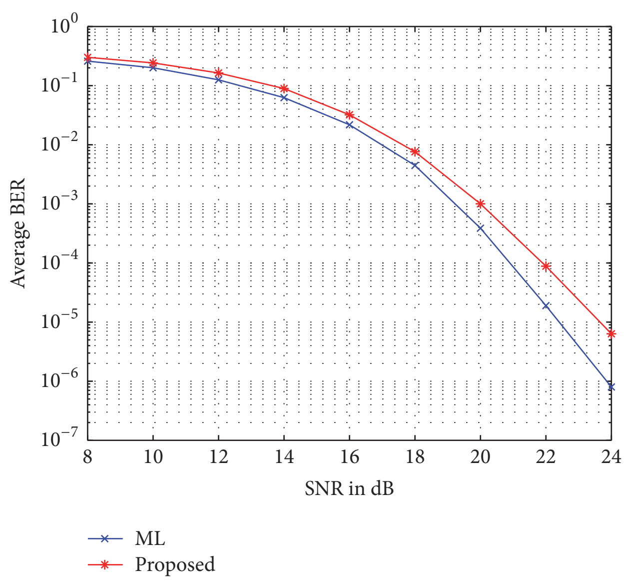

FIGURE 5: Average BER versus SNR for the UD codes, $\mathbf{C}^3_{8\times17}$.

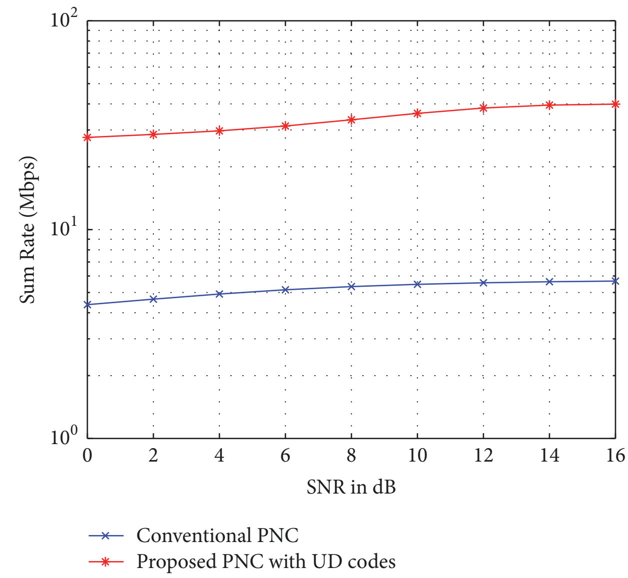

FIGURE 6: Sum rate versus SNR for $K = 8$ user nodes using UD codes, $\mathbf{C}^2_{4\times8}$.

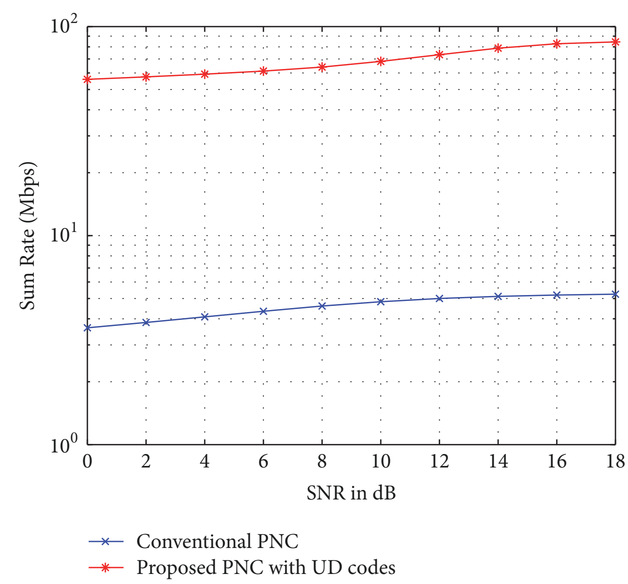

FIGURE 7: Sum rate versus SNR for $K = 17$ user nodes using UD codes, $\mathbf{C}^3_{8\times17}$.

applies the proposed FDA to decode its information. For comparison purposes, we compare FDA algorithm with the optimum ML decoder. In Figures 4 and 5, we plot the BER performance averaged over all the different users for the UD code sets of $\mathbf{C}^2_{4\times8}$ and $\mathbf{C}^3_{8\times17}$, respectively. For a BER of 10^{-3} the performance of FDA is only about 1 dB worse than the ML. In other words, our proposed FDA achieves near-ML performance without having an exponentially complex algorithm.

In order to show the advantage of PNC with UD codes compared to the conventional PNC, we first define the sum rate to be the number of correctly transmitted bits per unit time. Suppose that the bit rate of each node is 10 Mbps. In the case of conventional PNC $2(K-1)$ TSs are required compared to only 2 TSs for the PNC with UD code. Thus, the sum rates are $8/14\cdot(1-P_{e,\mathrm{PNC}})\cdot 10$ Mbps and $17/32\cdot(1-P_{e,\mathrm{PNC}})\cdot 10$ Mbps, where $P_{e,\mathrm{PNC}}$ is the error rate of conventional PNC, with $K = 8$ and $K = 17$ user nodes, respectively. Using the conventional PNC 80 Mbps and 170 Mbps are exchanged during 14 and 32 TSs. Meanwhile, the sum rates of the proposed PNC with UD code sets are $8/2\cdot(1-P_e)\cdot 10$ Mbps and $17/2\cdot(1-P_e)\cdot 10$ Mbps, where P_e is the error rate of the detector of the UD codes.

In Figures 6 and 7, we plot the sum rates for the cases of $K = 8$ and $K = 17$ user nodes, respectively. We can see from Figures 6 and 7 that the proposed scheme utilizing UD codes improves the sum rates by almost 7 and 16 times, respectively, compared to conventional PNC scheme. When the signal-to-noise ratio (SNR) is high enough, the sum rate is nearly enhanced by about $K-1$ times.

7. Conclusion

In this paper, we have proposed to apply a ternary uniquely decodable (UD) code sets to a multiway relay channel (MWRC) network where two or more users are able to exchange information with each other simultaneously through the help of a relay node. A key feature of the proposed scheme is that it utilizes a novel UD code sets in a transmission, which requires only two time slots (TSs) as opposed to $2(K-1)$ TSs for the conventional physical-layer network coding (PNC) scheme. We developed for the proposed UD code set a very simple decoding algorithm, which requires only a few logical comparisons. Simulation results in terms of bit error rate (BER) and sum rates demonstrate that the proposed decoder outperforms the conventional methods. The performance of the proposed low computational cost decoder is almost as good as the maximum-likelihood (ML) decoder.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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