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## Research Article

# Adaptive Neural Network Control for Nonlinear Hydraulic Servo-System with Time-Varying State Constraints

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An adaptive neural network control problem is addressed for a class of nonlinear hydraulic servo-systems with time-varying state constraints. In view of the low precision problem of the traditional hydraulic servo-system which is caused by the tracking errors surpassing appropriate bound, the previous works have shown that the constraint for the system is a good way to solve the low precision problem. Meanwhile, compared with constant constraints, the time-varying state constraints are more general in the actual systems. Therefore, when the states of the system are forced to obey bounded time-varying constraint conditions, the high precision tracking performance of the system can be easily realized. In order to achieve this goal, the time-varying barrier Lyapunov function (TVBLF) is used to prevent the states from violating time-varying constraints. By the backstepping design, the adaptive controller will be obtained. A radial basis function neural network (RBFNN) is used to estimate the uncertainties. Based on analyzing the stability of the hydraulic servo-system, we show that the error signals are bounded in the compacts sets; the time-varying state constrains are never violated and all singles of the hydraulic servo-system are bounded. The simulation and experimental results show that the tracking accuracy of system is improved and the controller has fast tracking ability and strong robustness.

#### 1. Introduction

Throughout history, the hydraulic servo-system has been always widely applied in many aspects, such as aerospace, aviation, navigation, weapon, mining, and metallurgy due to the advantages of high response, high power, large stiffness, strong robustness capability, small volume, and so on. However, serious nonlinear behavior, such as control input saturation [1], state constraint, valve opening, nonlinear friction [2], and model uncertainty [3] (load changes, the parameters variation and the element parameter uncertainty [4] caused by the abrasive, containing external disturbance [5, 6], leakage, and other uncertain nonlinear elements), restricts the development of high-performance closed loop system controller [7, 8].

Recently, in order to solve the uncertain nonlinear problem in the system, the most authors adopted the fuzzy logical system or adaptive neural network (NN) control [9, 10] to approximate the external disturbance [11] and model uncertainty [12, 13]. In [14, 15], the different stability problems of switched nonlinear systems are solved by using the fuzzy adaptive control. The paper [14] solved the discretetime switch system tracking control problem, and [15, 16] presented decentralized controller for switched uncertain nonlinear large-scale systems with dead zones based on observer [17]. The article [18] solved the control problem of the nonlinear system with dynamic uncertainties and input dead zone. Similarly, the authors in [19] used the NN to deal with the uncertain problem of nonlinear nonstrict feedback discrete-time systems [20]. The article [21] adopted general projection neural network (GPN) to iteratively solve a quadratic programming (QP) problem, which updates the algorithm of NN to develop the new application. Two neural networks which include the critic NN and the action NN are used to approximate the strategic utility function and to minimize both the strategic utility function and the tracking error in [22], respectively. The articles [23-26] adopt the adaptive NN in quantized nonlinear systems, nonlinear

time-delay systems, and uncertain nonlinear systems. These articles extend the theoretical application of neural network. In [27], an adaptive neural control scheme is presented to take the unknown output hysteresis and computational efficiency into account. The motion/force performances limits of the robotic system will be relaxed. The NN turns out to be a clever theory to solve the uncertain information of the systems from the above articles.

At present, the control problem of hydraulic servo-system has been an active research field by designing the adaptive control technique. The authors in [28] have presented an adaptive controller based on the robust integral of the error signal to deal with modeling uncertainties of the system, and the feedback gain is reduced. In [29], the robust adaptive control ensures uniform boundedness of the thruster assisted position mooring system which is in the transverse direction under the ocean current disturbance. The robust adaptive controller has been designed for single-rod nonlinear hydraulic servo-system where the element parameter uncertainty and nonlinear uncertainty behavior exist in [30]. The article in [31] presents output feedback nonlinear robust control based on an extended state observer (ESO) to solve mismatched modeling uncertainties problem, which is very important for high-accuracy tracking control of hydraulic servo-systems. In [32], considering the influence of friction on the system, an adaptive controller is designed to solve the problem of system friction and parameter uncertainty. In fact, due to the high bearing capacity and high stiffness properties of the hydraulic servo-system, so the state constraints problems are ignored in the environment and the interaction of measurement unit tests process, which lead to too large displacement, velocity, or acceleration causing damage to the measuring equipment during testing process [33]. In tests and experiments, if the initial conditions of the system do not match, it is possible that the displacement, velocity, or acceleration of the system is oversized. At the same time, the constraint for the system is a good way to achieve high accuracy tracking performance. However, none of the above articles considered constraints condition. And considering that the state variables vary with time, the constraint condition should also be considered the factor of time changing. Meanwhile, compared with constant constraints, the time-varying state constraints are more general in the actual systems.

In fact, some systems are often affected by constraints [34], such as temperature constraints in chemical reactions and the speed or acceleration constraints when some mechanical systems suffer physical failure [35]. The authors first present the theory of nonlinear system with output constrains and time-varying output constrains by using Barrier Lyapunov in [36, 37]. However, those articles do not consider the interference of unknown functions. In [38], considering a class of special nonlinear systems with the hysteretic output mechanism and the unmeasured states, adaptive neural output feedback control is presented to guarantee the prescribed convergence of the tracking error [39]. The article in [40] has studied adaptive neural control of uncertain stochastic nonlinear systems with dead zone and output constraint. The continuous stirred tank reactor [41], flexible crane system [42], and robot system [43, 44] all design

adaptive NN controller based on output constrains condition. It is essential that the theories of constraint are combined with the actual systems [45]. Similarly, the state constrains have also been researched by many scientific researchers. For example, the article in [46] presents adaptive fuzzy NN control using impedance learning for a constrained robot. The authors considering time-delay condition investigate the robotic manipulator system [47], nonlinear MIMO unknown systems [48], and common nonlinear systems [49] with full state constrains [50]. The authors further research the nonlinear pure-feedback systems, Nussbaum gain adaptive control, and ABLF adaptive controller to apply them on stochastic nonlinear systems and wheeled mobile robotic system by using the theory of integral barrier Lyapunov function or state constrains in [51], [52], [53], [54], [55], and [56], respectively.

Based on the above descriptions, this article presents an adaptive NN tracking control for hydraulic servo-system based on time-varying state constraint, and hydraulic system adaptive control is first designed with the time-varying state constraints. Finally, when the states of the system are forced to obey bounded time-varying constraint conditions, the high precision tracking performance of the system can be easily realized. Meanwhile, it can be proved that the tracking error signal converges to zero asymptotically and all singles of the hydraulic servo-system are bounded by using the Lyapunov analysis. The experimental results show the availability of the design controller.

# 2. System Descriptions and Problem Preliminaries

A class of hydraulic servo-system dynamics of the inertial load can be described as the following form:

$$m\ddot{x} = P_e A - \rho \dot{x} - d(x, \dot{x}, t), \qquad (1)$$

where x represents the displacement of the inertial load; m is the mass of moving parts;  $P_e = P_1 - P_2$ ,  $P_1$  and  $P_2$  are pressures of the left and right inside the hydraulic cylinder, respectively; A is the corresponding areas of the above-mentioned chamber;  $\rho$  represents the viscous friction coefficient;  $d(x, \dot{x}, t)$  is defined as the unconsidered disturbance.

Considering the compressibility of the hydraulic oil and ignoring leakage of hydraulic cylinder, the pressure dynamic equation of both chambers is as follows:

$$\frac{V_e}{4B_v}\dot{P}_e = -A\dot{x} - C_m P_e - w + Q,$$
 (2)

where  $V_e = V_{s1} + V_{s2}$  is the total effective cylinder volume;  $V_{s1} = V_{I1} + Ax$  and  $V_{s2} = V_{I2} - Ax$  represent the total volume of the left and the right chamber inside hydraulic cylinder, respectively;  $V_{I1}$  and  $V_{I2}$  are the corresponding the initial volume;  $B_v$  is the effective bulk modulus of hydraulic cylinder;  $C_m$  represents the total internal leakage coefficient of hydraulic cylinder; w represents the modeling error of both chambers;  $Q = (Q_1 + Q_2)/2$  is the load-flow of hydraulic cylinder;  $Q_1$  and  $Q_2$ , respectively, represent the supplied oil flow rate to the left chamber and the returned oil flow rate to the right chamber.

Consider that the flow rates  $Q_1$  and  $Q_2$  are the function of the spool displacement  $x_s$ . The flow equation of hydraulic cylinder is written as follows:

$$Q_{1} = k_{f} x_{s} h_{1} (x_{s}, P_{1}),$$

$$Q_{2} = k_{f} x_{s} h_{2} (x_{s}, P_{2})$$
(3)

with

$$h_{1}(x_{s}, P_{1}) = \begin{cases} (P_{s} - P_{1})^{1/2}, & x_{s} \ge 0, \\ (P_{1} - P_{r})^{1/2}, & x_{s} < 0, \end{cases}$$

$$h_{2}(x_{s}, P_{2}) = \begin{cases} (P_{2} - P_{r})^{1/2}, & x_{s} \ge 0, \\ (P_{s} - P_{2})^{1/2}, & x_{s} < 0, \end{cases}$$

$$(4)$$

where  $k_f$  denotes the flow gain coefficient;  $P_s$  is the supply pressure of the oil;  $P_r$  represents the return oil pressure.

We assume that the spool displacement  $x_s$  is direct proportion with control input voltage u in the situation of ignoring the high frequency; that is,  $x_s = k_s u$ ,  $k_s > 0$  is the small gain. Then, (3) can be rewritten as

$$Q_{1} = K_{f}uh_{1}(u, P_{1}),$$

$$Q_{2} = K_{f}uh_{2}(u, P_{2}),$$
(5)

where  $K_f = k_f k_s$  represents the total flow gain coefficient. Then, we can conclude that the load-flow of hydraulic cylinder is

$$Q = K_f u (P_s - \text{sgn}(u) P_e)^{1/2},$$
 (6)

where sgn(u) represents the step function as

$$sgn(u) = \begin{cases} 1, & u \ge 0, \\ -1, & u < 0, \end{cases}$$

$$P_s = P_1 + P_2, \qquad (7)$$

$$P_r = 0,$$

$$P_e = P_1 - P_2.$$

Define state variables  $x = [x, \dot{x}, \ddot{x}]^T = [x_1, x_2, x_3]^T$ ; the dynamics of the hydraulic servo-system (1) can be translated into the following the state space expression based on the pressure dynamic equation (2) and the flow equations (5)-(6):

$$\dot{x}_1 = x_2, 
\dot{x}_2 = x_3, 
\dot{x}_3 = -f_1 x_2 - f_2(x) + f_3 g u,$$
(8)

where

$$f_{1} = \frac{4A^{2}B_{v}}{mV_{e}} - \frac{\rho^{2}}{m},$$

$$f_{3} = \frac{4AB_{v}K_{f}}{mV_{e}},$$

$$g = \sqrt{\frac{(P_{s} - \text{sgn}(u)P_{e})}{2}}$$

$$f_{2}(x) = \frac{4AB_{v}C_{m}P_{e}}{mV_{e}} + \frac{4AB_{v}w}{mV_{e}} + \frac{\rho AP_{e}}{m^{2}} - \frac{\rho}{m}d(x)$$

$$+ \frac{\dot{d}(x)}{m}.$$
(9)

 $f_1(x)$ ,  $f_2(x)$ , and  $f_3(x)$  are unknown smooth parameter functions. In this paper, the state variable  $x_i$  is constrained in the following time-varying compact sets:

$$|x_i(t)| \le k_{b_i}(t), \quad i = 1, 2, 3,$$
 (10)

where  $k_{b_i}(t)$  represents the time-varying smooth functions.

Assumption 1 (see [37]). There exist the functions  $\overline{f}_3(x)$  and  $\underline{f}_3(x)$  which satisfy the inequality  $0 < \underline{f}_3 \le f_3 \le \overline{f}_3$ .

Assumption 2 (see [43]). There are functions  $Y_0(t)$ ,  $Y_1(t)$ , and  $Y_2(t)$ . We assume that the functions satisfy the inequalities  $|Y_{i-1}(t)| \leq |k_{b_i}(t)|$ , i=1,2,3. Then, the smooth desired trajectory  $y_d(t)$  and its time derivatives, respectively, satisfy  $|y_d(t)| \leq Y_0(t)$  and  $|y_d^{(i)}(t)| \leq Y_{i+1}(t)$ , i=1,2 for all t>0.

**Lemma 3** (see [54]). For any time-varying function  $k_a(t)$ , the following inequality holds for all error function z(t) in the interval  $|z(t)| \le k_a(t)$ :

$$\log \frac{k_a^2(t)}{k_a^2(t) - z^2(t)} \le \frac{z^2(t)}{k_a^2(t) - z^2(t)}.$$
 (11)

The above-mentioned unknown smooth functions  $f_1$ ,  $f_2(x)$ , and  $f_3$  are approximated by the radial basis function NN as

$$f_i(X) = W_i^T \Phi_i(X), \quad i = 1, 2, 3,$$
 (12)

where  $W_i \in R^{l_i}$  denotes the neural network weight vector and  $\Phi_i(X) = [\varphi_{i,1}, \ldots, \varphi_{i,l_i}] \in R^{l_i}$  are the basis function vectors with input vector  $X \in R^{n_i}$  and the NN node number  $l_i > 1$ . Gaussian functions  $\varphi_{i,j}(X)$  are chosen in the following forms:

$$\varphi_{i,j}(X) = \exp\left(\frac{-\|X - \mu_{i,j}\|^2}{\eta_{i,j}^2}\right),$$

$$i = 1, 2, 3, \quad j = 1, \dots, l_i,$$
(13)

where  $\mu_{i,j} = [\pi_{i,j,1}, \dots, \pi_{i,j,n_i}]^T$  denotes the center of NN function and  $\eta_{i,j}$  is the width of the Gaussian function.

Define the unknown optimal weight vector as the following form:

$$W_{i}^{*} = \arg\min_{W_{i} \in \mathbb{R}^{l_{i}}} \left[ \sup_{X \in \mathbb{R}^{n_{i}}} \left| W_{i}^{T} \Phi_{i}(X) - f_{i}(X) \right| \right],$$

$$i = 1, 2, 3.$$
(14)

Then, there exists the unknown approximation errors  $\varepsilon_i(X)$  which has supremum  $\overline{\varepsilon}_i$ ; that is,  $|\varepsilon_i(X)| \leq \overline{\varepsilon}_i$ . Equation (12) will be rewritten in the approximation equations as follows:

$$f_i(X) = W_i^{*T} \Phi_i(X) + \varepsilon_i(X), \quad i = 1, 2, 3.$$
 (15)

#### 3. The Adaptive NN Controller Design

Define the track trajectory  $y_d$  and the virtual controllers  $\alpha_1$  and  $\alpha_2$ ; we have the track error  $z_1 = x_1 - y_d$ . Then, let  $z_2 = x_2 - \alpha_1$  and  $z_3 = x_3 - \alpha_2$ . Choose the barrier Lyapunov function candidate as

$$V = \sum_{i=1}^{3} V_i + V_N,$$

$$V_i = \frac{1}{2} \log \frac{k_{a_i}^2}{k_{a_i}^2 - z_i^2}, \ V_N = \frac{1}{2} \sum_{j=1}^{3} \widetilde{W}_j^T \Theta_j^{-1} \widetilde{W}_j,$$
(16)

where  $\log(\cdot)$  represents the natural logarithm;  $k_{a_i}(t) = k_{b_i}(t) - D_i(t)$  is boundaries of error vector  $z_i$  from subsequent feasibility analysis;  $\Theta_j$ , j=1,2,3 are the positive gain matrices;  $\widetilde{W}_i$  is the neural network weight error with  $\widetilde{W}_i = \widehat{W}_i - W_i^*$ , and note that V is the positive definite and continuously differentiable in the set  $|z_i(t)| \leq k_{a_i}(t)$  for i=1,2,3.

Based on (16), the time derivative of V is

$$\dot{V} = \sum_{i=1}^{3} \dot{V}_{i} + \dot{V}_{N} 
= \sum_{i=1}^{3} \frac{z_{i}}{\left(k_{a_{i}}^{2} - z_{i}^{2}\right)} \left(\dot{z}_{i} - \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} z_{i}\right) + \sum_{j=1}^{3} \widetilde{W}_{j}^{T} \Theta_{j}^{-1} \dot{\widetilde{W}}_{j}.$$
(17)

Design the virtual controllers as

$$\alpha_1 = -\left(\gamma_1 + \overline{\gamma}_1(t)\right) z_1 + \dot{\gamma}_d,\tag{18}$$

$$\alpha_2 = -(\gamma_2 + \overline{\gamma}_2(t))z_2 + \dot{\alpha}_1 - \frac{k_{a_2}^2 - z_2^2}{k_{a_1}^2 - z_1^2}z_1,$$
 (19)

where the time-varying gains are given by

$$\overline{\gamma}_{i}(t) = \sqrt{\left(\frac{\dot{k}_{a_{i}}}{k_{a_{i}}}\right)^{2} + \delta_{i}}, \quad i = 1, 2, 3$$
(20)

for all positive constants  $\delta_i$  and  $\gamma_i$ . When any  $k_{a_i}$  is zero,  $\delta_i$  will ensure that the time derivatives of virtual controllers  $\alpha_i$  and controller u are bounded.

Substituting (8) and (18)–(20) into (17), we can obtain

$$\dot{V} = -\sum_{i=1}^{2} \left( \gamma_{i} + \overline{\gamma}_{i} \left( t \right) + \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} \right) \frac{z_{i}^{2}}{\left( k_{a_{i}}^{2} - z_{i}^{2} \right)} + \frac{z_{2}z_{3}}{\left( k_{a_{2}}^{2} - z_{2}^{2} \right)} 
+ \frac{z_{3}}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} \left( \dot{x}_{3} - \dot{\alpha}_{2} - \frac{\dot{k}_{a_{3}}}{k_{a_{3}}} z_{3} \right) 
+ \sum_{j=1}^{3} \widetilde{W}_{j}^{T} \Theta_{j}^{-1} \dot{\overline{W}}_{j},$$
(21)

where

$$\begin{split} \dot{\alpha}_{1} &= \frac{\partial \alpha_{1}}{\partial x_{1}} \dot{x}_{1} + \sum_{k=0}^{1} \left( \frac{\partial \alpha_{1}}{y_{d}^{(k)}} y_{d}^{(k+1)} + \frac{\partial \alpha_{1}}{k_{a_{1}}^{(k)}} k_{a_{1}}^{(k+1)} \right), \\ \dot{\alpha}_{2} &= \sum_{j=1}^{2} \frac{\partial \alpha_{2}}{\partial x_{j}} \dot{x}_{j} \\ &+ \sum_{k=0}^{2} \left( \frac{\partial \alpha_{2}}{y_{d}^{(k)}} y_{d}^{(k+1)} + \frac{\partial \alpha_{2}}{k_{a_{1}}^{(k)}} k_{a_{1}}^{(k+1)} + \frac{\partial \alpha_{2}}{k_{a_{2}}^{(k)}} k_{a_{2}}^{(k+1)} \right). \end{split}$$
 (22)

According to approximation equations (15), (21) becomes

$$\dot{V} = -\sum_{i=1}^{2} \left( \gamma_{i} + \overline{\gamma}_{i}(t) + \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} \right) \frac{z_{i}^{2}}{\left(k_{a_{i}}^{2} - z_{i}^{2}\right)} 
+ \sum_{j=1}^{3} \widetilde{W}_{j}^{T} \Theta_{j}^{-1} \dot{\overline{W}}_{j} + \frac{z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} \left[ -W_{1}^{*T} \Phi_{1}(X) x_{2} \right] 
- W_{2}^{*T} \Phi_{2}(X) + W_{3}^{*T} \Phi_{3}(X) gu + D(x, t) - \dot{\alpha}_{2} 
+ \frac{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)}{\left(k_{a_{2}}^{2} - z_{2}^{2}\right)} z_{2} - \frac{\dot{k}_{a_{3}}}{k_{a_{3}}} z_{3} ,$$
(23)

where D(x,t) represents the systematic disturbance compensation as the following form:

$$D(x,t) = -\varepsilon_1 x_2 - \varepsilon_2 + \varepsilon_3 gu.$$
 (24)

Choose the controller as follows:

$$u = \frac{1}{\widehat{W}_{3}^{T} \Phi_{3} g} \left( -\left(\gamma_{3} + \overline{\gamma}_{3}(t)\right) z_{3} + \widehat{W}_{1}^{T} \Phi_{1} x_{2} + \widehat{W}_{2}^{T} \Phi_{2}(x) + \dot{\alpha}_{2} - \frac{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)}{\left(k_{a_{2}}^{2} - z_{2}^{2}\right)} z_{2} - \frac{z_{3}}{2\left(k_{a_{2}}^{2} - z_{3}^{2}\right)} \right).$$
(25)

Adopting the equations  $W_i^* = \widehat{W}_i - \widetilde{W}_i$ , i = 1, 2, 3 and controller (25), the expression for (23) can be rewritten by the above controller as

$$\dot{V} = -\sum_{i=1}^{3} \left( \gamma_{i} + \overline{\gamma}_{i} \left( t \right) + \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} \right) \frac{z_{i}^{2}}{\left( k_{a_{i}}^{2} - z_{i}^{2} \right)} 
+ \widetilde{W}_{1}^{T} \left( \Theta_{1}^{-1} \dot{\widehat{W}}_{1} + \Phi_{1} \left( X \right) \frac{x_{2}z_{3}}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} \right) 
+ \widetilde{W}_{2}^{T} \left( \Theta_{2}^{-1} \dot{\widehat{W}}_{2} + \Phi_{2} \left( X \right) \frac{z_{3}}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} \right)$$

$$+ \widetilde{W}_{3}^{T} \left( \Theta_{3}^{-1} \dot{\widehat{W}}_{3} - \Phi_{3} \left( X \right) \frac{z_{3}gu}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} \right) 
+ D \left( x, t \right) \frac{z_{3}}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} - \frac{z_{3}^{2}}{2 \left( k_{a_{3}}^{2} - z_{3}^{2} \right)^{2}}.$$

Based on the above-mentioned definition of  $\overline{\gamma}_i(t)$ , we can obtain that

$$\overline{\gamma}_{i}(t) + \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} \ge 0, \quad i = 1, 2, 3.$$
 (27)

Introducing the projection algorithm of adaptive control, we design the adaptive law from (26) as follows:

$$\begin{split} \widehat{W}_{1} \\ &= \begin{cases} -\Theta_{1} \left[ \Phi_{1}(X) \frac{x_{2}z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} + \sigma_{1}\widehat{W}_{1} \right], & \Theta_{1} \leq 0, \ \widehat{W}_{1} > M_{f1}, \\ P_{f1}\left(\widehat{W}_{1}, \Phi_{1}\right), & \Theta_{1} > 0, \ \widehat{W}_{1} = M_{f1}, \end{cases} \end{aligned} \tag{28}$$

where  $P_{f1}(\widehat{W}_1, \Phi_1)$  represents projection operator as the following form:

$$P_{f1}(\widehat{W}_{1}, \Phi_{1}) = -\Theta_{1} \left[ \Phi_{1}(X) \frac{x_{2}z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} - \Phi_{1}(X) \frac{\widehat{W}_{1}^{T} \widehat{W}_{1} x_{2} z_{3}}{\left\|\widehat{W}_{1}\right\|^{2} \left(k_{a_{3}}^{2} - z_{3}^{2}\right)} + \sigma_{1} \widehat{W}_{1} \right].$$
(29)

 $M_{f1}$  is the minuteness positive constant to ensure the adaptive law  $\dot{\widehat{W}}_1 \geq 0$  in any situation.

Similarly, the other adaptive laws can obtain the forms as

$$\dot{\widehat{W}}_{2} = \begin{cases}
-\Theta_{2} \left[ \Phi_{2}(X) \frac{z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} + \sigma_{2}\widehat{W}_{2} \right], & \Theta_{2} \leq 0, \ \widehat{W}_{2} > M_{f2}, \\
P_{f2}\left(\widehat{W}_{2}, \Phi_{2}\right), & \Theta_{2} > 0, \ \widehat{W}_{2} = M_{f2},
\end{cases}$$
(30)

$$P_{f2}\left(\widehat{W}_{2}, \Phi_{2}\right) = -\Theta_{2}\left[\Phi_{2}\left(X\right) \frac{z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} - \Phi_{2}\left(X\right) \frac{\widehat{W}_{2}^{T} \widehat{W}_{2} z_{3}}{\left\|\widehat{W}_{2}\right\|^{2} \left(k_{a_{3}}^{2} - z_{3}^{2}\right)} + \sigma_{2}\widehat{W}_{2}\right],\tag{31}$$

$$\dot{\widehat{W}}_{3} = \begin{cases}
\Theta_{3} \left[ \Phi_{3}(X) \frac{z_{3}gu}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} - \sigma_{3}\widehat{W}_{3} \right], & \Theta_{3} \leq 0, \ \widehat{W}_{3} > M_{f3}, \\
P_{f3}\left(\widehat{W}_{3}, \Phi_{3}\right), & \Theta_{3} > 0, \ \widehat{W}_{3} = M_{f3},
\end{cases}$$
(32)

$$P_{f_3}(\widehat{W}_3, \Phi_3) = -\Theta_3 \left[ \Phi_3(X) \frac{z_3 g u}{\left(k_{a_3}^2 - z_3^2\right)} - \Phi_3(X) \frac{\widehat{W}_3^T \widehat{W}_3 z_3 g u}{\left\|\widehat{W}_3\right\|^2 \left(k_{a_3}^2 - z_3^2\right)} + \sigma_3 \widehat{W}_3 \right]. \tag{33}$$

In particular, when component  $\widehat{W}_{3,i}$  of  $\widehat{W}_3$  is a small constant  $m_{f3,i}$ , that is,  $\widehat{W}_3 = M_{f3}$ , we adopt the following component of adaptive law:

$$\dot{\widehat{W}}_{3,i} = \begin{cases} -\Theta_{3,i} \left[ \Phi_{3,i} (X) \frac{z_3 g u}{\left(k_{a_3}^2 - z_3^2\right)} + \sigma_3 \widehat{W}_{3,i} \right], \\ 0. \end{cases}$$
(34)

*Remark 4.* Based on the above description (34), we can make the following conclusion. If  $\widehat{W}_{3,i}=m_{f3,i}$ , we can get  $\dot{\widehat{W}}_{3,i}>0$ ,

that is,  $\widehat{W}_{3,i} \ge m_{f3,i} > 0$  for all any  $\widehat{W}_{3,i}$ . Then, there does not exist insignificance function for controller u.

Substituting (28), (30), and (32) into (26), we obtain

$$\dot{V} \leq -\sum_{i=1}^{3} \left( \gamma_{i} + \overline{\gamma}_{i} \left( t \right) + \frac{\dot{k}_{a_{i}}}{k_{a_{i}}} \right) \frac{z_{i}^{2}}{\left( k_{a_{i}}^{2} - z_{i}^{2} \right)} - \sum_{i=1}^{3} \sigma_{i} \widetilde{W}_{i}^{T} \widehat{W}_{i} 
+ D\left( x, t \right) \frac{z_{3}}{\left( k_{a_{3}}^{2} - z_{3}^{2} \right)} - \frac{z_{3}^{2}}{2\left( k_{a_{3}}^{2} - z_{3}^{2} \right)^{2}}.$$
(35)

Using Young's inequality in the last two terms of  $\dot{V}$ , we seem to easily get

$$-\sigma_{i}\widetilde{W}_{i}^{T}\widehat{W}_{i} = -\sigma_{i}\widetilde{W}_{i}^{T}\left(\widetilde{W}_{i} + W_{i}^{*}\right)$$

$$\leq -\sigma_{i} \|\widetilde{W}_{i}\|^{2} + \frac{\sigma_{i}}{2} \|-W_{i}^{*}\|^{2}$$

$$+ \frac{\sigma_{i}}{2} \|\widetilde{W}_{i}\|^{2}$$

$$= -\frac{\sigma_{i}}{2} \|\widetilde{W}_{i}\|^{2} + \frac{\sigma_{i}}{2} \|W_{i}^{*}\|^{2},$$

$$D\left(x, t\right) \frac{z_{3}}{\left(k_{a_{3}}^{2} - z_{3}^{2}\right)} \leq \frac{1}{2}D^{2}\left(x, t\right) + \frac{z_{3}^{2}}{2\left(k_{a_{3}}^{2} - z_{3}^{2}\right)^{2}}.$$
(36)

Based on definition (27) and using inequalities (36) and (11) in Lemma 3, (35) can be rewritten as

$$\dot{V} \leq -\sum_{i=1}^{3} \gamma_{i} \log \frac{k_{a_{i}}^{2}}{\left(k_{a_{i}}^{2} - z_{i}^{2}\right)} - \sum_{i=1}^{3} \frac{\sigma_{i}}{2} \left\|\widetilde{W}_{i}\right\|^{2} + \frac{1}{2} D^{2}(x, t) + \sum_{i=1}^{3} \frac{\sigma_{i}}{2} \left\|W_{i}^{*}\right\|^{2}.$$
(37)

**Theorem 5.** A class of hydraulic servo-system is described by (1)–(3) under Assumptions 1 and 2. If the initial displacement  $x_1(0)$ , the initial velocity  $x_2(0)$ , and the initial acceleration  $x_3(0)$  satisfy  $|x_i(0)| \le k_{b_i}(0)$ , i = 1, 2, 3, then, the proposed control method has the following properties.

(1) The error signals are bounded in the following sets:

$$z_i(t) \in \Omega_{z_i}, \quad i = 1, 2, 3,$$
 (38)

where  $\Omega_{z_i} := \{z_i \in \mathbb{R}: |z_i(t)| \le k_{a_i}(t)\sqrt{1-e^{-V(0)e^{-\psi t}-C/\psi}}\}$ , noting that the tracking error signal converges to zero asymptotically.

- (2) The time-varying state constrains are never violated; that is,  $|x_i(t)| \le k_{b_i}(t)$ ,  $\forall t \ge 0, i = 1, 2, 3$ .
  - (3) All singles of the hydraulic servo-system are bounded.

#### 4. Simulation Example

Simulation has been performed to demonstrate the performance of the proposed approach. The values of system parameters which are used in the simulation are given in Table 1.

The initial states of the hydraulic servo-system are given as  $x_1(0)=0$ ,  $x_1(0)=0$ , and  $x_1(0)=0$ . The time-varying constraint functions are  $k_{b_1}(t)=2\sin(5t)+40$ ,  $k_{b_2}(t)=2\sin(5t)+60$ , and  $k_{b_3}(t)=4\sin(5t)+150$ . The desired trajectory tracking periodic reciprocation of the displacement is given  $y_d=40\arctan(\sin(0.4\pi t))(1-e^{-t})$  with  $t\in[0,t_f]$  and  $t_f=50$  s. The modeling error disturbance of the

TABLE 1: The system parameters used in the simulation.

Name	Value
m(kg)	2200
$P_s$ (Pa)	20
$P_r$ (Pa)	0
$A(m^3)$	$2 \times 10^{-3}$
$\rho$ (Ns/m)	500
$V_{I1}$ (m <sup>3</sup> )	$5 \times 10^{-4}$
$V_{I2}$ (m <sup>3</sup> )	$5 \times 10^{-3}$
$B_{\nu}$ (Pa)	200
$C_m \left( \text{m}^5/\text{Ns} \right)$	$2 \times 10^{-5}$
$K_f$ (m <sup>3</sup> /Ns)	$8 \times 10^{-2}$

both chambers is  $w = \sin(t) + 2$ . We choose the following controller:

$$\alpha_{1} = -(\gamma_{1} + \overline{\gamma}_{1}(t))z_{1} + \dot{y}_{d},$$

$$\alpha_{2} = -(\gamma_{2} + \overline{\gamma}_{2}(t))z_{2} + \dot{\alpha}_{1} - \frac{k_{a_{2}}^{2} - z_{2}^{2}}{k_{a_{1}}^{2} - z_{1}^{2}}z_{1},$$

$$u = \frac{1}{\widehat{W}_{3}^{T}\Phi_{3}g} \left(-(\gamma_{3} + \overline{\gamma}_{3}(t))z_{3} + \widehat{W}_{1}^{T}\Phi_{1}x_{2} + \widehat{W}_{2}^{T}\Phi_{2}\right)$$

$$+ \dot{\alpha}_{2} - \frac{(k_{a_{3}}^{2} - z_{3}^{2})}{(k_{a_{2}}^{2} - z_{2}^{2})}z_{2} - \frac{z_{3}}{2(k_{a_{3}}^{2} - z_{3}^{2})},$$
(39)

where we choose design parameters as  $Y_0 = 20$ ,  $Y_1 = 10$ ,  $Y_2 = 50$ ,  $\gamma_1 = 10$ ,  $\gamma_2 = 3$ ,  $\gamma_3 = 6$ ,  $\delta_1 = 0.8$ ,  $\delta_2 = 0.7$ ,  $\delta_3 = 0.6$  the initial weight vector estimation  $\widehat{W}_1 = -100$ ,  $\widehat{W}_2 = 20$ ,  $\widehat{W}_3 = 10$ ,  $\Theta_1 = 3$ ,  $\Theta_2 = 2$ ,  $\Theta_3 = 2$ ,  $\sigma_1 = 2$ ,  $\sigma_2 = 0.3$ ,  $\sigma_3 = 0.5$ ,  $\Phi_1 = 0.2$ ,  $\Phi_2 = 0.4$ ,  $\Phi_3 = 0.06$ .

Figures 1–6 are illustrated to show the simulation results. Figures 1–3 show the better tracking trajectories performance of the system variable state and minor error curve of the errors on the time-varying constraints, respectively. It can be easily obtained that the time-varying state constrains are not violated. In Figure 4, we can clearly get the conclusion that the tracking errors are all bounded and the tracking error signal curve converges to zero. Figures 5 and 6 show the trajectories of estimation and the controller, respectively. We make the conclusion that the adaptive laws and the controller of the system are all ensured boundedness.

#### 5. Conclusion

An adaptive control algorithm for hydraulic servo-system considering time-varying state constraints has been designed in this paper. The uncertainty and time-varying state constraints problem of the system have been considered in the controller designing process. The uncertainties and the accurate estimation of disturbance have been solved by RBFNN. By designing barrier Lyapunov function to solve the time-varying state constraints problem of system, the stability of the control strategy has been proved. Besides, the proposed method has proved that the error signals are bounded in the

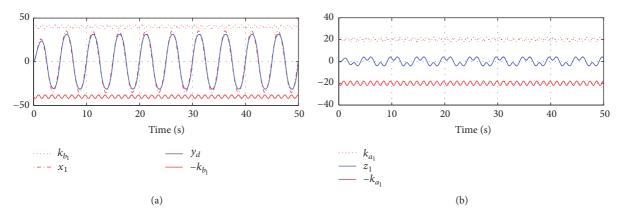


FIGURE 1: (a) Trajectories of  $x_1$  (dashed) and  $y_d$  (solid); (b) the trajectory of tracking error  $z_1$ .

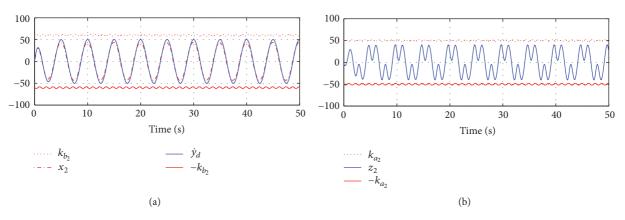


FIGURE 2: (a) Trajectories of  $x_2$  (dashed) and  $\dot{y}_d$  (solid); (b) the trajectory of tracking error  $z_2$ .

small compacts sets, noting that the tracking error signal converges to zero asymptotically; the time-varying state constrains are never violated and all singles of the hydraulic servo-system are bounded. The experimental results have showed the effectiveness of the proposed method.

#### **Appendix**

*Proof of Theorem 5.* Based on the state constrains (10) and the definition of errors  $z_i$ , the initial error signals are bounded between the sets as  $|z_i(0)| \le k_{a_i}(0)$ . From (37), the following can be obtained:

$$\dot{V} \le -\psi V + C,\tag{A.1}$$

where

$$\psi = \min\left\{2\gamma_1, 2\gamma_2, 2\right\},\,$$

$$C = \frac{1}{2}D^{2}(x,t) + \sum_{i=1}^{3} \frac{\sigma_{i}}{2} \|W_{i}^{*}\|^{2}.$$
 (A.2)

Let us multiply both sides of the above inequality by  $e^{\psi t}$ ; (A.1) can be rewritten as

$$\dot{V}e^{\psi t} + \psi V e^{\psi t} \le C e^{\psi t}. \tag{A.3}$$

And then we integrate both sides of inequality (A.3) over [0, t] to obtain

$$0 \le V(t) \le V(0) e^{-\psi t} + \frac{C}{\psi}.$$
 (A.4)

Substituting (16) into (A.4), we can get

$$\log \frac{k_{a_{i}}^{2}}{k_{a_{i}}^{2} - z_{i}^{2}} \le V(t) \le V(0) e^{-\psi t} + \frac{C}{\psi}.$$
 (A.5)

Thus, we have the following inequality based on the transitivity of the inequality from (A.5):

$$\frac{k_{a_i}^2}{k_{a_i}^2 - z_i^2} \le e^{V(0)e^{-\psi t} + C/\psi}.$$
 (A.6)

Then, (A.6) can be rewritten as

$$|z_{i}(t)| \le k_{a_{i}}(t) \sqrt{1 - e^{-V(0)e^{-\psi t} - C/\psi}}.$$
 (A.7)

Furthermore, it can be easy proved that the error signal  $z_i$  converges to zero asymptotically based on appropriate parameter design.

Based on Assumption 2, we can get  $|y_d(t)| \le Y_0(t)$  and  $|y_d^{(i)}(t)| \le Y_{i+1}(t)$ , i = 1, 2. Then, from  $|z_1| \le k_{a_1}$  and

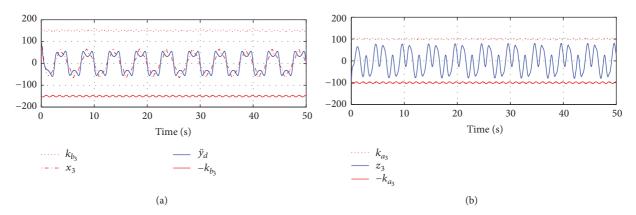


FIGURE 3: (a) Trajectories of  $x_3$  (dashed) and  $\ddot{y}_d$  (solid); (b) the trajectory of tracking error  $z_3$ .

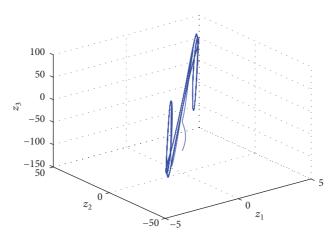


Figure 4: The phase portrait of tracking errors  $z_1$ ,  $z_1$ , and  $z_3$ .

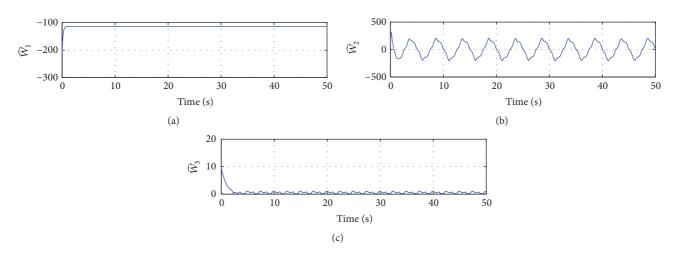


Figure 5: (a) The trajectory of  $\widehat{W}_1$ ; (b) the trajectory of  $\widehat{W}_2$ ; (c) the trajectory of  $\widehat{W}_3$ .

 $x_1=z_1+y_d$ , we can get the inequality  $|x_1(t)|\leq k_{a_1}(t)+Y_0(t)\leq k_{b_1}(t)$ . In (18), the victual controller  $\alpha_1$  can be bounded from the boundedness of  $x_1$  and  $\dot{y}_d$ . It can be seen that there exists the supremum  $\overline{\alpha}_1$  of  $\alpha_1$ . From  $|z_2|\leq k_{a_2}$  and  $x_2=z_2+\alpha_1$ , it has  $|x_2|\leq k_{a_2}+\overline{\alpha}_1\leq k_{b_2}$ . In the same way, we can get the boundedness supremum  $\overline{\alpha}_2$  based on the

definition of  $\alpha_2$  in (19). Thus, the state  $x_3$  can be proved by the function  $k_{b_3}$  from the above proof of boundedness. So the time-varying state constrains are never violated.

From (A.4) and adaptive laws in (28), (30), and (32), we can get that the neural network weight vector errors  $\widetilde{W}_i$  are bounded. Because the weight vectors  $W_i^*$  are bounded,

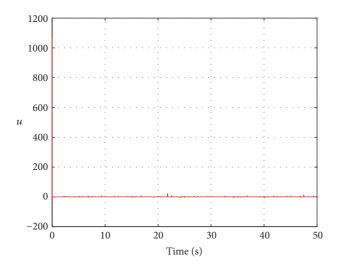


FIGURE 6: The trajectory of u.

the weight vector estimations  $\widehat{W}_i = \widetilde{W}_i + W_i^*$  are bounded. Based on the above identified process, the actual controller u consists of the bounded functions  $y_d$ ,  $\dot{y}_d$ ,  $y_d^{(2)}$ ,  $y_d^{(3)}$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $z_i$ , i=1,2,3. Then, it is easy to obtain that u is bounded. Thus, all the signals of the hydraulic servo-system are bounded.

The proof is completed.  $\Box$ 

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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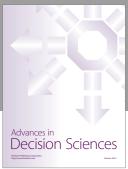
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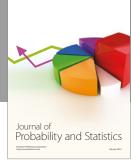
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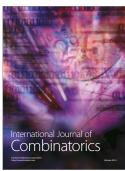








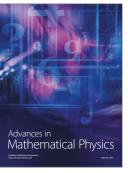






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