Evaluation of the Reliable Data Rates Supported by Multiple-Antenna Coded Wireless Links for QAM Transmissions

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***Abstract*—In this paper, we present some novel results about the reliable information-rate supported by point-to-point multiple-antenna Rayleigh-faded wireless links for coded transmissions that employ two-dimensional (QAM or PSK) data constellations. After deriving the symmetric capacity of these links, we present *fast-computable* analytical upper and lower bounds that are asymptotically exact *both* for high and low SNRs, and give rise to a reliable evaluation of the link capacity when perfect channel state information (CSI) is available at the receiver. Furthermore, asymptotically exact simple upper bounds are also presented for a tight evaluation of the outage probability.**

***Index Terms*—Information-rates, MIMO wireless data systems, multiple antennas, outage probability, Rayleigh-fading.**

I. Introduction and Motivations of the Work

THE GROWING demand for high-throughput wireless services experienced in the last several years motivates the design of digital transmission systems able to convey increasing data rates without substantial bandwidth-expansion. At the present, typical cellular wireless standards support data services at about 9–10 kb/s. But recently there has been interest in providing more sophisticated services at ISDN-compatible data rates exceeding 100 kb/s using the cellular spectrum [19, Ch. 7]. Since the wireless channel is inherently band-limited by multipath phenomena, bandwidth-efficient coding with diversity constitutes an effective means for coping with the deleterious effects of fading. Although recent progress of multi-element array (MEA) technology [23] makes wireless systems with multiple antennas at the receiver today quite common ([3], [10], [13], and references therein), several important contributions [1], [2], [9] have pointed out that space diversity at the transmitter can give rise to an extraordinary improvement in the reliable rates conveyable by wireless bandwidth-limited links when the receiver also employs space-diversity. More recently, in [14] and [15], interesting results have been reported for the case when the propagation coefficients modeling the multiple-antenna wireless link are unknown both at the transmitter and receiver, and the fading fluctuations are approximately piecewise constant. All these contributions point out the large performance improvements which may arise from the utilization of multiple transmit/receive antennas so that, motivated by these promising information-theoretic results, several coding strategies suitable for actual implementations have also been developed [5]–[7], [9], [16], [17].

Since the coded systems considered in the above cited contributions provide data-transmissions and then rely on *finite-size* QAM/PSK-type constellations, a natural question that is still unanswered concerns the reliable rates effectively supported by multiple-antenna point-to-point wireless systems which employ *finite-size* data-constellations. In this paper, we attempt to give an answer to this question. In particular, we consider a point-to-point multiple-antenna link affected by flat Rayleigh-distributed fadings and under the assumption of perfect CSI at the receiver we compute the (symmetric) Shannon capacity of the coded channel for data transmissions which employ two-dimensional QAM/PSK signal constellations. Since the formula for the capacity resists closed-form evaluation, and its computation requires multiple nested numerical integrations, we present (in Section III) some fast-computable upper and lower bounds which provide reliable (and asymptotically exact) evaluation of the capacity. Furthermore, since actual cellular wireless systems may be impaired by slow-variant (i.e., nonergodic) fadings that, in fact, makes meaningless the link capacity [12], [13, p. 2631], in Section V we investigate on the outage probability of the considered multiple-antenna systems. Numerical results and some concluding remarks are provided in Sections VI and VII.

II. Link Modeling and System Capacity

A. Multiple-Antenna MIMO Wireless System

By referring to the narrow-band system with t transmit and r receive antennas sketched in Fig. 1, let us assume that the t coded streams $\{x_i(n)\}$, $1 \le i \le t$, simultaneously generated by the transmit antennas are zero-mean mutually independent equal-power memoryless identically distributed stationary random sequences which take values on an assigned q-ary complex constellation $A_X \equiv \{\alpha_1, \ldots, \alpha_q\}$. The channel is assumed flat faded, and the complex path gains $g_{ji}(n)$ from the transmit antenna i to receive antenna j at the epoch n are modeled as a sample of a proper zero-mean complex random variable with unit-variance per real dimension. Therefore, the signal $y_j(n)$ output at time n by antenna j is modeled as

$$y_j(n) = \sum_{i=1}^{t} g_{ji}(n)x_i(n) + w_j(n), \qquad 1 \le j \le r \quad (1)$$

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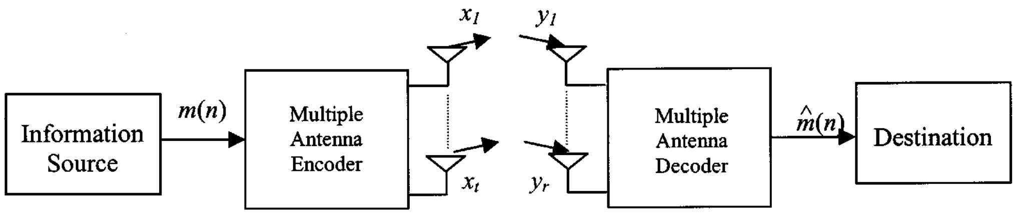

Fig. 1. A simplified block diagram for a point-to-point wireless data-link using t transmit antennas and r receive ones.

where the noise sequences $\{w_j(n) \in \mathcal{C}^1\}$, $1 \leq j \leq r$, are mutually independent zero-mean stationary white Gaussian complex processes with variance N_o per real dimension. Furthermore, we normalize to unity the total power radiated by the t antennas so that, after assuming as mutually independent the $t \cdot r$ path-gain sequences $\{g_{ij}(n)\}$, the resulting average $\mathrm{SNR}\overline{\gamma} \equiv E\{|\sum_{i=1}^{t} g_{ji}(n)x_i(n)|^2\}/E\{|w_i(n)|^2\}$[1] at the output of each receiving antenna is equal to $1/N_o$. For future convenience, we collect the antennas outputs in (1) into the r-dimensional complex vector $\underline{y}(n) \equiv [y_1(n) \cdots y_r(n)]^T$ and then rewrite the r scalar relationships in (1) in the following matrix form

$$\underline{y}(n) = G(n)\underline{x}(n) + \underline{w}(n) \tag{2}$$

where $\underline{w}(n) \equiv [w_1(n) \cdots w_r(n)]^T$ and the coded vector $\underline{x}(n) \equiv [x_1(n) \cdots x_t(n)]^T \in (A_X)^t$ is an $N \equiv q^t$-variate random variable with outcomes taking values on the expanded coding constellation $(A_X)^t$.[2] Finally, $G(n)$ in (2) is the $r \times t$ complex random matrix which collects the path gains $g_{ji}(n)$, $1 \leq j \leq r$, $1 \leq i \leq t$, at time n. We assume $G(n)$ known (i.e., tracked) at the receiver but not at the transmitter.

B. Validity Limits of the Assumed System Model

The matrix relationship in (2) captures the multiinput multioutput (MIMO) representation of the considered link. About the *flat* fading model here assumed, we note that this last may adequately describe application scenarios where the relative delays of the multiple copies of the transmitted signal arriving at the receiver are less than the signaling period T_S. In general, this assumption can be considered well met in indoor wireless applications and outdoor microcellular ones [19, Ch. 4].

As far as the mutual independence of the $t \times r$ path gains composing the matrix $G(n)$ in (2) is concerned, this assumption can be viewed, indeed, as an approximation which improves when the antenna spacing becomes large compared to the RF wavelength λ. The minimum required antenna spacing generally depends on the considered application scenarios. For example, a spacing below one wavelength λ may be adequate for indoor applications while outdoor cellular links planned in urban and suburban environments typically require an antenna spacing of about ten wavelengths at the base stations and three wavelengths at the mobile units [3], [23]. However, it has also been observed that the performance loss due to correlated receiving branches is very limited even for correlation values as high as 0.5 [3].

Lack of knowledge of the channel gains at the transmitter is typical of cellular mobile radio systems where a reliable fast feedback link from the receiver to the transmitter is not available. In these application scenarios, beamforming of the covariance matrix of the transmitted coded streams is not possible so that the assumption of equal powered transmitted streams naturally arises [1, Section 2], [4, Section III.B], [14, Appendix C].

Finally, a few words about the assumption of mutually independent, memoryless, and identically distributed coded streams are in order. From an information-theoretic point of view, these assumptions guarantee that the information-throughput conveyed by the MIMO channel in (2) is maximized, and formal proof of this property can be found, for example, in [1, Section 3], [4, Section III.B], and [14, Appendix C]. In practice, the codewords generated by the space-time block codes recently proposed in [16] and analyzed in [6] well match these assumptions.

C. The Symmetric Capacity of MIMO Links

By extending to the present case of MIMO channels a current taxonomy holding for the single-input single-output (SISO) ones [8, p. 350], we qualify as "symmetric capacity" of the MIMO channel in (2) the corresponding average mutual information evaluated for *equidistributed* t-variate input vector $\underline{x} \in (A_X)^t$.

Therefore, after introducing the additional assumption of *ergodic* behavior for the path-gain sequences $\{g_{ji}(n)\}$, $1 \leq j \leq r$, $1 \leq i \leq t$, the symmetric capacity $C_{t,r}^*$ for the MIMO channel in (2) with t transmit and r receive antennas can be computed via an application of the (usual) chain rule reported, for example, in [8, p. 361] for the more standard case of SISO channels as summarized below.

$$\begin{aligned} C_{t,r}^* &\equiv \lim_{M\to\infty} \frac{1}{M} I\left(\underline{X}^M; \underline{Y}^M, \underline{G}^M\right) \\ &= \lim_{M\to\infty} \frac{1}{M} I\left(\underline{X}^M; \underline{Y}^M | \underline{G}^M\right) \\ &= \lim_{M\to\infty} \frac{1}{M} \left[\sum_{i=1}^{M} I\left(\underline{y}(i); \underline{x}(i), \underline{G}(i)\right)\right] \\ &= I\left(\underline{y}^1; \underline{x}^1 | \underline{G}^1\right) \end{aligned} \tag{3}$$

where $I(\cdot;\cdot,\cdot)$ indicates the average mutual information functional (expressed in nats per channel-use)[3] and $\underline{X}^M$, $\underline{Y}^M$, $\underline{G}^M$ are subsequences of M elements picked out from $\{\underline{x}(n)\}$, $\{\underline{y}(n)\}$, and $\{\underline{G}(n)\}$, respectively. Therefore, as in

[1] $|\cdot|$ indicates the absolute value of scalar entities, while $\|\cdot\|$ denotes the norm of vectors and matrices.

[2] $(A_X)^t$ indicates the set given by the t-fold Cartesian product of A_X by itself.

[3] By referring to the MIMO model in (2), a channel use consists in the transmission of a vector $\underline{x}(\cdot)$ of t coded q-ary symbols.

[8, eqs. (4.6.14), (4.7.13)], the symmetric capacity $C^*_{t,r}$ can be computed by averaging over the fading statistics as

$$C^*_{t,r} \equiv E_G\left\{\tilde{C}^*_{t,r}(G)\right\} \equiv \int \tilde{C}^*_{t,r}(G)p(G)\,dG. \tag{4}$$

$\tilde{C}^*_{t,r}(G)$ in (4) is the symmetric capacity of the MIMO channel in (2) *conditioned* on the outcome G of the channel gain matrix in (2) and it is given by the following expression [see eqs. (A.1), (A.2) of Appendix A]:

$$\begin{aligned}\tilde{C}^*_{t,r}(G) &= t\lg q - \left(\frac{1}{q}\right)^t\left(\frac{\overline{\gamma}}{2\pi}\right)^r \sum_{\underline{x}\in(A_X)^t}\int_{\underline{y}\in C^r}\exp\left(-\frac{\overline{\gamma}}{2}\|\underline{y}-G\underline{x}\|^2\right)\\ &\quad\times\lg\left\{1+\sum_{\substack{\underline{x}'\in(A_X)^t\\ \underline{x}'\neq\underline{x}}}\exp\left[\frac{\overline{\gamma}}{2}\left(\|\underline{y}-G\underline{x}\|^2-\|\underline{y}-G\underline{x}'\|^2\right)\right]\right\}d\underline{y}\end{aligned} \tag{5}$$

while $p(G)$ indicates the corresponding joint pdf of the $t\times r$ random elements of the channel matrix which for Rayleigh-faded links assumes the simple form

$$p(G) = \left(\frac{1}{2\pi}\right)^{tr}\exp\left(-\frac{1}{2}Tr(G^HG)\right) \tag{6}$$

with $Tr\{G^HG\}$ being is the trace of the matrix G^HG.

Remark (Ergodic Assumption): In principle, the ergodic behavior of the path-gain sequences $\{g_{ji}(n)\}$ in (1) guarantees "information stability" (in the sense of [18, Section I]) for the channel in (2) so that the limiting operation in (3) is well posed. In practice, the channel can be assumed ergodic when the variability of the fading processes over the time interval T_W requested by the transmission of a *whole* codeword is sufficiently high. This means that the ergodic assumption can be considered met when the following inequality is satisfied [13, Section III.B]:

$$\begin{aligned}T_W &> T_{COH}(j,i)\\ &\equiv (B_D(j,i))^{-1}, \qquad 1\le j\le r,\ 1\le i\le t\end{aligned} \tag{7}$$

where $T_{COH}(j,i)$ is the coherence time of the fading path from transmit antenna i to receiving antenna j, while $B_D(j,i)$ is the corresponding Doppler-spread [19, ch. 4]. After dividing (7) by the system signaling period T_S, this last can be equivalently rewritten as

$$M > (T_S B_D(j,i))^{-1}, \qquad 1\le j\le r,\ 1\le i\le t \tag{8}$$

where $M\equiv T_W/T_S$ is the interval (in multiple of the signaling period) spanned by the transmission of an overall codeword. The right-hand side of (8) directly relates the minimum codeword length dictated by the ergodic assumption to the time variability of the fading processes so that (8) can be utilized for designing codes and interleavers effectively matched to the considered application scenarios.

By focusing now on the computational aspects related to the capacity evaluation, we note that a direct computation of $C^*_{t,r}$ via the above relationships (4), (5) appears very cumbersome even for small values of t and r and, in principle, it requires *at least* $r(t+1)$ nested numerical integrations over the complex plane together with a number of exponential and logarithmic operations growing as q^t. Furthermore, an examination of the integral formulas in (4) and (5) does not give direct insight into the performance limits of the considered MIMO coded channel with multiple antennas.

For these reasons, in the following sections we present some simple asymptotically tight bounds which allow us to point out the ultimate performance of the MIMO channel in (2). Furthermore, since it has been well understood that MIMO systems with an equal (finite) number of transmit and receive antennas offer the best tradeoff between performance gain and implementation complexity [1, Section 4], [2], [7, Section IV], [9, Section III], [10, Section I], in the following we focus on this important case and indicate as C^*_t the symmetric capacity of the MIMO coded channel in (2) with finite $t=r$.

III. The Proposed Upper and Lower Bounds on the Symmetric Capacity

In Appendix A, it is proven that the following limits hold for the conditional capacity in (5):

$$\widetilde{LB}^*_t(G) \le \tilde{C}^*_t(G) \le \widetilde{UB}^*_t(G) \tag{9}$$

where the above conditional bounds assume the forms reported below [see (A.3)–(A.11) and (A.12)–(A.14) of Appendix A]

$$\widetilde{UB}^*_t(G) \equiv t\lg q - \lg\left\{1+(q^t-1)\exp\left[-\overline{\gamma}\left(\frac{q^t}{q^t-1}\right)\|G\|^2\right]\right\} \tag{10}$$

$$\widetilde{LB}^*_t(G) \equiv t\lg q - \lg\left\{1+\frac{1}{q^t}\sum_{\underline{x}\in(A_X)^t}\sum_{\substack{\underline{x}'\in(A_X)^t\\ \underline{x}'\neq\underline{x}}}\times\exp\left[-\frac{\overline{\gamma}}{8}\|\underline{x}-\underline{x}'\|^2\Theta_{\min}\right]\right\}. \tag{11}$$

The term $\|G\|^2 \equiv \sum_{i=1}^t\sum_{j=1}^t|g_{ij}|^2$ in (10) is the squared Euclidean norm of the channel gain matrix G in (2), while $\Theta_{\min}$ in (11) indicates the minimum eigenvalue of the resulting Wishart-type complex random matrix GG^H (see [11, ch. 3–5] for the definition and the main properties of a complex Wishart matrix). Therefore, from the conditional bounds in (10) and (11), we obtain the following limits for the corresponding symmetric capacity (see Appendix B):

$$LB^*_t \le C^*_t \le UB^*_t \tag{12}$$

where the above bounds can be expressed as [see (B.1), (B.3), and (B.4) of Appendix B]

$$LB_t^* \equiv t\lg q - \lg\left\{1 + \frac{1}{q^t}\sum_{\underline{x}\in(A_X)^t}\sum_{\substack{\underline{x}'\in(A_X)^t\\ \underline{x}'\neq\underline{x}}} \times\left(1+\frac{\overline{\gamma}}{4t}\|\underline{x}-\underline{x}'\|^2\right)^{-1}\right\} \tag{13}$$

$$UB_t^* \equiv t\lg q - \sum_{k=1}^{\infty}(k-1)!\left(\frac{q^t-1}{q^t}\right)^k \times\left\{\sum_{m=1}^{k}\frac{(-1)^{m-1}}{m!(k-m)!}\left[1+2m\overline{\gamma}\left(\frac{q^t}{q^t-1}\right)\right]^{-t^2}\right\}. \tag{14}$$

The most appealing feature of the limits (13) and (14) is their simple structure which allows us to draw the conclusions about the ultimate performance of the MIMO channel (2) summarized in the following remarks.

Remark (Asymptotic Behavior of the Symmetric Capacity for high SNRs): Since for finite $t = r$ and large SNRs the bounds (13) and (14) collapse to the following asymptotic expressions:

$$LB_t^* \cong t\lg q - \frac{1}{\overline{\gamma}}\frac{4t}{q^t}\sum_{\underline{x}\in(A_X)^t}\sum_{\substack{\underline{x}'\in(A_X)^t\\ \underline{x}'\neq\underline{x}}}\|\underline{x}-\underline{x}'\|^{-2}, \quad \overline{\gamma}\gg 1 \tag{15}$$

$$UB_t^* \cong t\lg q - \left(\frac{1}{\overline{\gamma}}\right)^{t^2}\frac{(q^t-1)^{t^2+1}}{(2q^t)^{t^2}}, \qquad \overline{\gamma}\gg 1 \tag{16}$$

we conclude that for $\overline{\gamma}\to\infty$ both the above bounds approach $t\lg q$. Therefore, since C_t^* falls between LB_t^* and UB_t^* [see (12)], even the actual symmetric capacity of the system must reach the same limiting value so that we can write

$$\lim_{\overline{\gamma}\to\infty} C_t^* = t\lg q. \tag{17}$$

The above relationship shows that for large $\overline{\gamma}$ the symmetric capacity of the Rayleigh-faded MIMO channel in (2) scales *linearly* with the number $t = r$ of the transmit and receive antennas. However, due to the finite size q of the employed constellation, the ultimate value allowable C_t^* remains *finite and limited up to* $t\lg q$. Furthermore, the linear scaling in (17) exhibited by C_t^* with $t = r$ for large $\overline{\gamma}$ *holds in general for every two-dimensional QAM or PSK finite-size data-constellation*.

As far as the behaviors of the proposed bounds in (13) and (14) for fixed $t = r$ and growing $\overline{\gamma}$ is concerned, the asymptotic expressions in (15) and (16) lead to the conclusion that both these bounds approach the actual value of C_t^* for large $\overline{\gamma}$ and, therefore, they are *asymptotically exact*. Furthermore, from (12) it follows that the gap $t\lg q - C_t^*$ falls between $t\lg q - UB_t^*$ and $t\lg q - LB_t^*$, and, then, for large $\overline{\gamma}$ it can be limited as [see eqs. (15) and (16)]

$$\begin{aligned}\frac{1}{\overline{\gamma}}\frac{4t}{q^t}&\sum_{\underline{x}\in(A_X)^t}\sum_{\substack{\underline{x}'\in(A_X)^t\\ \underline{x}'\neq\underline{x}}}\|\underline{x}-\underline{x}'\|^{-2}\\ &\le t\lg q - C_t^*\\ &\ge \left(\frac{1}{\overline{\gamma}}\right)^{t^2}\frac{(q^t-1)^{t^2+1}}{(2q^t)^{t^2}}, \qquad (\overline{\gamma}\gg 1).\end{aligned} \tag{18}$$

Remark (Asymptotic Behavior of the Symmetric Capacity for Vanishing SNRs): Both the proposed bounds in (13) and (14) approach zero for vanishing SNRs so that they are tight and asymptotically exact *even* for small $\overline{\gamma}$. Furthermore, since the following asymptotic approximate expression holds for UB_t^*:

$$UB_t^* \cong 2t^2\overline{\gamma}, \qquad (\overline{\gamma}\to 0) \tag{19}$$

we can say that the symmetric capacity C_t^* approaches zero at most linearly for vanishing $\overline{\gamma}$.

Remark (Computational Aspects): From a practical point of view, the most appealing feature of the proposed bounds in (13) and (14) and the corresponding asymptotic expressions (15), (16), (19) is that their evaluation *does not require* numerical integrations. More in detail, the computation of the lower bound in (13) and the asymptotic relationships (15), (16), (19) is straightforward. As far as a reliable evaluation of the upper bound in (14) is concerned, we have experienced that, independent of the constellation size q and the number t of transmit/receive antennas, it can be achieved for moderate SNR values (e.g., for $\overline{\gamma}$ ranging from 5–6 dB to 11–12 dB) by retaining only the first 30 terms of the outer summation in (14).

IV. A Brief Comparison to Previous Results

When the coding alphabet is allowed to be *continuous* and only the *average* transmitted power is upper bounded, it is known that the capacity-achieving input distribution becomes Gaussian and the resulting capacity C_t (in nats/channel-use) of the Rayleigh-faded MIMO channel in (2) with perfect CSI at the receiver is given by the following relationship ([1], [2], [4], [9], [10] and references therein):

$$\begin{aligned}C_t &= E\left\{\lg\left[\det\left(I+\frac{\overline{\gamma}}{2t}GG^H\right)\right]\right\}\\ &\equiv \sum_{i=1}^{t}E\left\{\lg\left(1+\frac{\overline{\gamma}}{2t}\Theta_i\right)\right\}\end{aligned} \tag{20}$$

where Θ_i, $1\le i\le t$, in (20) indicates the ith eigenvalue of the random Wishart matrix GG^H. The main conclusion arising from (20) is that for $t = r$ and large SNRs the capacity C_t can be effectively approximated as in the following [1], [2], [4], [9], [10]:

$$C_t \cong t\lg\overline{\gamma} + O(t^{-1}), \qquad (\overline{\gamma}\gg 1) \tag{21}$$

where the term $O(t^{-1})$ vanishes for large t. As in (17), even the asymptotic relationship (21) shows that the system capacity scales linearly with the number t of the transmit/receive antennas. However, for large $\overline{\gamma}$, C_t^* in (17) remains limited up to

$t \lg q$, while C_t in (21) grows unbounded and the behavior of C_t^* and C_t is, in every case, very different.

V. Asymptotically Tight Upper Bounds on the Outage Probability for Interleaved Coded Packet Transmissions

When the time variability of the fading processes over the time interval requested by the transmission of a whole codeword is low, the above introduced ergodic assumption falls short and, as a consequence, the capacity (4) can no longer be considered a meaningful index of the reliable information-throughput conveyed by the MIMO channel in (2) (see, for example, [13, Section II.B] and references therein for additional details on this subject). In these environments, actual TDMA and CDMA wireless systems generally achieve time diversity by splitting a codeword of overall length M in L deeply interleaved packets built up by P coded symbols [12], [13, Section IV]. So doing, the fading can be assumed *time invariant* (i.e., static) over each packet but *independent* from one packet to the other [1], [12]–[14]. Therefore, after indicating as $\vec{G}^L \equiv [G_1, \ldots, G_L]$ the set of outcomes of the $t \times t$ matrix gains for the MIMO channel in (2) over the L transmitted packets, the resulting conditional mutual information [12], [13]: $I^L(\vec{G}^L) \equiv (1/L)\sum_{i=1}^{L} \tilde{C}_t^*(G_i)$ is a random variable whose distribution $P_{\text{out}}(\delta; L) \equiv P(I^L(\vec{G}^L) \leq \delta)$ is referred to as the outage probability[4] of the multiple-antenna data system in (2). Now, since (5) resists closed-form computations even for $t = r$, an analytical evaluation of $P_{\text{out}}(\delta; L)$ does not appear feasible. However, a suitable application of the Chernoff bound leads to the following *asymptotically tight* limit for the outage:

$$P_{\text{out}}(\delta; L) \leq \exp\{-sL[\Psi(s, \overline{\gamma}, t) - \delta]\} \tag{22}$$

where $s \geq 0$ is the Chernoff parameter and the real nonnegative $\Psi(s, \overline{\gamma}, t)$ functional in (22) assumes the form

$$\begin{aligned}\Psi(s, \overline{\gamma}, t) &\equiv -\frac{1}{s}\lg\left\{1 + \sum_{i=1}^{\infty}(-1)^i \binom{s}{i}\left(1 - \frac{1}{q^t}\right)^i \right.\\ &\qquad \left. \times \left[\sum_{k=0}^{i}\binom{i}{k}(-1)^k\left(1 + \frac{k\overline{\gamma}t}{4}d_{\min}^2\right)^{-1}\right]\right\}\end{aligned} \tag{23}$$

with $d_{\min}^2$ indicating the squared minimum Euclidean distance between two constellation points. Unfortunately, the optimization of the Chernoff parameter $s \geq 0$ in (22) and (23) must be carried out by trials; furthermore, from the power-series expansion in (23) for $\Psi(\cdot, \cdot, \cdot)$ it is not easy to obtain insight about the ultimate system performance. For these reasons, in the following subsections, we present the limit forms assumed by the Chernoff bound (22), (23) for large and small SNRs, respectively.

[4] δ is the so-called "outage-parameter," and for the multiple-antenna systems here considered with t transmit antennas it ranges from zero up to $t \lg q$.

A. Asymptotic Form of the Chernoff Bound on the Outage for Large SNRs

It can be proved that for large SNRs (i.e., for $\overline{\gamma}$ over 9–10 dB) the functional $\Psi(\cdot, \cdot, \cdot)$ becomes virtually independent of the Chernoff parameter s and it boils down to

$$\Psi(\overline{\gamma}, t) \cong t\lg q - \frac{4t^2}{\overline{\gamma}d_{\min}^2}, \qquad (\overline{\gamma} \gg 1). \tag{24}$$

Furthermore, in this condition it can be also viewed that an optimized value for s in (22) is

$$s = \frac{\overline{\gamma}d_{\min}^2}{4t^2}\lg\left(\frac{t\lg q}{\delta}\right) \tag{25}$$

so that the insertion of (24) and (25) in (22) leads to the following limiting expression for the outage probability of the considered multiple-antenna data system:

$$\begin{aligned}&P_{\text{out}}(\delta; L)\\ &\quad\leq \exp\left\{-\frac{\overline{\gamma}d_{\min}^2}{4t^2}\lg\left(\frac{t\lg q}{\delta}\right)L\left[t\lg q - \frac{4t^2}{\overline{\gamma}d_{\min}^2} - \delta\right]\right\},\\ &\qquad 0 \leq \delta < t\lg q - \frac{4t^2}{\overline{\gamma}d_{\min}^2}.\end{aligned} \tag{26}$$

Although the numerical computation of the above bound is straightforward, nevertheless it is *asymptotically tight* and, then, it *approaches zero* for large L and $\overline{\gamma}$, and also for small δ. These properties make the utilization of the proposed bound useful for a reliable evaluation of the outage probability when the actual values assumed from this last fall into the region of practical interest for wireless applications (i.e., around 10^{-3} [10, Section III]). The numerical plots reported in Section VI confirm this conclusion.

B. Asymptotic Form of the Chernoff Bound on the Outage for Small SNRs

For small SNRs, the power-series expansion in (23) reduces to $\Psi(s, \overline{\gamma}, t) \cong \overline{\gamma}/2t$, so that for vanishing $\overline{\gamma}$, the overall bound in (22) can be recast in the simple form

$$P_{\text{out}}(\delta; L) \leq \exp\{-sL[\overline{\gamma}/2t - \delta]\}, \qquad 0 \leq \delta < \overline{\gamma}/2t. \tag{27}$$

Furthermore, it can also be seen that, in this case, the optimized value for the Chernoff parameter s in (27) depends on $\overline{\gamma}$, t, and δ, and a good setting for it is given by the nonnegative maximum root of the following algebraic equation:

$$\frac{2\overline{\gamma}t}{(2t + s\overline{\gamma})^2} = \left(1 - \frac{\overline{\gamma}}{2t}s\right)\delta. \tag{28}$$

The bound in (27) is asymptotically tight, and for positive SNRs it approaches zero for vanishing δ.

VI. Numerical Results

Due to the limited sizes of actual portable handsets and nonlinear distortions typically introduced by power-efficient class-C RF amplifiers [19, ch. 5], wireless systems with two transmit/receive antennas which utilize constant-envelope

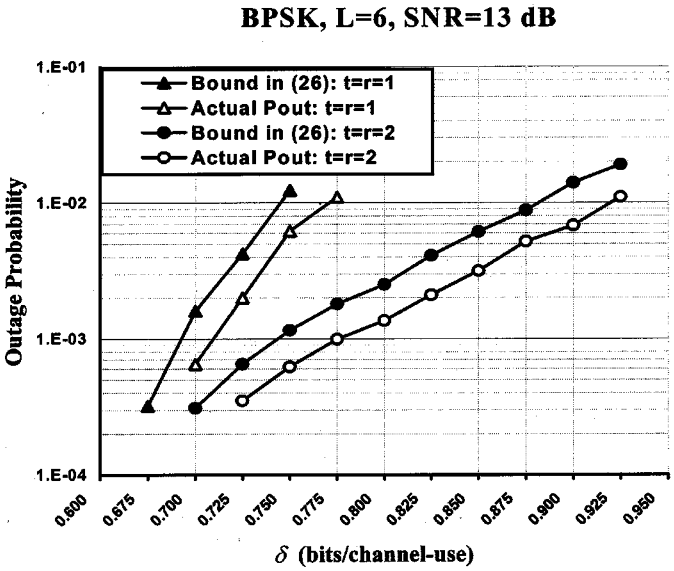

Fig. 2. P_{out} (in a $\log_{10}$ scale) versus δ for a BPSK coding constellation with $L = 6$ blocks at $\overline{\gamma} = 13$ dB. The plots for the two cases of $t = r = 1$ and $t = r = 2$ are reported.

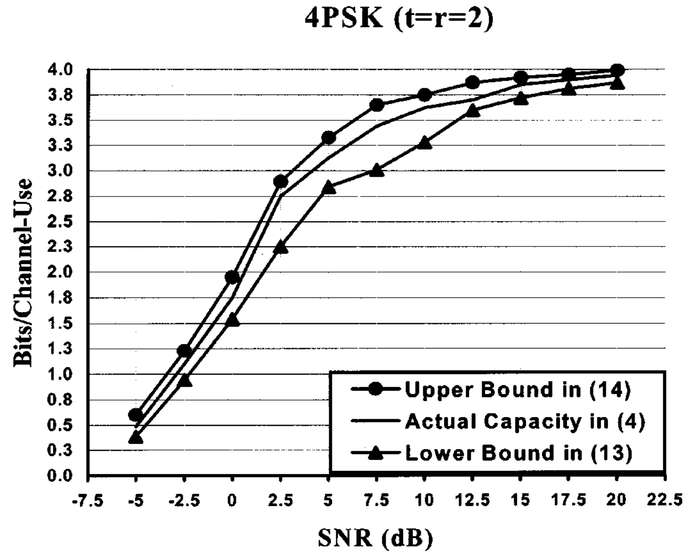

Fig. 4. Behaviors of LB_2^* in (13) and UB_2^* in (14) for a 4PSK constellation with $t = r = 2$. The corresponding values for the actual capacity C_2^* in (4) are also plotted.

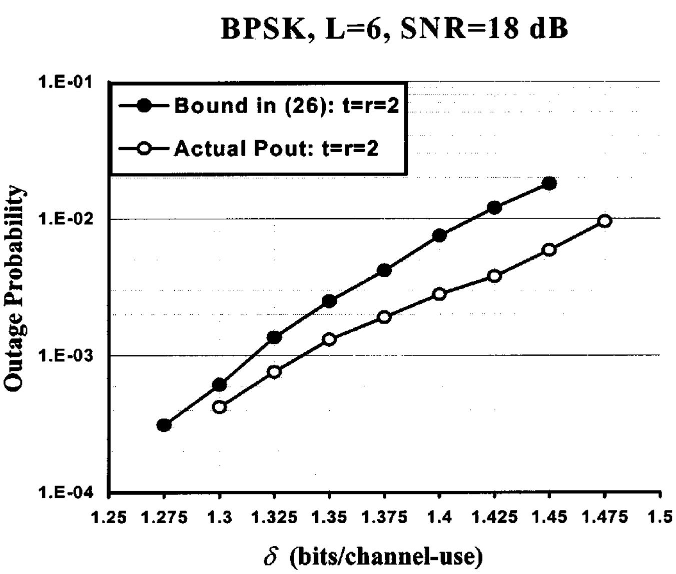

Fig. 3. P_{out} (in a $\log_{10}$ scale) versus δ for a BPSK coding constellation with $L = 6$ blocks at $\overline{\gamma} = 18$ dB for $t = r = 2$.

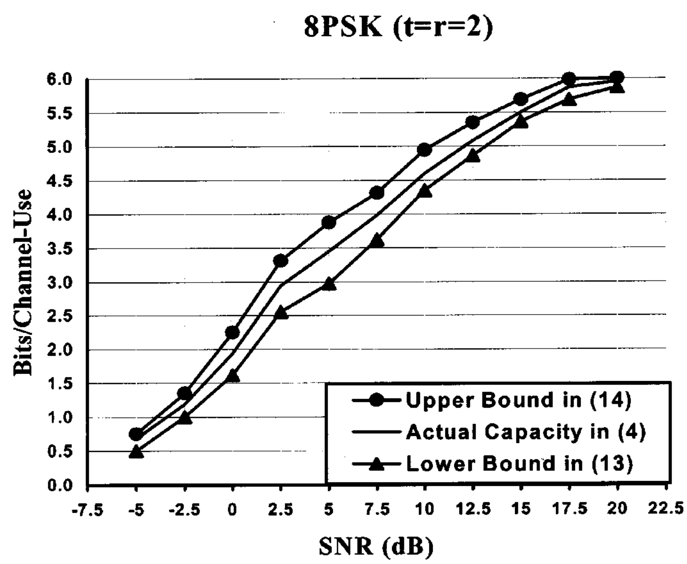

Fig. 5 The same as in Fig. 4 for a 8PSK coding constellation with $t = r = 2$.

PSK-type modulation constellations appear, at the present, the most appealing for practical wireless applications [7], [10], [19, ch. 7]. For this reason, in the sequel we focus on the numerical evaluation of the performance of such links.

From the plots for the outages of Figs. 2 and 3, we can draw some interesting insights. First, these plots confirm that the tightness of the bound (26) improves for small δ, and it gives rise to quite reliable evaluations of the actual outages for values of P_{out} of practical interest, typically falling below 10^{-2} [10]. Second, an examination of the plots of Fig. 2 shows that for $\overline{\gamma} = 13$ dB and at (measured) outage of 10^{-3} the throughput supported by the system with two transmit/receive antennas is about 10% higher than that conveyed by the corresponding single-antenna system. The improvement due to the utilization of two transmit/receive antennas becomes more remarkable when the SNR increases to 18 dB. In fact, the system with two transmit/receive antennas conveys throughputs exceeding 1.3 bits at an outage of 10^{-3}, while the information rates supported by the corresponding single-antenna link remain obviously limited up to 1 bit/channel-use. As far as the capacity is concerned, the numerical plots of Figs. 4 and 5 for PSK constellations agree with the asymptotic relationship in (17), and confirm that for high SNR $\overline{\gamma}$ the symmetric capacity in (4) *grows linearly* with the number $t = r$ of transmit/receive antennas. These curves also support the conclusion that the capacity limits proposed in (13) and (14) *are asymptotically exact both for small and large SNRs* and, in general, closely approach the actual system capacity for SNRs exceeding 10 dB

TABLE I
SUMMARY OF THE MAIN RELATIONSHIPS PRESENTED IN THE WORK

	Lower Bound	Upper Bound	High SNRs	Low SNRs
Symmetric Capacity	Eq.(13)	Eq.(14)	Eqs.(15),(16)	Eq.(19)
Outage Probability	————	Eqs.(22),(23)	Eq.(26)	Eqs.(27),(28)

and below 2–3 dB. In this regard, we also note that the actual wireless data systems work, for the most part, at SNRs typically exceeding 8–9 dB [1], [7], [10], [19, ch. 7], and an examination of the reported curves shows that over this range of SNRs the presented bounds generally differ from actual capacities within 8%–9%.

VII. CONCLUSION

From the main results presented in this paper (summarized in Table I), some practical guidelines emerge for the design of actual multiantenna data systems.

First, at high SNRs, the symmetric capacity of the link scales linearly with the number t of transmit/receive antennas, but for finite-size QAM/PSK constellations remains bounded as reported in (17). Second, for large SNR $\overline{\gamma}$, the gap $t\lg q - C_t^*$ vanishes essentially as $(1/\overline{\gamma})^{t^2}$, while for small $\overline{\gamma}$, the capacity approaches zero substantially as $t^2\overline{\gamma}$ (see Remarks 1 and 2 of Section III). Third, from the asymptotic form (26) of the bound on outage, we conclude that, for large $\overline{\gamma}$, the outage probability exponentially vanishes with the number L of the interleaved packets constituting the transmitted codeword according to the law $K_1 \exp(-K_2\overline{\gamma}L)$, where K_1, K_2 depend on the antenna number t and outage parameter δ as reported in (26).

Finally, we note that this paper focuses on systems without feedback link from the receiver to the transmitter. Some recent works consider application scenarios with feedback link, and address the optimization of the transmitted signal spectrum under the assumption of continuous Gaussian-shaped coding alphabet (among others, see, for example, [4], [13, Section II] and references therein). Unfortunately, the approaches pursued in these contributions are not directly applicable to the case of finite-size coding alphabet, and so the optimized beamforming of data constellations looks, indeed, like a nontrivial task, which is currently being investigated by the author.

APPENDIX A
DERIVATION OF CONDITIONAL CAPACITY IN (5) AND THE RELATED BOUNDS (10) and (11)

Since the t components of the transmit random vector $\underline{x}$ in (2) are mutually independent and equidistributed over a signal constellation of size q, the conditional symmetric capacity $\tilde{C}_{t,r}^*(G)$ of the MIMO channel (2) can be expressed as

$$\tilde{C}_{t,r}^*(G) \equiv H(\underline{x}) - H(\underline{x}|\underline{y}, G) = t\lg q - H(\underline{x}|\underline{y}, G). \quad \text{(A.1)}$$

Now, starting from the definitory relationship of the conditional entropy $H(\underline{x}|\underline{y}, G)$ in (A.1) and resorting to some applications of the Bayes' rule, we have the following developments:

$$\begin{aligned} &H(\underline{x}|\underline{y}, G) \\ &\quad \equiv -E_{\underline{X},\underline{Y}|G}\{\lg p(\underline{x}|\underline{y}, G)\} \\ &\quad = -E_{\underline{X}}\{E_{\underline{Y}|\underline{X},G}\{\lg p(\underline{x}|\underline{y}, G)\}\} \\ &\quad = E_{\underline{X}}\left\{\int_{\underline{y}} p(\underline{y}|\underline{x}, G)\lg\left[1 + \sum_{\substack{\underline{x}'\in(A_X)^t \\ \underline{x}'\neq\underline{x}}} \frac{p(\underline{y}|\underline{x}', G)}{p(\underline{y}|\underline{x}, G)}\right] d\underline{y}\right\} \\ &\quad = \frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t}\int_{\underline{y}} p(\underline{w} = \underline{y} - G\underline{x}) \\ &\qquad \times \lg\left[1 + \sum_{\substack{\underline{x}'\in(A_X)^t \\ \underline{x}'\neq\underline{x}}} \frac{p(\underline{w} = \underline{y} - \underline{x}'G)}{p(\underline{w} = \underline{y} - \underline{x}G)}\right] d\underline{y}. \end{aligned} \quad \text{(A.2)}$$

Therefore, after replacing the pdf $p(\underline{w})$ of the noise vector in (2) with the corresponding Gaussian expression, from (A.1) and (A.2) the relationship (5) for the conditional symmetric capacity directly arises.

For deriving the conditional upper bound in (10), we begin to note that the conditional entropy in (A.2) can be lower bounded as reported below.

$$\begin{aligned} &H(\underline{x}|\underline{y}, G) \\ &\quad \geq \frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t} E_{\underline{Y}|\underline{X},G}\left\{\lg\left[1 + (q^t-1)\prod_{\substack{\underline{x}'\in(A_X)^t \\ \underline{x}'\neq\underline{x}}}\right.\right. \\ &\qquad \left.\left. \times \left(\frac{p(\underline{w} = \underline{y} - \underline{x}'G)}{p(\underline{w} = \underline{y} - \underline{x}G)}\right)^{(q^t-1)^{-1}}\right]\right\} \\ &\quad = \frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t} E_{\underline{Y}|\underline{X},G}\left\{\lg\left[1 + (q^t-1)\right.\right. \\ &\qquad \times \exp\left(\frac{\overline{\gamma}}{2(q^t-1)}\sum_{\substack{\underline{x}'\in(A_x)^t \\ \underline{x}'\neq\underline{x}}}((\underline{y} - G\underline{x})^H(\underline{y} - G\underline{x})\right. \\ &\qquad \left.\left.\left. - (\underline{y} - G\underline{x}')^H(\underline{y} - G\underline{x}'))\right)\right]\right\} \end{aligned} \quad \text{(A.3)}$$

$$\geq \frac{1}{q^t} \sum_{\underline{x}\in(A_x)^t} \lg \Bigg[1+(q^t-1) \times \exp\Bigg(\frac{\overline{\gamma}}{2(q^t-1)} \sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\neq\underline{x}}} E_{\underline{Y}|\underline{X},G}\{(\underline{y}-G\underline{x})^H \times (\underline{y}-G\underline{x}) - (\underline{y}-G\underline{x}')^H(\underline{y}-G\underline{x}')\}\Bigg)\Bigg] \tag{A.4}$$

$$= \frac{1}{q^t} \sum_{\underline{x}\in(A_x)^t} \lg \Bigg[1+(q^t-1)\exp\Bigg(-\frac{\overline{\gamma}}{2(q^t-1)} \times \sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\neq\underline{x}}} (\underline{x}-\underline{x}')^H G^H G(\underline{x}-\underline{x}')\Bigg)\Bigg] \tag{A.5}$$

where the first half of (A.3) follows from an application of the usual arithmetic-geometric inequality [21, p. 1126] to the argument of the logarithmic term in (A.2); the second half of (A.3) stems from the Gaussianity of the pdf of the noise $\underline{w}$ and (A.4) has been obtained via an application of the Jensen's inequality to the $\cup$-convex logarithmic function in (A.3). Now, after indicating as $\Theta_{\max}$ the maximum eigenvalue of the Hermitian semidefinite-positive matrix $G^H G$, an exploitation of the basic properties of the so-called "Rayleigh quotient" [22, Section 8.2] leads to the inequality

$$(\underline{x}-\underline{x}')^H G^H G(\underline{x}-\underline{x}') \leq \|\underline{x}-\underline{x}'\|^2 \Theta_{\max}. \tag{A.6}$$

Therefore, after inserting (A.6) in (A.5), from (A.5) we obtain the chain of limits

$$H(\underline{x}|\underline{y}, G) \geq \frac{1}{q^t} \sum_{\underline{x}\in(A_x)^t} \lg \Bigg[1+(q^t-1) \times \exp\Bigg(-\frac{\overline{\gamma}\Theta_{\max}}{2(q^t-1)} \sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\neq\underline{x}}} \|\underline{x}-\underline{x}'\|^2\Bigg)\Bigg] \tag{A.7}$$

$$\geq \lg \Bigg[1+(q^t-1)\exp\Bigg(-\frac{\overline{\gamma}\Theta_{\max}}{2} \times \Bigg\{-\frac{1}{q^t(q^t-1)} \sum_{\underline{x}\in(A_x)^t} \sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\neq\underline{x}}} \|\underline{x}-\underline{x}'\|^2\Bigg\}\Bigg)\Bigg] \tag{A.8}$$

$$\geq \lg\left\{1+(q^t-1)\exp\left[-\overline{\gamma}\Theta_{\max}\left(\frac{q^t}{(q^t-1)}\right)\right]\right\} \tag{A.9}$$

where (A.7) follows from (A.6), and (A.8) derives from an application of Jensen's inequality to the $\cup$-convex logarithmic function appearing in (A.7). Furthermore, (A.9) stems from the so-called "Plotkin limit" [20, Section 3.7.1] which allows us to directly compute the average squared Euclidean distance into brackets $\{\cdots\}$ of (A.8) as $2q^t/(q^t-1)$. Although for $t=r$ the random matrix $G^H G$ is a Wishart type, nevertheless the pdf of its maximum eigenvalue $\Theta_{\max}$ resists closed-form evaluation and, in fact, may be computed only via numerical procedures [11, ch. 6]. Therefore, to bypass this handicap, we resort to known properties about the spectral norm of semidefinite positive Hermitian matrices [22, Sections 10.3 and 10.4] which allows us to upper-bound $\Theta_{\max}$ via the squared Euclidean norm $\|G\|^2 \equiv \sum_{i=1}^t \sum_{j=1}^t |g_{ij}|^2$ of G as

$$\Theta_{\max} \leq \|G\|^2. \tag{A.10}$$

Therefore, after introducing (A.10) in (A.9), we obtain

$$H(\underline{x}|\underline{y}, G) \geq \lg\left\{1+(q^t-1)\exp\left[-\left(\frac{q^t}{(q^t-1)}\right)\overline{\gamma}\|G\|^2\right]\right\} \tag{A.11}$$

and then the conditional upper bound (10) stems from the insertion in (A.1) of the lower bound (A.11).

Passing now to the derivation of the conditional lower bound in (11) for $t=r$, we begin to note that for any assigned channel matrix G, the resulting symmetric capacity $\tilde{C}_t^*(G)$ of the MIMO channel (2) is obviously lower-bounded by the corresponding symmetric cutoff rate $R_o^*(G)_t$ [8, Section 4.3], [20, Section 3.2] and then we can write

$$R_o^*(G)_t \leq \tilde{C}_t^*(G). \tag{A.12}$$

This parameter can be computed pursuing the *same approach* detailed in [8, Section 4.3] for SISO channels, which allows us to arrive to the following closed-form formula for the link model in (2):

$$R_o^*(G)_t = t\lg q - \lg\Bigg\{1+\frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t} \sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\neq\underline{x}}} \times \exp\left(-\frac{\overline{\gamma}}{8}\|G(\underline{x}-\underline{x}')\|^2\right)\Bigg\}, \quad \text{nats/channel-use.} \tag{A.13}$$

Unfortunately, it turns out that this last relationship is still too complex to be effectively exploited to derive closed-form lower bounds on the symmetric capacity C_t^*. Therefore, we proceed to somewhat simplify (A.13) by resorting to basic properties of the "Rayleigh quotient" of semidefinite-positive Hermitian square matrices [22, Section 8.2] which allows us to limit the squared norm present in (A.13) as

$$||\underline{x}-\underline{x}'||^2\Theta_{\min} \le ||G(\underline{x}-\underline{x}')||^2 \tag{A.14}$$

where $\Theta_{\min}$ is the minimum eigenvalue of $G^H G$. Hence, the conditional lower bound (11) directly stems from (A.12) and (A.13) after introducing in (A.13) the left-hand side of (A.14).

Appendix B
Derivation of the Bounds (13) and (14) on the Symmetric Capacity C_t^*

For the derivation of the unconditional bounds (13) and (14) from the corresponding conditional ones (11), (10), we must average these last over the statistics of the fading processes. As far as the upper bound (10) is concerned, from the Rayleigh assumption on the fading of Section II-A it follows that the term $||G||^2$ in (10) is a real nonnegative random variable described by the central chi-squared pdf with $2t^2$ degrees of freedom reported, for example, in [8, eq. 2.2.53]. Moreover, the developments reported below hold for the expectation of $\widetilde{UB}_t^*(G)$:

$$\begin{aligned} UB_t^*(G) &\equiv E\left\{\widetilde{UB}_t^*(G)\right\} \\ &= -E\left\{\lg\left[1+\left(\frac{q^t-1}{q^t}\right)\right.\right. \\ &\qquad \left.\left.\times\left[\exp\left(-\left(\frac{q^t-1}{q^t}\right)\overline{\gamma}||G||^2\right)-1\right]\right]\right\} \\ &= -\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{q^t-1}{q^t}\right)^k \\ &\quad \times E\left\{\left[\exp\left(-\left(\frac{q^t-1}{q^t}\right)\overline{\gamma}||G||^2\right)-1\right]^k\right\} \\ &= t\lg q-\sum_{k=1}^{\infty}(k-1)!\left(\frac{q^t-1}{q^t}\right)^k \\ &\quad \times\left\{\sum_{m=1}^{k}\frac{(-1)^m}{m!(k-m)!}\right. \\ &\qquad \left.\times E\left\{\left[\exp\left(-\left(\frac{q^t-1}{q^t}\right)m\overline{\gamma}||G||^2\right)\right]\right\}\right\} \end{aligned} \tag{B.1}$$

where the first part of (B.1) is a consequence of elementary algebraic manipulations on the argument of the logarithm in (10); while the second part stems from the power series expansion of the logarithmic function. Finally, the expectation in (B.1) over the pdf of $||G||^2$ can be evaluated by resorting to [21, eq. 3.381.4] which directly leads to the relationship (14) for UB_t^*.

As far as the lower bound (11) is concerned, we recall that for $t=r$ the matrix GG^H is complex Wishart [11, ch. 3] and its minimum eigenvalue $\Theta_{\min}$ in (11) is a nonnegative real random variable with exponential-type pdf given by [11, p. 62]

$$p(\Theta_{\min}) = \frac{t}{2}\exp\left(-\frac{t}{2}\Theta_{\min}\right), \qquad \Theta_{\min}\ge 0. \tag{B.2}$$

Unfortunately, the direct average of (11) over the above pdf leads to

$$\begin{aligned} &E\left\{\widetilde{LB}_t^*(G)\right\} \\ &= t\lg q - E\left\{\lg\left[1+\frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t}\sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\ne\underline{x}}}\right.\right. \\ &\qquad\left.\left.\times\exp\left(-\frac{\overline{\gamma}}{8}||\underline{x}-\underline{x}'||^2\Theta_{\min}\right)\right]\right\} \end{aligned} \tag{B.3}$$

and the above expectation resists analytical closed-form evaluation. However, an application of Jensen's inequality which exploits the ∩-convexity of the logarithmic function allows us to arrive at the following simpler (but looser) bound:

$$\begin{aligned} &E\left\{\widetilde{LB}_t^*(G)\right\} \\ &\ge t\lg q - \lg\left[1+\frac{1}{q^t}\sum_{\underline{x}\in(A_x)^t}\sum_{\substack{\underline{x}'\in(A_x)^t\\ \underline{x}'\ne\underline{x}}}\right. \\ &\qquad\left.\times E\left\{\exp\left(-\frac{\overline{\gamma}}{8}||\underline{x}-\underline{x}'||^2\Theta_{\min}\right)\right\}\right]. \end{aligned} \tag{B.4}$$

Hence, after evaluating the expectation in (B.4) over the pdf (B.2) according to [21, 3.310], from (B.4) we obtain the lower bound LB_t^* in (13).

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