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Existence and p-exponential stability of periodic solution for stochastic shunting inhibitory cellular neural networks with time-varying delays

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Abstract

In this paper, we investigate a class of stochastic shunting inhibitory cellular neural networks with time-varying delays. Applying integral inequality, some sufficient conditions on the existence and p-exponential stability of periodic solutions for stochastic shunting inhibitory cellular neural networks with time-varying delays are established. An example is presented to illustrate our main theoretical findings. Our results are new and complementary to previously known studies.

Keywords: Stochastic shunting inhibitory cellular neural networks, periodic solution, p-exponential stability, delay.

1. Introduction

Since shunting inhibitory cellular neural networks have been successfully applied to pattern recognition, image and signal processing, vision, and optimization, their dynamics has attracted many attentions. Numerous important results on the existence and uniqueness of equilibrium point, periodic solution, almost periodic solution, pseudo almost periodic solution, almost automorphic solution and anti-periodic solution have been reported. For example, Gao et al.[1] studied the existence and stability almost periodic solutions for cellular neural networks with time-varying delays in leakage terms on time scales, Liu and Shao[2] analyzed the almost periodic solutions for SICNNs with time-varying delays in the leakage terms, Liu[3] focused on the pseudo almost periodic solutions for neutral type CNNs with continuously distributed leakage delays, Armmaret al. [4] made a detailed discussion on the existence and uniqueness of pseudo almost periodic solutions of recurrent neural networks with time-varying coef-

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ficients and mixed delays, Abbas and Xia [5] investigated the almost automorphic solutions of impulsive cellular neural networks with piecewise constant argument, Li and Shu [6] dealt with the anti-periodic solutions to impulsive shunting inhibitory cellular neural networks with distributed delays on time scales. For details, we refer readers to papers [7,8,9,10,11,12,13,14].

In 1994, Haykin [15] pointed out that in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Neural networks could be stabilized or destabilized by some stochastic inputs [16]. Considering that neural networks are inevitably affected by the random fluctuations from the release of neurotransmitters and other probabilistic causes which is an important component in neural networks, we think that it is worth while to investigate the stochastic neural networks. Recently, there are many papers that deal with this aspect[17,18,19,20,21]. In this paper, we will consider the following stochastic shunting inhibitory cellular neural networks with time-varying delays

$$\begin{aligned} dx_{ij}(t) = \Big[& -a_{ij}(t)x_{ij}(t) + L_{ij}(t) \\ & - \sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t))x_{ij}(t) \\ & - \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) g_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t)\Big]dt \\ & + \sum_{D^{kl}\in N_r(i,j)} D_{ij}^{kl}(t)\sigma_{ij}(x_{ij}(t))dw_{ij}(t), t \geqslant t_0, \quad (1) \end{aligned}$$

where $i = 1,2,\cdots,m, j = 1,2,\cdots,n$, $\tau_{ij}(t) > 0$ denotes axonal signal transmission delay at time t, $C_{ij}(t)$ denotes the cell at the (i,j) position of the lattice at time t, the r-neighborhood $N_r(i,j)$ of $C_{ij}(t)$ is given as $N_r(i,j) = \{C_{ij}^{kl} : \max(|k-i|, |l-j| \leqslant r), 1 \leqslant k \leqslant m, 1 \leqslant l \leqslant n\}$, $x_{ij}(t)$ stands for the activity of the cell $C_{ij}(t)$, $L_{ij}(t)$ denotes the external input to $C_{ij}(t)$, $a_{ij}(t) > 0$ stands for the passive decay rate of the cell activity, $B_{ij}^{kl}(t) \geqslant 0$ and $C_{ij}^{kl}(t) \geqslant 0$ represent the connection or coupling strength of postsynaptic of activity of the cell transmitted to the cell $C_{ij}(t)$ at time t and $t-\tau_{kl}(t)$, respectively, the activity functions $f_{ij}(t, x_{kl}(t))$ and $g_{ij}(t, x_{kl}(t))$ are continuous functions representing the output or firing rate of the cell $C_{kl}(t)$ at time t and $t-\tau_{kl}(t)$, respectively, $w(t) = (w_{11}(t), w_{12}(t), \cdots, w_{mn}(t))^T$ is $m\times n$-dimensional Brownian motions defined on a complete probability space, $\sigma_{ij} \in C(R,R)$ is a Borel measurable function and $\sigma = (\sigma_{ij})_{mn\times mn}$ is a diffusion coefficient matrix.

Let $R^n(R_+^n)$ be the space of n-dimensional (nonnegative) real column vectors and R^{mn} be the space of $m\times n$-dimensional real column vectors. We denote $(\Omega, F, \{F_t\}_{t\geqslant 0}, P)$ by a complete probability space with a filtration $\{F_t\}_{t\geqslant 0}$, where F is a σ-algebra on a given set Ω, P is the probability measure and the filtration Ftt. satisfies the usual conditions, that is, $\{F_t\}_{t\geqslant 0}$ is right continuous and F_0 contains all P-null sets. Denote by $BC_{F_0}^b(R, R^{mn})$ the family of bounded F_0-measurable, R^{mn} valued random variables $x(t)$, that is, the value of $x(t)$ is an $m\times n$-dimensional real vector and can be decided from the values of $w(s)$ for $s \leqslant 0$. Then $BC_{F_0}^b(R, R^{mn})$ is a Banach space with the norm $||x|| = \sup_{0\leqslant t\leqslant\omega}(E|x(t)|_1^p)^{\frac{1}{p}}$, where $p > 1$ is an integer, $|x(t)|_1 = \max_{(i,j)}|x_{ij}(t)|$, and $E(.)$ stands for the correspondent expectation operator with respect to the given probability measure P. For convenience, for an ω-periodic continuous function $f: R\to R$, denote $f = \max_{0\leqslant t\leqslant\omega}|f(t)|, f = \min_{0\leqslant t\leqslant\omega}|f(t)|$, for any $\phi \in BC_{F_0}^b([-\tau,0], R^{mn})$, denote $[\phi(t)]_\tau^+ = (|\phi_{11}|_\tau, |\phi_{12}|_\tau, \cdots, |\phi_{mn}|_\tau)^T$, where $|\phi_{ij}|_\tau = \sup_{-\tau\leqslant s\leqslant 0}|\phi_{ij}(t+s)|$, $i = 1,2,\cdots,m, j = 1,2,\cdots,n$.

The initial value of system (1) takes the form

$$x_{ij}(s) = \varphi_{ij}(s), s\in[t_0-\tau, t_0], \qquad (2)$$

where $\varphi_{ij}(s) \in BC_{F_0}^b([t_0-\tau, 0], R), \tau = \max_{1\leqslant k\leqslant m, 1\leqslant l\leqslant n}\{\tau_{kl}\}, t_0 \in R$.

Throughout this paper, we make the assumption as follows.

(H1) For $i = 1,2,\cdots,m, j = 1,2,\cdots,n$, $a_{ij}(t), B_{ij}^{kl}(t), C_{ij}^{kl}(t), D_{ij}^{kl}(t)$ and $L_{ij}(t)$ are all ω-periodic continuous functions for all $t\in R$.

(H2) For $i = 1,2,\cdots,m, j = 1,2,\cdots,n$, there exist positive constants $L_{ijf}, g_{ijg}, \sigma_{ij\sigma}, M_f$ and M_g such that

$$|f_{ij}(t,u)-f_{ij}(t,v)| \leqslant L_{ijf}|u-v|, |f_{ij}(u)| \leqslant M_f,$$

$$|g_{ij}(t,u)-g_{ij}(t,v)| \leqslant L_{ijg}|u-v|, |g_{ij}(u)| \leqslant M_g,$$

$$|\sigma_{ij}(t,u)-\sigma_{ij}(t,v)| \leqslant L_{ij\sigma}|u-v|$$

for all $u,v,t \in R$.

The remainder of the paper is organized as follows: in Section 2, several definitions and some preliminary results which are useful in later section are introduced. some sufficient conditions for the the existence of periodic solutions of system (1) are derived in Section 3. In Section 4, the p-exponential stability of periodic solutions are analyzed. An examples are given to illustrate the feasibility and effectiveness of our results obtained in previous section in Section 5. A brief conclusion is drawn in Section 6.

2. Preliminaries

In this section, we shall recall several definitions and present some preliminary results which are necessary in later sections.

Definition 2.1 [22] *A stochastic process $x(t)$ is said to be periodic with period ω if its finite-dimensional distributions are periodic with period ω, that is, for any positive integer m and any moments of time $t_1,t_2,\cdots,t_m$, the joint distribution of the random variables $x(t_1+k\omega),x(t_2+k\omega),\cdots,x(t_m+k\omega))$ are independent of $k, k=\pm1,\pm2,\cdots$.*

Lemma 2.1 [23] *If $x(t)$ is an ω-periodic stochastic process, then its mathematical expectation and variance are ω-periodic.*

Definition 2.2 *A function $x(t)=(x_{11}(t),x_{12}(t),\cdots,x_{mn}(t))^T$ defined on $[t_0-\tau,\infty)$ is said to be a solution of (1) with initial condition (2) if the following conditions holds.*
(i) $x_{ij}(t)$ is absolutely continuous on $[t_0-\tau,\infty)$, $i=1,2,\cdots,m, j=1,2,\cdots,n$,
(ii) $x_{ij}(t)$ satisfies (1) for almost everywhere $t \in [t_0,\infty)$, $i=1,2,\cdots,m, j=1,2,\cdots,n$,
(iii) $x_{ij}(s)=\varphi_{ij}(s), s\in[t_0-\tau,t_0], i=1,2,\cdots,m, j=1,2,\cdots,n$.

Throughout this paper, we assume that (1) with initial condition (2) has a unique solution. Denote the solution of (1) by $x(t)=x(t,t_0,\varphi)$ for all $\varphi \in BC^b_{F_0}([t_0-\tau,t_0],R^{mn})$ and $t_0 \in R$.

Definition 2.3[22] *The solution $x(t,t_0,\varphi)$ of (1) is said to be*
(i) p-uniformly bounded, if for each $\alpha>0, t_0\in R$, there exists a positive constant $\theta=\theta(\alpha)$ which is independent of t_0 such that $||\varphi||^p \leqslant \alpha$ implies $E(x(t,t_0,\varphi)||^p) \leqslant \theta, t \geqslant t_0$;
(ii) p-point dissipative, if there exists a constant $N>0$ such that for any point $\varphi \in BC^b_{F_0}([-\tau,0],R^n)$, there exists $T(t_0,\varphi)$ such that $E(||x(t,t_0,\varphi)||^p) \leqslant N, t \geqslant t_0+T(t_0,\varphi)$.

Lemma 2.2[24] *In addition to (H1) and (H2), suppose that the solution of (1) is p-uniformly bounded and p-point dissipative for $p>2$, then (1) has an ω-periodic solution.*

Lemma 2.3[25] *For any $x \in R^n_+$ and $p>0$,*

$$|x|^p \leqslant n^{(\frac{p}{2}-1)\vee 0}\sum_{i=1}^{n} x_i^p, \left(\sum_{i=1}^{n} x_i\right)^p \leqslant n^{(p-1)\vee 0}\sum_{i=1}^{n} x_i^p.$$

Definition 2.4 [22] *The periodic solution $x(t,t_0,\varphi)$ with initial value $\varphi \in BC^b_{F_0}([-\tau,0],R^n)$ of (1) is said to be p-exponentially stable, if there are constants $\lambda>0$ and $M>0$ such that for any solution $y(t,t_0,\varphi_1)$ with initial value $\varphi_1 \in BC^b_{F_0}([-\tau,0],R^n)$ of (1) satisfies*

$$E(|x-y|_1^p) \leqslant M|varphi-\varphi_1||^p e^{-\lambda(t-t_0)}, t \geqslant t_0.$$

Lemma 2.4 [22] *Let $u(t) \in C(R,R^n_+)$ be a solution of the delay integral inequality*

$$\begin{cases} u(t) \leqslant M_1 e^{-\delta(t-t_0)}[\varphi]^+_\tau + \int_{t_0}^t e^{-C_1(t-s)}A_1 u(s)ds \\ \qquad + \int_{t_0}^t e^{-C_1(t-s)}B_1[u(s)]^+_\tau ds + J_1, t \geqslant t_0, \\ u(t) \leqslant \varphi(t), \forall\, t\in[t_0-\tau,t_0], \end{cases} \tag{3}$$

where $A_1,B_1,C_1,M_1 \in R^{n\times n}_+, J_1 \geqslant 0$ is a constant vector, $\varphi(t) \in C([t-\tau,t_0],R^n_+)$. If $\rho(\Pi)<1$, where $\Pi=C_1^{-1}(A_1+B_1)$, then there are constants $0<\lambda\leqslant\delta$ and $N \geqslant 1$ such that

$$u(t) \leqslant Nze^{-\lambda(t-t_0)} + (I-\Pi)^{-1}J_1, t \geqslant t_0,$$

where z satisfies $[\varphi]_\tau^+ \leqslant z$.

Lemma 2.5 [22] *Assume that all conditions of Lemma 2.4 hold. If* $J_1 = 0$, *then all solutions of inequality of (1) exponentially convergent to zero.*

In view of Lemma 2.4 and Lemma 2.5, we have the following results.

Lemma 2.6 [26] *Let* $u(t) \in C(R, R_+^n)$ *be a solution of the delay integral inequality*

$$\begin{cases} u(t) \leqslant M_1 e^{-\delta(t-t_0)}[\varphi]_\tau^+ + \int_{t_0}^t e^{-C_1(t-s)} A_1 u(s) ds \\ \quad + \int_{t_0}^t e^{-C_1(t-s)} B_1 [u(s)]_\tau^+ ds + J_1, t \geqslant t_0, \\ u(t) \leqslant \varphi(t), \forall\, t \in [t_0 - \tau, t_0], \end{cases} \tag{4}$$

where $A_1, B_1, C_1, M_1 \in R_+^{n\times n}, J_1 \geqslant 0$ *is a constant vector,* $\varphi(t) \in C([t-\tau, t_0], R_+^n)$. *If* $\frac{A_1+B_1}{C_1} < 1$, *then there are constants* $0 < \lambda \leqslant \delta$ *and* $N \geqslant 1$ *such that*

$$u(t) \leqslant Nze^{-\lambda(t-t_0)} + (1 - \frac{A_1+B_1}{C_1})^{-1} J_1, t \geqslant t_0,$$

where z satisfies $[\varphi]_\tau^+ \leqslant z$. *Moreover, if* $J_1 = 0$, *then all solutions of the inequality of (4) are exponentially convergent to zero.*

3. Existence of Periodic Solution

In this section, we discuss the existence of periodic solution of (1).

Theorem 3.1 *In addition to (H1) and (H2), assume further that*
(H3) there exists an integer $p > 2$ *such that* $\rho\delta^{-1} < 1$, *where* $\delta = \min_{(i,j)}\{\underline{a}_{ij}\}$, $\sigma = \left(\frac{p(p-1)}{2}\right)^{\frac{p}{2}}$,

$$\begin{aligned} \rho &= \max_{(i,j)} \Bigg\{ 5^{p-1} \left(\underline{a}_{ij}\right)^{1-p} \Bigg[\left(M_f \sum_{B^{kl}\in N_r(i,j)} \overline{B}_{ij}^{kl} \right)^p \\ &\quad + \left(M_g \sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl} \right)^p \Bigg] \\ &\quad + \sigma (mn)^{\frac{p}{2}} \left(\frac{2\underline{a}_{ij}(p-1)}{p-2} \right)^{1-\frac{p}{2}} \\ &\quad \times \left(\sum_{D^{kl}\in N_r(i,j)} \overline{D}_{ij}^{kl} \right)^p, \end{aligned}$$

then (1.1) has an ω*-periodic solution.*

Proof It follows from the method of variation parameter and (1) that for $t \geqslant t_0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$,

$$\begin{aligned} x_{ij}(t) &= x_{ij}(t_0) e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} - \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \\ &\Bigg[\sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(s) f_{ij}(s, x_{kl}(s)) x_{ij}(s) \\ &+ \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) g_{ij}(s, x_{kl}(s - \tau_{kl}(s))) x_{ij}(s) \\ &- L_{ij}(s) \Bigg] ds + \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \\ &\sum_{D^{kl}\in N_r(i,j)} D_{ij}^{kl}(s) \sigma_{ij}(x_{ij}(s)) dw_{ij}(s). \end{aligned} \tag{5}$$

Let

$$\begin{aligned} \Theta_{ij}^{(1)} &= x_{ij}(t_0) e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}, \\ \Theta_{ij}^{(2)} &= \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \\ &\sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(s) f_{ij}(s, x_{kl}(s)) x_{ij}(s) ds, \\ \Theta_{ij}^{(3)} &= \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s) \\ &\times g_{ij}(s, x_{kl}(s - \tau_{kl}(s))) x_{ij}(s) ds, \\ \Theta_{ij}^{(4)} &= \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} L_{ij}(s) ds, \\ \Theta_{ij}^{(5)} &= \int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \\ &\sum_{D^{kl}\in N_r(i,j)} D_{ij}^{kl}(s) \sigma_{ij}(x_{ij}(s)) dw_{ij}(s). \end{aligned}$$

Taking expectations and applying Lemma 2.3, we get

$$\begin{aligned} E|x_{ij}(t)|^p \leqslant\ & 5^{p-1} E(|\Theta_{ij}^{(1)}|^p + |\Theta_{ij}^{(2)}|^p \\ & + |\Theta_{ij}^{(3)}|^p + |\Theta_{ij}^{(4)}|^p + |\Theta_{ij}^{(5)}|^p), \end{aligned} \tag{6}$$

where $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$. Next, we will evaluate every term of (6). For the first term of (6), we have

$$E|\Theta_{ij}^{(1)}|^p = x_{ij}(t_0)e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \leqslant E|x_{ij}(t_0)e^{-\underline{a}_{ij}(t-t_0)}|^p \leqslant e^{-p\underline{a}_{ij}(t-t_0)}E|x_{ij}(t_0)|^p. \quad (7)$$

For the second term of (6), we have

$$\begin{aligned} E|\Theta_{ij}^{(2)}|^p &= E\left|\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(s)f_{ij}(s,x_{kl}(s))x_{ij}(s)ds\right|^p \\ &\leqslant E\left(\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)} \sum_{B^{kl}\in N_r(i,j)} \overline{B}_{ij}^{kl}(s)|f_{ij}(s,x_{kl}(s))| \times|x_{ij}(s)|ds\right)^p \\ &\leqslant E\left(\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)} \sum_{B^{kl}\in N_r(i,j)} \overline{B}_{ij}^{kl}(s)M_f|x_{ij}(s)|ds\right)^p \\ &= E\left(\int_{t_0}^t \left(e^{-\underline{a}_{ij}(t-s)}\right)^{\frac{p-1}{p}} \left(e^{-\underline{a}_{ij}(t-s)}\right)^{\frac{1}{p}} \times \sum_{B^{kl}\in N_r(i,j)} \overline{B}_{ij}^{kl}(s)M_f|x_{ij}(s)|ds\right)^p \\ &\leqslant E\left(\left(\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}ds\right)^{p-1} \int_{t_0}^t e^{-\underline{a}_{ij}(t-s)} \times\left(\sum_{B^{kl}\in N_r(i,j)} \overline{B}_{ij}^{kl}(s)M_f|x_{ij}(s)|\right)^p ds\right) \\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\left(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^p \times E||x_{ij}(s)|^p ds. \end{aligned} \quad (8)$$

For the third term of (6), we have

$$\begin{aligned} E|\Theta_{ij}^{(3)}|^p &= E\left|\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(s)g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))x_{ij}(s)ds\right|^p \\ &= E\left(\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{C^{kl}\in N_r(i,j)} |C_{ij}^{kl}(s)| |g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))||x_{ij}(s)|ds\right)^p \\ &\leqslant E\left(\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{C^{kl}\in N_r(i,j)} \overline{C}_{ij}^{kl}M_g|x_{ij}(s)|ds\right)^p \\ &\leqslant E\left(\left(\int_{t_0}^t e^{-a_{ij}(t-s)}ds\right)^{p-1}\int_{t_0}^t e^{-a_{ij}(t-s)} \times\left(\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}M_g|x_{ij}(s)|\right)^p ds\right) \\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-a_{ij}(t-s)}\left(\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}M_g\right)^p \times E|x_{ij}(s)|^p ds. \end{aligned} \quad (9)$$

For the fourth term of (6), we get

$$\begin{aligned} E|\Theta_{ij}^{(4)}|^p &= E\left|\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}L_{ij}(s)ds\right|^p \\ &\leqslant E\left|\int_{t_0}^t e^{-a_{ij}(t-s)}L_{ij}(s)ds\right|^p \\ &\leqslant \left(\frac{\overline{L}_{ij}}{\underline{a}_{ij}}\right)^p. \end{aligned} \quad (10)$$

For the fifth term of (6), we get

$$\begin{aligned} E|\Theta_{ij}^{(5)}|^p &= E\left|\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi} \sum_{D^{kl}\in N_r(i,j)} D_{ij}^{kl}(s)\sigma_{ij}(x_{ij}(s))dw_{ij}(s)\right|^p \\ &\leqslant \sigma\left[\int_{t_0}^t \left(e^{-p\underline{a}_{ij}(t-s)} \times E\left|\sum_{D^{kl}\in N_r(i,j)} (D_{ij}^{kl}(s))^2\sigma_{ij}^2(x_{ij}(s))\right|^{\frac{p}{2}}\right)^{\frac{2}{p}} ds\right]^{\frac{p}{2}} \\ &\leqslant \sigma(mn)^{\frac{p}{2}}\left[\int_{t_0}^t \left(e^{-p\underline{a}_{ij}(t-s)}\right.\right. \end{aligned}$$

$$\times E\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}|x_{ij}(s)|\right)^p\Bigg)^{\frac{2}{p}}ds\Bigg]^{\frac{p}{2}}$$

$$= \sigma(mn)^{\frac{p}{2}}\Bigg[\int_{t_0}^{t}\Bigg(e^{-(p-1)\underline{a}_{ij}(t-s)}e^{-\underline{a}_{ij}(t-s)}$$

$$\times E\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}|x_{ij}(s)|\right)^p\Bigg)^{\frac{2}{p}}ds\Bigg]^{\frac{p}{2}}$$

$$\leqslant \sigma(mn)^{\frac{p}{2}}\left(\int_{t_0}^{t}e^{-\frac{2\underline{a}_{ij}(p-1)}{p-2}(t-s)}ds\right)^{\frac{p}{2}-1}$$

$$\times\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}E\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}|x_{ij}(s)|\right)^p ds\right)$$

$$\leqslant \sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$

$$\times\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}E\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}|x_{ij}(s)|\right)^p ds\right)$$

$$\leqslant \sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$

$$\times\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\right)^p\right.$$

$$\left.\times E|x_{ij}(s)|^p ds\right). \tag{11}$$

It follows from (8)-(11) that

$$E|x_{ij}(t)|^p \leqslant 5^{p-1}\Bigg\{e^{-p\underline{a}_{ij}(t-t_0)}E|x_{ij}(t_0)|^p$$

$$+\left(\frac{\overline{L}_{ij}}{\underline{a}_{ij}}\right)^p+(\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}$$

$$\left(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^p E||x_{ij}(s)|^p ds$$

$$+(\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-a_{ij}(t-s)}\left(M_g\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\right)^p$$

$$\times E|x_{ij}(s)|^p ds$$

$$+\sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$

$$\times\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\right)^p\right.$$

$$\left.\times E|x_{ij}(s)|^p ds\right)\Bigg\}. \tag{12}$$

Define

$$V(t)=(v_{11}(t),V_{12}(t),\cdots,V_{mn}(t))^T, \tag{13}$$

where $V_{ij}(t) = E|x_{ij}(t)|^p, i = 1,2,\cdots,m, j = 1,2,\cdots,n$. It follows from (12) that

$$V_{ij}(t)\leqslant 5^{p-1}e^{-\delta(t-t_0)}V_{ij}(t_0)+\int_{t_0}^{t}e^{-\delta(t-s)}V_{ij}(s)ds+\iota, \tag{14}$$

where $\iota = \max_{(i,j)}\left\{\left(\frac{\overline{L}_{ij}}{\underline{a}_{ij}}\right)^p\right\}$. In view of (H3) and Lemma 2.4, we know that the solutions of (1) are p-uniformly bounded and the family of all solutions of (1) is p-point dissipative. By Lemma 2.2, we can conclude that (1) has an ω-periodic solution. The proof of Theorem 3.1 is completed.

4. p-exponential Stability of Periodic Solution

In this section, we will consider the p-exponential stability of periodic solutions of (1).

Theorem 4.1 *In addition to (H1)-(H2), assume further that*
(H4) there exists an integer $p > 2$ such that $(\rho_1 + \rho_2)\delta^{-1} < 1$, where

$$\rho_1 = \max_{(i,j)}\Bigg\{6^{p-1}\Bigg((\underline{a}_{ij})^{1-p}\Bigg[\left(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^p$$

$$+\left(NL_{ijf}\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^p\Bigg]$$

$$+\sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}}$$

$$\times\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\right)^p\Bigg)\Bigg\}$$

and

$$\rho_2 = \max_{(i,j)}\left\{6^{p-1}(\underline{a}_{ij})^{1-p}\left[\left(M_g\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\right)^p + \left(NL_{ijg}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\right)^p\right]\right\},$$

then the periodic solution of (1) is p-exponentially stable.

Proof Obviously, if (H4) holds, then (H3) is fulfilled. In view of Theorem 3.1, we know that (1) has an ω-periodic solution $x^*(t)=\{x^*_{ij}(t)\}$ with the initial condition $\varphi(t)=\{\varphi_{ij}(t)\}, i=1,2,\cdot,m, j=1,2,\cdots,n$. Thus $x^*(t)$ is p-uniform, namely, there is a constant $C_0>0$ such that $E|x^*_{ij}(t)|^p<C_0, i=1,2,\cdot,m, j=1,2,\cdots,n$. Assume that $x(t)=\{x_{ij}(t)$ is an arbitrary solution of (1) with the initial condition $\psi(t)=\{\psi_{ij}(t)\}, i=1,2,\cdot,m, j=1,2,\cdots,n$. Let

$$u(t)=\{u_{ij}(t)\}=\{x_{ij}(t)-x^*_{ij}(t)\}, \tag{15}$$

where $i=1,2,\cdot,m, j=1,2,\cdots,n$. Then for $i=1,2,\cdot,m, j=1,2,\cdots,n$ and $t\geqslant t_0$, we get

$$\begin{aligned}du_{ij}(t)=&\Big[-a_{ij}(t)u_{ij}(t)\\&-\sum_{B^{kl}\in N_r(i,j)}B^{kl}_{ij}(t)(f_{ij}(t,x_{kl}(t))x_{ij}(t)\\&-f_{ij}(t,x^*_{kl}(t))x^*_{ij}(t))\\&-\sum_{C^{kl}\in N_r(i,j)}C^{kl}_{ij}(t)(g_{ij}(t,x_{kl}(t-\tau_{kl}(t)))x_{ij}(t)\\&-g_{ij}(t,x^*_{kl}(t-\tau_{kl}(t)))x^*_{ij}(t))\Big]dt\\&+\sum_{D^{kl}\in N_r(i,j)}D^{kl}_{ij}(t)(\sigma_{ij}(x_{ij}(t))\\&-\sigma_{ij}(x^*_{ij}(t)))dw_{ij}(t)\end{aligned}\tag{16}$$

with this initial condition

$$\phi_{ij}(s)=\psi_{ij}(s)-\varphi_{ij}(s), s\in[-\tau,t_0],$$

where $i=1,2,\cdot,m, j=1,2,\cdots,n$. Applying the method of variation parameter, we have

$$\begin{aligned}u_{ij}(t)=&\;u_{ij}(t_0)e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}-\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\\&\times\Big[\sum_{B^{kl}\in N_r(i,j)}B^{kl}_{ij}(s)(f_{ij}(s,x_{kl}(s))x_{ij}(s)\\&-f_{ij}(s,x^*_{kl}(s))x^*_{ij}(s))\\&+\sum_{C^{kl}\in N_r(i,j)}C^{kl}_{ij}(s)(g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))x_{ij}(s)\\&-g_{ij}(s,x^*_{kl}(s-\tau_{kl}(s)))x^*_{ij}(s)\Big]ds\\&+\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\sum_{D^{kl}\in N_r(i,j)}D^{kl}_{ij}(s)(\sigma_{ij}(x_{ij}(s))\\&-\sigma_{ij}(x^*_{ij}(s)))dw_{ij}(s)\\=&\;u_{ij}(t_0)e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}-\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\\&\times\Big[\sum_{B^{kl}\in N_r(i,j)}B^{kl}_{ij}(s)(f_{ij}(s,x_{kl}(s))u_{ij}(s)\\&(f_{ij}(s,x_{kl}(s))-f_{ij}(s,x^*_{kl}(s))x^*_{ij}(s))\\&+\sum_{C^{kl}\in N_r(i,j)}C^{kl}_{ij}(s)(g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))u_{ij}(s)\\&+(g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))\\&-g_{ij}(s,x^*_{kl}(s-\tau_{kl}(s))))x^*_{ij}(s))\Big]ds\\&+\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\sum_{D^{kl}\in N_r(i,j)}D^{kl}_{ij}(s)(\sigma_{ij}(x_{ij}(s))\\&-\sigma_{ij}(x^*_{ij}(s)))dw_{ij}(s),\end{aligned}\tag{17}$$

where $i=1,2,\cdot,m, j=1,2,\cdots,n$ and $t\geqslant t_0$. Let

$$\begin{aligned}&\Phi^{(1)}_{ij}=u_{ij}(t_0)e^{-\int_{t_0}^t a_{ij}(\xi)d\xi},\\&\Phi^{(2)}_{ij}=\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\\&\times\sum_{B^{kl}\in N_r(i,j)}B^{kl}_{ij}(s)f_{ij}(s,x_{kl}(s))u_{ij}(s)ds,\\&\Phi^{(3)}_{ij}=\int_{t_0}^t e^{-\int_{t_0}^t a_{ij}(\xi)d\xi}\sum_{B^{kl}\in N_r(i,j)}B^{kl}_{ij}(s)\end{aligned}$$

$$\times(f_{ij}(s,x_{kl}(s))-f_{ij}(s,x_{kl}^*(s))x_{ij}^*(s)ds,$$

$$\Phi_{ij}^{(4)}=\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(s)g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))u_{ij}(s)ds,$$

$$\Phi_{ij}^{(5)}=\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(s)(g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))-g_{ij}(s,x_{kl}^*(s-\tau_{kl}(s))))x_{ij}^*(s)ds,$$

$$\Phi_{ij}^{(6)}=\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{D^{kl}\in N_r(i,j)}D_{ij}^{kl}(s)(\sigma_{ij}(x_{ij}(s))-\sigma_{ij}(x_{ij}^*(s)))dw_{ij}(s).$$

Taking expectations and applying Lemma 2.3, we have

$$E|u_{ij}(t)|^p \leqslant 6^{p-1}E(|\Phi_{ij}^{(1)}|^p+|\Phi_{ij}^{(2)}|^p+|\Phi_{ij}^{(3)}|^p+|\Phi_{ij}^{(4)}|^p+|\Phi_{ij}^{(5)}|^p+|\Phi_{ij}^{(6)}|^p). \quad (18)$$

where $i=1,2,\cdots,m$, $j=1,2,\cdots,n$. Applying the similar method in the proof of Theorem 3.1, we get

$$E|\Phi_{ij}^{(1)}|^p \leqslant e^{-p\underline{a}_{ij}(t-t_0)}E|u_{ij}(t_0)|^p,$$

$$\begin{aligned}E|\Phi_{ij}^{(2)}|^p &= E\left|\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{B^{kl}\in N_r(i,j)}B_{ij}^{kl}(s)f_{ij}(s,x_{kl}(s))u_{ij}(s)ds\right|^p\\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\left(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^p\times E||u_{ij}(s)|^pds,\end{aligned}$$

$$\begin{aligned}E|\Phi_{ij}^{(3)}|^p &\leqslant E\left|\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{B^{kl}\in N_r(i,j)}B_{ij}^{kl}(s)(f_{ij}(s,x_{kl}(s))-f_{ij}(s,x_{kl}^*(s))x_{ij}^*(s)ds\right|^p\\ &\leqslant E\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\times\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}C_0L_{ijf}|u_{kl}(s)|ds\right)^p\\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\times\left(C_0L_{ijf}\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\right)^pE|u_{kl}(s)|^pds,\end{aligned}$$

$$\begin{aligned}E|\Phi_{ij}^{(4)}|^p &\leqslant E\left|\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(s)\times g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))u_{ij}(s)ds\right|^p\\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\left(M_g\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\right)^pE||u_{kl}(s-\tau_{kl}(s))|^pds,\end{aligned}$$

$$\begin{aligned}E|\Phi_{ij}^{(5)}|^p &\leqslant E\left|\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(s)(g_{ij}(s,x_{kl}(s-\tau_{kl}(s)))-g_{ij}(s,x_{kl}^*(s-\tau_{kl}(s))))x_{ij}^*(s)ds\right|^p\\ &\leqslant (\underline{a}_{ij})^{1-p}\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\times\left(C_0L_{ijg}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\right)^p\times E|u_{kl}(s-\tau_{kl}(s))|^pds,\end{aligned}$$

$$\begin{aligned}E|\Phi_{ij}^{(6)}|^p &\leqslant E\left|\int_{t_0}^{t}e^{-\int_{t_0}^{t}a_{ij}(\xi)d\xi}\times\sum_{D^{kl}\in N_r(i,j)}D_{ij}^{kl}(s)(\sigma_{ij}(x_{ij}(s))-\sigma_{ij}(x_{ij}^*(s)))dw_{ij}(s)\right|^p\\ &\leqslant \sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}}\left(\int_{t_0}^{t}e^{-\underline{a}_{ij}(t-s)}\left(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\right)^p\right.\end{aligned}$$

$$\times E|u_{ij}(s)|^p ds\Big).$$

Then we have

$$\begin{aligned}
E|u_{ij}|^p &\leqslant 6^{p-1}\Big\{e^{-p\underline{a}_{ij}(t-t_0)}E|u_{ij}(t_0)|^p \\
&+(\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\Big(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\Big)^p \\
&\times E||u_{ij}(s)|^p ds \\
&+(\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\Big(C_0L_{ijf}\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\Big)^p \\
&\times E|u_{kl}(s)|^p ds \\
&+(\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\Big(M_g\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\Big)^p \\
&\times E||u_{kl}(s-\tau_{kl}(s))|^p ds \\
&+(\underline{a}_{ij})^{1-p}\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\Big(C_0L_{ijg}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\Big)^p \\
&\times E|u_{kl}(s-\tau_{kl}(s))|^p ds \\
&+\sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}} \\
&\times\Big(\int_{t_0}^t e^{-\underline{a}_{ij}(t-s)}\Big(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\Big)^p \\
&\times E|u_{ij}(s)|^p ds\Big)\Big\}. \qquad (19)
\end{aligned}$$

Define

$$W(t)=(W_{11}(t),W_{12}(t),\cdots,W_{mn}(t))^T, \qquad (20)$$

where $W_{ij}(t) = E|u_{ij}(t)|^p, i = 1,2,\cdots,m, j = 1,2,\cdots,n$. It follows from (19) that

$$\begin{aligned}
W_{ij}(t) \ \leqslant\ & 6^{p-1}e^{-\delta(t-t_0)}W_{ij}(t_0) \\
&+\int_{t_0}^t e^{-\delta(t-s)}\rho_1 W_{ij}(s)ds \\
&+\int_{t_0}^t e^{-\delta(t-s)}\rho_2|W_{ij}(s)|_\tau^+ ds, \quad (21)
\end{aligned}$$

where

$$\begin{aligned}
\rho_1 \ =\ & \max_{(i,j)}\Big\{6^{p-1}\Big((\underline{a}_{ij})^{1-p}\Big[\Big(M_f\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\Big)^p \\
&+\Big(C_0L_{ijf}\sum_{B^{kl}\in N_r(i,j)}\overline{B}_{ij}^{kl}\Big)^p\Big] \\
&+\sigma(mn)^{\frac{p}{2}}\left(\frac{2\underline{a}_{ij}(p-1)}{p-2}\right)^{1-\frac{p}{2}} \\
&\times\Big(\sum_{D^{kl}\in N_r(i,j)}\overline{D}_{ij}^{kl}\Big)^p\Big)\Big\}
\end{aligned}$$

and

$$\begin{aligned}
\rho_2 \ =\ & \max_{(i,j)}\Big\{6^{p-1}(\underline{a}_{ij})^{1-p}\Big[\Big(M_g\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\Big)^p \\
&+\Big(C_0L_{ijg}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\Big)^p\Big]\Big\}.
\end{aligned}$$

In view of (H4) and Lemma 2.5, we can conclude that the periodic solution $x^*(t)$ of (1) is p-exponentially stable. The proof of Theorem 4.1 is completed.

Remark 4.1*In Zhao and Zhang*[27]*, Zhao and Zhang investigated the almost periodic solution of the following shunting inhibitory cellular neural networks with variable coefficients and time-varying delays*

$$\begin{aligned}
&\dot{x}_{ij}(t)=-a_{ij}(t)x_{ij}(t) \\
&-\sum_{B^{kl}\in N_r(i,j)}B_{ij}^{kl}(t)f_{ij}(t,x_{kl}(t))x_{ij}(t) \\
&-\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(t)g_{ij}(t,x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) \\
&+L_{ij}(t). \qquad (22)
\end{aligned}$$

By applying Dini derivative, they established some criteria on the existence and local exponential stability of (22). All the results in Zhao and Zhang[27] *can not be applicable to system (1) to obtain the the existence and p-exponential stability of periodic solutions. This implies that the results of this article are essentially new.*

5. An Example with Its Numerical Simulations

Example 5.1 Let $i = j = 2$. Consider the following stochastic shunting inhibitory cellular neural networks with time-varying delays

$$\begin{aligned} dx_{ij}(t) &= \Big[-a_{ij}(t)x_{ij}(t) + L_{ij}(t) \\ &- \sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t))x_{ij}(t) \\ &- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) g_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t)\Big] dt \\ &+ \sum_{D^{kl}\in N_r(i,j)} D_{ij}^{kl}(t)\sigma_{ij}(x_{ij}(t))dw_{ij}(t), \end{aligned} \tag{23}$$

where $f_{ij}(t,x) = \sin\frac{1}{5}x + \frac{4}{5}x, g_{ij}(t,x) = \sin\frac{1}{3}x + \frac{2}{3}x$ and

$$\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.3+0.02\cos\frac{\pi}{2}t & 0.4+0.02\sin\frac{\pi}{2}t \\ 0.4+0.01\cos\frac{\pi}{2}t & 0.3+0.01\cos\frac{\pi}{2}t \end{bmatrix},$$

$$\begin{bmatrix} B_{11}(t) & B_{12}(t) \\ B_{21}(t) & B_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.02|\cos\frac{\pi}{4}t| & 0.03\sin\frac{\pi}{4}t \\ 0.01\sin\frac{\pi}{4}t & 0.02\cos\frac{\pi}{4}t \end{bmatrix},$$

$$\begin{bmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.03|\sin\frac{\pi}{4}t| & 0.03\cos\frac{\pi}{4}t \\ 0.02\cos\frac{\pi}{4}t & 0.02\sin\frac{\pi}{4}t \end{bmatrix},$$

$$\begin{bmatrix} D_{11}(t) & D_{12}(t) \\ D_{21}(t) & D_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.02|\cos\frac{\pi}{4}t| & 0.01\sin\frac{\pi}{4}t \\ 0.01\cos\frac{\pi}{4}t & 0.02\sin\frac{\pi}{4}t \end{bmatrix},$$

$$\begin{bmatrix} L_{11}(t) & L_{12}(t) \\ L_{21}(t) & L_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.2\cos\frac{\pi}{2}t & 0.1\sin\frac{\pi}{2}t \\ 0.3\cos\frac{\pi}{2}t & 0.2\sin\frac{\pi}{2}t \end{bmatrix},$$

$$\begin{bmatrix} \sigma_{11}(u) & \sigma_{12}(u) \\ \sigma_{21}(u) & \sigma_{22}(u) \end{bmatrix} = \begin{bmatrix} 0.2\sin u & 0.2\sin u \\ 0.2\cos u & 0.1\sin u \end{bmatrix},$$

$$\begin{bmatrix} \tau_{11}(t) & \tau_{12}(t) \\ \tau_{21}(t) & \tau_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.01|\sin t| & 0.02|\cos u| \\ 0.02|\sin u| & 0.01|\cos u| \end{bmatrix}.$$

Then $L_{ijf} = L_{ijg} = M_f = M_g = 1 (i,j = 1,2), \tau = 0.02, L_{11\sigma} = L_{22\sigma} = 0.1, L_{12\sigma} = L_{21\sigma} = 0.2$ and

$$\begin{bmatrix} \sum_{B^{kl}\in N_1(1,1)} \overline{B}_{11}^{kl} & \sum_{B^{kl}\in N_1(1,2)} \overline{B}_{12}^{kl} \\ \sum_{B^{kl}\in N_1(2,1)} \overline{B}_{21}^{kl} & \sum_{B^{kl}\in N_1(2,2)} \overline{B}_{22}^{kl} \end{bmatrix} = \begin{bmatrix} 0.08 & 0.12 \\ 0.04 & 0.08 \end{bmatrix},$$

$$\begin{bmatrix} \sum_{C^{kl}\in N_1(1,1)} \overline{C}_{11}^{kl} & \sum_{C^{kl}\in N_1(1,2)} \overline{C}_{12}^{kl} \\ \sum_{B^{kl}\in N_1(2,1)} \overline{C}_{21}^{kl} & \sum_{C^{kl}\in N_1(2,2)} \overline{C}_{22}^{kl} \end{bmatrix} = \begin{bmatrix} 0.12 & 0.12 \\ 0.08 & 0.08 \end{bmatrix},$$

$$\begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} = \begin{bmatrix} 0.28 & 0.38 \\ 0.39 & 0.29 \end{bmatrix}, \begin{bmatrix} \overline{L}_{11} & \overline{L}_{12} \\ \overline{L}_{21} & \overline{L}_{22} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}.$$

Take $p = 3, r = 1$. Then we have $\delta \approx 0.2302, \sigma_1 \approx 0.0704, \sigma_2 \approx 0.0462$. It is easy to check that all the conditions in Theorem 3.1 and Theorem 4.2 are fulfilled. Hence we can conclude that then (23) has a 4-periodic solution, which is 3-exponentially stable. The results are shown in Figs. 1-2.

6. Conclusions

In this paper, a class of stochastic shunting inhibitory cellular neural networks with time-varying delays are considered. We establish some sufficient conditions ensuring the existence and p-exponential stability of periodic solutions for stochastic shunting inhibitory cellular neural networks with time-varying delays by using integral inequalities. Comparisons between our results and the previous results show that our results complement the earlier publications and are completely new. An example is presented to illustrate our main theoretical findings. Our results play an important key in designing of shunting inhibitory cellular neural networks. The obtained results show that under some appropriate circumstances, stochastic shunting inhibitory cellular neural networks with time-varying delays can display sustainable periodic oscillatory phenomenon. These periodic oscillatory phenomenon can help us to process visual information quickly and effectively[28,29]. Also periodic oscillatory phenomenon can be helpful for us to predict pathological brain states, which is important to diagnose disease in medical science[30,31,32].

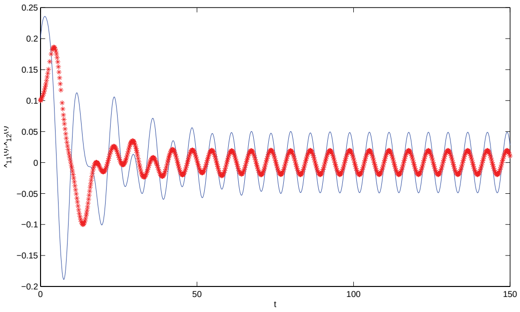

Fig. 1. Transient response of state variables $x_{11}(t)$ and $x_{12}(t)$, where the blue line stands for $x_{11}(t)$ and the red line stands for $x_{12}(t)$.

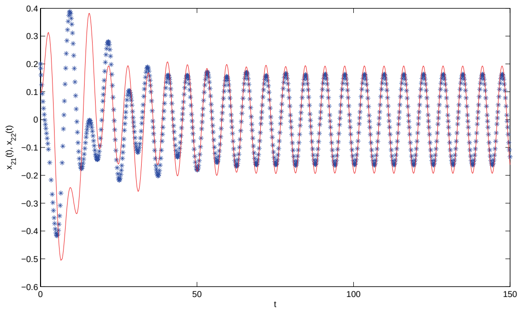

Fig. 2. Transient response of state variables $x_{21}(t)$ and $x_{22}(t)$, where the blue line stands for $x_{21}(t)$ and the red line stands for $x_{22}(t)$.

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