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Applications of Cumulants to Array Processing—Part VI: Polarization and Direction of Arrival Estimation with Minimally Constrained Arrays

Egemen Gönen and Jerry M. Mendel

***Abstract*—A fourth-order statistics-based method is presented for joint estimation of polarization and direction of arrival parameters of as many as $M-1$ narrowband signals with an M-element array having $M-3$ elements that are of *arbitrary* and *unknown* response and geometry and a subarray consisting of *three* short dipole antennas configured in a certain fashion. The method is computationally efficient and offers considerable savings in hardware over a recently published second-order statistics-based method.**

***Index Terms*— Antenna arrays, cumulants, direction of arrival estimation, higher order statistics, polarization, short dipole.**

I. INTRODUCTION

The problems of estimating direction-of-arrival and polarization parameters of diversely polarized multiple cochannel signals have been considered in various works [2], [6]–[8], [10]. In all of these methods, it was assumed that the antenna array manifold is either *known* or obtained through array calibration. In [7], Li and Compton used ESPRIT [11] to estimate direction-of-arrival and polarization parameters of multiple signals. ESPRIT does not require a known array manifold or calibration; however, it is applicable only to antenna arrays having a special structure called *displacement invariance*. As a consequence, in Li and Compton's method, to estimate parameters of at most $M-1$ signals, it is required that the array be a $2M$ element ULA consisting of M-pairs of crossed dipoles.

In this correspondence, we show that using fourth-order statistics, both directions-of-arrival and polarization parameters of at most $M-1$ multiple cochannel signals can be estimated using an M-element array having $M-3$ elements that are of *arbitrary* and *unknown* response and geometry and a subarray consisting of *three* short dipole antennas displaced in space and configured in a certain fashion. Our approach is different from existing ones in a way that it is applicable to minimally constrained arrays. There is no comparable method in the literature. For a totally linear array, our method requires 50% less hardware than Li and Compton's.

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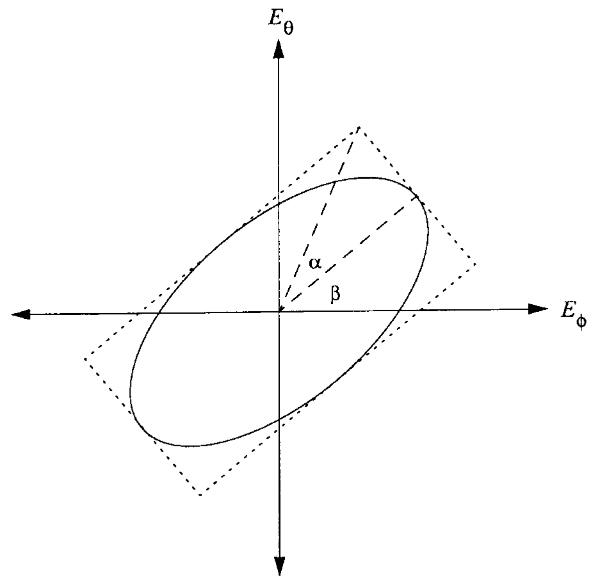

Fig. 1. Typical polarization ellipse.

In Section II, the problem is formulated. We propose a solution in Section III. Section IV provides a simulation experiment. Conclusions are presented in Section V.

II. FORMULATION OF THE PROBLEM

Suppose there are P elliptically polarized wavefronts $\{s_1(t), \cdots, s_P(t)\}$ from statistically independent non-Gaussian sources impinging on a planar array of M antennae from directions $\{\phi_1, \cdots, \phi_P\}$ in the same plane as the array. Let $\boldsymbol{r}(t)$ be the M-vector representing the signal received by the antenna array. Then, $\boldsymbol{r}(t)$ is expressed by the measurement equation $\boldsymbol{r}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) = \sum_{p=1}^{P} \boldsymbol{a}_p s_p(t) + \boldsymbol{n}(t)$, where $\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_P]$ is an $M \times P$ steering matrix whose columns represent the (unknown) responses of the subarray to the incoming wavefronts, and $\boldsymbol{s}(t)$ is the P vector of the sources signals $\{s_i(t)\}_{i=1}^{P}$. Our assumptions are as follows.

1) $\{s_i(t)\}_{i=1}^{P}$ are non-Gaussian, statistically independent, and have nonzero fourth-order cumulants.
2) $\boldsymbol{n}(t)$ is a Gaussian noise process that may have arbitrary and unknown cross-statistics and is statistically independent of $\boldsymbol{s}(t)$.
3) The columns of $\boldsymbol{A}$ are linearly independent for the given direction of arrival and polarization parameters; this nonambiguity assumption is common in array processing.

Polarization of a transverse electromagnetic (TEM) wave is characterized by the ellipse traced by the extremity of its electric field vector as time progresses. Polarization is classified as linear, circular, or elliptical. If the electric field vector as a function of time is always directed along a line, the field is said to be linearly polarized. Linear and circular polarizations are special cases of elliptical polarization. A typical polarization ellipse is shown in Fig. 1. The polarization ellipse is defined by two constants, namely, the ellipticity angle α and the orientation angle β. For a given polarization, specified by α and β, the electric field vector can be written [1] as $\boldsymbol{e} = E_\phi\ \boldsymbol{e}_\phi + E_\theta\ \boldsymbol{e}_\theta$, where its components E_ϕ and E_θ, are given by $E_\phi = E\ \cos\gamma$ and

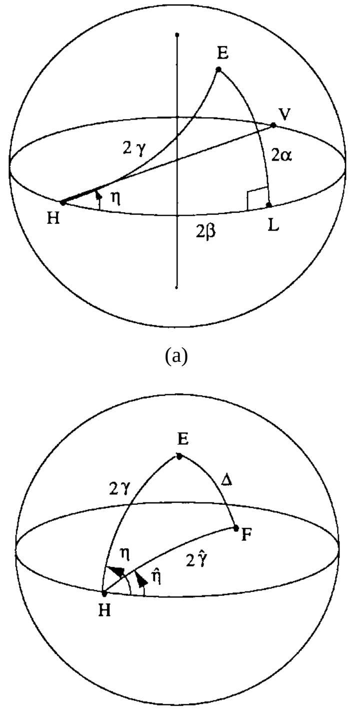

Fig. 2. (a) Poincare sphere. Any polarization $(\alpha,\, \beta)$ is represented by a point on the Poincare sphere with coordinates $(\alpha,\, \beta)$. The relationship between $(\gamma,\, \eta)$ and $(\alpha,\, \beta)$ is easily seen on the sphere. The points L, H, V, and E correspond to linear, horizontal, vertical, and elliptical polarizations, respectively. (b) Polarization error on the Poincare sphere. E represents the actual value of polarization. F represents the estimate of E.

$E_\theta = E \sin \gamma e^{j\eta}$ in which E is the electric field amplitude, and the parameters α and β can be expressed in terms of γ and η as $\tan 2\beta = \tan 2\gamma \cos \eta$ and $\sin 2\alpha = \sin 2\gamma \sin \eta$. The ranges of $\alpha,\, \beta,\, \gamma$ and η are defined as $-\pi/4 \le \alpha \le \pi/4,\, 0 \le \beta < \pi,\, 0 \le \gamma \le \pi/2$ and $-\pi \le \eta < \pi$. The polarization parameters are conveniently displayed on the Poincare sphere [1] as in Fig. 2(a).

The electric field vector of the wave arriving at the array from direction ϕ (measured with respect to a suitable reference direction) can be expressed in rectangular coordinates as $\boldsymbol{e} = -E \cos \gamma \sin \phi \boldsymbol{e}_x + E \cos \gamma \cos \phi \boldsymbol{e}_y - E \sin \gamma e^{j\eta} \boldsymbol{e}_z$. Consequently, a plane wave impinging on the array is characterized uniquely by the four parameters $\{\phi,\, E,\, \alpha,\, \beta\}$.

The problem of interest is to estimate the parameters $\{\phi_p,\, \alpha_p,\, \beta_p\}_{p=1}^P$ of the source signals given N snapshots received by the array. The parameters $\{E_p\}_{p=1}^P$ are not needed because they are not useful for discriminating the sources.

III. New Solution

Consider an M-element array consisting of three short dipole antennas and an $M-3$ element arbitrary subarray. Assume that two of the dipoles are crossed and that the third dipole is placed in parallel to either of the other two at a known distance, as shown in Fig. 3. The other $M-3$ elements may have *arbitrary* and *unknown* responses and locations.

Since the dipoles are assumed to be short, the measurement from each dipole is proportional to the electric field component along the dipole [7]; therefore, the measurement from the first and third dipoles will be proportional to the z component of the electric field, whereas the measurement from the second dipole will be proportional to the x component of the electric field. By considering the separation between the first and third sensors, the received signals at the first three sensors are given $r_1(t) = -\sum_{p=1}^P \sin \gamma_p e^{j\eta_p} s_p(t)$, $r_2(t) = -\sum_{p=1}^P \cos \gamma_p \sin \phi_p s_p(t)$, and $r_3(t) = -\sum_{p=1}^P \sin \gamma_p e^{j\eta_p} e^{-j(2\pi d/\lambda) \cos \phi_p} s_p(t)$, where $s_p(t) = E_p a_p(t) e^{j(w_c t + \theta_p)}$, in which $a_p(t)$ is the modulating signal, and θ_p is the carrier phase. The modulating signal $a_p(t)$ is assumed to be non-Gaussian, which is a valid assumption for communication signals.

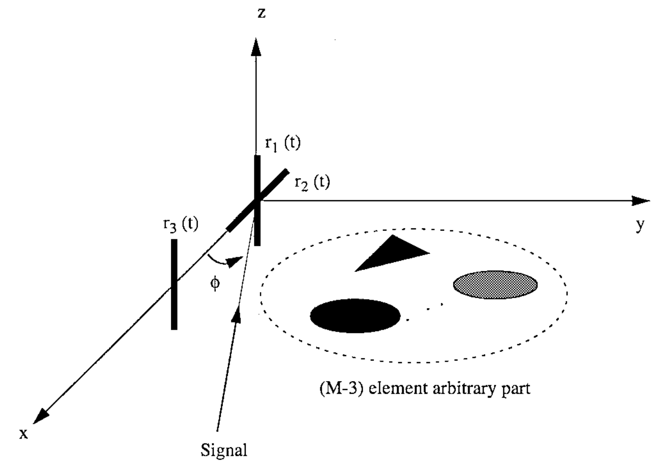

Fig. 3. Array structure.

The geometry of the assumed three-element subarray leads to three fourth-order statistics-based invariance properties that may be exploited by the ESPRIT algorithm to jointly estimate the arrival angles and polarizations. We use fourth-order cumulants as they are typically the least-order nonzero cumulants of communication signals. Odd-order cumulants of communication signals are generally zero because these types of signals are symmetrically distributed.

Before presenting our approach, we adopt the following notation for fourth-order cumulant matrices. Given two scalar processes $x_1(t)$ and $x_2(t)$ and an M-vector process $\boldsymbol{y}(t)$, we define $\text{cum}(x_1(t),\, x_2(t),\, \boldsymbol{y}(t),\, \boldsymbol{y}^H(t))$ as the $M \times M$ matrix whose ijth entry is $\text{cum}(x_1(t),\, x_2(t),\, y_i(t),\, y_j^*(t))$, where $y_i(t)$ and $y_j(t)$ are the ith and jth components of $\boldsymbol{y}(t)$, respectively.

Consider first an $M \times M$ fourth-order cumulant matrix of the signals $r_1(t)$ and $\boldsymbol{r}(t)$ formed as follows:

$$
\begin{aligned}
\boldsymbol{C}_0 &\triangleq \text{cum}(r_1(t), r_1^*(t), \boldsymbol{r}(t), \boldsymbol{r}^H(t)) \\
&= \sum_{i=1}^P \sum_{j=1}^P \sum_{k=1}^P \sum_{l=1}^P \text{cum}(\sin \gamma_i e^{j\eta_i} s_i(t) \\
&\quad \sin \gamma_j e^{-j\eta_j} s_j^*(t), \boldsymbol{a}_k s_k(t), \boldsymbol{a}_l^H s_l^*(t)) \\
&= \sum_{i=1}^P \text{cum}(\sin \gamma_i e^{j\eta_i} s_i(t), \sin \gamma_i e^{-j\eta_i} s_i^*(t), \boldsymbol{a}_i s_i(t), \boldsymbol{a}_i^H s_i^*(t)) \\
&= \sum_{i=1}^P \sin^2 \gamma_i \boldsymbol{a}_i \boldsymbol{a}_i^H \mu_{4,i} \\
&= \boldsymbol{A}\boldsymbol{\Lambda}\boldsymbol{A}^H
\end{aligned} \tag{1}
$$

where $\{\mu_{4,i}\}_{i=1}^P$ are the fourth-order cumulants of the source signals that are assumed to be nonzero, and $\boldsymbol{\Lambda} \triangleq \text{diag}\{\mu_{4,1} \sin^2 \gamma_1,\, \cdots,\, \mu_{4,P} \sin^2 \gamma_P\}$, which is nonsingular, provided $\gamma_i \neq 0$[1] $i = 1,\, \cdots,\, P$. In deriving (1), we used the

[1]Note that the requirement of $\gamma_i \neq 0$ is merely a result of the geometry displayed in Fig. 3, which was chosen for demonstration purposes. In fact, if the third sensor is placed in parallel with the second sensor, the case of $\gamma_i = 0$

cumulant properties $[\mathbf{CP1}]$, $[\mathbf{CP3}]$, $[\mathbf{CP5}]$ and $[\mathbf{CP6}]$ in [9], the facts that cumulants of Gaussian processes are zero, and that cumulants of independent processes are delta functions.

Consider next the $M \times M$ cumulant matrices $\boldsymbol{C}_1 \triangleq \text{cum}(r_1(t), r_2^*(t), \boldsymbol{r}(t), \boldsymbol{r}^H(t)) = \boldsymbol{A\Phi}_1\boldsymbol{\Lambda A}^H$ and $\boldsymbol{C}_2 \triangleq \text{cum}(r_1(t), r_3^*(t), \boldsymbol{r}(t), \boldsymbol{r}^H(t)) = \boldsymbol{A\Phi}_2\boldsymbol{\Lambda A}^H$, where $\boldsymbol{\Phi}_1 \triangleq \text{diag}\{(\sin\phi_1/\tan\gamma_1)e^{j\eta_1}, \cdots, (\sin\phi_P/\tan\gamma_P)e^{j\eta_P}\}$, and $\boldsymbol{\Phi}_2 \triangleq \text{diag}\{e^{j(2\pi d/\lambda)\cos\phi_1}, \cdots, e^{j(2\pi d/\lambda)\cos\phi_P}\}$. These equations were derived in a similar way to the derivation of $\boldsymbol{C}_0$.

The cumulant matrices $\boldsymbol{C}_0, \boldsymbol{C}_1$, and $\boldsymbol{C}_2$ possess two invariance structures characterized by $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$, which allow us to jointly estimate the polarization parameters and arrival angles of the incident waves. The diagonal matrix $\boldsymbol{\Phi}_2$ contains the arrival angles, whereas $\boldsymbol{\Phi}_1$ contains both the arrival angles and polarization parameters; hence, these parameters can be extracted from estimates of $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$. The problem is to estimate $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ using the cumulant matrices $\boldsymbol{C}_0, \boldsymbol{C}_1$, and $\boldsymbol{C}_2$. The solution to this problem is based on the idea of rotational invariance of the underlying signal subspace, which is the basis of the ESPRIT algorithm [11]. In ESPRIT, the rotational invariance of the signal subspace is induced by the translational invariance of the array, i.e., an identical copy of the array that is displaced in the space is needed. On the other hand, in our cumulant-based algorithm, the same invariance is obtained with no need for an identical copy. In ESPRIT, the signal subspace is extracted from the eigendecomposition of the covariance matrix of the concatenated measurements from the main array and its copy. Here, the signal subspace is obtained from the singular value decomposition of the $3M \times M$ concatenated matrix

$$\boldsymbol{C} \triangleq \begin{bmatrix} \boldsymbol{C}_0 \\ \boldsymbol{C}_1 \\ \boldsymbol{C}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A\Phi}_1 \\ \boldsymbol{A\Phi}_2 \end{bmatrix} \boldsymbol{\Lambda A}^H \tag{2}$$

and the matrices $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ are extracted from the signal subspace. Algorithms for solving problems exploiting models similar to (2) have been developed [13].

The polarization parameters can be determined using $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$; however, to do so, we must first find the correct pairing of the diagonal elements of $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ so that the ith diagonal elements of $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ contain parameters that belong only to the ith source ($i = 1, \cdots, P$). The pairing can be done as in [7]. Reordering the elements of $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ so that they are paired correctly, we obtain two diagonal matrices $\overline{\boldsymbol{\Phi}}_1$ and $\overline{\boldsymbol{\Phi}}_2$. The arrival angles $\phi_i, i = 1, \cdots, P$ can be determined from $\overline{\boldsymbol{\Phi}}_2$ as $\phi_i = \arccos\big((\lambda/2\pi d)\text{angle}(\overline{\boldsymbol{\Phi}}_2(i, i))\big)$, where $\overline{\boldsymbol{\Phi}}_2(i, i)$ is the ith diagonal element of $\overline{\boldsymbol{\Phi}}_2$.

The polarization parameters η_i and $\gamma_i i = 1, \cdots, P$ can then be determined as $\eta_i = \text{angle}(\overline{\boldsymbol{\Phi}}_1(i, i))$ and $\gamma_i = \arctan(\sin\phi_i/|\overline{\boldsymbol{\Phi}}_1(i, i)|)$. Finally, the polarization parameters α_i and β_i are obtained from η_i and γ_i using the relationship $\tan 2\beta = \tan 2\gamma \cos\eta$ and $\sin 2\alpha = \sin 2\gamma \sin\eta$.

IV. Simulation Experiment

This experiment demonstrates our joint DOA and polarization estimation method and evaluates its error performance. We assume four statistically independent sources having diverse polarizations. The array has five elements with the configuration in Fig. 4. The first three elements are short dipole antennas; the other two are assumed omnidirectional sensors. The arrival angles and polarization parameters of the first and second sources are arbitrarily chosen as in Table I. The measurements are contaminated by circularly symmetric white Gaussian noise that is independent of the signals with SNR = 10 dB. Our method was used to estimate the direction of arrival (ϕ) and polarization parameters (α, β) of the signals. The number of snapshots was 2000, and we performed a 100-run Monte Carlo experiment.

The means and standard deviations of the DOA and polarization estimates obtained by averaging 100 Monte Carlo runs are shown in Table II. Observe that means of the estimated parameters are very close to their actual values, and the standard deviations are low.

can be resolved; however, in this case, there would be a new requirement that $\gamma_i \neq 90$.

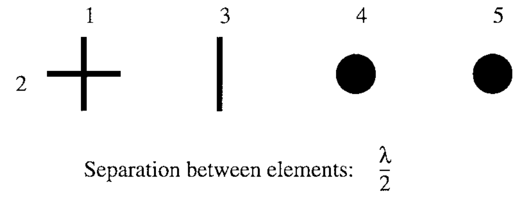

Fig. 4. Array used in the simulation.

TABLE I
ARRIVAL ANGLES AND POLARIZATION PARAMETERS (IN DEGREES) OF THE SOURCES

Source No.	*DOA*	α	β
1	−60	40	70
2	−20	10	50
3	50	−10	30
4	70	−30	10

TABLE II
SAMPLE MEANS AND STANDARD DEVIATIONS OF THE BEARING AND POLARIZATION PARAMETER ESTIMATES. SNR =10 db

	DOA		α		β	
Source No.	*mean*	*std*	*mean*	*std*	*mean*	*std*
1	−59.9	0.66	40.1	0.98	70.4	3.26
2	−19.9	0.66	9.7	1.17	49.65	4.25
3	49.7	1.74	−10.12	1.10	29.16	1.83
4	69.60	3.20	−29.75	1.0	9.89	2.34

V. Conclusions

We have presented a new method to estimate both arrival angles and polarization parameters of narrowband cochannel signals that is applicable to any arbitrary array of unknown geometry and response, provided there exists a subarray consisting of three dipoles arranged in a fashion described in the correspondence and depicted in Fig. 3. With our method, parameters of $M - 1$ signals can be estimated using an M-element array. This represents a 50% savings in hardware over the recently published method in [7]. Our solution requires estimation of three cumulant matrices followed by steps that are much like those of the ESPRIT algorithm.

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On the Virtual Array Concept for the Fourth-Order Direction Finding Problem

Pascal Chevalier and Anne Férréol

***Abstract*—For more than a decade, fourth-order (FO) direction finding (DF) methods have been developed for non-Gaussian signals. Recently, it has been shown, through the introduction of the *virtual cross-correlation* (VCC) concept, that the use of FO cumulants for the DF problem increases the effective aperture of an arbitrary antenna array, which eventually introduces the *virtual array* concept. The purpose of this correspondence is first to present this virtual array (VA) concept through an alternative way that is easier and more direct to handle than the VCC tool and, second, to present further results associated with this concept, not only for arrays with space diversity but also for arrays with angular and/or polarization diversity.**

***Index Terms*—Angular and polarization diversity, fourth-order direction finding, virtual array, virtual cross-correlation.**

I. Introduction

Up to the middle of the 1980's, the DF methods exploited only the information contained in the second-order (SO) statistics of the observations. However, for more than a decade, DF methods exploiting the information contained in the FO statistics of the data have been developed for non-Gaussian signals. Most of these techniques, such as the fourth MUSIC [1], [8] or the fourth ESPRIT methods [5], are FO extensions of SO techniques, although a new concept of higher order (HO) DF has been presented recently in [3].

However, although promising for some applications, relatively few papers have been devoted to the performance analysis of these FO DF methods. Among these scarce papers, we find, in particular, [1], [2], and [8], which present either analytic or simulation results about the performance of the fourth MUSIC method.

Recently, a new light on these methods and on their potential performance has been given in [7] where it has been shown, for arrays with space diversity and through the introduction of the *virtual cross-correlation* (VCC) concept, that the use of FO cumulants for the DF problem increases the effective aperture of an arbitrary antenna array; this eventually introduces the *virtual array* (VA) concept. This new concept allows physical interpretations of rather abstract HO algebraic results and makes it possible to predict performances of the FO DF methods. Nevertheless, although it is very interesting and really pertinent, the VCC concept may seem to be relatively difficult to handle by nonspecialists.

In this context, the purpose of this correspondence is first to present the VA concept by an alternative way that is easier and more direct to handle than that using the VCC tool and, second, to present further results associated with this concept, concerning not only arrays with space diversity but also arrays with polarization and/or angular diversity. Note that some of the results presented in this correspondence have been presented for the first time in [4], whereas the existence of [7] was not known by the authors.

II. Hypotheses and Notations

In this correspondence, we consider an array of N narrowband (NB) sensors, and we call $\boldsymbol{x}(t)$ the vector of the complex amplitudes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of P stationary and statistically independent NB sources corrupted by a noise. Under these assumptions, the observation vector can be written as

$$\boldsymbol{x}(t) = \sum_{i=1}^{P} m_i(t)\boldsymbol{a}(\theta_i, \varphi_i) + b(t) \triangleq A\boldsymbol{m}(t) + \boldsymbol{b}(t) \tag{1}$$

where

- $\boldsymbol{b}(t)$ noise vector;
- $\boldsymbol{m}(t)$ vector whose components $m_i(t)$ are the complex amplitudes of the sources;
- θ_i azimuth of the source i (Fig. 1);
- φ_i elevation angle of the source i (Fig. 1);
- A $(N \times P)$ matrix of the source steering vectors $\boldsymbol{a}(\theta_i, \varphi_i)$, which contains in particular the information about the direction of arrival of the sources.

In particular, the component n of vector $\boldsymbol{a}(\theta_i, \varphi_i)$, which is noted as $a_n(\theta_i, \varphi_i)$, can be written, in the general case of an array with space, angular, and polarization diversity, as [6]

$$\begin{aligned} a_n(\theta_i, \varphi_i) = f_n(\theta_i, \varphi_i, p_i)\exp\{j2\pi[x_n\cos(\theta_i)\cos(\varphi_i) \\ + y_n\sin(\theta_i)\cos(\varphi_i) + z_n\sin(\varphi_i)]/\lambda\} \end{aligned} \tag{2}$$

where λ is the wavelength, (x_n, y_n, z_n) are the coordinates of sensor n of the array, and $f_n(\theta_i, \varphi_i, p_i)$ is a complex number corresponding to the response of sensor n to a unit electric field coming from the direction (θ_i, φ_i) and having the state of polarization p_i (characterized by two angles in the wave plane as shown in Section IV-C) [6]. Let us recall that an array of sensors has space

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