Asymptotic Tracking Position Control for Nonlinear Systems using BacksteppingTechnique

S. Md. Rozali[a,*], M.Nizam Kamarudin[a], M. F. Rahmat[b], A. Rashid Husain[b]

[a]Faculty of Electrical Engineering
Universiti Teknikal Malaysia Melaka, Malacca, Malaysia
[b]Faculty of Electrical Engineering
Universiti Teknologi Malaysia
Johore, Malaysia

Abstract

Backstepping is a technique developed for designing stabilizing controls for a special class of nonlinear dynamical systems. It is built from subsystems that radiate out from an irreducible system which can be stabilized using some other method. This method is based directly on the mathematical model of the examined system. It is developed by introducing new variables into it in a form depending on the state variables, controlling parameters and stabilizing functions. The stabilizing functions compensate for nonlinearities exists in the system which affect the stability of its operation. Backstepping can be used for tracking and regulation problem. With the aid of Lyapunov stability design, this paper presents control approach for asymptotical tracking. Electrohydraulic actuator system is chosen as numerical example for the designed controller. The performance of backstepping controller is then compared with unity feedback controller applied to the same system. The results show that backstepping controller produced better output tracking then unity feedback controller.

1. Introduction

Usually, linearization method is used in ordinary feedback-based systems to eliminate the nonlinearities existing in the system. On the other hand, backstepping method makes it possible to create additional nonlinearities and introduce them into the control process to cancel out undesirable nonlinearities from the system. The backstepping method permits to obtain global stability in the cases when the feedback linearization method only secures local stability[1]. The fundamental concept of backstepping method is introduced by Kristic et al in their book[2]. This book focused on adaptive and nonlinear control design of SISO system and some of MIMO system. The extension of this book, the approach focusing on the stabilisation problem in stochastic nonlinear systems is developed[3]. The backstepping control method is also presented in [4]. [4]explained this technique in detail for regulating and tracking problem.

---

* Corresponding author. *E-mail address:* sahazati@utem.edu.my

The backstepping method was used in numerous applications such as flight trajectory control, in the industrial systems, electric machines, nonlinear systems of wind turbine-based power production and robotic system. Besides, backstepping also is the effective tool in adaptive control design for estimating parameters[5] and optimal control problems. This is because,thebackstepping control algorithms allow the developing of robust nonlinear controller. The observer based on backstepping technique is designed for force control of electrohydraulic actuator system[6]. This control strategy guarantees the convergence of the tracking error. This control technique also used as observer to control DC servo motor[7]. This control approach for this system is developed by combining backstepping observer with adaptive and sliding mode controller. Backstepping approach is also proposed to design controller for electro-hydraulic active suspension system[8].Besides, this technique enhanced the power system stability by using particle swarm optimization tool to obtain the best value for its tuning parameters[9]. It is proved that the technique improve the transient stability and damping presented in the system. Other than that, backstepping technique is also used to design controller for permanent magnet synchronous motor[10]. The performance of backstepping controller also depends on its gain or controller parameter. In this paper, the controller parameters are tuned manually.

2. Backstepping Controller

Backstepping is one of the recursive design controller. The controller is designed by step back toward the control input starting with the scalar equation which is separated from it by the largest number of integrations. Because of this recursive structure, the design process of this controller starts at the known-stable system and back out new controllers that progressively stabilize each outer subsystem. The process will terminates when the final external control is reached. Without uncertainties, backstepping can be used to force a nonlinear system to behave like a linear system in a new set of coordinates[2]. The advantage of backstepping is that is has the availability to avoid cancellations of useful nonlinearities and pursue the objectives of stabilization and tracking rather than that of linearization method. In addition, backstepping approaches relaxes the matching conditions on perturbations. It means that the perturbation doesn't have to appear in the equation that contains the input of the system.For tracking problem, backstepping always use the error between the actual and desired input to start the design process. Consider a general class of nonlinear affine system

$$\dot{x} = F(x) + G(x)u \tag{1}$$

Where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are states vector and control vector respectively. $F(\cdot)$ and $G(\cdot)$ are smooth vector fields of appropriate dimensions. The main idea of backstepping is such that every step of backstepping adds a new step in the construction of a control lyapunov function (CLF) by the augmentation of the starting CLF with a term which penalises the error between a new state variable and its desired value. Thus, finally the method will guarantee the references asymptotic stable tracking and a transient performance. The construction of CLF in backstepping can be made negative definite by a variety of control laws rather than by a specific control law.Referring to definition in [4], for system

$$\dot{x} = f(x) \tag{2}$$

Where

$f: D \longrightarrow \mathbb{R}^n$is a locally Lipschitz map from a domain $D \subset \mathbb{R}^n$ into $\mathbb{R}^n$.

Let $x = 0$ be an equilibrium point for (3) and $D \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V: D \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$V(0) = 0 \text{ and } V(x) > 0 \text{ } in \text{ } D - \{0\} \tag{3}$$

and

$$\dot{V}(x) < 0 \text{ } in \text{ } D - \{0\} \tag{4}$$

Then $x = 0$ is asymptotically stable.

3. Numerical example

By considering state-space equation of electrohydraulic actuator (EHA) system as given by

$$\dot{x}_1 = x_2 \tag{5a}$$

$$\dot{x}_2 = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{S}{m}x_3 \tag{5b}$$

$$\dot{x}_3 = -\frac{S}{k_c}x_2 - \frac{k_1}{k_c}x_3 + \frac{c}{k_c}\sqrt{\frac{P_a - x_3}{2}}k_v u \tag{5c}$$

With

$$c = c_d w \sqrt{\frac{2}{\rho}},$$

where
x_1 = displacement of the load,
x_2 = load velocity,
x_3 = pressure difference $(p_1 - p_2)$ between the cylinder chambers caused by load.

Define the error for each state x_1, x_2 and x_3 respectively as

$$e_1 = x_1 - x_{1d} \tag{6a}$$
$$e_2 = x_2 - x_{2d} \tag{6b}$$
$$e_3 = x_3 - x_{3d} \tag{6c}$$

Where
x_{1d} = reference input,
x_{1d} and x_{2d} = virtual control,
e_i = error for each state, $\forall i = 1,2,3$.

Let the Lyapunov function

$$V_1 = \frac{\rho_1}{2}e_1^2 \tag{7}$$

The derivative of (7) along the system (5) is

$$\dot{V}_1 = \rho_1 e_1(e_2 + x_{2d} - \dot{x}_{1d}) \tag{8}$$

Choose virtual control variable as

$$x_{2d} = \dot{x}_{1d} - k_1 e_1 \tag{9}$$

Thus, $\dot{V}_1$ becomes

$$\dot{V}_1 = -k_1\rho_1 e_1^2 + \rho_1 e_1 e_2 \tag{10}$$

A new Lyapunov function V_2 is defined as

$$V_2 = V_1 + \frac{\rho_2}{2}e_2^2 \tag{11}$$

Taking the derivative of (11) along the system (5) gives

$$\dot{V}_2 = -k_1\rho_1 e_1^2 + e_2\left[\rho_1 e_1 - \frac{\rho_2}{m}(kx_1 + fx_2 - Se_3 - Sx_{3d}) - \rho_2\dot{x}_{2d}\right] \tag{12}$$

Choose

$$x_{3d} = \frac{1}{S}[kx_1 + fx_2 - \frac{\rho_1}{\rho_2}me_1 + mx_{2d}' - \frac{k_2}{\rho_2}me_2] \tag{13}$$

Thus, V_2' becomes

$$V_2' = -k_1\rho_1 e_1^2 - k_2 e_2^2 + \frac{\rho_2}{m}Se_2 e_3 \tag{14}$$

Finally V_3 is defined as

$$V_3 = V_2 + \frac{\rho_3}{2}e_3^2 \tag{15}$$

Taking derivative of (15) along the system (5)

$$V_3' = -k_1\rho_1 e_1^2 - k_2 e_2^2 + e_3[\frac{\rho_2}{m}Se_2 - \frac{\rho_3}{.}Sx_2 \ldots - \frac{\rho_3}{.}k_1 x_3 + \frac{\rho_3}{.}C\sqrt{\frac{P_a - x_3}{.}}k_v u - \rho_2 x_{3d}'] \tag{16}$$

By assuming equation (5) with nonsaturating load, the best and asymptotically stabilized position tracking of equation (5) with respect to the desired input can be achieved by the feedback

$$u = \frac{k_c}{\rho_3 ck_v}\sqrt{\frac{2}{P_a - x_3}}[-\frac{\rho_2}{m}Se_2 + \frac{\rho_3}{k_c}Sx_2 + \frac{\rho_3}{k_c}k_1 x_3 \ldots + \rho_3 x_{3d}' - k_3 e_3] \tag{17}$$

Thus, (18) becomes

$$V_3' = -k_1\rho_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{18}$$

Equation (10), (14) and (18) proved the stability of the system through the designed controller. This is because all these equations are negative definite which fulfilled the requirement for the system to be stable. k_1, k_2 and k_3 are the controller parameters.

The tasks of stabilization, tracking and disturbance rejection lead to a number of control problems. Besides, there are additional goals for the design such as meeting certain requirements on the transient response or certain constraints on the control input. When model uncertainty is taken into consideration, issues of sensitivity and robustness come into play. A robust control design tries to meet the control objective for any model within range of uncertainty. The tracking problem is focused in this paper so that the output of the system followed the reference input given.

4. Simulation Result

Electrohydraulic actuator system (EHA) which is represented by equation (5) and control signal in equation (17) is injected with step and sine input. Parameter for EHA system and controller is given in Table 1[11].

Table 1. Parameter Of Eha System And Backstepping Controller

Load at the EHS rod	$0.33Ns^2/cm$
Piston area, S	$10cm^2$
Coefficient of viscous friction, f	$27.5Ns/cm$
Coefficient of aerodynamic elastic force, k	$1000N/cm$
Valve port width, w	$0.05cm$
Supply pressure, P_a	$2100N/cm^2$
Coefficient of volumetric flow of the valve port, c_d	0.63
Coefficient of internal leakage between the cylinder chambers, k_l	$2.38 \times 10^{-3} cm^5/Ns$
Coefficient of servo valve, k_v	$0.017cm/V$
Coefficient involving bulk modulus and EHA volume, k_c	$2.5 \times 10^{-4} cm^5/N$
Oil density, ρ	$8.87 \times 10^{-7} Ns^2/cm^4$
Controller parameter, k_1	400

Controller parameter, k_2	40
Controller parameter, k_3	4000
Controller parameter, ρ_1	400
Controller parameter, ρ_2	0.033
Controller parameter, ρ_3	1

Fig. 1 and Fig. 2 respectively illustrate the position x_1 for the system with step and sine input given while Fig. 3 and Fig. 4 show the tracking error, e with each input. In order to observe the performance of EHA in equation (5), backstepping controller with feedback control signal in equation (17) is compared with unity feedback signal applied on the same system. The performance of EHA system with unity feedback signal is shown in Fig. 5 and Figure 6 with step and sine input respectively. Fig. 7 and Fig. 8 revealed position tracking error for each input given to system.
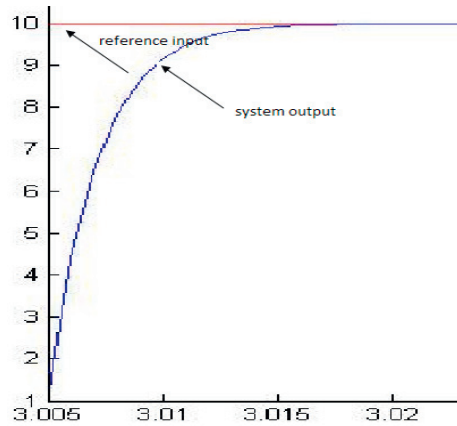
Fig. 1. Position x_1 in response to step input
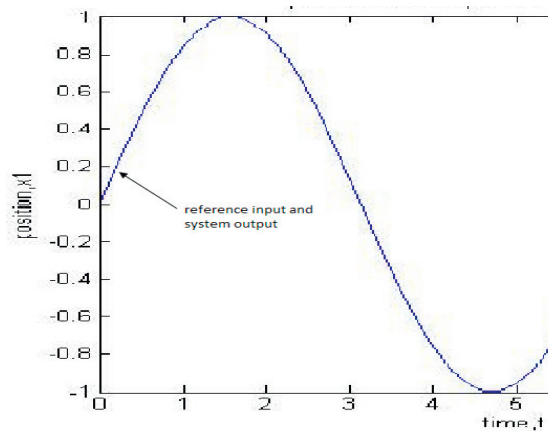
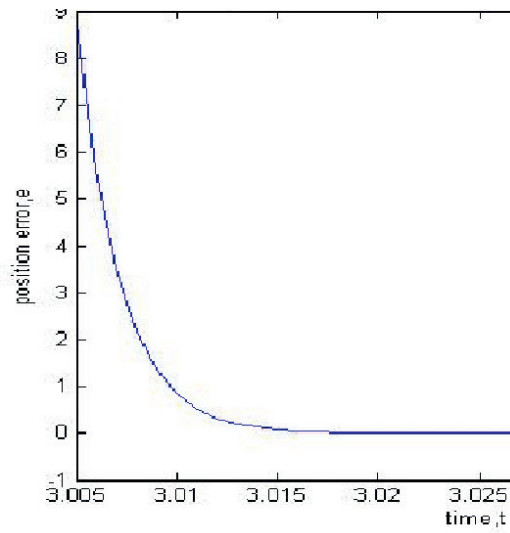Fig. 2. Position x_1 in response to sine input

Fig. 3. Position tracking error e in response to step input
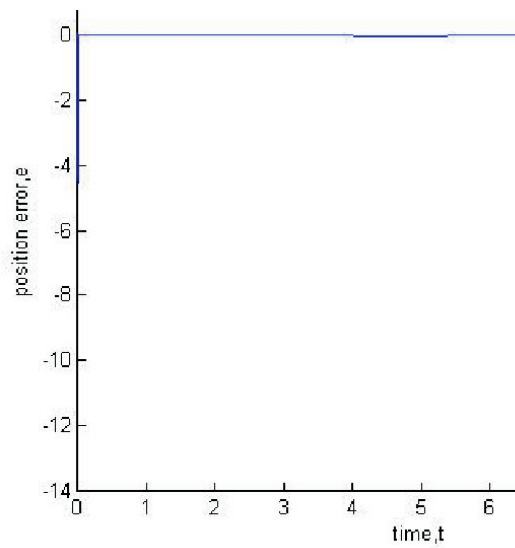

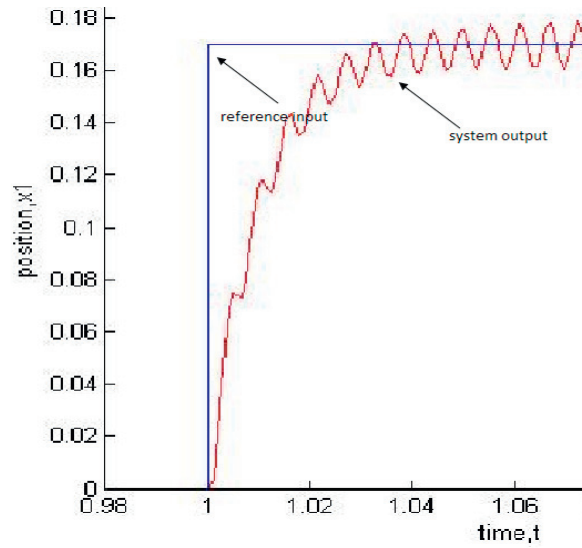Fig. 4. Position tracking error e in response to sine input
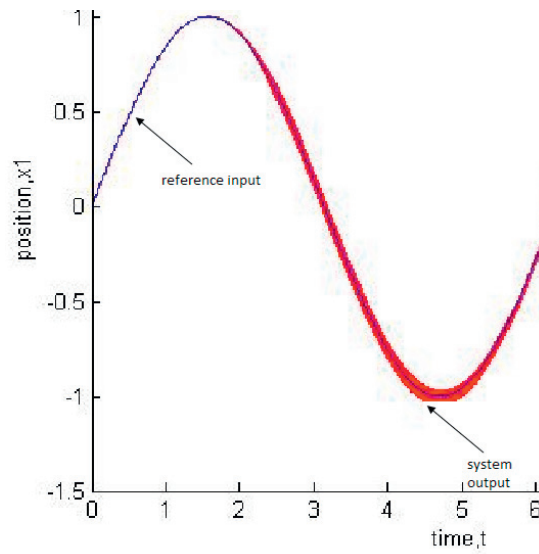
Fig. 5. Position x_1 in response to step input

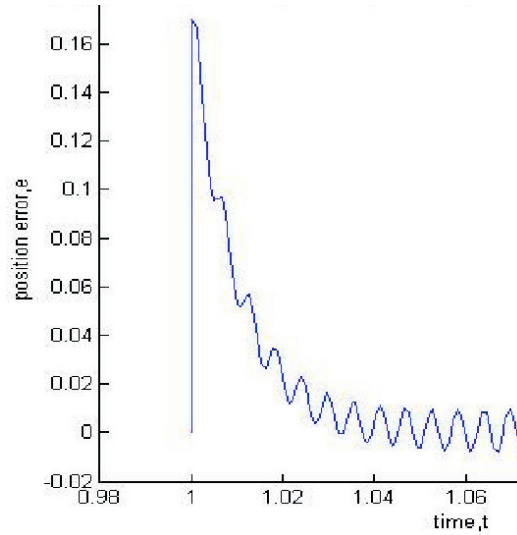Fig. 6. Position x_1 in response to sine input

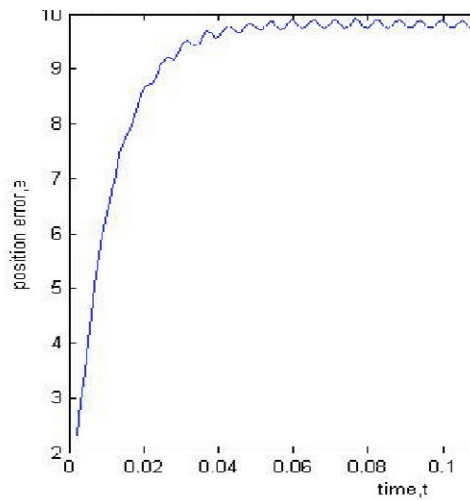Fig. 7. Position tracking error **e** in response to step input

Fig. 8. Position tracking error **e** in response to sine input

5. Conclusion

Backstepping controller improves the tracking performance of the system compared to unity feedback controller in terms of tracking error. Via Lyapunov stability and design technique, backstepping control ensures the output globally asymptotically tracks a reference signal with smaller error.

Acknowledgment

References

[1] M. T. Anna Witkowska, Roman Smierzchalski, "A Backstepping Approach To Ship Course Control," *International Journal of Applied Mathematic, Computer and Science,* vol. 17, pp. 73-85, 2007.

[2] K. I. Kristic M, Kokotovic P.V., "Nonlinear and Adaptive Control Design," 1995.

[3] D. H. Kristic m, "Stabilisation of Nonlinear Uncertain Systems," 1998.

[4] H. K.Khalil. (2002). *Nonlinear Systems (Third ed.)*.

[5] Z. E. Fang y., Queiroz M.S. and Dawson D.M., "Global Output Feedback Control of Dynamically Positioned Surface Vessels : An Adaptive Control Approach," *Mechatronics,* vol. 14, pp. 341-356, 2004.

[6] P. Nakkarat and S. Kuntanapreeda, "Observer-based backstepping force control of an electrohydraulic actuator," *Control Engineering Practice,* vol. 17, pp. 895-902, 2009.

[7] B. M. D. A.farrokh Payam, "Nonlinear Sliding Mode Controller for Sensorless Speed Control of DC Servo Motor Using Adaptive Backstepping Observer," *IEEE Transactions,* 2006.

[8] J.-P. K. Claude Kaddissi, Maarouf Saad, "Draw by Wire control of an Electro-Hydraulic Active Suspension a Backstepping Approach," *IEEE Conference on Control Applications,* 28-31 August,2005.

[9] A. Karimi*, et al.*, "Power system stability enhancement using backstepping controller tuned by particle swarm optimization technique," in *Power Engineering Society General Meeting, 2005. IEEE*, 2005, pp. 1388-1395 Vol. 2.

[10] Y. Ming*, et al.*, "Adaptive backstepping controller design for permanent magnet synchronous motor," in *Intelligent Control and Automation (WCICA), 2010 8th World Congress on*, 2010, pp. 4968-4972.

[11] I. Ursu*, et al.*, "Backstepping design for controlling electrohydraulic servos," *Journal of the Franklin Institute,* vol. 343, pp. 94-110, 2006.