

## A hybrid method for solving multi-objective global optimization problems

C. Gil · A. Márquez · R. Baños · M. G. Montoya ·  
J. Gómez

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**Abstract** Real optimization problems often involve not one, but multiple objectives, usually in conflict. In single-objective optimization there exists a global optimum, while in the multi-objective case no optimal solution is clearly defined but rather a set of optimums, which constitute the so called Pareto-optimal front. Thus, the goal of multi-objective strategies is to generate a set of non-dominated solutions as an approximation to this front. However, most problems of this kind cannot be solved exactly because they have very large and highly complex search spaces. The objective of this work is to compare the performance of a new hybrid method here proposed, with several well-known multi-objective evolutionary algorithms (MOEA). The main attraction of these methods is the integration of selection and diversity maintenance. Since it is very difficult to describe exactly what a good approximation is in terms of a number of criteria, the performance is quantified with adequate metrics that evaluate the proximity to the global Pareto-front. In addition, this work is also one of the few empirical studies that solves three-objective optimization problems using the concept of global Pareto-optimality.

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C. Gil (✉) · A. Márquez · R. Baños · M. G. Montoya  
Dpt. de Arquitectura de Computadores y Electrónica, Universidad de Almería, Cañada de San Urbano, 04120 Almería, Spain  
e-mail: cgil@ace.ual.es

J. Gómez  
Dpt. Lenguajes y Computación, Universidad de Almería, Cañada de San Urbano, 04120 Almería, Spain  
e-mail: jgomez@ual.es

A. Márquez  
e-mail: amarquez@ace.ual.es

R. Baños  
e-mail: rbanos@ace.ual.es

M. G. Montoya  
e-mail: mari@ace.ual.es

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## 1 Introduction

The aim of global optimization (GO) is to find the best solution of decision models, in presence of multiple local solutions [1]. In recent years, researchers have begun to apply different GO techniques to problems that occur in the analysis and solution of multi-objective linear and non linear programming problems [2,3]. However, when the problem has several objective functions, the notion of optimum changes. Classical optimization methods (including multi-criteria decision-making methods) suggest converting the multi-objective optimization problem (MOP) into a single-objective optimization problem by combining the objectives in a single mathematical function [4]. However, in last two decades most papers dealing with MOPs used the concept of Pareto-dominance [5], where the goal is to find trade-off solutions (widely known as Pareto-optimal solutions) rather than a single solution. In the absence of any further information, none of these Pareto-optimal solutions is considered to be better than the others. Generating the Pareto-optimal set in complex problems is computationally expensive and often infeasible, but using heuristic methods can allow to obtain a representative sample of it.

Conceptually, evolutionary algorithms (EA) [5] base their operation on maintaining a set of solutions (individuals) which evolve in successive iterations (generations) trying to optimize an objective function. In the multi-objective context, it is necessary to design strategies that obtain good solutions in all the objectives. The main advantage of the multi-objective EAs (MOEAs) [6,7] is their ability to find multiple Pareto-optimal solutions in one single run. Over the past decade, a number of MOEAs have been suggested [8–16]. The interest in the design and implementation of hybrid meta-heuristics has increased remarkably in recent years [17]. However, to date there has been little research effort dedicated to the hybridization of meta-heuristics for multi-objective optimization. Some studies [15,16] have shown that combining ideas of different methods allows the quality of the solutions to be improved. Although many MOEAs have adequately demonstrated their ability to solve two-objective problems, there are few studies in problems with three or more objectives. For instance, Deb et al. [18] compared the performance of two Pareto-based MOEAs in some three-objective optimization problems. This paper presents a new hybrid method, msPESA, whose performance is evaluated versus other well-known MOEAs in several test functions of two and three objectives.

The remainder of the paper is organized as follows. Section 2 gives some general Multi-objective optimization concepts, and a brief overview of some well known MOEAs found in the literature. Section 3 describes the new hybrid method here proposed. Section 4 details the test problems and performance metrics used to evaluate the performance of the MOEAs described in Sects. 2 and 3. Experimental results are provided and analyzed in Sect. 5. Finally, we outline the conclusions of this paper in Sect. 6.