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IIR system identification using differential evolution with wavelet mutation

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In this paper an improved version of Differential Evolution (DE) technique called Differential Evolution with Wavelet Mutation (DEWM) is applied to the infinite impulse response (IIR) system identification problem. Instead of fixed value of scaling factor in standard DE, an iteration dependent scaling factor governed by the wavelet function during the mutation process is adopted in the proposed technique. This modification in the mutation process ensures not only the faster searching in the multidimensional search space but also the solution produced is very close to the global optimal solution. Apart from this, the proposed technique DEWM has alleviated from inherent drawbacks of premature convergence and stagnation, unlike Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). The simulation results obtained for some well known benchmark examples justify the efficacy of the proposed system identification approach using DEWM over GA, PSO and DE in terms of convergence speed, plant coefficients and mean square error (MSE) values produced for both the same order and reduced order models of adaptive IIR filters.

1. Introduction

A filter is a frequency selective device, designed and used to extract or enhance the useful portion of information from the signal according to the set values of design parameters. An adaptive system also behaves like a filter with the exception of iteration based coefficient values due to incorporation of adaptive algorithm to cope up with ever changing environmental condition and/or unknown system parameters. The adaptive algorithm varies the filter characteristic by manipulating or varying the filter coefficient values according to the performance criterion of the system. In most of the cases error between input and output signals of the unknown system is considered as the important performance criterion and adaptive filter works toward the minimization of error signal with the proper adjustment of the filter coefficients. Design of such adaptive filter may be alternatively considered as system identification problem. Adaptive filter has got a wide scope of applications in different fields such as communication, sonar, navigation, control, biomedical engineering, seismology, radar and many more. In these fields different types of applications are noticed, namely system identification, inverse system identification, prediction and array processing etc.

Finite impulse response (FIR) and infinite impulse response (IIR) filters are the two types of digital filters. For IIR filter, due to recursive nature, present output depends not only on present input but also the previous inputs and outputs. But in case of FIR filter, the present and past inputs are required to calculate the present output. Hence, more design complexity and larger memory space are demanded for IIR filter optimization problem. But an IIR filter requires lower order compared to FIR filter [1]. In the present work adaptive IIR filter is considered for identifying/modelling an unknown plant.

Previously, as a classical approach of adaptive filtering, Least Mean Square (LMS) technique and its variants are used extensively as optimization tools for adaptive filter. This high acceptance of classical optimization technique is due to the low complexity and simplicity of implementation. But the main drawback of LMS technique is its slow convergence speed to reach the optimal solution. Several measures have been reported to increase the speed [2,3].

In adaptive IIR filtering applications, non-differentiable and multimodal nature of cost function is a major point of concern. Classical optimization methods such as least mean square

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technique are gradient based optimization methods. They are incapable to handle such optimization problems due to following inherent deficiencies:

- Requirement of continuous and differentiable cost function,
- Usually converges to the local optimum solution or revisits the same suboptimal solution,
- Incapable to search the large problem space,
- Requirement of the piecewise linear cost approximation (linear programming),
- Highly sensitive to starting points when the number of solution variables is increased and as a result the solution space is also increased.

Because of the above shortfalls of classical optimization methods, heuristic and meta-heuristic evolutionary search algorithms have got attention for adaptive filtering optimization problems. Different evolutionary optimization techniques aptly used are as follows: genetic algorithm (GA) is inspired by the Darwin's "Survival of the Fittest" strategy [4]; human searching nature is mimicked in seeker optimization algorithm (SOA) [5]; the cat swarm optimization (CSO) is based upon the behaviour of cat's tracing and seeking of an object [6]; bee colony algorithm (BCA) is based upon honey searching behaviour of the bee swarm [7,8]; gravitational search algorithm (GSA) is motivated by the gravitational laws and laws of motion [9]; food searching behaviour is mimicked in bacterial foraging algorithm [10] and swarm intelligence is mimicked in particle swarm optimization (PSO) and its variants [11–20]. Conventional PSO has mimicked the behaviour of bird flocking or fish schooling [1,11,15,16,30,31]; in quantum behaved PSO (QPSO) quantum behaviour of particles in a potential well is adopted in conventional PSO algorithm [18]; in PSO with Quantum Infusion (PSO-QI), a hybridized version of PSO and QPSO in which fast convergence property of PSO and the property of convergence to a lower average error of QPSO have been combined to enhance the performance [13]. In Adaptive Inertia Weight PSO (AIW-PSO), a modified Versoria function is introduced to alter inertia weight of the basic PSO for the improvement of convergence speed and optimization efficiency of standard PSO [14]. To increase the randomness by the process of mutation, a random vector is introduced in the basic QPSO for the enhancement of global search ability [15]. Biological evolutionary strategy is adopted in the development of differential evolution (DE) algorithm [21,22].

Naturally, it is a vast area of research continuously being carried out. In this paper, the capability of global searching and finding near optimum result of GA, PSO, DE and DEWM is investigated thoroughly for GA, PSO, DE and DEWM in identifying the unknown IIR system with the help of optimally designed adaptive IIR filters of same order and reduced order as well. GA is a probabilistic heuristic search optimization technique developed by Holland [23].

PSO is swarm intelligence based algorithm developed by Eberhart et al. [24,25]. Several attempts have been taken towards the system identification problem with basic PSO and its modified versions [11–20]. The key advantage of PSO is its simplicity in computation and a few number of steps are required in the algorithm.

The DE algorithm was first introduced by Storn and Price in 1995 [21]. Like GA, it is a randomized stochastic search technique enriched with the operations of crossover, mutation and selection but unlike GA, stagnation and entrapment to local minima are not associated to it [22].

It has been realized that GA is incapable for local searching [22] in a multidimensional search space and GA, PSO and DE suffer from premature convergence and are easily trapped to suboptimal solution [8,26,27]. So, to enhance the performance of optimization algorithm in global search (exploration stage) as well as local search (exploitation stage), wavelet mutation in association with DE called differential evolution with wavelet mutation (DEWM) is prescribed by authors as an alternative technique for handling IIR system identification problem. The optimal FIR filter design problem using DEWM was reported in Ref. [28].

In this paper the performances of all the optimization algorithms are analyzed with four benchmarked IIR plants and adaptive filters of same and reduced orders. Simulation results obtained with the proposed DEWM technique are compared to those of real coded genetic algorithm (RGA), PSO, and DE to demonstrate the effectiveness and better performance of the proposed technique for achieving the global optimal solution in terms of filter coefficients and the mean square error (MSE) of the adaptive system identification problem.

The rest of the paper is organized as follows: in Section 2, mathematical expression of an adaptive IIR filter and the objective function are formulated. In Section 3, different evolutionary techniques under consideration, namely, RGA, PSO, DE and DEWM are discussed briefly for adaptive IIR filter design problem. In Section 4, comprehensive and demonstrative sets of data and illustrations are given to make a floor of comparative study among different algorithms. Finally, Section 5 concludes the paper.

2. Design formulation

The main task of the system identification is to vary the parameters of the adaptive IIR filter iteratively using evolutionary algorithms unless and until the filter's output signal matches to the output signal of unknown system when the same input signal is applied simultaneously to both the adaptive filter and unknown plant under consideration. In other way, it can be said that in the system identification, the optimization algorithm searches iteratively for the adaptive IIR filter coefficients such that the filter's input/output relationship matches closely to that of the unknown system. The basic block diagram for system identification using adaptive IIR filter is shown in Fig. 1.

This section discusses the design strategy of IIR filter. The input–output relation is governed by the following difference equation [1]:

$$y(p) + \sum_{k=1}^{n} a_k y(p-k) = \sum_{k=0}^{m} b_k x(p-k) \tag{1}$$

where $x(p)$ and $y(p)$ are the filter's input and output, respectively and $n(\geq m)$ is the filter's order. With the assumption of coefficient $a_0 = 1$, the transfer function of the adaptive IIR filter is expressed as given in Eq. (2).

$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{n} a_k z^{-k}} \tag{2}$$

In this design approach the unknown plant of transfer function $H_s(z)$ is to be identified with the adaptive IIR filter $H_{af}(z)$ in such a way so that the outputs from both the systems match closely for the given input.

In this transfer function, filter order is n and $n \geq m$. In the system identification problem mean square error (MSE) of time samples, J is considered as the objective function, also known as error fitness function, expressed as in Eq. (3).

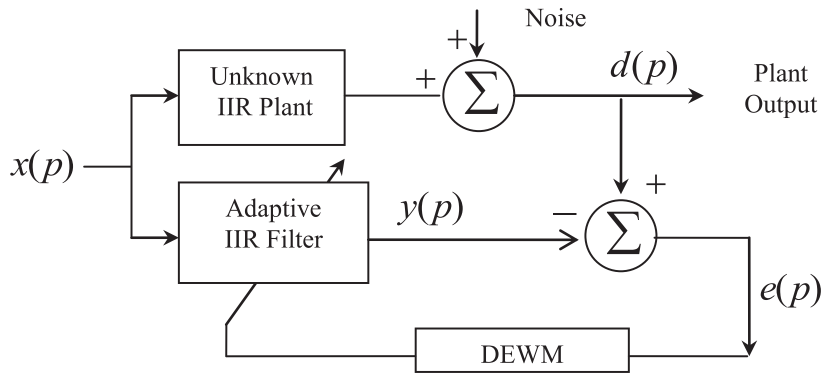

Fig. 1. Adaptive IIR filter for system identification.

$$J = \frac{1}{N_s}\sum_{p=1}^{N} e^2(p) \tag{3}$$

where the error signal is $e(p) = d(p) - y(p)$; $d(p)$ is the response of the unknown plant; $y(p)$ is the response of the adaptive IIR filter and N is the number of samples. The main objective of any evolutionary algorithm considered in this work is to minimize the value of the error fitness J with proper adjustment of coefficient vector ω of the transfer function of the adaptive filter so that output responses of filter and plant match closely and hence the error fitness function is minimized.

Here $\omega = [a_0 a_1 \ldots a_n b_0 b_1 \ldots b_m]^T$.

3. Evolutionary algorithm employed

Evolutionary algorithms stand upon the platform of meta-heuristic optimization methods, which are characterized as stochastic, adaptive and learning in order to produce intelligent optimization schemes. Such schemes have the potential to adapt to their ever changing dynamic environment through the previously acquired knowledge. Few such efficient algorithms have been discussed here for the identification of some benchmarked IIR systems.

3.1. Real coded genetic algorithm (RGA)

Standard Genetic Algorithm (also known as real coded GA) is mainly a probabilistic search technique, based on the principles of natural selection and evolution built upon the Darwin's "Survival of the Fittest" strategy [23]. Each encoded chromosome that constitutes the population is a solution to the unknown system under study. These solutions may be good or bad, but are tested rigorously through the genetic operations such as crossover and mutation to evolve a near global optimal solution to the problem at hand. Chromosomes are constructed over some particular alphabet $\{0, 1\}$, so that chromosomes' values are uniquely mapped onto the real decision variable domain. Each chromosome is evaluated by the fitness function or objective function of the corresponding optimization problem. Each chromosome has a probability of selection and has to take part in the genetic operation based upon the Roulette's wheel strategy. In the genetic operations, crossover and mutation bring the variation in alleles of gene in the chromosome population along with the alleviation of trapping to local optimal solution.

Steps of RGA as implemented for the optimization of coefficient vector ω are as follows [29,30]:

Step 1: Initialize the real coded chromosome strings (ω) of n_p population, each consisting of a number of numerator and denominator filter coefficients b_k and a_k, respectively. Coefficients are generated in random manner within the range of maximum ($h_{\max}$) and minimum ($h_{\min}$) values, ($h_{\min} = -2$, $h_{\max} = 2$); number of samples = 128; maximum iteration cycles = 200 (for Examples 1 and 3) and 300 (for Examples 2 and 4).
Step 2: Decoding of the strings and evaluation of the cost function J according to (3).
Step 3: Selection of the elite strings in order of increasing J values from the minimum value.
Step 4: Copying the elite strings over the non selected strings.
Step 5: Crossover and mutation generate offspring.
Step 6: Genetic cycle updating.
Step 7: The iteration stops when the maximum number of cycles is reached. The grand minimum error and its corresponding chromosome string or the desired solution having same or reduced number of coefficients of the adaptive IIR filter are finally obtained.

3.2. Particle swarm optimization (PSO)

PSO is a flexible, robust, population based stochastic search algorithm with attractive features of simplicity in implementation and ability to quickly converge to a reasonably good solution. Additionally, it has the capability to handle larger search space and non-differential objective function, unlike traditional optimization methods. Eberhart et al. [24,25] developed PSO algorithm to simulate random movements of bird flocking or fish schooling.

The algorithm starts with the random initialization of a swarm of individuals, which are known as particles within the multidimensional problem search space, in which each particle tries to move toward the optimum solution, where the next movement is influenced by the previously acquired knowledge of particle best and global best positions once achieved by the individual and the entire swarm, respectively. The features incorporated within this simulation are velocity matching of individuals with the nearest neighbour, elimination of ancillary variables and inclusion of multidimensional search and acceleration by distance. Instead of the presence of direct recombination operators, modifications of acceleration and position supplement the recombination process in PSO. Due to the aforementioned advantages and simplicity, PSO has been applied to different fields of practical optimization problems.

To some extent, adaptive IIR filter design with PSO is already reported in Refs. [11–18]. A brief idea about the algorithm for a D-dimensional search space with n_p particles that constitutes the flock is presented here. Each ith particle is described by a position vector as: $S_i = (s_{i1}, s_{i2}, \ldots, s_{iD})^T$ and the velocity is expressed by $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})^T$. The best position that the ith particle has reached previously $pbest_i = (p_{i1}, p_{i2}, \ldots, p_{iD})^T$, and the group best is expressed as $gbest = (p_{g1}, p_{g2}, \ldots, p_{gD})^T$.

The maximum and minimum velocities are V_{max}, V_{min}, respectively. $V_{max} = (v_{max\,1}, v_{max\,2}, \ldots, v_{max\,D})^T$ and $V_{min} = (v_{min\,1}, v_{min\,2}, \ldots, v_{min\,D})^T$.

The positive constants C_1, C_2 are related with accelerations and $rand_1$, $rand_2$ lie in the range [0, 1]. The inertia weight w is a constant chosen carefully to obtain fast convergence to optimum result. k denotes the iteration number.

The basic steps of the PSO algorithm are as follows [31]:

Step 1: Initialize the real coded particles (ω) of n_p population, each consisting of a number of numerator and denominator filter coefficients b_k and a_k respectively; dimension of the search space, D, which is equal to the number of adaptive filter coefficients, need to be optimized; minimum and maximum values of adaptive filter coefficients, $h_{min} = -2$, $h_{max} = 2$; number of samples = 128; maximum iteration cycles = 200 (for Examples 1 and 3)/300 (for Examples 2 and 4).

Step 2: Compute the error fitness value J for the current position S_i of each particle.

Step 3: Each particle can remember its best position $(pbest)$ which is known as cognitive information and that would be updated with each iteration.

Step 4: Each particle can also remember the best position the swarm has ever attained $(gbest)$ and is called social information and would be updated in each iteration.

Step 5: Velocity and position of each particle are modified according to Eqs. (4) and (5), respectively [24].

$$V_i^{(k+1)} = w^*V_i^{(k)} + C_1{}^*rand_1{}^*\left\{pbest_i^{(k)} - S_i^{(k)}\right\} + C_2{}^*rand_2{}^*\left\{gbest_i^{(k)} - S_i^{(k)}\right\} \quad (4)$$

where $V_i = V_{max}$ for $V_i > V_{max}$
$= V_{min}$ for $V_i < V_{min}$

$$S_i^{(k+1)} = S_i^{(k)} + V_i^{(k+1)} \quad (5)$$

Step 6: The iteration stops when maximum number of cycles is reached. The grand minimum error value and its corresponding particle or the desired solution having same or reduced number of coefficients of the adaptive IIR filter are finally obtained.

3.3. Differential evolution (DE) algorithm

The DE algorithm was first introduced by Storn and Price in 1995 [21]. The crucial idea behind DE algorithm is a scheme for generating trial parameter vectors and adds the weighted difference between two population vectors to a third one. Like any other evolutionary algorithm, DE algorithm aims at evolving a population of n_p, D-dimensional parameter vectors, so-called individuals, which encode the candidate solutions, i.e.,

$$\overrightarrow{x_{i,g}} = \{x_{1,i,g}, x_{2,i,g}, \ldots, x_{D,i,g}\}, \quad (6)$$

where $i = 1, 2, 3, \ldots, n_p$. The initial population (at $g = 0$) should cover the entire search space as much as possible by uniformly randomizing individuals within the search constrained by the prescribed minimum and maximum parameter bounds: $\vec{x}_{min} = \{x_{1,min}, \ldots, x_{D,min}\}$ and $\vec{x}_{max} = \{x_{1,max}, \ldots, x_{D,max}\}$.

For example, the initial value of the jth parameter of the ith vector is

$$x_{j,i,0} = x_{j,min} + rnd^*(x_{j,max} - x_{j,min}), \text{ where } j = 1, 2, 3, \ldots, D \quad (7)$$

The random number generator, rnd returns a uniformly distributed random number from the range [0,1]. After initialization, DE enters a loop of evolutionary operations: mutation, crossover, and selection.

i) Mutation

Once initialized, DE mutates and recombines the population to produce new population. For each trial vector $x_{i,g}$ at generation g, its associated mutant vector $\vec{v}_{i,g} = \{v_{1,i,g}, v_{2,i,g}, \ldots, v_{D,i,g}\}$ can be generated via certain mutation strategy. Five most frequently used mutation strategies in the DE codes are listed as follows:

$$\text{“DE/rand/1”}: \vec{v}_{i,g} = \vec{x}_{r_1',g} + F\left(\vec{x}_{r_2',g} - \vec{x}_{r_3',g}\right) \quad (8)$$

$$\text{“DE/best/1”}: \vec{v}_{i,g} = \vec{x}_{best,g} + F\left(\vec{x}_{r_1',g} - \vec{x}_{r_2',g}\right) \quad (9)$$

$$\text{“DE/rand-to-best/1”}: \vec{v}_{i,g} = \vec{x}_{i,g} + F\left(\vec{x}_{best,g} - \vec{x}_{i,g}\right) + F\left(\vec{x}_{r_1',g} - \vec{x}_{r_2',g}\right) \quad (10)$$

$$\text{“DE/best/2”}: \vec{v}_{i,g} = \vec{x}_{best,g} + F\left(\vec{x}_{r_1',g} - \vec{x}_{r_2',g}\right) + F\left(\vec{x}_{r_3',g} - \vec{x}_{r_4',g}\right) \quad (11)$$

$$\text{“DE/rand/2”}: \vec{v}_{i,g} = \vec{x}_{r_1',g} + F\left(\vec{x}_{r_2',g} - \vec{x}_{r_3',g}\right) + F\left(\vec{x}_{r_4',g} - \vec{x}_{r_5',g}\right) \quad (12)$$

The indices r_1', r_2', r_3', r_4', r_5' are mutually exclusive integers randomly chosen from the range $[1, n_p]$ and all are different from the base index i. These indices are randomly generated once for each mutant vector. The scaling factor F is a positive control parameter for scaling the difference vector. $x_{best,g}$ is the best individual vector with the best fitness value in the population at generation ‘g’.

ii) Crossover

To complement the differential mutation search strategy, crossover operation is applied to increase the potential diversity of the population. The mutant vector $v_{i,g}$ exchanges its components with the target vector $x_{i,g}$ to generate a trial vector:

$$\vec{u}_{i,g} = \{u_{1,i,g}, u_{2,i,g}, \ldots, u_{D,i,g}\} \quad (13)$$

In the basic version, DE employs the binomial (uniform) crossover defined as

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if } (rnd_{i,j} \le C_r \text{ or } j = j_{rand}) \\ x_{i,j,g} & \text{otherwise} \end{cases} \quad (14)$$

where $j = 1, 2, \ldots, D$; $rnd_{i,j}$ returns a uniformly distributed random number from within the range [0,1]. The crossover rate C_r is user-specified constant within the range (1, 0), which controls the fraction of parameter values copied from the mutant vector. j_{rand} is a randomly chosen integer in the range [1, D]. The binomial crossover operator copies the jth parameter of the mutant vector $\vec{v}_{i,g}$ to the corresponding element in the trial vector $\vec{u}_{i,g}$ if $rnd_{i,j} \le C_r$ or $j = j_{rand}$; otherwise, it is copied from the corresponding target vector $\overrightarrow{x_{i,g}}$.

iii) Selection

To keep the population size constant over subsequent generations, the next step of the algorithm calls for selection to determine whether the target or the trial vector survives to the next generation, i.e., at $g = g + 1$. The selection operation is described as (15).

$$\vec{x}_{i,g+1} = \begin{cases} \vec{u}_{i,g} & \text{if } f(\vec{u}_{i,g}) \le f(\vec{x}_{i,g}) \\ \vec{x}_{i,g} & \text{otherwise} \end{cases} \tag{15}$$

where $f(x)$ is the objective/cost function to be minimized. So, if the new vector yields an equal or lower value of the objective function, it replaces the corresponding target vector in the next generation; otherwise the target is retained in the population. Hence, the population either gets better (with respect to the minimization of the objective function) or remains the same in fitness status, but never deteriorates.

The above three steps are repeated generation after generation until some specific termination criteria are satisfied.

3.3.1. *Control parameter selection of DE*

Proper selection of control parameters is very important for the success and performance of an algorithm. The optimal control parameters are problem-specific. Therefore, the set of control parameters that best fit each problem is to be chosen carefully. Values of F lower than 0.5 may result in premature convergence, while values greater than 1 tend to slow down the convergence speed. Large populations help maintaining diverse individuals, but also slow down convergence speed. In order to avoid premature convergence, F or n_p should be increased or .. should be decreased. Larger values of F result in larger perturbations and better probabilities to escape from local optima, while lower C_r preserves more diversity in the population, thus avoiding local optima.

Algorithmic steps of DE are as follows:

Step 1. *Generation of initial population*: Set the generation counter $g = 0$ and randomly initialize D-dimensional n_p individuals (parameter vectors/target vectors), $\vec{x}_{i,g} = \{x_{1,i,g}, x_{2,i,g}, \ldots, x_{D,i,g}\}$; where D is equal to the number of adaptive filter coefficients; need to be optimized; minimum and maximum values of adaptive filter coefficients ($h_{\min} = -2$, $h_{\max} = 2$); and $i = 1, 2, 3, \ldots, n_p$. The initial population (at $g = 0$) should cover the entire search space as much as possible by uniformly randomizing individuals within the search constrained by the prescribed minimum and maximum parameter bounds: $\vec{x}_{\min} = \{x_{1,\min}, \ldots, x_{D,\min}\}$ and $\vec{x}_{\max} = \{x_{1,\max}, \ldots, x_{D,\max}\}$.

Number of samples = 128; maximum iteration cycles = 200 (for Examples 1 and 3)/300 (for Examples 2 and 4).

Step 2. *Mutation*: For $i = 1$ to n_p, generate a mutated vector, $\vec{v}_{i,g} = \{v_{1,i,g}, v_{2,i,g}, \ldots, v_{D,i,g}\}$ corresponding to the target vector $\vec{x}_{i,g}$ via any one of 5 mutation strategies mentioned earlier.

In this work, Eq. (10) is chosen as the best mutation strategy determined after some experimentation. $F = 0.5$.

Step 3. *Crossover*: Generation of a trial vector $\vec{u}_{i,g}$ for each target vector $\vec{x}_{i,g}$ where $\vec{u}_{i,g} = \{u_{1,i,g}, u_{2,i,g}, \ldots, u_{D,i,g}\}$.

for $i = 1$ to n_p; $j_{rand} = [rnd^{*}D]$; for $j = 1$ to D.

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if } (rnd_{i,j} \le C_r \text{ or } j = j_{rand}) \\ x_{i,j,g} & \text{otherwise.} \end{cases}$$

'rnd_{ij}' is an uniformly distributed random number generated within [0,1]. The crossover rate C_r is user-specified constant within the range [1,0], which controls the fraction of parameter values copied from the mutant vector. j_{rand} is a randomly chosen integer in the range [1, D]. The binomial crossover operator copies the jth parameter of the mutant vector $\vec{v}_{i,g}$ to the corresponding element in the trial vector $\vec{u}_{i,g}$ if $rnd_{i,j} \le C_r$ or $j = j_{rand}$; otherwise, it is copied from the corresponding target vector, $\overrightarrow{x_{i,g}}$.

Step 4. *Selection*: for $i = 1$ to n_p, $\vec{x}_{i,g+1}$

$$= \begin{cases} \vec{u}_{i,g} & \text{if } f(\vec{u}_{i,g}) \le f(\vec{x}_{i,g}) \\ \vec{x}_{i,g} & \text{otherwise.} \end{cases}$$

Increment the generation count $g = g + 1$.

The limitations of RGA, PSO and DE are that they may be influenced by parameter convergence and stagnation problem [8,22,26,27]. To overcome stagnation and suboptimal convergence problems associated with RGA, PSO and DE; this paper adopts the modified DE algorithm known as differential evolution with wavelet mutation algorithm (DEWM) for the purpose of finding optimal set of adaptive IIR filter coefficients.

3.4. *Differential evolution with wavelet mutation (DEWM)*

3.4.1. *Basic wavelet theory: a concept*

Certain seismic signals can be modelled by combining translations and dilations of an oscillatory function with a finite duration called a "wavelet". Wavelet transform can be divided in two categories: continuous wavelet transform and discrete wavelet transform. The continuous wavelet transform $W_{a,b}(x)$ of function $f(x)$ with respect to a mother wavelet $\psi(x) \in L^2(\Re)$ is given by the following equation [26,32,33].

$$W_{a,b}(x) = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{+\infty} f(x)\psi^{*}_{a,b}(x)\mathrm{d}x \tag{16}$$

where $\psi_{a,b}(x) = (1/\sqrt{a})\psi(x - b/a)$; $x \in \Re$, $a, b \in \Re$, $a > 0$

In Eq. (16), (*) denotes the complex conjugate, a is the dilation (scale) parameter, and b is the translation (shift) parameter. It is to be noted that a controls the spread of the wavelet and b determines its control position. A set of basis functions $\psi_{a,b}(x)$ is derived from scaling and shifting the mother wavelet. The basis function of the transform is called the daughter wavelet. The mother wavelet has to satisfy the following admissibility condition.

$$C_\psi = 2\pi \int_{-\infty}^{+\infty} \frac{\left|\overline{\psi}(\omega)\right|^2}{\omega} \mathrm{d}\omega < \infty \tag{17}$$

where $\overline{\psi}(\omega)$ is the Fourier transform of $\psi(\omega)$ and is given by the following equation (18).

$$\overline{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x) \times e^{-j\omega x}\mathrm{d}x \tag{18}$$

Most of the energy $\psi(x)$ is confined to a finite domain and is bounded.

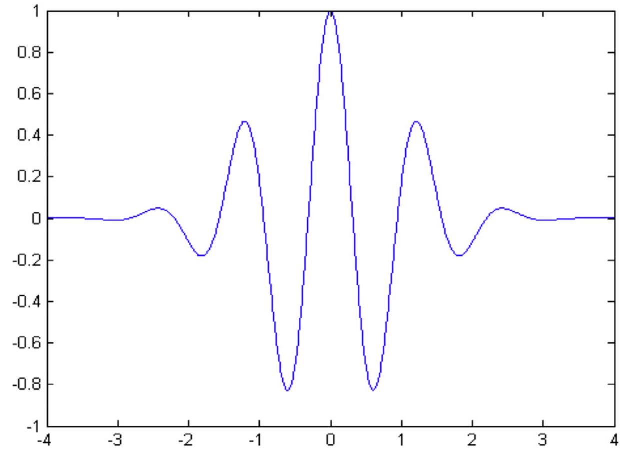

Fig. 2. Morlet wavelet.

3.4.2. Association of wavelet based mutation with DE (DEWM)

It is proposed that every element of the particle of the population will mutate. Among the population, a randomly selected ith particle and its jth element (within the limits $(S_{j,\min}, S_{j,\max})$) at the kth iteration ($S_{i,j}^{(k)}$; vector $S_i^{(k)}$ same as $\vec{x}_{i,g}$/updated $\vec{v}_{i,g}$ of (10)) will undergo mutation as given in the following equation (19).

$$S_{i,j}^{(k)} = \begin{cases} S_{i,j}^{(k)} + \sigma \times \left(S_{j,\max} - S_{i,j}^{(k)}\right), & \text{if } \sigma > 0 \\ S_{i,j}^{(k)} + \sigma \times \left(S_{i,j}^{(k)} - S_{j,\min}\right), & \text{if } \sigma \le 0 \end{cases} \tag{19}$$

where $\sigma = \psi_{a,0}(x) = (1/\sqrt{a})\psi(x/a)$; Eq. (19) represents the new mutation strategy by which DEWM differs from DE; otherwise all other steps of DE and DEWM are the same. A Morlet wavelet (mother wavelet) is defined in the following equation (20). It is also shown in Fig. 2.

$$\psi(x) = e^{\frac{-x^2}{2}} \cos(5x) \tag{20}$$

Thus,

$$\sigma = \frac{1}{\sqrt{a}} e^{\frac{-\left(\frac{x}{a}\right)^2}{2}} \cos\left(5\left(\frac{x}{a}\right)\right) \tag{21}$$

Different dilated Morlet wavelets are shown in Fig. 3. From this Figure, it is clear that as the dilation parameter a increases, the amplitude of $\psi_{a,0}(x)$ will be scaled down. In order to enhance the searching performance in the fine tuning stage, this property will be utilized in mutation operation. As over 99% of the total energy of the mother wavelet function is contained in the interval $[-2.5, 2.5]$, x can be randomly generated from $[-2.5 \times a, 2.5 \times a]$ [26,32,33]. The value of the dilation parameter a is set to vary with the value of k/K in order to meet the fine tuning purpose; where k is the current iteration cycle and K is the maximum number of iteration cycles. In order to perform a local search when k is large, the value of a should increase as k/K increases to reduce the significance of the mutation. Hence, a monotonic increasing function governing a and k/K may be written as given in the following equation:

$$a = e^{-\ln(g_1)\times\left(1-\frac{k}{K}\right)^{\xi_{\omega m}} + \ln(g_1)} \tag{22}$$

where $\xi_{\omega m}$ is the shape parameter of the monotonic increasing function, and g_1 is the upper limit of the parameter a. The value of a is between 1 and 10,000. The magnitude of mutation operator σ decreases as a increases towards g_1 with increasing iteration cycle, hence, resulting in appreciable mutation during early search or exploration stage and fine tuning (i.e., lesser mutation) during local search or exploitation stage near the end of maximum iteration cycles. A perfect balance between the exploration of new regions and the exploitation of the already sampled regions in the search space is expected in DEWM. This balance, which critically affects

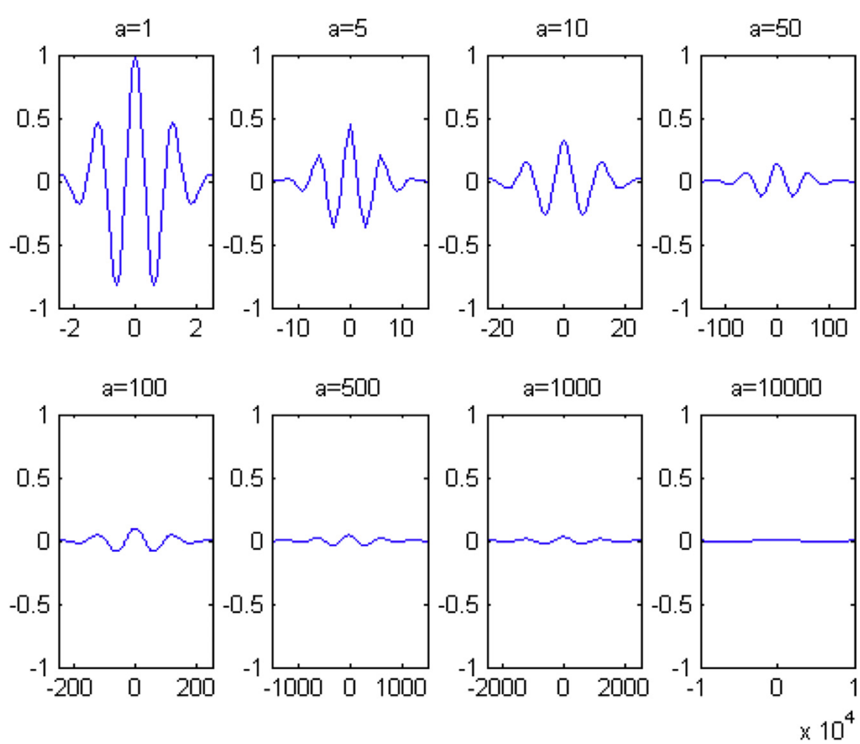

Fig. 3. Morlet wavelet dilated by different values of parameter a.

Table 1
Control parameters of RGA, PSO, DE and DEWM.

Parameters	RGA	PSO	DE	DEWM
Population size	120	25	25	25
Iteration cycles	200/300	200/300	200/300	200/300
Crossover rate	1	–	–	–
Crossover	Two point crossover	–	–	–
Mutation rate	0.01	–	–	–
Mutation	Gaussian mutation	–	–	–
Selection	Roulette	–	–	–
Selection probability	1/3	–	–	–
C_1, C_2	–	2.05, 2.05	–	–
$v_i^{\min}$, $v_i^{\max}$	–	0.01, 1.0	–	–
$w_{\max}$, $w_{\min}$	–	1.0, 0.4	–	–
C_r, F	–	–	0.3, 0.5	0.3, 0.5
$\xi_{\omega m}$, g_1	–	–	–	2.0, 1000

Table 3
MSE value for Example 1 (Case 1).

Run	RGA	PSO	DE	DEWM
1	0.0529	0.0051	9.4261e-005	8.5220e-006
2	0.0733	0.0013	8.2321e-005	5.8399e-006
3	0.0472	0.0028	3.7453e-004	2.6659e-006
4	0.0139	0.0054	9.0273e-004	7.8974e-006
5	0.0464	0.0016	3.7453e-004	6.2787e-005

Table 4
Statistical analysis of MSE (dB) for Example 1 (Case 1).

MSE statistics	RGA	PSO	DE	DEWM
Best	−18.5699	−28.8606	−40.8449	−55.7416
Worst	−11.349	−22.6761	−30.4444	−42.0213
Mean	−13.8559	−25.5896	−36.0153	−50.3637
Variance	6.0642	6.3817	15.6950	20.5908
Standard deviation	2.4626	2.5262	3.9617	4.5377

the performance of DEWM, is governed by the right choices of the control parameters, e.g., the swarm size (..), the probability of mutation (p_m), and the shape parameter of WM ($\xi_{\omega m}$). Changing the parameter $\xi_{\omega m}$ will change the characteristics of the monotonic increasing function of WM. The dilation parameter a will take a value to perform fine tuning faster as $\xi_{\omega m}$ increases. A larger value of $\xi_{\omega m}$ is to be used to increase the step size (σ) for the early mutation. Rigorous sensitivity analysis with respect to the dependence of a on (k/K), $\xi_{\omega m}$ and g_1 was performed to determine the individual best values of $\xi_{\omega m}$ and g_1. The individual best values of $\xi_{\omega m}$ and g_1 are 2.0 and 1000, respectively. In general, if the optimization problem is smooth and symmetric, it is easier to find the solution, and the fine tuning can be done in early iteration cycle.

Steps of DEWM algorithm are as follows:

Step 1: Initialize D dimensional population or swarm of, n_p particle vectors where D is equal to the number of adaptive filter coefficients, need to be optimized; maximum iteration cycles = 200 (for Examples 1 and 3)/300 (for Examples 2 and 4); minimum and maximum values of filter coefficients ($h_{\min} = -2$, $h_{\max} = 2$); number of samples = 128; wavelet mutation parameters: $g_1 = 1000$, $\xi_{\omega m} = 2$, crossover ratio, $C_r = 0.3$.
Step 2: Generate initial n_p particle vectors of adaptive filter, randomly within limits; computation of initial error cost functions (cost) of the total population n_p.
Step 3: Computation of the initial population based best solution (h_{gbest}) vector.
Step 4: Compute the wavelet parameters 'a' as per (22); compute $x = 2.5^*a$ if rnd (0,1) $\geq$ 0.5, otherwise, $x = -2.5^*a$; compute σ as per (21); updating the particle vectors as per the new mutation formula (19), named as h_{wm} and checking against the limits of the filter coefficients.
Step 5: Formation of trial vectors: If C_r is $\geq rand\,(1)$, a trial vector (T) is formed by h_{wm} vector, otherwise by previous h vector; this is done for the total population; then, computation of costs ($\text{cost}_{\text{Trial}}$) for all trial vectors.
Step 6: Selection: If cost of h vector is $\geq\text{cost}_{\text{Trial}}$ of T vector, then, a selected vector h_{select} is formed by T vector otherwise by h vector; this is done for the whole population.
Step7: Compute the costs ($\text{cost}_{\text{select}}$) of all h_{select} vectors and update the h_{gbest} vector; replace all h_{select} vectors as h vectors and all $\text{cost}_{\text{select}}$ values as re-initialized cost values.
Step 8: Iteration continues from Step 4 till the maximum iteration cycles or the convergence of minimum $\text{cost}_{\text{select}}$ values; finally, h_{gbest} is the vector of optimal adaptive IIR filter coefficients, which are used for identifying the unknown IIR plant.

4. Simulation results and discussions

Extensive MATLAB simulation studies have been performed for the performance comparison of four algorithms namely, RGA, PSO, DE and DEWM for the unknown system identification optimization problems. The values of the control parameters used for RGA, PSO, DE, and DEWM are given in Table 1. All optimization programs were run in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

The simulation studies have been carried out on four different benchmarked examples and for all examples, two different cases are studied, one with the same order filter and the other with the reduced order filter. For each case, independent 50 independent runs were performed using all four algorithms for analyzing the

Table 2
Optimized coefficients for Example 1 (Case 1).

Run	RGA		PSO		DE		DEWM	
	b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
1	0.2888	−0.6827	0.0645	−0.4585	0.0435	−0.3871	0.0501	−0.3997
	−1.0373	0.1755	−0.9596	0.0912	−1.1509	0.2679	−1.1327	0.2522
2	−0.0764	−0.4239	0.0131	−0.3583	0.0590	−0.4108	0.0490	−0.3976
	−0.5655	−0.2752	−1.1530	0.2691	−1.1237	0.2436	−1.1356	0.2542
3	−0.0892	−0.4069	0.0982	−0.4637	0.0659	−0.4270	0.0486	−0.3976
	−0.6710	−0.1705	−1.0705	0.1955	−1.1050	0.2277	−1.1334	0.2520
4	−0.0247	−0.2479	0.0134	−0.4012	0.0223	−0.3734	0.0522	−0.4040
	−1.3363	0.4315	−1.0177	0.1454	−1.1340	0.2519	−1.1268	0.2464
5	0.2536	−0.6584	0.0614	−0.4384	0.0659	−0.4270	0.0479	−0.3927
	−0.9675	0.1112	−1.0541	0.1798	−1.1050	0.2277	−1.1478	0.2653

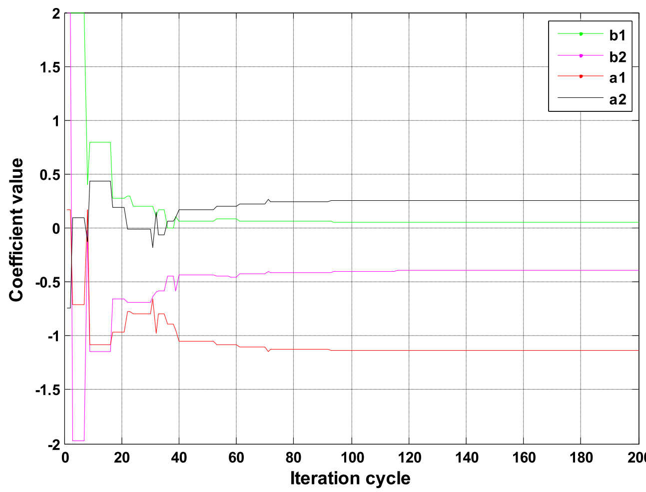

Fig. 4. Coefficient convergence profile of DEWM for Example 1 (Case 1).

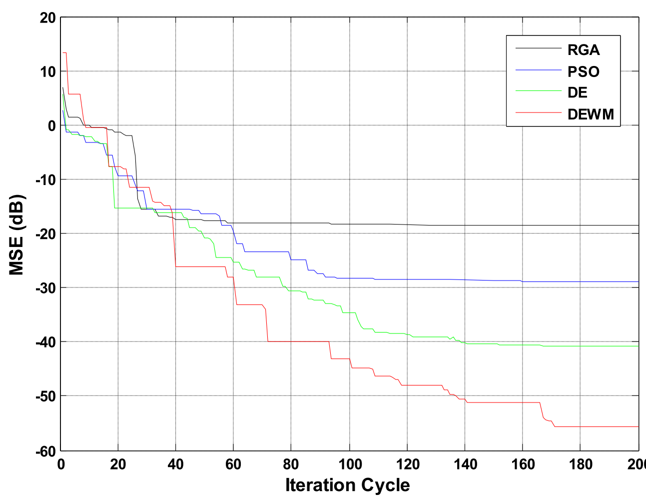

Fig. 5. Algorithm convergence profile for Example 1 (Case 1).

Table 6
MSE value for Example 1 (Case2).

Run	RGA	PSO	DE	DEWM
1	0.4495	0.2397	0.0681	0.0058
2	0.3431	0.2297	0.0794	0.0042
3	0.3723	0.2373	0.0955	0.0052
4	0.2736	0.2021	0.0623	0.0077
5	0.6260	0.2418	0.0439	0.0046

Table 7
Statistical analysis of MSE (dB) for Example 1 (Case2).

MSE statistics	RGA	PSO	DE	DEWM
Best	−5.6288	−6.9443	−13.5754	−23.7675
Worst	−2.0343	−6.1654	−10.2000	−21.1351
Mean	−4.0145	−6.3897	−11.7002	−22.6961
Variance	1.4592	0.0826	1.2763	0.8344
Standard deviation	1.2080	0.2874	1.1297	0.9134

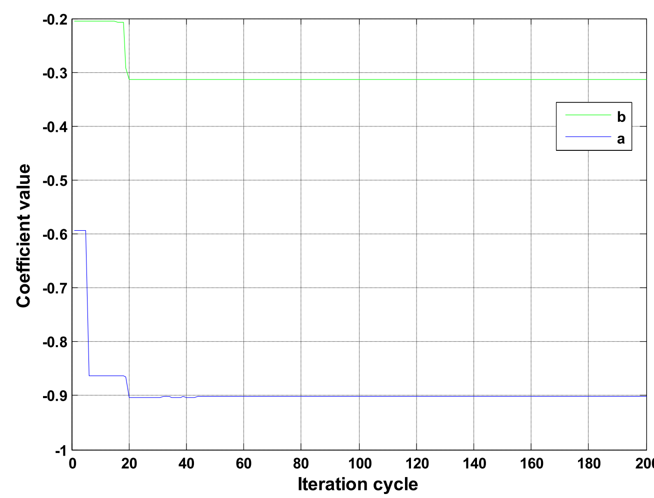

Fig. 6. Coefficient convergence profile of DEWM for Example 1 (Case 2).

consistency and usefulness of the results obtained. The results for the best 5 runs are reported in this work.

4.1. Example 1

In this example, a second order IIR plant is considered and is taken from Refs. [5,6,8,9,12,15,18–20,22]. The transfer function is shown in Eq. (23).

$$H_s(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.131z^{-1} + 0.25z^{-2}} \tag{23}$$

1. Case 1

This second order plant $H_s(z)$ can be modelled using second order IIR filter $H_{af}(z)$. Hence the transfer function of the adaptive IIR filter model is assumed by (24).

$$H_{af}(z) = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{24}$$

In Eq. (24), b_1, b_2, a_1 and a_2 are the numerator and denominator coefficients, respectively. Tables 2 and 3 show the optimized coefficients and MSE values obtained over the five best independent

Table 5
Optimized coefficients for Example 1 (Case2).

Run	RGA		PSO		DE		DEWM	
	b	*a*	*b*	*a*	*b*	*a*	*b*	*a*
1	−0.4336,	−0.7181	−0.3278,	−0.8998	−0.3330,	−0.8867	−0.2207,	−0.9249
2	−0.1634,	−0.9449	−0.2948,	−0.9096	−0.3242,	−0.9031	−0.3142,	−0.9023
3	−0.3877,	−0.7963	−0.3174,	−0.9123	−0.3064,	−0.8509	−0.2900,	−0.9121
4	−0.4091,	−0.8787	−0.3239,	−0.9142	−0.3289,	−0.8994	−0.1483,	−0.9513
5	−0.7182,	−0.8092	−0.3153,	−0.9038	−0.3104,	−0.9098	−0.3289,	−0.9014

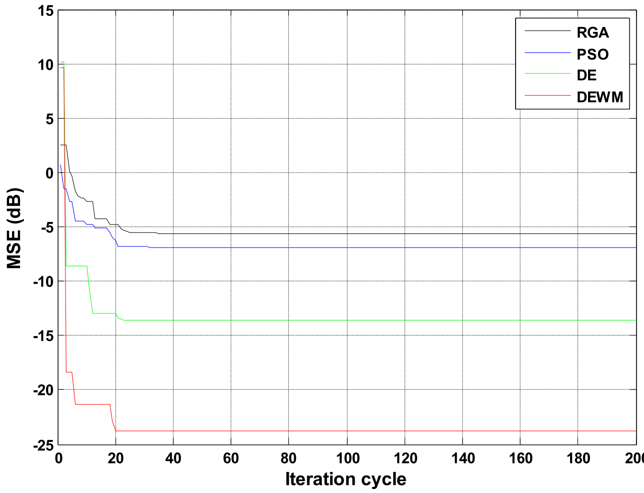

Fig. 7. Algorithm convergence profile for Example 1 (Case 2).

runs for four optimization techniques, namely, RGA, PSO, DE, and DEWM, respectively. Statistically analyzed results, reported in Table 4 provide a platform of judgement to identify the best optimization technique. It is observed that all MSE (dB) values obtained by the DEWM are lower as compared to others and the lowest MSE (dB) value of −55.7416 dB is achieved using DEWM. It is also noticed from Table 2 that the optimized coefficients obtained with DEWM are more accurate in approximating the coefficients of the unknown plant.

Coefficient convergence profile is shown in Fig. 4 for the best run (run 3 in Table 3) which produces the lowest MSE for the proposed optimization technique DEWM in unknown IIR system identification problem. Finally, optimized coefficient values obtained after 200 iteration cycles can also be tallied with the reported coefficient values in Table 2. The algorithm convergence characteristics for the

Table 9
MSE value for Example 2 (Case 1).

Run	RGA	PSO	DE	DEWM
1	0.1393	0.0286	0.0237	2.8617e-005
2	0.2406	0.0537	0.0048	7.7061e-005
3	0.1024	0.0420	0.0190	4.0690e-004
4	0.2872	0.0642	0.0088	7.1474e-005
5	0.1503	0.0619	0.0111	6.0337e-005

Table 10
Statistical analysis of MSE (dB) for Example 2 (Case 1).

MSE statistics	RGA	PSO	DE	DEWM
Best	−9.8970	−15.4363	−23.1876	−45.4338
Worst	−5.4182	−11.9246	−16.2525	−33.9051
Mean	−7.6586	−13.1824	−19.3509	−40.8246
Variance	2.6672	1.6891	6.0763	14.2990
Standard deviation	1.6331	1.2997	2.4650	3.7814

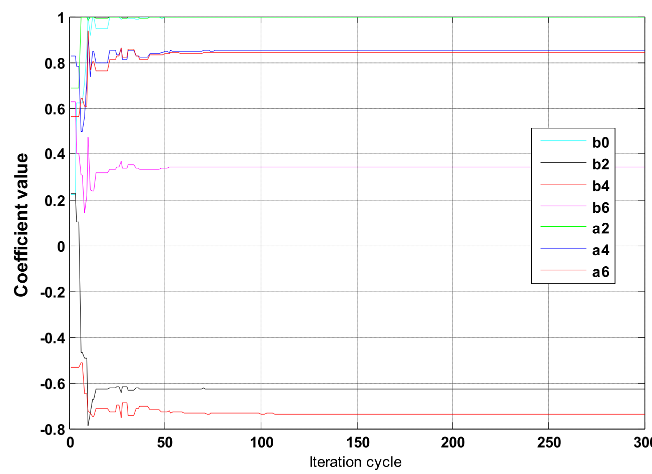

Fig. 8. Coefficient convergence profile of DEWM for Example 2 (Case 1).

Table 8
Optimized coefficients for Example 2 (Case 1).

Run	RGA		PSO		DE		DEWM	
	b_0	b_2	b_0	b_2	b_0	b_2	b_0	b_2
	b_4	b_6	b_4	b_6	b_4	b_6	b_4	b_6
	a_2	a_4	a_2	a_4	a_2	a_4	a_2	a_4
	a_6		a_6		a_6		a_6	
1	0.9310	−0.1148	0.9308	−0.4207	0.9155	−0.2397	1.0000	−0.6267
	0.1204	−0.1874	−0.1991	0.1162	−0.1617	−0.0977	−0.7357	0.3429
	0.6074	0.0078	0.7917	0.4188	0.5922	0.4339	0.9969	0.8521
	−0.2221		0.3106		0.0767		0.8420	
2	0.6139	−0.1314	0.8717	−0.3294	0.9358	−0.1877	0.9939	−0.1991
	0.5322	−0.3720	−0.0215	−0.0841	−0.6138	0.1874	−0.5851	0.1883
	0.4983	−0.1153	0.6586	0.2894	0.5630	0.8824	0.5688	0.8567
	−0.4428		0.0249		0.4936		0.4830	
3	0.9935	0.1947	0.9877	0.1551	1.0000	−0.3383	0.9938	−0.0145
	0.1964	−0.1032	0.0655	−0.2683	−0.2830	0.0431	−0.5279	0.1440
	−0.0516	0.4113	0.1764	0.3803	0.6812	0.5070	0.3595	0.8807
	−0.4227		−0.2953		0.2648		0.3329	
4	0.5028	0.2695	0.9162	0.1397	0.9419	0.4739	1.0000	−0.0932
	0.0443	−0.2906	0.1329	−0.2267	−0.0934	−0.1772	−0.5299	0.1437
	0.1570	0.3649	0.2003	0.4068	−0.1054	0.6380	0.4636	0.8433
	−0.3789		−0.2201		−0.2876		0.3820	
5	0.6399	0.1936	0.9842	−0.7823	0.9667	0.6263	0.9999	−0.1149
	−0.1298	−0.0238	−0.9822	0.3705	−0.0208	−0.2205	−0.5376	0.1528
	0.1647	0.6216	0.9985	0.9399	−0.2671	0.6424	0.4850	0.8434
	−0.0082		0.9025		−0.4070		0.4018	

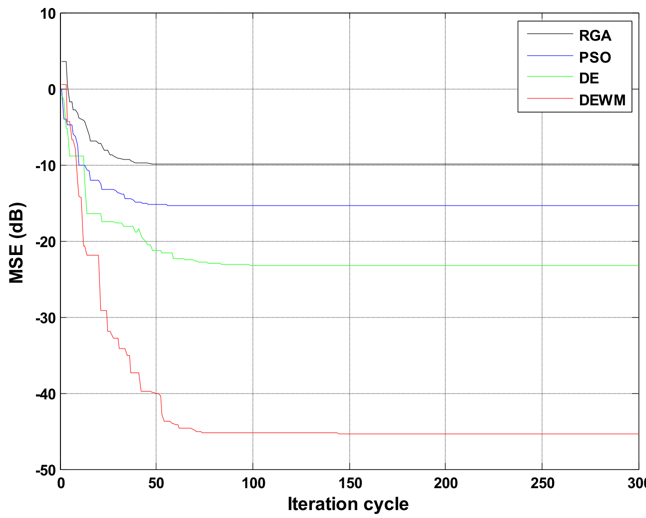

Fig. 9. Algorithm convergence profile for Example 2 (Case 1).

same order IIR model using RGA, PSO, DE, and DEWM, as shown in Fig. 5, provide a qualitative measure of the performance of the four algorithms. From Fig. 5, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at 171st iteration cycle without entrapment to suboptimal solution, unlike others.

Table 12
MSE value for Example 2 (Case2).

Run	RGA	PSO	DE	DEWM
1	0.5104	0.2238	0.0526	0.0016
2	0.5304	0.1915	0.0758	4.0185e-004
3	0.4682	0.2636	0.0656	0.0011
4	0.5185	0.2325	0.0358	0.0016
5	0.3520	0.2168	0.0336	0.0016

Table 13
Statistical analysis of MSE (dB) for Example 2 (Case2).

MSE statistics	RGA	PSO	DE	DEWM
Best	−4.5346	−7.1783	−14.7366	−33.9594
Worst	−2.7540	−5.7906	−11.2033	−27.9588
Mean	−3.2715	−6.4891	−13.0044	−29.4844
Variance	0.4325	0.2018	1.9579	5.4036
Standard deviation	0.6576	0.4493	1.3993	2.3246

2. Case 2

In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a second order plant as in Eq. (23) is modelled by a first order IIR filter given in Eq. (25).

Table 11
Optimized coefficients for Example 2 (Case2).

Run	RGA		PSO		DE		DEWM	
	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1
	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3
	b'_4	b'_5	b'_4	b'_5	b'_4	b'_5	b'_4	b'_5
	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2
	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4
	a'_5		a'_5		a'_5		a'_5	
1	0.9601	−0.5228	0.8256	−0.1669	0.9489	−0.3308	0.9943	−0.0158
	0.4939	−0.4814	−0.3022	−0.1525	0.1527	−0.2845	0.0562	0.2218
	0.0485	0.2293	0.3542	0.2980	−0.1127	0.2054	−0.2344	−0.1387
	−0.5557	0.4872	−0.0694	−0.5724	−0.2701	−0.2116	0.0004	−0.3020
	−0.6541	−0.1409	−0.1381	−0.2700	−0.0902	−0.6341	0.1871	−0.5818
	−0.0050		0.2340		0.2900		−0.1792	
2	1.1007	−0.0739	0.8479	−0.0830	0.8884	−0.2491	0.9770	−0.0105
	0.6934	0.1519	−0.2502	−0.0361	0.2485	0.0545	0.3338	−0.0115
	0.1998	0.0409	0.0351	0.2926	−0.0386	−0.1347	−0.3370	0.0097
	0.0694	0.2199	0.0340	−0.6859	−0.2275	−0.1387	−0.0011	−0.0180
	0.3775	−0.2902	−0.2400	−0.2091	0.1628	−0.6724	−0.0038	−0.8405
	0.0910		0.2590		−0.0134		0.0063	
3	0.5935	−0.1381	1.2239	−0.6695	1.0407	0.4450	0.9117	0.4657
	0.1643	−0.3046	−0.8226	0.5371	−0.6995	−0.3966	0.2746	0.1562
	0.3040	0.1382	0.2998	−0.0096	0.1895	0.1798	−0.2482	−0.1105
	−0.1755	−0.5417	−0.4348	−0.9576	0.3570	−0.9780	0.5349	−0.0471
	−0.1124	−0.2760	0.6025	0.0959	−0.3663	0.0540	−0.0266	−0.8053
	0.2381		−0.1470		0.0430		−0.4239	
4	0.9659	−0.7096	0.9252	−0.5404	0.9258	0.2240	1.0005	0.0943
	0.6918	−0.5039	0.3061	−0.6424	−0.0739	−0.0223	0.0697	−0.0748
	0.5122	−0.4622	0.1333	0.2117	−0.0487	0.0677	−0.2501	0.0209
	−0.7352	0.2400	−0.4254	−0.1841	0.2895	−0.4072	0.1027	−0.2573
	−0.1393	−0.1788	−0.3832	−0.3898	−0.1292	−0.4759	−0.1019	−0.6267
	−0.1191		0.4401		−0.1185		0.0042	
5	0.9690	0.0424	0.8390	−0.2955	0.9835	0.4533	0.9880	−0.0097
	0.2493	0.4077	−0.2772	0.0731	0.3121	−0.0977	0.1300	−0.3025
	0.2574	0.2316	0.3500	−0.2949	−0.3410	0.0277	−0.2927	0.1662
	0.1030	−0.0419	−0.6174	−0.4508	0.4791	−0.0880	−0.0279	−0.2480
	0.4907	−0.0465	0.5137	−0.3982	−0.2768	−0.7762	−0.2661	−0.6354
	0.1998		0.0105		−0.1463		0.2736	

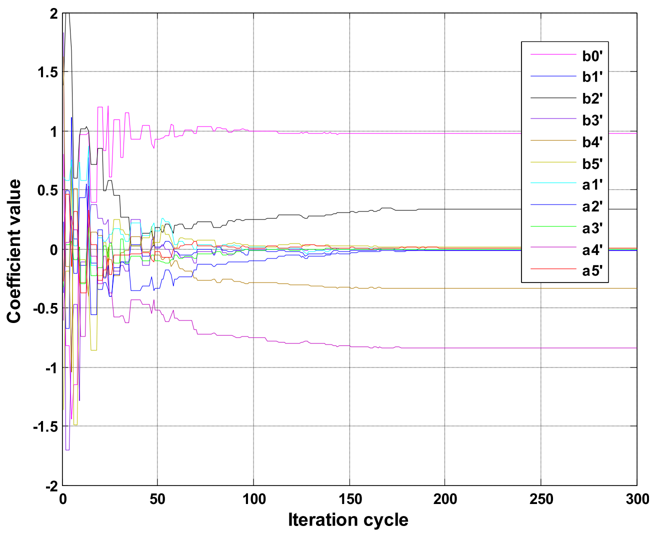

Fig. 10. Coefficient convergence profile of DEWM for Example 2 (Case 2).

$$H_{af}(z) = \frac{b}{1 + az^{-1}} \tag{25}$$

In Eq. (25), b and a are the numerator and denominator coefficients, respectively. Optimized coefficients and MSE values obtained over the best five independent runs for four optimization techniques, namely, RGA, PSO, DE and DEWM are shown in Tables 5 and 6, respectively. Statistically analyzed results for MSE (dB) values are reported in Table 7. From Table 7 it is observed that all MSE (dB) values obtained by the DEWM are lower as compared to others and the lowest MSE value of −23.7675 dB is achieved using DEWM. Consistency of results is established with the small values of variance and standard deviation for all concerned algorithms for unknown system identification problem.

Coefficient convergence profile for the best run (run 2 in Table 6) of the proposed optimization technique, DEWM is shown in Fig. 6 and optimized coefficient values obtained after 200 iteration cycles can also be tallied with the reported coefficient values in Table 5. The convergence characteristics for the reduced order model using RGA, PSO, DE, and DEWM, as shown in Fig. 7 provide a qualitative measure of the performance of the four algorithms. From Fig. 7, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at the 20th iteration cycle without entrapment to suboptimal solution, unlike others.

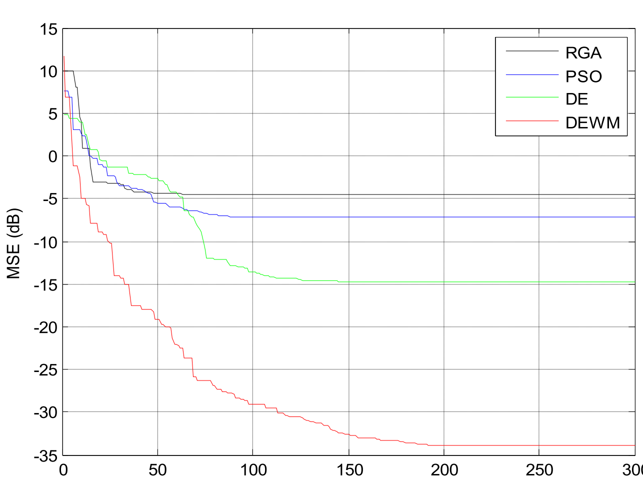

Fig. 11. Algorithm convergence profile for Example 2 (Case 2).

4.2. Example 2

In this example, a sixth order IIR plant is considered from Refs. [8,13] and the transfer function is shown in (26).

$$H_s(z) = \frac{1 - 0.4z^{-2} - 0.65z^{-4} + 0.26z^{-6}}{1 - 0.77z^{-2} - 0.8498z^{-4} + 0.6486z^{-6}} \tag{26}$$

1. Case 1

This sixth order plant $H_s(z)$ can be modelled using sixth order IIR filter $H_{af}(z)$. Hence the transfer function of the adaptive IIR filter model is assumed by

$$H_{af}(z) = \frac{b_0 + b_2z^{-2} + b_4z^{-4} + +b_6z^{-6}}{1 - a_2z^{-2} - a_4z^{-4} + a_6z^{-6}} \tag{27}$$

In Eq. (27), $b_0 \ldots b_6$ and $a_2 \ldots a_6$ are the numerator and denominator coefficients, respectively. Tables 8 and 9 show the optimized coefficients and MSE values obtained over the best five independent runs for four optimization techniques, namely, RGA, PSO, DE and DEWM, respectively. It is also noticed from Table 8 that the optimized coefficients obtained with DEWM are more accurate in approximating the coefficients of the unknown plant. Statistically analyzed results of MSE (in dB) are reported in Table 10. From Table 10 it is observed that all MSE (dB) values obtained by the DEWM are lower as compared to others and the lowest MSE value of −45.4338 dB is achieved using DEWM.

Coefficient convergence profile is shown in Fig. 8 for the best run (run 1 in Table 9) which produces the lowest MSE for the proposed optimization technique DEWM in unknown IIR system identification problem. Settled values of coefficients obtained after 300 iteration cycles can also be verified with the reported coefficient values in Table 8 for the concerned run. The convergence characteristics for the same order model using RGA, PSO, DE and, DEWM, as shown in Fig. 9 provide a qualitative measure of the performance of the four algorithms. From Fig. 9, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at the 77th iteration cycle without entrapment to suboptimal solution, unlike others.

2. Case 2

In this case the sixth order plant as in Eq. (26) is modelled by a fifth order IIR filter presented in Eq. (28).

$$H_{af}(z) = \frac{b_0' + b_1'z^{-1} + b_2'z^{-2} + b_3'z^{-3} + b_4'z^{-4} + b_5'z^{-5}}{1 + a_1'z^{-1} - a_2'z^{-2} + a_3'z^{-3} - a_4'z^{-4} + a_5'z^{-5}} \tag{28}$$

In Eq. (28), $b_0' \ldots b_5'$ and $a_1' \ldots a_5'$ are the numerator and denominator coefficients, respectively. Tables 11 and 12 show the optimized coefficients and MSE values obtained over the best five independent runs for four optimization techniques, namely, RGA, PSO, DE and DEWM, respectively. Statistically analyzed results for MSE (dB) are reported in Table 13. It is observed that all MSE (dB) values obtained by the DEWM are lower as compared to others and the lowest MSE value of −33.9594 dB is achieved using DEWM.

Coefficient convergence profile for the best run (run 2 in Table 12) of the proposed optimization technique DEWM is shown in Fig. 10 and optimized coefficient values obtained after 300

Table 14
Optimized coefficients for Example 3 (Case 1).

Run	RGA		PSO		DE		DEWM	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2		b_2		b_2		b_2	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3		a_3		a_3		a_3	
1	−0.4997	0.2674	−0.2710	0.2062	−0.2830	0.3661	−0.2986	0.3926
	−0.2969		−0.2721		−0.4683		−0.4934	
	0.5833	0.2049	0.9890	0.0908	1.1291	−0.3135	1.1755	−0.4496
	−0.0036		−0.2556		−0.0174		0.0729	
2	−0.4646	−0.2778	−0.3493	0.3568	−0.2680	0.3139	−0.3078	0.4159
	0.0476		−0.5155		−0.4277		−0.5150	
	−0.3786	0.6997	0.8477	−0.1698	1.1267	−0.3171	1.2155	−0.5463
	0.4009		0.0888		−0.0063		0.1291	
3	−0.1010	−0.1050	−0.2683	0.2123	−0.2612	0.3437	−0.2992	0.3953
	−0.0780		−0.4383		−0.4699		−0.4983	
	1.0792	0.0150	0.7581	0.0093	1.3106	−0.7430	1.1853	−0.4789
	−0.2497		0.0032		0.2451		0.0924	
4	−0.3753	−0.1301	−0.2861	0.2902	−0.3325	0.4388	−0.3029	0.4041
	−0.2632		−0.4866		−0.5287		−0.5008	
	−0.0093	0.2352	0.9197	−0.3881	1.0930	−0.3416	1.1870	−0.4702
	0.4266		0.2430		0.0380		0.0824	
5	0.0202	−0.0720	−0.2938	0.3465	−0.2904	0.3898	−0.3017	0.3998
	−0.4208		−0.4609		−0.5024		−0.5019	
	0.2354	0.4981	0.9737	−0.0461	1.2697	−0.6848	1.1792	−0.4701
	0.0177		−0.1377		0.2195		0.0888	

iteration cycles can also be tallied with the reported coefficient values as shown in Table 11. The convergence characteristics for the reduced order model using RGA, PSO, DE and DEWM, as shown in Fig. 11 provide a qualitative measure of the performance of the four algorithms. From Fig. 11, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at the 192nd iteration cycle without entrapment to suboptimal solution, unlike others.

4.3. Example 3

In this example, a third order IIR plant is considered from Refs. [5,12,15,18] and the transfer function is shown in Eq. (29).

$$H_s(z) = \frac{-0.3 + 0.4z^{-1} - 0.5z^{-2}}{1 - 1.2z^{-1} + 0.5z^{-2} - 0.1z^{-3}} \tag{29}$$

Table 15
MSE value for Example 3 (Case 1).

Run	RGA	PSO	DE	DEWM
1	0.1162	0.0330	0.0037	2.1626e-004
2	0.2257	0.0234	0.0047	3.1700e-004
3	0.1168	0.0422	0.0071	3.8177e-005
4	0.1385	0.0248	0.0038	8.3466e-005
5	0.2469	0.0109	0.0034	7.9325e-005

Table 16
Statistical analysis of MSE (dB) for Example 3 (Case 1).

MSE statistics	RGA	PSO	DE	DEWM
Best	−9.3479	−19.6257	−24.6852	−44.1820
Worst	−6.0748	−13.7469	−21.4874	−34.9894
Mean	−7.9597	−16.1102	−23.5944	−39.5225
Variance	1.9945	3.9329	1.3243	10.8608
Standard deviation	1.4123	1.9831	1.1508	3.2956

1. Case 1

This third order plant $H_s(z)$ can be modelled using third order IIR filter $H_{af}(z)$. Hence the transfer function of the model is assumed by

$$H_{af}(z) = \frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 - a_1z^{-1} - a_2z^{-2} - a_3z^{-3}} \tag{30}$$

In Eq. (30), $b_0 \ldots b_2$ and $a_1 \ldots a_3$ are the numerator and denominator coefficients, respectively. Tables 14 and 15 show the optimized coefficients and MSE values obtained over the best five independent runs for four optimization techniques, namely, RGA, PSO, DE, and DEWM, respectively. Statistically analyzed results for MSE (dB) are reported in Table 16. It is observed that the MSE values obtained by the DEWM are the lowest as compared to others and the lowest MSE value of −44.1820 dB is achieved. It is also noticed from Table 14 that the optimized coefficients obtained with DEWM

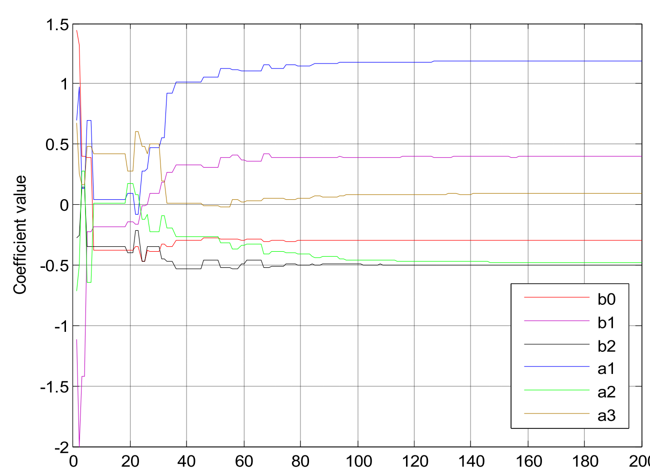

Fig. 12. Coefficient convergence profile of DEWM for Example 3 (Case 1).

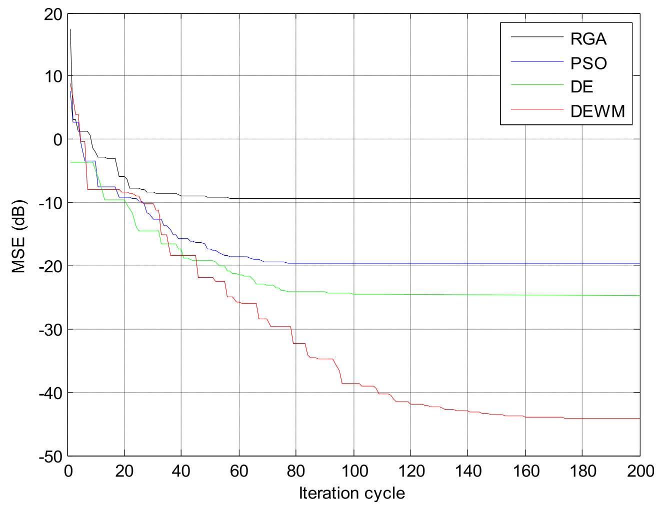

Fig. 13. Algorithm convergence profile for Example 3 (Case 1).

is more accurate in approximating the coefficients of the unknown plant.

Coefficient convergence profile is shown in Fig. 12 for the best run (run 3 in Table 15) which produces the lowest MSE for the proposed DEWM based IIR system identification problem. Finally, optimized coefficient values obtained after 200 iteration cycles can also be tallied with the reported coefficient values in Table 14. The algorithm convergence characteristics for the same order model, using RGA, PSO, DE, and DEWM, as shown in Fig. 13, provide a qualitative measure of the performance of the above mentioned algorithms. From Fig. 13, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at 174th iteration cycle without entrapment to suboptimal solution, unlike others.

2. Case 2

In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a third order plant as in Eq. (29) is modelled by a second order IIR filter given in Eq. (31).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1}}{1 - a'_1 z^{-1} - a'_2 z^{-2}} \tag{31}$$

In Eq. (31), .. are the numerator and denominator coefficients, respectively. Tables 17 and 18 show the optimized coefficients and MSE values obtained over the best five independent runs for four optimization techniques, namely, RGA, PSO, DE, and DEWM, respectively. Statistically analyzed results for MSE (dB) are reported in Table 19. It is observed that all MSE (dB) values obtained by DEWM are lower as compared to others and the lowest MSE value of −26.7778 dB is achieved. Consistency of results for DEWM is established with the smallest value of variance and standard deviation for the rest algorithms in IIR system identification problem.

Table 18
MSE value for Example 3 (Case 2).

Run	RGA	PSO	DE	DEWM
1	0.1625	0.0164	0.0168	0.0021
2	0.4407	0.0165	0.0080	0.0026
3	0.1624	0.0312	0.0207	0.0023
4	0.1715	0.0265	0.0197	0.0026
5	0.2022	0.0234	0.0171	0.0026

Table 19
Statistical analysis of MSE (dB) for Example 3 (Case 2).

MSE statistics	RGA	PSO	DE	DEWM
Best	−7.8941	−17.8516	−20.9691	−26.7778
Worst	−3.5586	−15.0585	−16.8403	−25.8503
Mean	−6.7887	−16.5621	−18.0563	−26.1423
Variance	2.7299	1.2430	2.2420	0.1435
Standard deviation	1.6523	1.1149	1.4973	0.3788

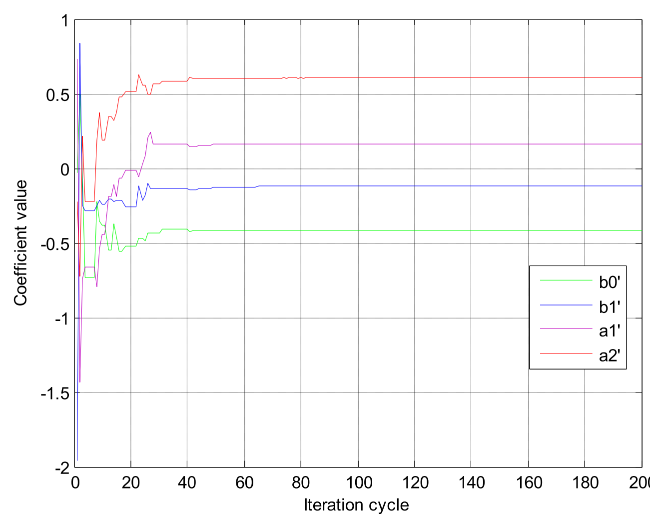

Fig. 14. Coefficient convergence profile of DEWM for Example 3 (Case 2).

Table 17
Optimized coefficients for Example 3 (Case 2).

Run	RGA		PSO		DE		DEWM	
	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1
	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2
1	−0.2516	−0.0352	−0.3785	0.0287	−0.3958	−0.0609	−0.4179	−0.1206
	1.0023	−0.1344	0.5228	0.3178	0.2322	0.5599	0.1603	0.6057
2	−0.2439	−0.0136	−0.3942	−0.0457	−0.3958	−0.0420	−0.2421	−0.0485
	−0.7190	−0.3431	0.2289	0.5675	0.3321	0.4728	0.6305	0.2294
3	−0.3996	−0.0575	−0.4346	−0.0945	−0.3959	−0.1141	−0.4020	−0.0745
	0.2748	0.5313	0.2312	0.5528	0.1750	0.6194	0.2322	0.5656
4	−0.3254	−0.1106	−0.4025	−0.0858	−0.4012	−0.1041	−0.4406	−0.0631
	0.2312	0.5666	0.2236	0.5573	0.1983	0.5796	0.2493	0.5560
5	−0.3797	−0.0305	−0.4204	0.0456	−0.3955	−0.1663	−0.3608	−0.0641
	0.2975	0.5239	0.4335	0.3923	0.0634	0.6824	0.2305	0.5897

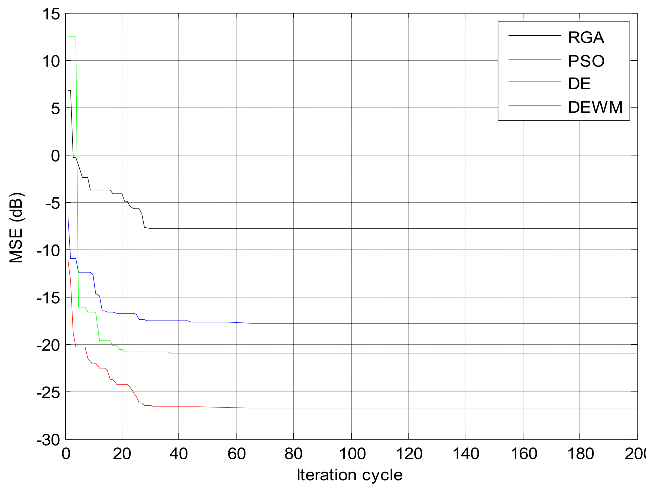

Fig. 15. Algorithm convergence profile for Example 3 (Case 2).

Coefficient convergence profile for the best run (run 1 in Table 18) of the DEWM is shown in Fig. 14 and optimized coefficient values obtained after 200 iteration cycles can also be tallied with the reported coefficient values as in Table 17. The algorithm convergence characteristics for the reduced order model for the best runs using RGA, PSO, DE, and DEWM, as shown in Fig. 15, provide a qualitative measure of the performance of the four algorithms. From Fig. 15, it can be observed that the proposed optimization technique DEWM has converged to the lowest MSE level at the 56th iteration cycle without entrapment to suboptimal solution unlike others.

4.4. Example 4

In this example, a third order IIR plant is considered from Refs. [6,13,15] and the transfer function is given in (32).

Table 21
MSE value for Example 4 (Case 1).

Run	RGA	PSO	DE	DEWM
1	0.2472	0.0424	0.0078	1.1848e-004
2	0.3033	0.0549	0.0084	6.9920e-005
3	0.4271	0.0424	0.0076	4.6051e-005
4	0.1861	0.0464	0.0071	7.5623e-005
5	0.3442	0.0408	0.0064	1.1157e-004

Table 22
Statistical analysis of MSE (dB) for Example 4 (Case 1).

MSE statistics	RGA	PSO	DE	DEWM
Best	−7.3025	−13.8934	−21.9382	−43.3676
Worst	−3.6947	−12.6043	−20.7572	−39.2635
Mean	−5.3754	−13.4570	−21.2907	−40.9846
Variance	1.5230	0.2155	0.1594	2.2298
Standard deviation	1.2341	0.4642	0.3993	1.4933

$$H_s(z) = \frac{-0.2 - 0.4z^{-1} + 0.5z^{-2}}{1 - 0.6z^{-1} + 0.25z^{-2} - 0.2z^{-3}} \quad (32)$$

1. Case 1

This third order plant $H_s(z)$ can be modelled using third order IIR filter $H_{af}(z)$. Hence the transfer function of the model is assumed by Eq. (33).

$$H_{af}(z) = \frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 - a_1z^{-1} - a_2z^{-2} - a_3z^{-3}} \quad (33)$$

In Eq. (33), $b_0 \ldots b_2$ and $a_1 \ldots a_3$ are the numerator and denominator coefficients, respectively. Tables 20 and 21 show the optimized coefficients and MSE values obtained over the best five independent runs for different optimization techniques, namely,

Table 20
Optimized coefficients for Example 4 (Case 1).

Run	RGA		PSO		DE		DEWM	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2		b_2		b_2		b_2	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3		a_3		a_3		a_3	
1	−0.0471	−0.1900	−0.1274	−0.6637	−0.1946	−0.5377	−0.2014	−0.3990
	0.1822		−0.2175		0.1560		0.5092	
	−0.2163	−0.1193	−0.5264	−0.2188	−0.0344	−0.3199	0.6199	−0.2569
	−0.4565		−0.0656		−0.0725		0.2206	
2	−0.2862	−0.0699	−0.3173	−0.2746	−0.1810	−0.4074	−0.2014	−0.4066
	0.2352		0.2298		0.4712		0.4896	
	−0.0325	0.1102	0.3573	−0.3622	0.4841	−0.1393	0.5721	−0.2444
	−0.0694		−0.0650		0.0662		0.1829	
3	−0.4886	−0.5664	−0.1274	−0.6637	−0.2000	−0.5083	−0.2019	−0.4039
	−0.3977		−0.2175		0.2875		0.4848	
	0.0645	−0.1555	−0.5264	−0.2188	0.1816	−0.2333	0.5723	−0.2529
	−0.0460		−0.0656		0.0278		0.1861	
4	−0.0832	−0.2618	−0.2383	−0.3220	−0.1971	−0.3862	−0.1983	−0.4082
	0.2736		0.3107		0.4217		0.4932	
	−0.0752	−0.4531	0.2965	−0.1804	0.5844	−0.3930	0.5731	−0.2404
	−0.2080		−0.1633		0.2439		0.1827	
5	0.3577	−0.6459	−0.3814	−0.6817	−0.2014	−0.4853	−0.1965	−0.4145
	0.2523		−0.1458		0.3401		0.4970	
	0.3976	0.1358	−0.4187	−0.3618	0.2515	−0.2123	0.5805	−0.2484
	0.1909		−0.1811		0.0110		0.1956	

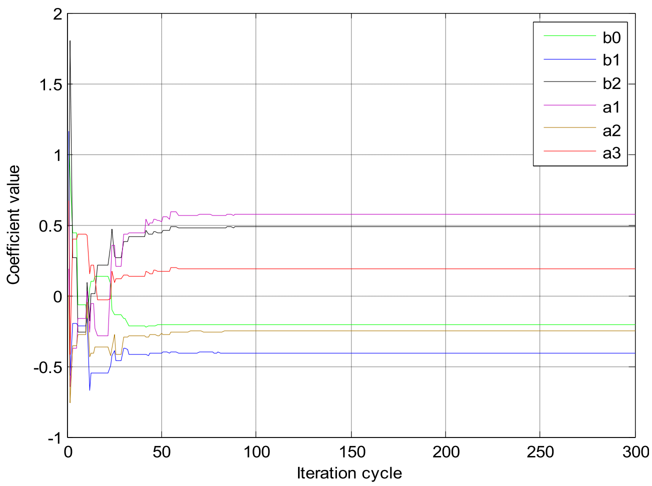

Fig. 16. Coefficient convergence profile of DEWM for Example 4 (Case 1).

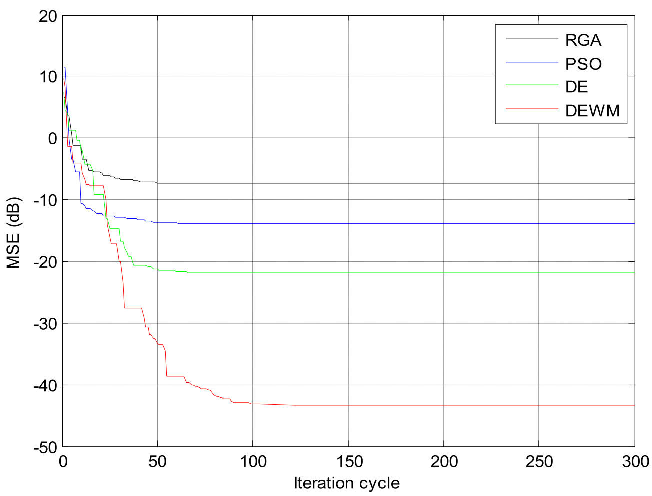

Fig. 17. Algorithm convergence profile for Example 4 (Case 1).

RGA, PSO, DE, and DEWM, respectively. Statistically analyzed results for MSE (dB) are reported in Table 22. It is observed that all MSE (dB) values obtained by the DEWM are lower as compared to others and the lowest MSE value of −43.3676 dB is achieved. It is also noticed from Table 20 that the optimized coefficients obtained with DEWM are more accurate in approximating the coefficients of the unknown plant.

Table 24
MSE value for Example 4 (Case 2).

Run	RGA	PSO	DE	DEWM
1	0.3336	0.0747	0.0075	6.0850e-004
2	0.3289	0.0527	0.0050	9.1221e-004
3	0.2416	0.0490	0.0071	4.0182e-004
4	0.3274	0.0430	0.0053	9.0427e-004
5	0.2273	0.0322	0.0051	9.7319e-004

Table 25
Statistical analysis of MSE (dB) for Example 4 (Case 2).

MSE statistics	RGA	PSO	DE	DEWM
Best	−6.4340	−14.9214	−23.0103	−33.9597
Worst	−4.7677	−11.2668	−21.2494	−30.1180
Mean	−5.4099	−13.1467	−22.2857	−31.4142
Variance	0.5378	1.4176	0.5733	2.1395
Standard deviation	0.7333	1.1906	0.7572	1.4627

Coefficient convergence profile is shown in Fig. 16 for the best run (run 3 in Table 21) which produces the lowest MSE for the proposed optimization technique DEWM in IIR system identification problem. Finally, optimized coefficient values obtained after 300 iteration cycles can also be tallied with the reported coefficient values in Table 20. The algorithm convergence characteristics for the same order model using RGA, PSO, DE, and DEWM, as shown in Fig. 17 provide a qualitative measure of the performance of the above mentioned algorithms. From Fig. 17, it is observed that the DEWM has converged to the lowest MSE level at 112th iteration cycle without entrapment to suboptimal solution unlike others.

2. Case 2

In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a third order plant as in Eq. (32) is modelled by a second order IIR filter presented in Eq. (34).

$$H_{af}(z) = \frac{b_0^{'} + b_1^{'} z^{-1}}{1 - a_1^{'} z^{-1} - a_2^{'} z^{-2}} \tag{34}$$

In Eq. (34), $b_0^{'}$, $b_1^{'}$, $a_1^{'}, a_2^{'}$ are the numerator and denominator coefficients, respectively. Tables 23 and 24 show the optimized coefficients and MSE values obtained over best 5 independent runs for different optimization techniques, namely, RGA, PSO, DE, and DEWM, respectively. Statistically analyzed results for the MSE (dB) are reported in Table 25. From Table 24 it is observed that all MSE

Table 23
Optimized coefficients for Example 4 (Case 2).

Run	RGA		PSO		DE		DEWM	
	$b_0^{'}$ / $a_1^{'}$	$b_1^{'}$ / $a_2^{'}$	$b_0^{'}$ / $a_1^{'}$	$b_1^{'}$ / $a_2^{'}$	$b_0^{'}$ / $a_1^{'}$	$b_1^{'}$ / $a_2^{'}$	$b_0^{'}$ / $a_1^{'}$	$b_1^{'}$ / $a_2^{'}$
1	−0.3826	0.2049	−0.3818	−0.6029	−0.2082	−0.5021	−0.3671	−0.6107
	0.7783	−0.1056	−0.0899	−0.1757	−0.1511	−0.3326	−0.1045	−0.1719
2	0.0804	−0.1459	−0.0970	−0.5176	−0.2131	−0.6077	−0.0898	−0.4776
	−1.1377	−0.6174	−0.1073	−0.2041	−0.1845	−0.3889	−0.4824	−0.5456
3	−0.4050	−1.0205	−0.1387	−0.5367	−0.2379	−0.5751	−0.0599	−0.5708
	−0.2940	−0.0713	−0.4544	−0.3710	−0.0967	−0.2676	−0.2372	−0.3089
4	0.0673	−0.1082	−0.1615	−0.5649	−0.2117	−0.5627	−0.0747	−0.5115
	−0.6519	−0.1326	−0.4358	−0.3394	−0.1595	−0.3961	−0.3180	−0.2549
5	−0.3296	0.1456	−0.1270	−0.5104	−0.2071	−0.5682	−0.2569	−0.5010
	1.0482	−0.6442	−0.2241	−0.3377	−0.1661	−0.3931	−0.2330	−0.2031

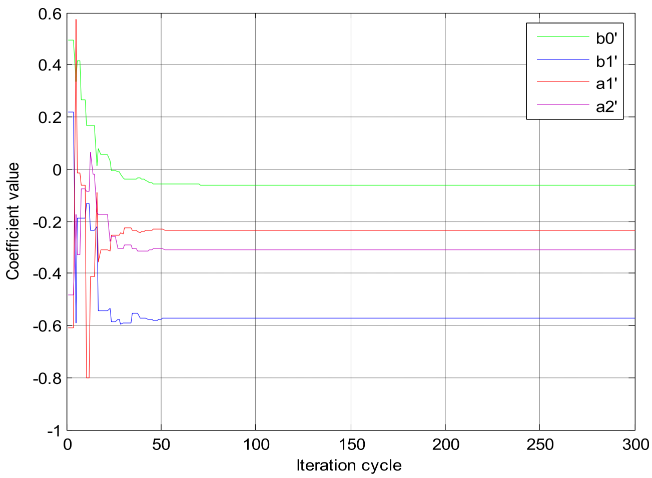

Fig. 18. Coefficient convergence profile of DEWM for Example 4 (Case 2).

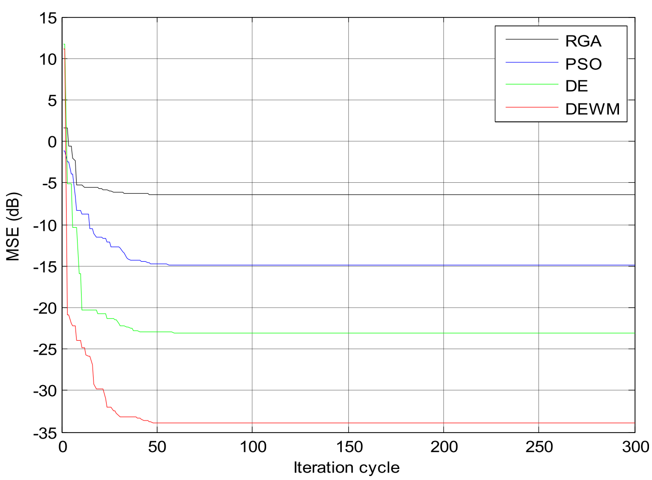

Fig. 19. Algorithm convergence profile for Example 4 (Case 2).

(dB) values obtained by the DEWM are lower as compared to others and the lowest MSE value of −33.9597 dB has been achieved.

Coefficient convergence profile for the best run (run 3 in Table 24) of the DEWM is shown in Fig. 18 and optimized coefficient values obtained after 300 iteration cycles can be tallied with the reported coefficient values as in Table 23. The algorithm convergence characteristics for the reduced order model using RGA, PSO, DE, and DEWM, as shown in Fig. 19, provide a qualitative measure of the performance of the four algorithms. From Fig. 19 it can be observed that the DEWM has converged to the lowest MSE level at the 52nd iteration cycle without entrapment to suboptimal solution, unlike others.

Performance of the DEWM is compared to other reported results with the examples cited in this paper for IIR system identification problem. Dai et al. used reduced model for Example 1 with SOA and a MSE value of 8.2773e-2 is reported in Ref. [5]. Panda et al. in Ref. [6] applied CSO and a MSE level of 6.36395e-5 and 0.0175154 are achieved for same and reduced order models, respectively. ABC algorithm is applied for the reduced order model by Karaboga in Ref. [8] and a MSE level of 0.0706 is reported. In Ref. [9], Rashedi et al. proposed GSA for the reduced order model and the best MSE value of 0.172 is reported in Ref. [9]. Chen et al. suggested PSO for the reduced order model and MSE of 0.275 is reported in Ref. [12]. A modified version of PSO, MuQPSO is proposed by Fang et al. for the reduced order model and the lowest level of MSE 0.206 is reported in Ref. [15]. Again Fang et al. suggested QPSO for the reduced order model and the best MSE level of 0.173 is reported in Ref. [18]. In Ref. [22], Karaboga has proposed DE algorithm and MSE level of 0.0685 for the reduced order model is reported in Ref. [22]. Majhi et al. [19] and Durmus and Gun [20] suggested PSO technique for same and reduced order models with MSE levels of −38 dB [19] and 0.015 [20], respectively. In this paper, the authors have suggested DEWM technique for the same and reduced order models with the best MSE levels of −55.7416 dB and −23.7675 dB, respectively.

For Example 2, Karaboga suggested ABC algorithm for the reduced order model and the best MSE level of 0.0144 is reported in Ref. [8]. Luitel et al. in Ref. [13] proposed PSO-QI for same and reduced order models with the best MSE levels of 7.984e-4 and

Table 26
Performance comparison of different reported MSE values.

Example	Reference	Proposed algorithm	MSE value	
			Same order	Reduced order
Example 1	Dai et al. [5]	SOA	NR[a]	8.2773e-2
	Panda et al. [6]	CSO	6.36395e-5	0.0175154
	Karaboga [8]	ABC	NR[a]	0.0706
	Rashedi et al. [9]	GSA	NR[a]	0.172
	Chen et al. [12]	PSO	NR[a]	0.275
	Fang et al. [15]	MuQPSO	NR[a]	0.206
	Fang et al. [18]	QPSO	NR[a]	0.173
	Karaboga [22]	DE	NR[a]	0.0685
	Majhi et al. [19]	PSO	−38 dB	NR[a]
	Durmus et al. [20]	PSO	NR[a]	0.015
	Present work	DEWM	2.6659e-6 (=−55.7416 dB)	0.0042 (=−23.7675 dB)
Example 2	Karaboga [8]	ABC	NR[a]	0.0144
	Luitel et al. [13]	PSO-QI	7.984e-4	0.001
	Present work	DEWM	2.8617e-5 (=−45.4338 dB)	4.0185e-004 (=−33.9594 dB)
Example 3	Dai et al. [5]	SOA	NR[a]	5.1821e-3
	Chen et al. [12]	PSO	NR[a]	−17.4036 dB
	Fang et al. [15]	MuQPSO	NR[a]	0.01374
	Fang et al. [18]	QPSO	NR[a]	0.013
	Present work	DEWM	3.8177e-5 (=−44.1820 dB)	0.0021 (=−26.7778 dB)
Example 4	Panda et al. [6]	CSO	6.35201e-5	0.001393846
	Luitel et al. [13]	PSO-QI	7.791e-4	0.004
	Fang et al. [15]	MuQPSO	2.041e-3	NR[a]
	Present work	DEWM	4.6051e-5 (=−43.3676 dB)	4.0182e-4 (=−33.9597 dB)

[a] NR: not reported in the refereed literature.

0.001, respectively. In this paper, DEWM yields MSE values of −45.4338 dB and −33.9594 dB for same and reduced order models, respectively.

For Example 3, Dai et al. suggested SOA technique for reduced order model and the best MSE of 5.1821e-3 is reported in Ref. [5]. Chen et al. in Ref. [12] also suggested PSO for reduced order model with best MSE level of −17.4036 dB. Fang et al. proposed MuQPSO in Ref. [15] and QPSO in Ref. [18] for reduced order model with the best MSE levels of 0.01374 and 0.013, respectively. In this paper, for the same and reduced order models, DEWM yields the best MSE values of −44.1820 dB and −26.7778 dB, respectively.

For Example 4, Panda et al. suggested CSO technique [6] for same and reduced order models with MSE values of 6.35201e-5 and 0.001393846, respectively. Luitel et al. also suggested MSE values of 7.791e-4 and 0.004 for same and reduced order models, respectively; with PSO-QI technique as reported in Ref. [13]. Fang et al. in Ref. [15], suggested MuQPSO for same order model with the best MSE level of 2.041e-3. In this paper, for same and reduced order models, DEWM yields MSE levels of −43.3676 dB and −33.9597 dB, respectively. All information given above for the comparative study are presented in Table 26.

5. Conclusions

In this paper, an approach of applying proposed DEWM algorithm for finding optimal set of adaptive IIR filter coefficients for same order and reduced order models is shown for unknown system identification problem. Morlet wavelet function is adopted to bring reducing mutation in chromosome in DEWM optimization technique. The adaptation of wavelet based mutation strategy brings a noticeable improvement in mimicking the unknown plant in terms of producing error fitness value and algorithm convergence profile. No doubt, complexity of the basic DE algorithm is increased with this mutation strategy, which has resulted in longer computation time for finding optimal solution but the advantages obtained in terms of quality output, have outweighed the disadvantage encountered with algorithm complexity. So, from the simulation study it is established that the proposed optimization technique DEWM for adaptive filtering is efficient in finding optimal solution in multidimensional search space where the rest algorithms are entrapped to suboptimal solution and hence it can be concluded that the proposed technique is good enough to handle such system identification problem.

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