Entropy In The Present And Early Universe: New Small Parameters And Dark Energy Problem

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Abstract

It is demonstrated that entropy and its density play a significant role in solving the problem of the vacuum energy density (cosmological constant) of the Universe and hence the dark energy problem. Taking this in mind, two most popular models for dark energy - Holographic Dark Energy Model and Agegraphic Dark Energy Model - are analyzed. It is shown that the fundamental quantities in the first of these models may be expressed in terms of a new small dimensionless parameter. It is revealed that this parameter is naturally occurring in High Energy Gravitational Thermodynamics and Gravitational Holography (UV-limit). On this basis the possibility of a new approach to the problem of Quantum Gravity is discussed. Besides, the results obtained on the uncertainty relation of the pair "cosmological constant - volume of space-time", where the cosmological constant is a dynamic quantity, are reconsidered and generalized up to the Generalized Uncertainty Relation

1 Introduction

The Dark Energy Problem is one of the key problems in a modern theoretical physics [1]–[5]. The vacuum energy is still the major candidate to play a role of this energy. Provided the Dark Energy is actually the vacuum energy, the indicated problem is reduced to better insight into the essence of the vacuum energy. This problem has attracted the attention of researchers

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fairly recently with understanding that a cosmological constant determining the vacuum energy density is still nonzero, despite its smallness. As is known, the cosmological constant Λ has been first introduced in the works of A.Einstein [6] who has used it as a antigravitational term to obtain solutions for the equations of the General Relativity (GR) in the stationary case. However, when A.Friedmann has found the solutions for GR in case of expanding Universe [7] and E.Hubble has derived an extension of the latter, A.Einstein refused from the cosmological constant considering its introduction to be erroneous [8].
But the situation was not so simple. In [9] it has been stated that any contribution into the vacuum energy acts exactly as the cosmological constant Λ and the Vacuum Energy Density is proportional to Λ. The principal problem of the cosmological constant resides in the fact that its experimental value is smaller by a factor of 10^{123} than that derived using a Quantum Field Theory (QFT) [10],[11].
And the theories actively developed at the present time (e.g., superstring theory, loop quantum gravity, etc.)offer a modified quantum theory including, in particular, the fundamental length at Planck's scale. The estimates of Λ obtained on the basis of these theories may be greatly differing from the initial ones derived from standard QFT.
In this paper some of the properties of the Vacuum Energy Density are studied within the scope of a Quantum Field Theory with UV cutoff (minimal length). Such a theory arises in the Early Universe in all the models without exception since the fundamental length (probably on the order of Planck's but not necessarily) is acknowledged to be of a crucial importance in this case It is shown that for this case the experimental and theoretical values are close and may be expressed in terms of a new small parameter introduced in physics at Planck's scales. Here some explanation is needed. The point is that a Quantum Field Theory with minimal length (QFTML) or, what is the same, UV-cutoff is always originating as a deformation of QFT. This deformation is understood as an extension of some theory with the use of one or several additional parameters in such a way that the initial theory shows itself in the limiting process [12]. One of such extensions generated by an additional small dimensionless parameter, in terms of which the Dark Energy Problem is formulated and successfully solved, is described in this paper. In so doing entropy of the Universe and its dynamics play a significant role. Additionally, within the scope of a dynamic approach to Λ, its behavior associated with the Generalized Uncertainty Principle is studied

for the pair "cosmological constant - volume of space-time". In what follows, there is no differentiation between the notions of the cosmological constant Λ and Vacuum Energy Density ρ_{vac}. Besides, it is demonstrated that a new small parameter occurs in High Energy Gravitational Thermodynamics and Gravitational Holography (UV-limit)as well. On this basis the possibility for a new approach to the problem of Quantum Gravity is discussed.

2 Vacuum Energy Density and Most Popular Modern Dark Energy Models

As noted in Introduction, the Vacuum Energy is a major candidate for the Dark Energy. At the same time, due to a factor of 10^{123} distinction between the experimental value ρ_{vac}^{exp} [1] and the value ρ_{vac}^{QFT} calculated using standard QFT [10]– $\rho_{vac}^{QFT} \approx m_p^4$

$$\frac{\rho_{vac}^{exp}}{\rho^{QFT}} \approx 10^{-123}, \qquad (1)$$

interpretation of Dark Energy as a Vacuum Energy presents great difficulties. But there are several methods enabling one to obviate the difficulties. We can name two most popular and acknowledged approaches.

2.1 Holographic Dark Energy Models

The basic relation for this model is the "energy" inequality [13]– [15]

$$E_{\overline{\Lambda}} \leq E_{BH} \rightarrow l^3 \rho_{\overline{\Lambda}} \leq m_p^2 l. \qquad (2)$$

Here $\rho_{\overline{\Lambda}} = \overline{\Lambda}^4$ – vacuum energy density with the UV cutoff $\overline{\Lambda}$ and l is the length scale (IR cutoff) of the system. For the equality in (2) we have the **holographic energy density**

$$\rho_{\overline{\Lambda}} \sim \frac{m_p^2}{l^2} \sim \frac{1}{(l_{\rm p} l)^2}. \qquad (3)$$

Also, from (2) we can get the "entropic" inequality (entropy bound)

$$S_{\overline{\Lambda}} \leq (m_p^2 A)^{3/4}, \qquad (4)$$

where $A = 4\pi l^2$ is the area of this system in the spherically symmetric case. The number of works devoted to the Holographic Dark Energy Models, beginning from the first publication [13], is ever growing [16] to relieve us from citing the whole list.

2.2 Agegraphic Dark Energy Models

Agegraphic Dark Energy Models became the subject of study only two years ago [17]. These relations were based on the result of Károlyházy for quantum fluctuations of time [18]–[20]

$$\delta t = \lambda t_{\rm p}^{2/3} t^{1/3}. \tag{5}$$

Using the uncertainty relation of "energy-time" in the flat space

$$\Delta E \sim t^{-1}, \tag{6}$$

we can obtain the **agegraphic energy density** [21], [15]

$$\rho_{\boldsymbol{T}} \sim \frac{\Delta E}{(\delta t)^3} \sim \frac{m_{\rm p}^2}{\boldsymbol{T}^2}, \tag{7}$$

where $\boldsymbol{T}$ is an age of the Universe.
The number of publications associated with models of this type is constantly increasing too [22]. This is caused by their relative simplicity and by a sufficiently good coincidence of the agegraphic energy density $\rho_{\boldsymbol{T}}$ with ρ_{vac}^{exp}.

3 Dark Energy Problem and Quantum Theory with UV Cutoff

By Holographic Dark Energy Models (explicitly) and by Agegraphic Dark Energy Models (implicitly) it is implied that QFT, where they are valid, is actually QFT with the UV cutoff or fundamental length.
As it has been repeatedly demonstrated earlier, a Quantum Mechanics of the Early Universe (Plank Scale) is a Quantum Mechanics with the Fundamental Length (QMFL) [23]. The main approach to framing of QFT with UV cutoff is that associated with the Generalized Uncertainty Principle (GUP) [24],[25],[26] and with the corresponding Heisenberg algebra deformation produced by this principle [27]–[30].

Besides, QMFL has been framed first using the deformed density matrix and then in the produced corresponding Heisenberg algebra deformation [31]–[40], the density matrix deformation $\rho(\alpha)$ in QMFL being a starting object called the density pro-matrix and the deformation parameter (additional parameter) $\alpha = l_{min}^2/x^2$, where x is the measuring scale and $l_{min} \sim l_p$. As indicated in this paper, the deformation parameter α is varying within the limits $0 < \alpha \leq 1/4$. Moreover $\lim\limits_{\alpha\to 0}\rho(\alpha) = \rho$, where ρ is the density matrix in the well-known Quantum Mechanics (QM), and the following condition must be fulfilled:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0\alpha^2 + ... \tag{8}$$

The explicit form of the above-mentioned deformation gives an exponential ansatz:

$$\rho^*(\alpha) = exp(-\alpha)\sum_i \omega_i |i><i|, \tag{9}$$

where all $\omega_i > 0$ are independent of α and their sum is equal to 1.
In the corresponding deformed Quantum Theory (denoted as QFT^α) for average values we have

$$<B>_\alpha = exp(-\alpha) <B>, \tag{10}$$

where $<B>$ - average in well-known QFT [36],[37] denoted as QFT^α. All the variables associated with the considered α - deformed quantum field theory are hereinafter marked with the upper index $^\alpha$.
Note that the deformation parameter α is absolutely naturally represented as a ratio between the squared UV and IR limits

$$\alpha = (\frac{UV}{IR})^2, \tag{11}$$

where UV is fixed and IR is varying.
As follows from the holographic principle [41]–[44], maximum entropy that can be stored within a bounded region $\Re$ in 3-D space must be proportional to the value $A(\Re)^{3/4}$, where $A(\Re)$ is the surface area of $\Re$. Of course, this is associated with the case when the region $\Re$ is not an inner part of a particular black hole. Provided a physical system contained in $\Re$ is not bounded by the condition of stability to the gravitational collapse, i.e. this system is simply non-constrained gravitationally, then according to the conventional

QFT $S_{\max}(\Re) \sim V(\Re)$, where $V(\Re)$ is the bulk of $\Re$. However in the Holographic Principle case, as it has been demonstrated originally by G. 't Hooft [41] and later by other authors (for example R. V. Buniy and S. D. H. Hsu [45]), we have

$$S_{\max}(\Re) \leq \frac{A(\Re)^{3/4}}{{l_p}^2}. \tag{12}$$

In terms of the deformation parameter α, the principal values of the Vacuum Energy Problem may be simply and clearly defined. Let us begin with the Schwarzschild black holes, whose semiclassical entropy is given by

$$S = \pi R_{Sch}^2/l_p^2 = \pi R_{Sch}^2 m_p^2 = \pi \alpha_{R_{Sch}}^{-1}, \tag{13}$$

with the assumption that in the formula for α $R_{Sch} = x$ is the measuring scale and $l_p = 1/m_p$. Here R_{Sch} is the adequate Schwarzschild radius, and $\alpha_{R_{Sch}}$ is the value of α associated with this radius. Then, as it has been pointed out in [46], in case the Fischler- Susskind cosmic holographic conjecture [47] is valid, the entropy of the Universe is limited by its "surface" measured in Planck units [46]:

$$S \leq \frac{A}{4} m_p^2, \tag{14}$$

where the surface area $A = 4\pi R^2$ is defined in terms of the apparent (Hubble) horizon

$$R = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{15}$$

with curvature k and scale a factors.
Again, interpreting R from (15) as a measuring scale, we directly obtain(14) in terms of α:

$$S \leq \pi \alpha_R^{-1}, \tag{16}$$

where $\alpha_R = l_p^2/R^2$. Therefore, the average entropy density may be found as

$$\frac{S}{V} \leq \frac{\pi \alpha_R^{-1}}{V}. \tag{17}$$

Using further the reasoning line of [46] based on the results of the holographic thermodynamics, we can relate the entropy and energy of a holographic system [48],[49]. Similarly, in terms of the α parameter one can easily estimate the upper limit for the energy density of the Universe (denoted here by ρ_{hol}) [50]:

$$\rho_{hol} \leq \frac{3}{8\pi R^2} m_p^2 = \frac{3}{8\pi} \alpha_R m_p^4, \tag{18}$$

that is drastically differing from the one obtained with well-known QFT

$$\rho^{QFT} \sim m_p^4. \tag{19}$$

Here by ρ^{QFT} we denote the energy vacuum density calculated from well-known QFT (without UV cutoff) [10]. Obviously, as α_R for R determined by (15) is very small, actually approximating zero, ρ_{hol} is by several orders of magnitude smaller than the value expected in QFT – ρ^{QFT}.
Since $m_p \sim 1/l_p$, the right-hand side of (18) is actually nothing else but the right-hand side of (3) in Holographic Dark Energy Models (subsection 2.1). Thus, in Holographic Dark Energy Models the principal quantity, **holographic energy density** $\rho_{\overline{\Lambda}}$ (3), may be estimated in terms of the deformation parameter α.
In fact, the upper limit of the right-hand side of (18) is attainable, as it has been demonstrated in [50] and indicated in [46]. The "overestimation" value of r for the energy density ρ^{QFT}, compared to ρ_{hol}, may be determined as

$$r = \frac{\rho^{QFT}}{\rho_{hol}} = \frac{8\pi}{3} \alpha_{\mathbf{R}}^{-\mathbf{1}} = \frac{8\pi}{3} \frac{R^2}{l_p^2} = \frac{8\pi}{3} \frac{S}{S_p}, \tag{20}$$

where S_p is the entropy of the Plank mass and length for the Schwarzschild black hole. It is clear that due to smallness of α_R the value of α_R^{-1} is on the contrary too large. It may be easily calculated (e.g., see [46])

$$r = 5.44 \times 10^{122} \tag{21}$$

in a good agreement with the astrophysical data.
Naturally, on the assumption that the vacuum energy density ρ_{vac} is involved in ρ as a term

$$\rho = \rho_M + \rho_{vac}, \tag{22}$$

where ρ_M - average matter density, in case of ρ_{vac} we can arrive to the same upper limit (right-hand side of the formula (18)) as for ρ.

4 Some Comments on a Dynamic Character of Cosmological Constant and GUP

Generally speaking, Λ is referred to as a constant just because it is such in the equations, where it occurs: Einstein equations [6]. But in the last few years the dominating point of view has been that Λ is actually a dynamic quantity, now weakly dependent on time [51]–[53]. It is assumed therewith that, despite the present-day smallness of Λ or even its equality to zero, nothing points to the fact that this situation was characteristics for the early Universe as well. Some recent results [54]–[57] are rather important pointing to a potentially dynamic character of Λ. Specifically, of great interest is the Uncertainty Principle derived in these works for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda \, \Delta V \sim \hbar, \tag{23}$$

where Λ is the vacuum energy density (cosmological constant). It is a dynamic value fluctuating around zero; V is the space-time volume. Here the volume of space-time V results from the Einstein-Hilbert action [55]:

$$S_{EH} \supset \Lambda \int d^4x \sqrt{-g} = \Lambda V. \tag{24}$$

In this case "the notion of conjugation is well-defined, but approximate, as implied by the expansion about the static Fubini–Study metric" (Section 6.1 of [54]). Unfortunately, in the proof per se (23), relying on the procedure with a non-linear and non-local Wheeler–de-Witt-like equation of the background-independent Matrix theory, some unconvincing arguments are used, making it insufficiently rigorous (Appendix 3 of [54]). But, without doubt, this proof has a significant result, though failing to clear up the situation.
Let us attempt to explain (23)(certainly at an heuristic level) using simpler and more natural terms involved with the other, more well-known, conjugate pair (E, t) - "energy - time". We use the designations of [54],[55]. In this way a four-dimensional volume will be denoted, as previously, by V.
Just from the start, the Generalized Uncertainty Principle (GUP) is used. Then a change over to the Heisenberg Uncertainty Principle at low energies will be only natural. As is known, the Uncertainty Principle of Heisenberg at Planck's scales (energies) may be extended to the Generalized Uncertainty Principle. To illustrate, for the conjugate pair "momentum-coordinate" (p, x)

this fact has been noted in many works [24]–[28]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_p^2 \frac{\triangle p}{\hbar}. \qquad (25)$$

In [33],[39] it is demonstrated that the corresponding Generalized Uncertainty Relation for the pair "energy - time" may be easily obtained from

$$\Delta t \geq \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar}, \qquad (26)$$

where l_p and t_p represent Planck length and time, respectively.
Now we assume that in the space-time volume $\int d^4x\sqrt{-g} = V$ the temporal and spatial parts may be separated (factored out) in the explicit form:

$$V(t) \approx t\overline{V}(t), \qquad (27)$$

where $\overline{V}$ - spatial part V. For the expanding Universe such an assumption is quite natural. Then it is obvious that

$$\Delta V(t) = \Delta t\overline{V}(t) + t\Delta\overline{V}(t) + \Delta t\Delta\overline{V}(t). \qquad (28)$$

Now we recall that for the inflation Universe the scaling factor is $a(t) \sim e^{Ht}$. Consequently, $\Delta\overline{V}(t) \sim \Delta t^3 f(H)$, where $f(H)$ is a particular function of Hubble's constant. From (26) it follows that

$$\Delta t \geq t_{min} \sim t_p. \qquad (29)$$

However, it is suggested that, even though Δt is satisfying (29), its value is sufficiently small in order that ΔV be contributed significantly by the terms containing Δt to the power higher than the first. In this case the main contribution on the right-hand side of (28) is made by the first term $\Delta t\overline{V}(t)$ only. Then, multiplying the left- and right-hand sides of (26) by $\overline{V}$, we have

$$\Delta V \geq \frac{\hbar\overline{V}}{\Delta E} + \alpha' t_p^2 \frac{\Delta E\overline{V}}{\hbar} = \frac{\hbar}{\Delta\Lambda} + \alpha' t_p^2 \overline{V}^2 \frac{\Delta\Lambda}{\hbar}. \qquad (30)$$

It is not surprising that a solution of the quadratic inequality (30) leads to a minimal volume of the space-time $V_{min} \sim V_p = l_p^3 t_p$ since (25) and (26) result in minimal length $l_{min} \sim l_p$ and minimal time $t_{min} \sim t_p$, respectively.
(30) is of interest from the viewpoint of two limits:

1)IR - limit: $t \to \infty$
2)UV - limit: $t \to t_{min}$.
In the case of IR-limit we have large volumes $\overline{V}$ and V at low $\Delta\Lambda$. Because of this, the main contribution on the right-hand side of (30) is made by the first term, as great $\overline{V}$ in the second term is damped by small t_p and $\Delta\Lambda$. Thus, we derive at

$$\lim_{t\to\infty} \Delta V \approx \frac{\hbar}{\Delta\Lambda} \tag{31}$$

in accordance with (23) [54]. Here, similar to [54], Λ is a dynamic value fluctuating around zero.
And for the case (2) $\Delta\Lambda$ becomes significant

$$\lim_{t\to t_{min}} \overline{V} = \overline{V}_{min} \sim \overline{V}_p = l_p^3; \lim_{t\to t_{min}} V = V_{min} \sim V_p = l_p^3 t_p. \tag{32}$$

As a result, we have

$$\lim_{t\to t_{min}} \Delta V = \frac{\hbar}{\Delta\Lambda} + \alpha_\Lambda V_p^2 \frac{\Delta\Lambda}{\hbar}, \tag{33}$$

where the parameter α_Λ absorbs all the above-mentioned proportionality coefficients.
For(33) $\Delta\Lambda \sim \Lambda_p \equiv \hbar/V_p = E_p/\overline{V}_p$.
It is easily seen that in this case $\Lambda \sim M_p^4$, in agreement with the value obtained using a naive (i.e. without super-symmetry and the like) quantum field theory [11],[10]. Despite the fact that Λ at Planck's scales (referred to as $\Lambda(UV)$) (33) is also a dynamic quantity, it is not directly related to well-known Λ (23),(31) (called $\Lambda(IR)$) because the latter, as opposed to the first one, is derived from Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \left(-\Lambda g_{\mu\nu} + T_{\mu\nu}\right). \tag{34}$$

However, Einstein's equations (34) are not valid at the Planck scales and hence $\Lambda(UV)$ may be considered as some high-energy generalization of the conventional cosmological constant, leading to $\Lambda(IR)$ in the low-energy limit.
In conclusion, it should be noted that the right-hand side of (25),(26) in fact is a series. Of course, a similar statement is true for (33) as well.
Then, we obtain a system of the Generalized Uncertainty Relations for the

Early Universe (Plancks scales) in the symmetric form as follows:

$$\left\{\begin{array}{lll}\Delta x & \geq & \frac{\hbar}{\Delta p}+\alpha^{\prime}\left(\frac{\Delta p}{p_{pl}}\right)\frac{\hbar}{p_{pl}}+... \\ \\ \Delta t & \geq & \frac{\hbar}{\Delta E}+\alpha^{\prime}\left(\frac{\Delta E}{E_{p}}\right)\frac{\hbar}{E_{p}}+... \\ \\ \Delta V & \geq & \frac{\hbar}{\Delta \Lambda}+\alpha_{\Lambda}\left(\frac{\Delta \Lambda}{\Lambda_{p}}\right)\frac{\hbar}{\Lambda_{p}}+...\end{array}\right. \tag{35}$$

The latter of relations (35) may be important when finding the general form for $\Lambda(UV)$, low-energy limit $\Lambda(IR)$, and also may be a step in the process of constructing future quantum-gravity equations, the low-energy limit of which is represented by Einstein's equations (34).

It should be noted that a system of inequalities (35) may be complemented by the Generalized Uncertainty Relation in Thermodynamics [33],[39],[58]. Let us consider the thermodynamics uncertainty relations between the inverse temperature and interior energy of a macroscopic ensemble

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U}, \tag{36}$$

where k is the Boltzmann constant.

N.Bohr [59] and W.Heisenberg [60] have been the first to point out that such kind of uncertainty principle should take place in thermodynamics. The thermodynamic uncertainty relations (36) were proved by many authors and in various ways [61]; their validity does not raise any doubts. Nevertheless, relation (36) was proved in view of the standard model of the infinite-capacity heat bath encompassing the ensemble. But it is obvious from the above inequalities that at very high energies the capacity of the heat bath can no longer be assumed infinite at the Planck scale. Indeed, the total energy of the pair heat bath - ensemble may be arbitrary large but finite merely as the Universe is born at a finite energy. Hence the quantity that can be interpreted as a temperature of the ensemble must have the upper limit and so does its main quadratic deviation. In other words the quantity $\Delta(1/T)$ must be bounded from below. But in this case an additional term should be introduced into (36) [33],[39],[58]:

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U}+\eta\, \Delta U, \tag{37}$$

where η is a coefficient. Dimension and symmetry reasons give

$$\eta \sim \frac{k}{E_p^2} \ .$$

As in the previous cases, inequality (37) leads to the fundamental (inverse) temperature [33],[39],[58].

$$T_{max} = \frac{\hbar}{\Delta t_{min} k} \sim \frac{\hbar}{t_p k}, \quad \beta_{min} = \frac{1}{kT_{max}} = \frac{\Delta t_{min}}{\hbar}. \tag{38}$$

In the recently published work [62]) the black hole horizon temperature has been measured with the use of the Gedanken experiment. In the process the Generalized Uncertainty Relations in Thermodynamics (37) have been derived also. Expression (37) has been considered in the monograph [63] within the scope of the mathematical physics methods.
Besides, note that one of the first studies of the cosmological constant within the scope of the Heisenberg Uncertainty Principle has been presented in several works [64] – [66] demonstrating the inference: **"vacuum fluctuation of the energy density can lead to the observed cosmological constant" [64]**. In these works, however, no consideration has been given to GUP, whereas UV-cutoff has been derived artificially.

5 Gravitational Thermodynamics and Gravitational Holography in Low and High Energy

In the last decade a number of very interesting works have been published. We can primary name the works of T.Padmanbhan [65]–[76], where gravitation, at least for the spaces with horizon, is directly associated with thermodynamics and the results obtained demonstrate a holographic character of gravitation. Of the greatest significance is a pioneer work written by T.Jacobson [48]. For black holes the association has been first revealed by Bekenstein and Hawking [77],[78], who related the black-hole event horizon temperature to the surface gravitation. T.Padmanbhan, in particular in [75], has shown that this relation is not accidental and may be generalized for the spaces with horizon. As all the foregoing results have been obtained in a semiclassical approximation, i.e. for sufficiently low energies, the problem arises: how these results are modified when going to higher energies. In the context of this paper, the problem may be stated as follows: since we have some infra-red (IR) cutoff L and ultraviolet (UV) cutoff l, we naturally have a problem how the above-mentioned results on Gravitational Thermodynamics are changed for

$$L \rightarrow l. \tag{39}$$

According to Section 3 of this paper, they should become dependent on the deformation parameter α. After all, in the already mentioned Section 3 (11) α is indicated as nothing else but

$$\alpha = \frac{l^2}{L^2}. \tag{40}$$

In fact, in several papers [79]–[85] it has been demonstrated that thermodynamics and statistical mechanics of black holes in the presence of GUP (i.e. at high energies) should be modified. To illustrate, in [84] the Hawking temperature modification has been computed in the asymptotically flat space in this case in particular. It is easily seen that in this case the deformation parameter α arises naturally. Indeed, modification of the Hawking temperature is of the following form(formula (10) in [84]):

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar r_+}{2\alpha'^2 l_p^2}[1-(1-\frac{4\alpha'^2 l_p^2}{r_+^2})^{1/2}], \tag{41}$$

where d is the space-time dimension, and r_+ is the uncertainty in the emitted particle position by the Hawking effect, expressed as

$$\Delta x_i \approx r_+ \tag{42}$$

and being nothing else but a radius of the event horizon; α' – dimensionless constant from GUP. But as we have $2\alpha' l_p = l_{min}$, in terms of α (41) may be written in a natural way as follows:

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar \alpha_{r_+}^{-1}}{\alpha' l_p}[1-(1-\alpha_{r_+})^{1/2}], \tag{43}$$

where α_{r_+}- parameter α associated with the IR-cutoff r_+. In such a manner T_{GUP} is only dependent on the constants including the fundamental ones and

on the deformation parameter α.
The dependence of the black hole entropy on α may be derived in a similar way. For a semiclassical approximation of the Bekenstein-Hawking formula [77],[78]

$$S = \frac{1}{4}\frac{A}{l_p^2}, \tag{44}$$

where A – surface area of the event horizon, provided the horizon event has radius r_+, then $A \sim r_+^2$ and (44) is clearly of the form

$$S = \sigma\alpha_{r_+}^{-1}, \tag{45}$$

where σ is some dimensionless denumerable factor. The general formula for quantum corrections [83] given as

$$S = \frac{A}{4l_p^2} - \frac{\pi\alpha'^2}{4}\ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const} \ , \tag{46}$$

where the expansion coefficients $c_n \propto \alpha'^{2(n+1)}$ can always be computed to any desired order of accuracy [83], may be also written as a power series in $\alpha_{r_+}^{-1}$ (or Laurent series in α_{r_+})

$$S = \sigma\alpha_{r_+}^{-1} - \frac{\pi\alpha'^2}{4}\ln\left(\sigma\alpha_{r_+}^{-1}\right) + \sum_{n=1}^{\infty} c_n \left(\sigma\alpha_{r_+}^{-1}\right)^{-n} + \text{const} \tag{47}$$

Note that here no consideration is given to the restrictions on the IR-cutoff

$$L \leq L_{max} \tag{48}$$

and to those corresponding the extended uncertainty principle (EUP) that leads to a minimal momentum [84]. This problem will be considered separately in further publications of the author.
A black hole is a specific example of the space with horizon. It is clear that for other horizon spaces [75] a similar relationship between their thermodynamics and the deformation parameter α should be exhibited.
Quite recently, in a series of papers, and specifically in [67]–[73], it has been shown that Einstein equations may be derived from the surface term of the GR Lagrangian, in fact containing the same information as the bulk term.
And as Einstein-Hilbert's Lagrangian has the structure $L_{EH} \propto R \sim (\partial g)^2 + \partial^2 g$, in the customary approach the surface term arising from $L_{surf} \propto \partial^2 g$

has to be canceled to get Einstein equations from $L_{bulk} \propto (\partial g)^2$ [74]. But due to the relationship between L_{bulk} and L_{surf} [69]–[71],[74], we have

$$\sqrt{-g} L_{suf} = -\partial_a \left(g_{ij} \frac{\partial \sqrt{-g} L_{bulk}}{\partial(\partial_a g_{ij})} \right). \tag{49}$$

In such a manner one can suggest a holographic character of gravity in that the bulk and surface terms of the gravitational action contain identical information. However, there is a significant difference between the first case, when variation of the metric g_{ab} in $L_{\rm bulk}$ leads to Einstein equations, and the second case, associated with derivation of the GR field equations from the action principle using only the surface term and virtual displacements of horizons [66], whereas the metric is not treated as a dynamic variable [74].
In the case under study, it is assumed from the beginning that we consider the spaces with horizon. It should be noted that in the Fischler-Susskind cosmic holographic conjecture it is implied that the Universe represents spherically symmetric space-time, on the one hand, and has a (Hubble) horizon (15), on the other hand. But proceeding from the results of [67]– [74], an entropy boundary is actually given by the surface of horizon measured in Planck's units of area [70]:

$$S = \frac{1}{4} \frac{A_R}{l_p^2}, \tag{50}$$

where A_R is the horizon area corresponding to the Hubble horizon R (15).
To sum it up, an assumption that space-time is spherically symmetric and has a horizon is the only natural assumption held in the Fischler-Susskind cosmic holographic conjecture to support its validity. Thus the arguments in support of the Fischler-Susskind cosmic holographic conjecture are given on the basis of the results obtained lately on Gravitational Holography and Gravitational Thermodynamics.
It should be noted that Einstein's equations may be obtained from the proportionality of the entropy and horizon area together with the fundamental thermodynamic relation connecting heat, entropy, and temperature [48]. In fact [67]– [74], this approach has been extended and complemented by the demonstration of holographicity for the gravitational action (see also [75]).And in the case of Einstein-Hilbert gravity, it is possible to interpret Einstein's equations as the thermodynamic identity [76]:

$$TdS = dE + PdV. \tag{51}$$

The above-mentioned results in the last paragraph have been obtained at low energies, i.e. in a semiclassical approximation. Because of this, the problem arises how these results are changed in the case of high energies? Or more precisely, how the results of [48],[67]– [76] are generalized in the UV-cutoff? It is obvious that, as in this case all the thermodynamic characteristics become dependent on the deformation parameter α, all the corresponding results should be modified (deformed) to meet the following requirements:
(a) to be clearly dependent on the deformation parameter α at high energies;

(b) to be duplicated, with high precision, at low energies due to the limiting transition $\alpha \to 0$.

(c) let us clear up what is meant by the adequate α-deformation of Einstein's equations (General Relativity) and by the Holographic Principle [41]–[44].
The problem may be more specific.
As, according to [48],[75],[76] and some other works, gravitation is greatly determined by thermodynamics and at high energies the latter is a deformation of the classical thermodynamics, it is interesting whether gravitation at high energies (or what is the same, quantum gravity or Planck scale)is being determined by the corresponding deformed thermodynamics. The formulae (43) and (47) are elements of the high-energy α-deformation in thermodynamics, a general pattern of which still remains to be formed. Obviously, these formulae should be involved in the general pattern giving better insight into the quantum gravity, as they are applicable to black mini-holes (Planck black holes) which may be a significant element of such a pattern. But what about other elements of this pattern? How can we generalize the results [48],[75],[76]when the IR-cutoff tends to the UV-cutoff (formula (39))? What are modifications of the thermodynamic identity (51) in a high-energy deformed thermodynamics and how is it applied in high-energy (quantum) gravity? What are the aspects of using the Generalized Uncertainty Relations in Thermodynamics [33],[39],[58] (37),(37)in this respect? It is clear that these relations also form an element of high-energy thermodynamics.
By authors opinion, the methods developed to solve the problem of point (c) and elucidation of other above-mentioned problems may form the basis for a new approach to solution of the quantum gravity problem. And one of the keys to the **quantum gravity** problem is a better insight into the **high-energy thermodynamics**.

6 QFT with UV-Cutoff for Different Approaches and Some Comments

I. As shown by numerous authors (to start with [29]), the Quantum Mechanics with the fundamental length (UV cutoff) generated by GUP is in line with the following deformation of Heisenberg algebra

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) \tag{52}$$

and

$$\Delta x_{\min} \approx \hbar\sqrt{\beta} \sim l_p. \tag{53}$$

In the recent works [86] it has been demonstrated that the Holographic Principle is an outcome of this approach, actually being integrated in the approach.
We can easily show that the deformation parameter β in (52),(53) may be expressed in terms of the deformation parameter α (see Section 3 of the text) that has been introduced in the approach associated with the density matrix deformation. Indeed, from (52),(53) it follows that $\beta \sim \mathbf{1/p^2}$, and for $x_{\min} \sim l_p$, β corresponding to $x_{\min}$ is nothing else but

$$\beta \sim 1/P_{pl}^2, \tag{54}$$

where P_{pl} is Planck's momentum: $P_{pl} = \hbar/l_p$.
In this way β is changing over the following interval:

$$\lambda/P_{pl}^2 \leq \beta < \infty, \tag{55}$$

where λ is a numerical factor and the second member in (52) is accurately reproduced in momentum representation (up to the numerical factor) by $\alpha = l_{min}^2/l^2 \sim l_p^2/l^2 = p^2/P_{pl}^2$

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) = i\hbar(1+a_1\alpha+a_2\alpha^2+...). \tag{56}$$

As indicated in the previous Section (formula (45)), parameter α has one more interesting feature:

$$\alpha_l^{-1} \sim l^2/l_p^2 \sim S_{BH}. \tag{57}$$

Here α_l is the parameter α corresponding to l, S_{BH} is the black hole entropy with the characteristic linear size l (for example, in the spherically symmetric case $l = R$ - radius of the corresponding sphere with the surface area A), and

$$A = 4\pi l^2, S_{BH} = A/4l_p^2 = \pi\alpha_l^{-1}. \tag{58}$$

This note is devoted to the demonstration of the fact that in case of the holographic principle validity in terms of the new deformation parameter α in QFT^{α}, considered above and introduced as early as 2002 [87]–[89], all the principal values associated with the Vacuum (Dark) Energy Problem may be defined simply and naturally. At the same time, there is no place for such a parameter in the well-known QFT, whereas in QFT with the fundamental length, specifically in QFT^{α}, it is quite natural [31],[32],[34], [36],[37],[39].

II. It should be noted that smallness of α_R (Section 3) leads to a very great value of r in (20),(21). Besides, from (20) it follows that there exists some minimal entropy $S_{min} \sim S_p$, and this is possible only in QFT with the fundamental length.

III.This Section is related to Section 3 in [65] as well as to Sections 3 and 6 in [66]. The constant l_Λ introduced in these works is such that in the case under consideration $\Lambda \equiv l_\Lambda^{-2}$ is equivalent to R, i.e. $\alpha_R \approx \alpha_{l_\Lambda}$ with $\alpha_{l_\Lambda} = l_p^2/l_\Lambda^2$. Then expression on the right-hand side of (18) is the major term of the formula for ρ_{vac}, and its quantum corrections are nothing else but a series expansion in terms of α_{l_Λ} (or α_R)

$$\rho_{\rm vac} \sim \frac{1}{l_p^4}\left(\frac{l_p}{l_\Lambda}\right)^2 + \frac{1}{l_p^4}\left(\frac{l_p}{l_\Lambda}\right)^4 + \cdots = \alpha_{l_\Lambda} m_p^4 + ... \tag{59}$$

In the first variant presented in [65] and [66] the right-hand side of (59) (formulas (12),(33)) in [65] and [66], respectively)reveals an enormous additional term $m_p^4 \sim \rho_{QFT}$ for renormalization. As indicated in the previous Section, it may be, however, ignored because the gravity is described by a pure surface term. And in the case under study, owing to the Holographic Principle, we may proceed directly to (59). Moreover, in QFT^{α} there is no need in renormalization as from the start we are concerned with the ultraviolet-finiteness. Moreover, a series expansion of (59) in terms of α is a complete analog of the expansion in terms of the same parameter, redetermining the measuring procedure in $QMFL^{\alpha}$ [32],[34],[36],[39]:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a'_0\alpha^2 + ... \tag{60}$$

As indicated in [40], the same expansion may be used to obtain quantum corrections to the semiclassical Bekenstein-Hawking formula (50) for the black hole entropy.

IV.Besides, the Heisenberg's algebra deformations are introduced due to the involvement of minimal length in quantum mechanics. These deformations are stable in the sense of [90]. But this is not true for the unified algebra of Heisenberg and Poincare. This algebra does not carry the indicated immunity. It is suggested that the Lie algebra for the interface of the gravitational and quantum realms is in its stabilized form. Now it is clear that such a stability should be raised to the status of a physical principle. In a very interesting work of Ahluwalia - Khalilova [90] it has been demonstrated that the stabilized form of the Poincare-Heisenberg algebra [91], [92] carries three additional parameters: "a length scale pertaining to the Planck/unification scale, a second length scale associated with cosmos, and a new dimensionless constant with the immediate implication that 'point particle' ceases to be a viable physical notion. It must be replaced by objects which carry a well-defined, representation space dependent, minimal spatiotemporal extent".

Thus, within the scope of a Quantum Field Theory with the UV cutoff (fundamental length), closeness of the theoretical and experimental values for ρ_{vac} is adequately explained. In this case an important role is played by new parameters appearing in the corresponding Heisenberg Algebra deformation. Specifically, by the new small dimensionless parameter α, in terms of which one can adequately interpret both the smallness of ρ_{vac} and its modern experimental value. Besides, it is shown that the Generalized Uncertainty Principle (GUP) may be an instrument in studies of a dynamic character of the cosmological constant Λ.

7 Conclusion

In conclusion it should be noted that in a series of the authors works [31]–[40] a minimal α-deformation of QFT has been formed. By minimal it is meant that no space-time noncommutativity was required, i.e. there was no requirement for noncommutative operators associated with different spatial coordinates

$$[X_i, X_j] \neq 0, i \neq j. \quad (61)$$

However, all the well-known deformations of QFT associated with GUP (for example, [27]–[29]) contain (61) as an element of the corresponding deformed Heisenberg algebra. Because of this, it is necessary to extend (or modify) the above-mentioned minimal α-deformation of QFT –QFT^{α} [31]–[40] to some new deformation $\widetilde{QFT}^{\alpha}$ compatible with GUP, as it has been noted in [93]. Besides, in this paper consideration has been given to QFT with a minimal length, i.e. with the UV-cutoff. Consideration of QFT with a minimal momentum (or IR-cutoff) (48) necessitates an adequate extension of α-deformation in QFT with the introduction of new parameters significant in the IR-limit. Proceeding from point (c) of Section 5, the problem may be stated as follows:

(c) provided α-deformation of GR describes the ultraviolet (quantum-gravity) limit of GR, it is interesting to examine the deformation type describing adequately the infrared limit of GR. It seems that some indications of a nature of such deformation may be found from the works devoted to the infrared modification of gravity [94],[95].

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