

A predictive analytics approach to reducing avoidable hospital readmission

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Abstract

Hospital readmission has become a critical metric of quality and cost of healthcare. Medicare anticipates that nearly \$17 billion is paid out on the 20% of patients who are readmitted within 30 days of discharge. Although several interventions such as transition care management and discharge reengineering have been practiced in recent years, the effectiveness and sustainability depends on how well they can identify and target patients at high risk of rehospitalization. Based on the literature, most current risk prediction models fail to reach an acceptable accuracy level; none of them considers patient's history of readmission and impacts of patient attribute changes over time; and they often do not discriminate between planned and unnecessary readmissions. Moreover, the effect of different time intervals that defines readmissions has not been looked at before. In this study, we tackle such drawbacks by developing and validating a predictive analytics framework for avoidable readmissions. We further assert that the government-endorsed 30-day time window that is used to count and report readmissions is not appropriate for chronic conditions such as chronic obstructive pulmonary disease. The proposed methods and tools are demonstrated with real world datasets from four hospitals of the Veterans Health Administration system.

Keywords: readmission, predictive analytics, patient flow, phase-type distribution, Markov chain.

1 Introduction

Hospital readmission is disruptive to patients and costly to healthcare systems. Unnecessary return to hospitals shortly after discharge has been increasingly perceived as a marker of the quality of care that patients receive during hospital admission (Chassin, Loeb, Schmaltz, & Wachter, 2010). About one in five Medicare fee-for-service beneficiaries, totaling over 2.3 million patients, are rehospitalized within 30 days after discharge, incurring an annual cost of \$17 billion, which constitutes nearly 20% of Medicare's total payment (Jencks, Williams, & Coleman, 2009). However, it is reported by the Medicare Payment Advisory Commission (MedPAC) that about 75% of such readmissions can and should be avoided because they often result from a fragmented healthcare system that leaves discharged patients with preventable flaws such as hospital-acquired infections and other complications, poor planning for follow up care

transitions, inadequate communication of discharge instructions, and failure to reconcile and coordinate medications (Medicare Payment Advisory Commission, 2007). Variations in both medical and surgical readmission rates by different hospitals and different geographic regions also indicate that some centers (or regions) perform better than others at containing readmission rates (Jencks et al., 2009; Tsai, Joynt, Orav, Gawande, & Jha, 2013). Studies also show that the adjusted readmission rate in the US is among the highest ranking in comparison to European countries (Westert, Lagoe, Keskimäki, Leyland, & Murphy, 2002).

In addition, effective October 2012, as directed by Patient Protection and Affordable Care Act (PPACA, also called Obamacare), the Centers for Medicare and Medicaid Services (CMS) started to cut reimbursement funds for hospitals that have excessive 30-day readmission rates for heart failure, myocardial infarction, and pneumonia patients. This included 2,213 US hospitals with approximately \$280 million funds nationwide, which constitutes 1% of the total Medicare budget. Moreover, this cut will grow to 2% and 3% for FY 2014 and 2015, respectively. As a result, numerous intervention programs have been proposed by policymakers and healthcare organizations to reduce rehospitalizations and improve quality and access to care (Hansen, Young, Hinami, Leung, & Williams, 2011).

While it would be perfect to include all patients in a transitional care intervention, due to their resource intensive nature on one hand and hospital supplies constraints on the other, it is inevitable to target and deliver such efforts to those subgroups that are at greater risk. Nevertheless, identifying patients at increased risk of readmission is challenging and calls for advanced analytics tools that help to stratify risk into clinically relevant classes and provide information early enough during the hospitalization. Various methods have been proposed in recent years to predict hospital readmission but most of them do not yield acceptable predictive accuracy, or they are based on patient factors that are not typically collected during clinical care (Kansagara et al., 2011). Furthermore, a few methods have tried to distinguish avoidable readmission from all other types of readmissions (van Walraven, Bennett, Jennings, Austin, & Forster, 2011), but it remains a disagreement how to systematically define and identify those readmissions that can be prevented based on credible clinical criteria.

Another important aspect of readmission studies is related to the choice of timeframe used to count the number of readmissions. Although the CMS establish 30-day cutoff point for the three acute conditions (heart failure, myocardial infarction, and pneumonia), researchers have considered other periods from two weeks (Reed, Pearlman, & Buchner, 1991) to 180 days (Benbassat & Taragin, 2000) for certain surgical and medical conditions. Moreover, with the new chronic and surgical conditions to be penalized in the next few years, questions regarding the suitability of the 30 day time window remain to be explored.

In this paper, we propose a predictive analytics framework that enables medical decision makers to characterize and (more accurately) predict avoidable readmissions, and to investigate the effects of differ-

ent patient risk factors on the likelihood of rehospitalization. The goals of our study are three-fold: 1) to develop and internally validate an administrative algorithm for characterizing avoidable readmissions from all types of readmissions, 2) to address the difficulties around selection of an appropriate timeframe that defines readmissions for chronic conditions, and 3) to create and validate a simple and real-time re-admission risk prediction model that can produce more desirable prediction accuracy than the literature. The proposed methods and tools are evaluated using a wide range of electronic health records from four Veteran Affairs (VA) hospitals in the State of Michigan.

The remainder of this paper is arranged as follows. Section 2 introduces our study design and describes patient variables and health outcomes. In Section 3, we outline methods and measures for the three stated research goals. Detailed analytics and results of applying the proposed methods to the data are presented in Section 4. Finally, concluding remarks and future research directions are posed in the last section.

2 Study design and data environment

The dataset used in this retrospective cohort study is provided by the Veteran Health Administration (VHA), which is the largest single medical system in the United States, with 152 medical centers and nearly 1400 outpatient clinics. We analyze inpatient administrative records gathered from four medical facilities in the State of Michigan, namely, Ann Arbor, Battle Creek, Detroit, and Saginaw, to identify all hospitalizations for Heart Failure (HF), Acute Myocardial Infarction (AMI), Pneumonia (PN), and Chronic Obstructive Pulmonary Disease (COPD) from Fiscal Year 2011 to FY12. Cohorts are marked with ICD-9-CM (International Classification of Diseases, Ninth Revision, Clinical Modification) codes, similar to the coding utilized by the CMS for calculating hospital readmission rates. There were no major changes in the hospital bed supplies, and in the patient admission/discharge processes through that period of time. During a hospital stay, patients may move to different acute wards within the hospital and their episodes of care are carefully tracked with standard computerized means. We use additional data files for patients with chronic conditions as well as patients exposed to environmental hazards such as Agent Orange, to effectively illustrate those impacts on the risk of readmission.

The original set contains 7200 randomly selected records which correspond to 2985 distinct adult patients with principal (or secondary) discharge diagnoses of HF, AMI, PN, and COPD. General exclusions include: (1) Hospital admissions within 24 hours of index discharge, (2) Hospitalizations with a length of stay less than 24 hours (observation stays) or followed by a death, (3) Patients transferred to another acute care facility, (4) Patients discharged against medical advice. To count readmissions in the last month of FY12, the first month of FY13 is taken into account. In additions, we omit stays in long term care, nurs-

ing home, psychiatry, rehabilitation, and hospice wards. However, as we are interested in modeling the effect of patient's related factor changes (over time) on the risk of readmission, unlike most studies in the literature (Au et al., 2012; Joynt, Orav, & Jha, 2011; Shulan, Gao, & Moore, 2013), we do not exclude recurrent (re)admissions of the same patient from the analyses. We also design both external and internal model validations by using stratified split sample and bootstrap resampling methods.

2.1 Controlled variables

We aggregate patient level data files with provider and station levels in order to obtain various types of risk factors for this study. To achieve a better picture of the data environment, we further arrange them into five groups: (1) Demographics: age at discharge, sex, race, and marital status; (2) Socioeconomic: means tested income, and insurance status (Medicare, Medicaid, private, none); (3) Utilization: length of stay of the index hospitalization (LOS), treating facility, source of admission (direct from home, outpatient clinic, transition from any of the four VA hospital, VA Nursing Home Care Unit (NHCU), and VA domiciliary), primary care provider, enrollment priority, and average distance (between patient's home zip code and the zip code of the facility he/she got admitted); (4) Service based: Agent Orange status, Prisoner Of War (POW) status, and radiation status; and (5) Comorbidity and severity: Diagnosis Related Group (DRG), Hierarchical Condition Category (HCC), and Care Assessment Need (CAN) score. The variables were selected based on the relevant medical literature and confirmed by a group of Veteran Affairs (VA) health professionals.

The enrollment priority is a priority level assigned according to the veteran's severity of service-connected disabilities and the VA means test. The DRG is a validated reimbursement classification scheme exploited to identify the cost of services that a hospital renders. In its basic version, the groups are organized with respect to their similarities in patient diagnosis, age, sex, and the presence of complications or comorbidities; then a measure of cost is attached to each group (Fetter, Shin, Freeman, Averill, & Thompson, 1980). HCCs have been used *ad hoc*, mainly for case-mix and risk adjustment in healthcare utilization and payment systems. Each HCC group forms a set of clinically and cost-similar conditions reflecting hierarchies among related diseases as defined by the ICD-9-CM codes (Pope et al., 2004). We create dummy variables for both the DRG and HCC variables in the regression studies; that is, if a patient is a member of the category, he or she is given a 1 on this variable; otherwise the score remains zero. The CAN score is a general illness severity score that reflects the likelihood of admission or death within a specified time period, and it works somewhat similar to diagnostic cost group (DxCG) risk score (Sales et al., 2003). The score is commonly expressed as a percentile ranging from 0 (lowest risk) to 99 (highest risk) and it shows how a VA patient is compared with others pertaining to the chances of hospitalization or death. It is interesting to note that all predictor variables except LOS are real time and would be availa-

ble before patient discharge, so they can be employed in planning for pre-discharge (transitional care) intervention programs.

2.2 Study outcomes

The main outcome is rate of avoidable readmission after discharge. Unlike the large literature that studies only the *occurrence* of readmission by logistic (or probit) regression methods (Berry et al., 2013; Pracht & Bass, 2011; Shulan et al., 2013), our current approach is a hybrid of both *occurrence* and *timing* of readmission, which enables us to directly incorporate the effect of partially known information (censored observations) into the risk of readmission. For hospitalizations after HF, AMI, and PN, we follow the CMS approach and define the readmission time interval as 30 days; if no consecutive admission is occurred within 30 days after the most recent admission, the outcome is flagged as censored. For COPD patients, however, we do not adopt the 30-day cutoff, and instead develop an approach to optimally determine the interval that best stratifies the quickly-readmitted and slowly-readmitted patient groups. We further modify the approach introduced by Goldfield et al. (2008), to distinguish between avoidable and unpreventable outcomes. The most common causes of readmission for the four cohorts as well as their changes over time are also investigated as secondary health outcomes.

3 Methods and measures

In this section, we first describe an algorithm built on administrative data to characterize avoidable readmissions from all outcomes (Section 3.1). Then an analytical approach based on Coxian Phase type (PH) distributions is introduced to determine the optimal readmission time interval for chronic diseases and particularly COPD patients (Section 3.2). Next, we propose our risk prediction model that enables one to evaluate and compare the effects of relevant patient factors on the risk of readmission, and to quantify the likelihood of a given future patient's being re-hospitalized (Section 3.3). Finally, we outline some criteria for comparing the performance of our proposal with existing ones (Section 3.4).

3.1 Identifying potentially avoidable readmissions

One of the main difficulties that makes the study of hospital readmission rather sophisticated is that no consensus yet exists on how to separate among so called “planned” readmissions (e.g., scheduled at or soon after the time of discharge) and those that might be prevented by implementing better transitional care programs. Different methods consider distinct outcomes and result in different readmission rates. Here we classify the existing approaches into two broad categories:

- Methods designed to detect and exclude planned hospitalizations from the outcome of interest. Examples in this group include the well-known CMS method (Horwitz et al., 2011) and SQLape (Striving for Quality Level and Analyzing of Patient Expenses) (Halfon et al., 2002), which is a validated hospital comparison system practiced in Switzerland.
- Methods intended to label avoidable readmissions from all index hospitalizations, such as 3M Health Information Division Potentially Preventable Readmission measure (Goldfield et al., 2008).

The CMS approach takes specific index stays and uses unplanned all-cause readmission rate as the primary outcome by removing all non-acute readmissions as well as readmissions for maintenance chemotherapy. It employs AHRQ (Agency for Healthcare Research and Quality) Clinical Classification System (CCS) to identify planned procedures that contain an inpatient stay, along with the conditions that are acute or are complications of care and associated with the planned procedures. Also consistent with National Quality Forum (NQF) standards, the approach performs risk adjustment for case mix (patient comorbidity) and service mix (types of conditions/procedures cared for by the hospital) with the help of CMS Condition Category groupers.

The 3M approach, on the other hand, considers all types of index hospitalization then decides on whether a readmission is avoidable with regard to clinical relationships between the reasons of admission and readmission. To this end, All Patient Refined Diagnosis Related Group (APR DRG) codes are first utilized to classify the patient cohorts; then a group of physicians evaluate the association between the initial admission and its following returns, and define preventable readmissions as those returns having a clinical relevance with the first hospitalization. Moreover, the approach makes use of APR DRG based Severity of Illness (SOI) measures to adjust for case mix risk factors.

Both approaches are built (and validated) on administrative data, and they are mainly used for the purposes of (1) profiling hospitals with respect to their readmission rates and (2) adjusting Medicare or Medicaid payments to low-performing medical centers. However, according to a recent study in the VHA (Mull et al., 2013), they show only a moderate correlation in specifying the readmission rates, which is found related to the preventability element of the 3M method.

In this study, since our goal is more to develop and validate a risk prediction model that can be used for clinical applications (rather than hospital profiling and payment adjustment), we derive a hybrid approach adopting both the CMS and 3M rationales to choose from the patient outcomes. In a nutshell, we first apply the CMS method to exclude those planned procedures that are followed by a non-acute or a non-complication of care condition; then the 3M procedure is implemented on the remaining indices in order to extract potentially avoidable readmissions (PARs). However, we modify the exclusion criteria of both methods and implement VHA definitions of eligible discharge. To increase the overall precision of the proposal, we also got help from three reviewers to judge all cases identified, after completing each

constituent algorithm. Moreover, instead of the APR DRG system, the newly-developed Diagnostic Cost Group Hierarchical Condition Category, version 21 (DCG/HCC v21) was utilized to assess the clinical relationship between each readmission and its initial admission(s) (Pope et al., 2004). We chose the DCG/HCC risk adjustment system because 1) it is a part of models that have been used and evolved over two decades of research; 2) it has special adjustments for elderly beneficiaries as well as patients with chronic conditions; and 3) it is recalibrated regularly according to recent modifications on diagnosis and expenditure data.

The algorithm, which we call Potentially Avoidable Readmission (PAR), can then be stated as follows:

Step 1 (general inclusion/exclusion)

- I. Identify HF, AMI, PN, and COPD cohorts based on principal (or secondary) discharge diagnoses, and eliminate all other conditions. Merge records of the same patient if he/she had multiple hospitalizations on the same day to the same medical unit. This applies to both medical and surgical patients.
- II. Establish readmission time interval (henceforth T) and categorize each entry as either admission or readmission. Also, define *eligible admissions* as all admissions that are at risk of having a readmission.
- III. Exclude:
 - a) From the admission set, cases whose discharge status is “death,” since they cannot have any readmission. These correspond to stand-alone admissions.
 - b) From the admission set, cases whose discharge status are “transfer” to another acute care facility, except the four hospitals studied. The reason is that the hospital cannot affect a patient’s consequent care under such circumstances. If transferred among the four hospitals, however, the final discharging hospital is considered responsible for any readmissions.
 - c) From the admission set, cases whose discharge status is “against medical advice.” Because in such cases, the planned treatment(s) could not be fulfilled and thus they do not represent a quality-of-care signal.
 - d) From the readmission set, those entries that fall within 24 hours of their prior index discharge. This is consistent with the VHA operations policies.
 - e) From the readmission set, cases in which any of the CMS planned procedures are conducted if not followed by an acute or a complication-of-care discharge condition category. Examples of such procedures include peripheral vascular bypass, heart valve, kidney transplant, mastectomy, colorectal resection, and maintenance chemotherapy [see (Horwitz et al., 2011) for the full list].

- f) From the readmission set, AMI patients hospitalized for a percutaneous coronary intervention (PCI) or coronary artery bypass graft (CABG), except those that are diagnosed for HF, AMI, unstable angina, arrhythmia, and cardiac arrest.
 - g) From both admission and readmission sets, hospitalizations in long-term care, palliative care, nursing home, aftercare of convalescence, psychiatry, rehabilitation, and hospice wards; or for fitting of prostheses and adjustment devices.
 - h) From both admission and readmission sets, stays for special conditions with high mortality risk, for which chances of post-discharge death is much higher than chances of being readmitted. These include, but are not limited to, patients with malignant neoplasm without specification of site; and medical patients with cancers of breast, skin, colon, upper digestive tract, lung, liver, pancreas, head, neck, brain, and fracture of neck of femur (hip). This is consistent with the CMS approach.
 - i) From both admission and readmission sets, records that are related to major or metastatic malignancies, multiple trauma, burns, neonatal, obstetrical, Human Immunodeficiency Virus (HIV), and eye care. The rationale is that these conditions usually require specialized follow-up cares and are often not avoidable. This is consistent with the 3M approach.
 - j) From both admission and readmission sets, patients not enrolled in the VA and thus lacking sufficient historical data for the 12 months prior to the index admission. The logic is that the information is required to adjust for the case-mix and comorbidities.
 - k) From both admission and readmission sets, records with inconsistent and/or error components such as age and gender discrepancies, invalid HCC assignment, discharge date that preceded the admission date, disagreements between the patient's VA status and its service-based attribute values, hospitalizations charged for less than \$200 or greater than \$4 million, and records with distances longer than 3000 miles.
- IV. Calculate *eligible admissions* as all records remaining in the admission set. Note that, situations described in a), b), and c), i.e., "death," "transfer," or "against-medical-advice" may happen to both admission and readmission entries.

Step 2 (labeling PARs)

- V. Mark records from the readmission set that have a clinical relationship with their initial admissions as defined by one of the eight following categories:
 - a) Readmissions for an ambulatory care-sensitive condition as specified by the AHRQ (National Quality Measures Clearinghouse).
 - b) Medical readmissions for repeated happening or extension of the reason for the initial (or a closely-related) condition.

- c) Medical readmissions for an acute decompensation of a chronic condition that relates back to the care given in the course or immediately after the initial admission (e.g., a return hospitalization for diabetes by an initially diagnosed AMI patient).
- d) Medical readmissions for acute medical complications acquired during or soon after the first admission (e.g., a readmission for addressing a urinary tract infection of a patient originally hospitalized for hernia repair).
- e) Readmissions for a mental health or substance abuse condition that follows an admission for a non-mental health or non-substance abuse condition.
- f) Readmissions for mental health or substance abuse reason following a hospitalization for a mental health or substance abuse reason.
- g) Surgical readmissions to deal with repeated happenings or extensions of the condition causing the initial hospitalization (e.g., a readmission for appendectomy surgery of a patient who was initially admitted for abdominal pain and fever).
- h) Surgical readmissions to tackle a medical or surgical complication resulting during the initial admission or in the post-discharge course (e.g., a readmission for treating a post-operative wound resulting from an initial hospitalization for a bowel resection).

Step 3 (clinical panel review)

- VI. All exclusions from step 1 and marked PARs in step 2 are reviewed by three physicians, and final decision about the outcomes was made by a majority of vote scheme.

Step 4 (calculating PAR rate)

- VII. Define a PAR series as a sequence of one or more PARs that are all clinically associated with a similar initial admission. In this way, the succeeding PARs are always assessed for having a clinical relationship in reference to the very first admission (which starts the sequence), not with the intermediate PARs. As a result, the total time interval encompassing a PAR series can be larger than T .
- VIII. Update the eligible admission set by reclassifying cases in the readmission set that are NOT found to be PARs (i.e., not having clinical relationship with their prior admissions) and at the same time, do not fall in “death”, “transfer”, or “against-medical-advice” categories.
- IX. Calculate PAR rate as $\frac{\# \text{PAR Series}}{\# \text{Eligible Admissions}}$.

The DCG/HCC system is used throughout the algorithm to assess the clinical association between an initial admission and its PAR series. In other words, we first apply the CMS HCC model to assign HCC

codes for all indices; then the reviewers examine the HCC codes of an initial admission and all of its related PARs to judge whether the readmission(s) could have been avoidable. If available, we also take into account other sources of information such as clinical visits between admission and readmission, and communication with the patient, patient's family and primary care physician assigned, to help on the avoidability assessment of the PARs.

The readmission time interval introduced in II is defined as 30 days for HF, AMI, and PN cohorts. For the COPD, we follow an analytical approach that is outlined in the next section.

3.2 Determining readmission time interval for chronic conditions

It is clear that, similar to the type of readmission (planned vs. avoidable), the length of time window between the dates of initial discharge and index readmission affects the ratio of avoidable readmissions (see sections VIII and IX in the PAR algorithm). The longer the interval is the more readmissions will be recognized and the more money the insurer could save under the bundled policy; however, the chances that a readmission has a clinical relevance with its initial admission become diminished. Although the CMS and health policy makers adopt a 30-day time window for profiling hospitals and public reporting, other intervals from two weeks (Reed et al., 1991) to 180 days (Benbassat & Taragin, 2000) are examined in different situations from certain types of surgery to a specific clinical condition. Researchers also raise the issue of improper interval selection in the literature (Hasan, 2001). In this study, to decide on the appropriate timeframe for COPD patients, first we examine the patterns of empirical readmission rates over days after discharge and recommend a graphical-based estimate; then we develop an analytical framework founded on Coxian phase-type distribution to determine the optimal cut off time defining the readmission.

3.2.1 Graphical based approach

The trend of COPD readmission rate over days following discharge is shown in Figure 1. Consider that the vertical axis displays percentage of patients not readmitted. As shown, the rate is high in the first weeks after discharge, then it levels off and becomes constant, before rising again near 60 days. The rate of (all-type) readmission is 17.3% for the CMS-endorsed 30-day time window. However, inspecting the plot, we find that the slope of the readmission curve becomes stable around the 39th day, so we suggest that 39-day interval may be more appropriate choice for counting COPD readmissions. We believe this finding is also clinically justifiable because chronic conditions, as opposed to acute conditions, are getting worse over an extended amount of time so those readmissions that occur even after the 30th day may also be associated with the quality of the “inpatient” care and should thus be considered for transitional care intervention programs.

3.2.2 Phase-type distribution

Phase-type (PH) distributions comprise a rich class of probability distributions that have been exploited extensively in various applications of stochastic modeling such as financial engineering, teletraffic modeling, drug kinetics, biostatistics, and survival analysis. The distribution is created by one or more inter-related Poisson processes on nonnegative real numbers, which can be represented as the time to reach an absorbing state in a finite-state continuous time homogeneous Markov chain. Despite its prevalence in other areas, the number of applications of PH distribution in healthcare literature is limited, with most publications focusing on modeling patients' length of stay (Fackrell, 2009).

Inspired by the observations from Figure 1, we assert that the rate of COPD readmission is not constant and changes over time. Further, using all-cause inpatient data from the same VA facilities, it was previously demonstrated that the (mean) hazard of readmission over time is influenced by a set of relevant patient factors including source of patient admission and treatment specialty (Shams, Ajorlou, & Yang, 2013). Therefore, it is desired to define the readmission timeframe in accordance with avoidability level and representativeness of quality care. We also recognize that the determination of the interval should not be based on the (instantaneous) risk or hazard of readmission, as the hazard (in the terminology of survival analysis) is a time-dependent conditional probability function that changes with both time and the patient's case mix. On the other hand, since bias may be introduced when using the graphical inspection method, the approach taken should be able to objectively stratify the patients into clinically distinct groups according to avoidability strength of their readmission.

Considering these characteristics, here we undertake a patient flow approach and develop a conceptual framework for the movements of patients after release from the hospital (see Figure 2). It is assumed that discharged patients travel between two major states (Short-Stay and Long-Stay) in their community before being returned to the hospital. In other words, patients begin their post-discharge period from the Short-Stay (SS) group consisting of m sequential transient phases; then they are either readmitted to the hospital at the rate of λ_{SS} or move to the Long-Stay (LS) group with rate λ_m . Patients entering in the LS group remain another r (transient) phases in the community before going back to the hospital at the rate of λ_{LS} . Here, consistent with the CMS and MedPAC logic, we ascertain that readmission from the SS group is a marker of poor quality of *inpatient* care possibly because of premature discharge and poorly-designed process of inpatient care, whereas readmission from the LS group represents deficient quality of post-acute and *outpatient* follow-up care. Note that the rates are not identical within or between the two groups.

The current framework results in a special case of order $m+r$ Coxian PH distribution, which is represented by an absorbing continuous-time Markov chain (CTMC) with $m+r$ transient states and one

absorbing state (Hospital). Then the dynamics of the underlying finite-state stochastic process $\{X(t); t \geq 0\}$ is governed by the transition intensity matrix $\mathbf{A} = \{\alpha_{hj}\}; h, j \in E$ where $E = \{1, 2, \dots, m+r\}$ and

$$\begin{aligned}\alpha_{hj}(t) &= \lim_{\Delta t \downarrow 0} \frac{P[X(t+\Delta t) = j \mid X(t) = h]}{\Delta t}, \\ \alpha_{hh}(t) &= -\sum_{h \neq j} \alpha_{hj}(t).\end{aligned}\tag{1}$$

Hence, the random variable time to readmission T is equal to the time spent in the above CTMC until absorption in the Hospital state, which is also known as the *sojourn* time (Stroock, 2005). In this case, the probability density function f , the survival function S , and the k -th noncentral moment of T can be expressed by

$$f(t) = \boldsymbol{\pi} \exp(\mathbf{Q}t)(-\mathbf{Q}\mathbf{1})\tag{2}$$

$$S(t) = \boldsymbol{\pi} \exp(\mathbf{Q}t)\mathbf{1}\tag{3}$$

$$m_k = (-1)^k k! \mathbf{Q}^{(-k)} \mathbf{1}, \quad k = 1, 2, \dots\tag{4}$$

where $\boldsymbol{\pi}$ is the row vector of the initial probabilities over the transient states, \mathbf{Q} is the $(m+r) \times (m+r)$ transient partition of the intensity matrix, and $\mathbf{1}$ represents an $(m+r) \times 1$ column vector of 1's. Here, $\exp(\mathbf{A})$ denotes the matrix exponential of a square matrix \mathbf{A} (Golub & Van Loan, 2012). Based on the transition flow diagram shown, the Coxian PH distribution is represented by $PH(\boldsymbol{\pi}, \mathbf{Q})$ where $\boldsymbol{\pi} = (1, 0, \dots, 0)$ and $\mathbf{Q} = \{q_{hj}\}$ can be simplified as:

$$\begin{aligned}q_{h,h+1} &= \lambda_h; \quad h = 1, 2, \dots, m+r-1 \\ q_{h,h} &= -\lambda_h; \quad h = \{1, 2, \dots, m-1, m+1, m+2, \dots, m+r-1\} \\ q_{m,m} &= -(\lambda_{SS} + \lambda_m), \quad q_{m+r,m+r} = -\lambda_{LS}.\end{aligned}\tag{5}$$

It is worth mentioning that the actual states of the Markov model are not observable; that is, we do not know the state (SS or LS) from which the patients absorb (readmit) to the hospital. In addition, the phases within each major state (SS and LS) do not carry any practical interpretations, but time spent in each phase follows an exponential distribution. Therefore, the PDF of time-to-readmission (2) can be viewed as a mixture of two generalized Erlang distributions (McLachlan & Peel, 2004) and then is reduced to

$$f(t) = pf_{SS}(t) + (1-p)f_{LS}(t) \quad (6)$$

where f_{SS} and f_{LS} are the PDFs of the time-to-readmission for the Short-Stay and Long-Stay groups, with shape parameters m and $m+r$ respectively, and p is the probability of a patient being in the Short-Stay group, which can be obtained by $\frac{\lambda_{SS}}{\lambda_{SS} + \lambda_m}$.

Following the methods discussed, we propose that the optimal cut-off point for the readmission time window is the point that separates the two components in (6), which, as mentioned earlier, are corresponding to the readmission from inpatient and outpatient care spells one-to-one. Thus, the solution t^* to

$$pf_{SS}(t) = (1-p)f_{LS}(t) \quad (7)$$

will give the optimal readmission timeframe. In order to solve (7), we need to estimate the $2(m+r)-1$ unknown parameters in (5) using approaches such as maximum likelihood (Asmussen, Nerman, & Olsson, 1996), moment matching (Johnson, 1993), or probabilistic clustering (Reinecke, Krauß, & Wolter, 2012) that best fit with empirical data. Observing the time to readmission data $\mathbf{t} = (t_1, t_2, \dots, t_N)$, in the current paper, we use the EM (Expectation-Maximization) algorithm to maximize the general log-likelihood function $\sum_{i=1}^N \log(\boldsymbol{\pi} \exp(\mathbf{Q}t) (-\mathbf{Q}\mathbf{1}))$ with EMpht software (Asmussen). Further, by altering the number of phases, we select the models that best compromise model parsimony and goodness of fit based on both Akaike's Information Criterion (AIC) (Akaike, 1974) and Bayesian Information Criterion (BIC) (Schwarz, 1978). Details about model fitting and inference are given in Section 4.3.

3.3 Predicting potentially avoidable readmissions

In the interest of reducing avoidable readmissions, it is necessary to note that in most cases, including all patients in the intervention programs is neither possible nor economically feasible. Thus to exploit the full potentials of such plans and raise their sustainability, it is beneficial to target patient subsets that are at higher risk of being readmitted. In this regard, predictive modeling approaches turned out to be promising not only in readmission risk prediction models (Kansagara et al., 2011), but also in other healthcare research areas such as hospital profiling based on patient health outcomes (Krumholz et al., 2006), chronic disease management programs (Bayerstadler, Benstetter, Heumann, & Winter, 2013), and patient no-show problems (Alaeddini, Yang, Reddy, & Yu, 2011). Employing advanced statistical and/or ma-

chine learning algorithms, such models typically try to predict the probability of an outcome given a set amount of health data including administrative, claim, or even medical laboratory data.

Generally, risk of readmission needs to be assessed in two different episodes of the intervention programs, namely, pre-discharge and post-discharge. In the former, controlled variables that can be reasonably achieved before hospital discharge (for instance initial diagnosis) is fed into the model and the results are used to identify high-risk subgroups to receive the pre-discharge interventions like patient education and medication reconciliation. The latter, in contrast, make use of relatively all captured information such as LOS of the index hospitalization or principal diagnosis at discharge, and pinpoint high-risk cohorts to be assigned to post-discharge interventions like follow-up telephone calls and timely ambulatory visits. Also according to the type and timing of data gathered, predictive models can be applied for either profiling hospitals based on rate of readmission or predicting the chance of rehospitalization for a given patient.

Suppose D and δ denote the time to readmission and the readmission status (1= readmitted within D days of the discharge, 0= Otherwise). Two modeling families with distinct objectives can be thought of for predicting patient readmission. The first group, which we call classification models, focus on readmission indicator δ and try to estimate it by first learning an algorithm based on inputted features and known class labels. These methods mostly use *algorithmic* models and treat the data mechanism as unknown (black box). Such models are usually prone to overfitting the training dataset in the case of small sample sizes and/or repeated measurements. Nonetheless, they are computationally fast and easy to implement with minimal assumptions about the underlying mechanism that generates the data.

The second group, which we name timing-based models, concentrate on D and try to relate some of its probability functions to a given set of covariates in parametric (accelerated failure time models) or semi-parametric (proportional hazards models) fashion. As opposed to the first class, these methods are rather *data* models: the parametric timing-based methods have distributional assumptions for D , and the semi-parametric ones have proportionality assumptions of the covariate effects. Nevertheless, they are capable of dealing with small (to medium) samples and also are able to update readily to take in new observations with minimum structural changes. In addition, models of this class have nice probabilistic interpretations of the outcome and they can incorporate correlations among the observations in the modeling process. Examples of the first group in readmission studies includes logistic regression (Shulan et al., 2013), random forest (Au et al., 2012), and support vector machines (Zhang, Yoon, Khasawneh, Srihari, & Poranki, 2013), while the second group includes the Cox proportional hazard model (Hernandez et al., 2010), and the lognormal frailty model (Shams et al., 2013). Although each model has its own merits and specific applicability, overall, it is found that most of the described techniques perform poorly in terms of discrimination power and predictive accuracy (Kansagara et al., 2011).

Besides, according to current problems in readmission reduction programs, no consensus exists on the chosen set of patient (and system) factors that deemed related to the likelihood of readmission. This usually happens in studies comprising dissimilar health care settings (e.g., Medicare versus private health insurance) or diverse patient populations (e.g., VA versus Non-VA). For instance, in a study of UK inpatients, the popular LACE measure (Length of stay, Acuity of the admission, Charlson comorbidity score (Charlson, Pompei, Ales, & MacKenzie, 1987), and Emergency department visits within six months) which works well in a number of UK populations, is found to be mediocre in predicting 30-day readmission rates. Therefore, we believe that in our study, a data-driven patient-centered approach should be developed to guide the decisions upon the readmission intervention programs. To this end, we do not impose *a priori* candidate variables to the modeling process nor do we limit our analysis to the previously-selected risk factors from other studies.

Beyond these aspects, in order to fill the gaps in the literature and satisfy specific requirements of our modeling environment, we determined that a desired PM proposal should:

- Be able to handle censored observations, which are common in time-to-event data.
- Have the means to deal with repeated measure and recurrent event cases that may happen in longitudinal event history analysis.
- Incorporate patients' past history of readmission into the modeling framework.
- Carry information about the timing of readmission.
- Manage many relevant risk factors without having computational or inferential problems such as overspecification or multicollinearity.
- Not be overly complex but should be computationally effective and easy to implement.
- Be as stable as possible in a complex data environment.
- Discriminate very high from very low risk patients and be comparable (or superior) to the existing methods in terms of such performance indices as *c*-statistics.

To fulfill the mentioned criteria, as well as to take advantages of both data models and algorithmic models, we propose our modeling methodology in the next section.

3.3.1 Phase-type Survival Forest

Decision trees are powerful and flexible non-parametric classifiers that use *inductive* inference for exploratory knowledge discovery. Due to their simple-to-comprehend and intuitive representation of information, decision trees have gained lots of attention in many disciplines such as computational biology, health informatics, medical imaging, and biomedical engineering (Breiman, 1993). A survival tree (Davis & Anderson, 1989) is a special kind of classification and regression tree (CART) for survival data that

partitions the covariate space recursively to build groups (nodes in the tree) of subjects that are homogeneous with respect to the outcome of interest. This is typically done by maximizing a measure of node homogeneity like Wasserstein metric between the survival functions (Gordon & Olshen, 1985) or logrank statistics (Ciampi, Thiffault, Nakache, & Asselain, 1986). A regular algorithm begins at the root node with all records and exhaustively searches all potential binary splits with the attributes, then picks the best one according to a splitting criterion such as a homogeneity measure. However, this process may lead to a large tree that usually overfits the data. To alleviate, a pruning mechanism is employed to find a subtree of appropriate size, or alternatively an ensemble approach (such as bagging or random forest) can be worked with which avoids the problem of selecting a single tree. Random Forest (RF; Breiman, 2001) is a popular ensemble method that grows many (de-correlated) classification trees by bootstrapping a training sample and then producing a label that is the mode of all votes from the individual trees.

Following what was proposed in section 3.2.2, here we develop a special case of Breiman’s RF, a phase-type survival forest (PHSF), that 1) uses the PH likelihood (with censored observations) as its splitting criterion for each tree grown, and 2) deals with repeated measure and recurrent readmission situations by performing bootstrap sample at a subject (patient) level.

- **Slitting criterion**

We chose minimization of the weighted average information criterion (WIC; Wu & Sepulveda, 1998) as the splitting criterion to induce individual trees. WIC is a weighted average of BIC and bias-corrected AIC (Hurvich & Tsai, 1989) which works better than (or at least as well as) other criteria in both small and large sample sizes. Recalling (2) and (3), the full log-likelihood function with censored observations becomes

$$L = \sum_{i=1}^N \alpha_i \log(f(t_i)) + (1 - \alpha_i) \log(S(t_i)), \quad (8)$$

where α_i is an indicator which equals 1, if t_i is a complete time for the i -th hospitalization, and becomes zero if t_i is a censored case (that is, no readmission occurs before the end of study). Notice that, unlike section 3.2.2, censorship may occur for the three acute conditions (and the COPD) if no readmission is seen within 30 (or T) days of discharge. The WIC is calculated as

$$\text{WIC}(d) = -2L + d + \left\{ \frac{d \left(((\log(N) - 1) \log(N)) (N - (d + 1))^2 + 2N(N + (d + 1))) \right)}{(2N + (\log(N)(N - (d + 1))))(N - (d + 1))} \right\} \quad (9)$$

in which $d = 2(m + r) - 1$, is the PH number of degrees of freedom, and N is the total number of sampled records.

In such a manner, at each node of a tree, if covariate ℓ has G values and breaks the node into partition set $(\ell_1, \ell_2, \dots, \ell_G)$, then the total WIC for the split can be expressed by the sum of singular WICs of every sub-group partitioned by the covariate:

$$\text{WIC}_{\text{full}}(d_{\text{full}}) = \sum_{g=1}^G \text{WIC}_{\ell_g}(d_{\ell_g}). \quad (10)$$

Therefore, the *information gain* is defined as the improvement made in the WIC after splitting the node:

$$IG_{\ell} = (\text{WIC}_R(d_R)) - \text{WIC}_{\text{full}}(d_{\text{full}}), \quad (11)$$

where R stands for the node before partition (i.e., the parent node). Beginning from the root node, at every single node, we apply one covariate at a time and record the gain for partitioning by that covariate. Then, we repeat this with other attributes and select a split that minimizes the WIC the most (or yields the largest gain) to recursively partition into child nodes. Also, if no positive gain can be obtained at a node by any possible split, the node becomes a terminal node.

- **Forest development**

As mentioned before, since patients (can) have multiple records in the dataset, repeated measures and recurrent events are likely. In this case, the bootstrapped samples are dependent and chances of having correlated observations in the in-bag training set are high. Consequently, trees grown may be correlated and overfitting is plausible. To alleviate this problem, we developed the PHSF algorithm (Algorithm 1) that performs subject-specific bootstrapping rather than using traditional replicate-based bootstrap. According to the algorithm, a subject classification is calculated as the label with the maximum number of votes cast across all records for that subject among all trees. The PHSF is able to produce predictions for specific replicates of a subject, and it can also be reduced to the original random forest if no replicate per subject is available. Consistent with the rule-of-thumb, subject-level bootstrapping performed in the algorithm ensures that about 63% of the subjects (rather than replicates) are elected in-bag. In this way subjects with more replicates cannot dominate the training process.

Considering that the PHSF generates proportions of votes for each class, similar to *data* models, we can have an (unbiased) estimate of the probability that a patient is readmitted. Further, like the original

Algorithm 1. Phase-type Survival Forest

- I. For $b = 1$ to B :
 - a) Take a bootstrap sample (i.e., a random sample chosen with replacement) of size S at the *subject* level (patient) from the training data. Assuming n_j records per patient j ,
 $N = n_1 + n_2 + \dots + n_S$.
 - b) Grow an *unpruned* tree T_b on each bootstrap by repeating the following steps, until no improvement is made in (11).
 - i. Select v' variables at random from the whole v variables. Normally v' should be much less than v , such as \sqrt{v} or even 1.
 - ii. Following the splitting criterion introduced, pick the best variable among the v' , and split the node into two child nodes.
- II. Output the ensemble of trees $\{T_b\}_1^B$.

To make a prediction for a new patient x :

Let $\hat{C}_b(x_{(i)})$ be the class prediction of the b th tree for replicate i of the patient. Then

$$\hat{C}_{\text{PHSF}}(x) = \text{majority vote} \left\{ \hat{C}_b(x_{(i)}) \right\}_1^B.$$

version of the random forest, the out-of-bag (OOB) data (which includes about one-third of all subjects) is used to get a running unbiased estimate of the classification error in both replicate and subject levels. Finally, like Breiman's RF algorithm, we use the permutation-based measure to get a raw importance score for variable v as:

$$I_v = \frac{\sum_{b=1}^B (p_{c,b} - p_{c,b}^v)}{B}. \quad (12)$$

In the formula, $p_{c,b}$ is the proportion of correctly classified replicates out of total OOB replicates in a given tree, and $p_{c,b}^v$ is the proportion of OOB replicates classified right after variable v is randomly permuted across all OOB cases.

3.4 Performance evaluation measures

We compare the PHSF algorithm with four popular classification methods: Breiman's Random Forest, Logistic Regression, Neural Network (NN), and Support Vector Machine (SVM). To evaluate the predictive accuracy of the approaches, we use sensitivity (also called true positive rate or recall), specificity,

positive predictive value (or precision), negative predictive value, F-measure, Matthews correlation coefficient, and the area under the receiver operating characteristic curve (AUROC). Denoting True Positive (Negative) and False Positive (Negative) outcomes as TP (TN) and FP (FN), the predictive measures are computed as

$$\begin{aligned}
\text{sensitivity} &= TP / (TP + FN) \\
\text{specificity} &= TN / (FP + TN) \\
\text{PPV} &= TP / (TP + FP) \\
\text{NPV} &= TN / (TN + FN) \\
F &= 2TP / (2TP + FP + FN) \\
\text{MCC} &= \frac{TP \times TN - FP \times FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}
\end{aligned} \tag{13}$$

Sensitivity (specificity) determines the proportions of actual positives (negatives) that are correctly classified as such. Positive predictive value (PPV) measures the proportion of positive predictions that are true positive, while negative predictive value (NPV) indicates the proportions of negative test outcomes that are true negatives. The F-score can be interpreted as a harmonic mean of precision and recall, with a best value at 1 and a worst at zero. The Matthews correlation coefficient (MCC) is a correlation coefficient between observed and predicted binary tuples which works well in class-imbalanced situations where the classes can be of very different sizes. Expectedly, it returns a real value in $[-1,+1]$, in which +1 represents a perfect prediction, zero means no better than random prediction, and -1 shows total disagreement between prediction and observation. The ROC curve is a graphical tool that exemplifies the performance of a classifier by plotting, at various cut-off points for the predicted risk, the sensitivity vs. one minus the specificity. In other words, moving along the ROC from bottom-left to top-right trades off false positives for false negatives. The AUROC (or c -statistics) can then be defined as the proportion of times a given classifier correctly discriminates a random pair of patients with and without readmission. Stated differently, an AUROC of 0.77 indicates that a patient with readmission is credited with a higher prediction value than a patient without readmission 77% the time, for a random pair of patients with and without readmission. A value of 0.5 for the AUROC shows that the model does not work better than a random pick; values between 0.7–0.8 suggest a modest separation ability; and values bigger than 0.8 imply good discriminative power. A simple approach to calculating AUROC for binary classification is presented by Hand and Till (2001) as

$$\hat{A} = \frac{S_0 - n^+(n^- + 1) / 2}{n^+n^-}, \tag{14}$$

Where n^+ and n^- are the number of positive and negative entries, respectively, and $S_0 = \sum r_i$, in which r_i is the rank of the i -th positive example in the ranked list.

4 Results and analyses

In this section, we describe steps to implement the proposed methods with the VHA data. First, data preprocessing is presented in Section 4.1. Then we perform the PAR algorithm to recognize avoidable readmissions from all other types of outcome (Section 4.2). After that, we fit the proposed Coxian PH distribution to COPD time-to-readmission data and find the optimal cut-off for the time interval (Section 4.3). Finally, predictive modeling with the PHSF algorithm as well as details about model calibration and validation are presented in Section 4.4.

4.1 Data preprocessing

Since real world data generally contain missing values, noise (e.g., errors and outliers), and inconsistent records, data preprocessing is essential for ensuring valid analytics. In this regard, added to what is given in the PAR algorithm (section II, part k), we perform the following tasks:

- a) Create attributes for time-to-readmission, readmission status, LOS, and sequence of (repeated) readmissions using admission and discharge dates.
- b) In predictive model building, we use the default replacement method of Breiman’s algorithm for missing values (Breiman, 2001). For univariate analysis, however, missing values are imputed with the hot-deck method (Ford, 1983).
- c) Identify and remove extreme records (outliers) with Local Outlier Factor (LOF) technique (Breunig, Kriegel, Ng, & Sander, 2000).
- d) Correct error records and flawed data combinations (e.g., POW: Yes, Veteran: No).
- e) Discretize attributes like distance into three levels (near: below 25 miles, middle: between 25 and 50 miles, far: above 50 miles) by k -means clustering.

Following these steps, the number of records is reduced to 6975 with 2813 distinct patients.

4.2 Potentially avoidable readmissions rates

Using 30-day (and T -day; see Section 4.3) readmission timeframes for the three acute conditions (and COPD), we begin by classifying all records to admissions and readmissions. Removing instances from the admission and readmission sets that meet one or more exclusion criteria (see section III of the PAR

algorithm), we initially identify total of 5,449 eligible admissions and 968 readmissions. Of the 968 readmissions, 173 cases were found not clinically related to their prior admissions (see PAR algorithm, section V), from which 27 cases are fitted in either “death,” “transfer,” or “against-medical-advice” groups and thus be dropped. The remaining 146 readmissions were then reclassified as eligible admissions, resulting in 5,595 eligible admissions. Hence, we end up having 795 PARs, from which 596 examples belong to a PAR series with only one PAR, and 78 match to a PAR series with two or more PARs. Consequently, the total number of unique PAR series becomes 674, and the PAR rate (see section IX of the PAR algorithm) is found to be 12.05 percent. Following the same approach, rates of PAR for HF, AMI, PN, and COPD are 13.26, 12.47, 11.16, and 11.33 percent.

The facility-adjusted PAR rates vary from 12.37% to 13.69% for HF; 11.83%–13.16% for AMI; 10.74%–11.93% for PN, and 10.68%–12.13% for COPD. From all HF avoidable readmissions, 86.3% are readmitted once, 11.4% are readmitted twice and 2.3% are readmitted three or more times. These rates are (81.7%; 14.6%; 3.7%), (88.4%; 10.9%; 0.7%), and (83.2%; 14.7%; 2.1%) for AMI, PN, and COPD respectively.

The pattern of PAR rates for the three acute conditions and the COPD during cumulative periods after discharge (days 0–7, 0–15, 0–21, and 0–30) are outlined in Table 1. As shown, of all 30-day avoidable readmissions, nearly 58% of the HF, 55% of the AMI, and 60% of the PN cohorts occurred within 15 days after hospital discharge. In other words, the majority of acute (avoidable) readmissions happens relatively soon after discharge, and they remain common even after two weeks of discharge. However, only around 58% of all COPD (avoidable) readmissions take place until the third week after discharge, and nearly 22% is left beyond the 30 day interval, which supports our argument about the inappropriateness of the 30-day timeframe for the chronic disease. This finding can be of great value for health professionals when they plan to allocate inpatient and early outpatient intervention programs to both acute and chronic conditions.

The mean (standard deviation) patient age of the readmitted cohort is 78.6 years (3.5 years) for HF, 80.3 years (4.1 years) for AMI, 79.3 years (2.9 years) for PN, and 76.2 years (3.1 years) for COPD. Frequent comorbid conditions among readmissions are coronary artery disease (CAD), atrial fibrillation, and diabetes for the HF cohort; anemia, congestive heart failure, and vascular disease with complications for the AMI cohort; chronic obstructive pulmonary disease, congestive heart failure, and cardiorespiratory failure and shock for the PN cohort; and chronic bronchitis, pneumonia, and diabetes mellitus for the COPD cohort.

The most common diagnoses of 30-day (or *T*-day; see Section 4.3) readmission after HF, AMI, PN, and COPD hospitalizations are displayed in Table 2. It appears that after admission for HF and AMI, readmissions happen mostly for heart failure (39.6% and 28.3% of readmissions, respectively), but follow-

ing hospitalizations for PN and COPD, patients get readmitted because of COPD (21.4% and 32.5%, in turn). Also, the top five readmission diagnoses contribute to 63.2% of all readmissions after HF, 59.4% of all readmissions after AMI, 55.6% of all readmissions after PN, and 65.1% of all readmissions after COPD.

Further, we realized that the most frequent reasons for avoidable readmissions in all conditions are related to “recurrence or extension of the reason (Section V, part b)” and “medical complications (Section V, part d)”, with an average of 54.7% and 23.2% through all the hospitals. As expected, in none of the acute and chronic conditions is the proportion of non-clinically related readmissions over 15.4 percent.

Next, we compared percentages of readmissions calculated by our method (PAR) to those of the 3M method for the three acute conditions in the four hospitals (Figure 3). In our method, consistent with the literature (Medicare Payment Advisory Commission, 2007), we observe that a greater proportion of all readmissions can be avoided in the first two weeks after discharge, but the contribution declines as time passes. Compared to the 3M approach, our method considers (slightly) fewer rehospitalizations as being avoidable and produces lower rates of readmission throughout all periods after discharge. A probable reason for this may be related to the CMS- and VHA-specific exclusions of our method, which is not found in the 3M approach. Besides, almost the same trend (not shown here) is seen for the COPD readmissions but over an extended time interval following discharge.

4.3 Optimal COPD readmission timeframe

In this section, we fit the proposed Coxian PH distribution to COPD time-to-readmission data in order to find the optimal cut-off point that defines avoidable readmissions. Using empirical data, we first examine the percentile distribution of times in Table 3.

As shown, the median time-to-(avoidable) readmission is about 36 days and the distribution is (highly) right skewed, with more than half of patients readmitted after the 30th day from discharge. This implies that, unlike acute conditions, poor quality of inpatient care for chronic conditions may reveal itself after 30 days from discharge. So setting the 30-day as a fixed timeframe for both acute and chronic conditions may not be appropriate.

We then applied the EMpht software to estimate the phase-type generator \mathbf{Q} in (5) using COPD time-to-readmission data from FY 11–12. In brief, the program starts with an initial guess $\mathbf{Q}^{(0)}$ (for the non-zero elements in (5)) and proceeds with a number of iterations of the EM algorithm to increase the log-likelihood function $\sum_{i=1}^N \log(\boldsymbol{\pi} \exp(\mathbf{Q}t)(-\mathbf{Q}\mathbf{1}))$. Fixing the maximum number of iterations at 5000

runs and Runge-Kutta step length at $\frac{0.1}{\max|\mathbf{Q}_{ii}|}$ (in which $\max|\mathbf{Q}_{ii}|$ is the largest absolute value of the diagonal element of the last estimate of \mathbf{Q}), we configure different Coxian PH structures by modifying the order of the sub-class Markov processes (i.e., parameters m and r). This way, we start with $m=1$, examine various levels of r from 1 to 10, and pick the best in terms of AIC and BIC; then we repeat this for $m=2$ until $m=10$. We stop the search if the log-likelihood does not improve in any intermediate level. Also due to non-identifiability of the parametrization of the phase-type distribution (O’Cinneide, 1990), we do several fits starting with various initial values produced in previous runs. The results of best fits at each level of m are then summarized in Table 4.

It is apparent that there is no improvement in the fits after the fourth phase of the Short-Stay group (i.e., $m=4$). Hence an order 6 of the Coxian PH distribution with 4 and 6 phases for the Short-Stay and Long-Stay groups respectively is considered to most suitably represent the time-to-readmission process of the COPD patients in the dataset. Note that we do not show fits after ($m=5$) as they provide no further enhancements.

The estimates of the intensity rates in (5) along with their standard errors are calculated in Table 5. Given the small amounts of error, we see that the parameters are well estimated with EMpht. Also note that λ_1 , λ_2 , and λ_3 belong to the Short-Stay group, while λ_4 and λ_5 are related to the Long-Stay group. According to the table, the probability of a patient being in the Short-Stay group is calculated as

$$p = \frac{1.26}{1.26+1.58} = 0.4437, \text{ which means that the COPD patients spend about 44.37\% of their time in the}$$

Short-Stay group in the community before returning to the hospital.

Then, in order to solve (7) and obtain the optimal COPD readmission timeframe, we need to derive the PDFs f_{SS} and f_{LS} . This can be done based on a convolution of a set of independent exponential variables (X_i) as follows (Ross, 2009):

$$\begin{aligned}
f_{SS} &= f_{X_1+X_2+\dots+X_m}(x) & (15) \\
&= \int_0^x \int_0^{t_1} \int_0^{t_2} \dots \int_0^{t_{m-1}} f_{X_1}(t_1) f_{X_2}(t_2 - t_1) \\
&\quad \times \dots \times f_{X_m}(x - t_{m-1}) dt_1 dt_2 \dots dt_{m-1} \\
&= \left[\prod_{i=1}^m (\lambda_i + \lambda_{i0}) \right] \\
&\quad \times \sum_{j=1}^m \frac{e^{-(\lambda_j + \lambda_{j0})x}}{\prod_{\substack{k=1 \\ k \neq j}}^m (\lambda_k + \lambda_{k0}) - (\lambda_j + \lambda_{j0})} \\
f_{LS} &= f_{X_1+X_2+\dots+X_{m+r}}(x) = \left[\prod_{i=1}^{m+r} (\lambda_i + \lambda_{i0}) \right] \\
&\quad \times \sum_{j=1}^{m+r} \frac{e^{-(\lambda_j + \lambda_{j0})x}}{\prod_{\substack{k=1 \\ k \neq j}}^{m+r} (\lambda_k + \lambda_{k0}) - (\lambda_j + \lambda_{j0})}
\end{aligned}$$

Note that the escape rate λ_{i0} is zero for all phases except m and $m+r$, that is,

$$\begin{aligned}
\lambda_{m0} &= \lambda_{SS}, \quad \lambda_{m+r0} = \lambda_{LS} \\
\lambda_{i0} &= 0; \quad i = \{1, 2, \dots, m-1, m+1, m+2, \dots, m+r-1\}
\end{aligned} \tag{16}$$

Finally, by substituting (15) in (7) with $p = 0.4437$, we compute t^* (or T in the PAR algorithm) as 42 days, which is pretty close to, but more accurate than what we observe in Figure 1. Thus, for the COPD cohort we should utilize a 42-day timeframe to count the correct number of avoidable readmissions in our study.

4.4 Predicting the risk of avoidable readmissions

In this section, we first perform some descriptive analytics on the patient risk factors for the two cohorts within each condition. Afterwards, steps for predictive modeling along with details about model validation and calibration are described. We then end by doing performance evaluation and comparison studies.

4.4.1 Descriptive Analytics

Baseline patient characteristics in PAR and No-readmission cohorts are displayed in Table 6 (for Heart Failure and Acute Myocardial Infarction) and Table 7 (for Pneumonia and COPD). The presence of any significant difference between the cohorts was also tested using univariate logistic regression and the re-

sults are shown in terms of *P*-Values [missing values are imputed by the hot-deck method (Ford, 1983)]. Since the same patient could have several avoidable readmissions during the study period, we used generalized estimation equation to adjust for serial correlations among readmissions of the same patient.

During the study, a total of 5,595 eligible admissions were made in the four VA hospitals, out of which about 14.21% were followed by an unnecessary rehospitalization. Note that this rate is different from what is reported in Section 4.2 (which is 12.05%) because here we count each readmission separately rather than as members of a PAR series. In all conditions, the populations are generally male (>86%), married (>51%), older (>67 years), and live within 25 miles of a VA facility (>60%). More than 21% in all conditions do not have private insurance or insurance through Medicare or Medicaid programs. More than half of patients in all conditions are admitted directly from their home and more than 50% have one to four past-year hospitalizations. On average, the care assessment score is higher in respiratory diseases (near 69) compared to circulatory conditions (about 66). Almost 18% of the patients are also diagnosed with more than ten HCCs (not shown in the tables). Note that in the attribute “source of admission,” class ‘transfer’ is related to those patients who are transferred only among the four VA hospitals, and ‘Other’ is related to some other admission sources such as observation/examination, non-VA hospitals not under VA auspices, community nursing homes under (or not under) VA auspices, non-veteran hospitals, etc. Further, priority groups 1, 2, and 3 are generally assigned to veterans with service-connected disabilities of > 50%, [30%, 50%), and [20%, 30%), respectively. Other groups are as follows: 4, catastrophically disabled veterans; 5, low income or Medicaid; 6, Agent Orange or Gulf War veterans; 7, non-service connected with income being below HUD; and 8, non-service connected with income being above HUD. For each condition, patient comorbidities are identified with the help of Comorbidity Software (Kaboli et al., 2012), using ICD-9-CM and DRG codes from the index hospitalization and any admission in the 12 months prior.

It is observed that patients who are subsequently readmitted are elderly and usually have a greater number of comorbidities. Male patients have on average a greater chance to be readmitted in HF and COPD cohorts rather than females, but this cannot be generalized since the VA sample here contains only about 8% female patients. The analysis shows that length of stay is not generally associated with odds of avoidable readmission, when patient and facility characteristics are not controlled for. However, after adjusting for the case-mix and service-mix (not shown here), the relation tends to be negative (about 7.3% increase for each in-hospital day lower than expected), which implies that shorter individual LOS is generally connected with higher risk of readmission. Therefore, consistent with (Kaboli et al., 2012), we observe that significant reduction in LOS, without simultaneously improving inpatient care, is more likely to result in premature discharge and rehospitalization. Further, enrollment priority turns out to be highly linked with odds of readmission in all conditions, especially when it comes to catastrophically disabled

veterans (increases of .2% in AMI to 10.9% in HF). Furthermore, the odds of avoidable readmissions are significantly higher in patients exposed to ionizing radiation and Agent Orange in all conditions. Among the comorbid conditions, having diabetes and cancer increases the chance of readmission, as does having mental disorders and substance abuse (with harsher effect in circulatory conditions).

4.4.2 Predictive modeling with PHSF

Following Algorithm 1, we used the entire set of patient risk factors to develop a readmission prediction model. Additionally, we create two more covariates, namely, “sequence” (see Section 4.1) and “Charlson comorbidity index” (Charlson et al., 1987) and entered them into the analysis. For non-categorical variables in the candidate set (i.e., age, LOS, CAN score, sequence, and Charlson index), we evaluated different cut-off points to split the dataset into binary partitions and explore the optimal cutpoint that most discriminates high vs. low risk using operating characteristic curves (with whole dataset). We then used this ROC-generated cutpoint for further analyses. Also for categorical features with more than two classes (like race), following Friedman, Hastie, and Tibshirani (2009), we optimally select a series of binary splits (instead of multiway splits) that produce the best discrimination results.

We begin with the baseline model that uses all sampled data points (5,595 records) in the subject-specific bootstrapped PHSF and we let the forest internally perform cross-validation using OOB samples during each run. The number of trees and the number of variables to try at each split are set to 6,000 and 5, respectively. Also we set the cutpoints with respect to minimizing the WIC criterion (see Section 3.1.1) as follows: Age, 68 (years); LOS, 5 (days); CAN score, 66; sequence, 3; and Charlson index, 4.5. Results of variable importance are summarized in Table 8 (Sig. stands for significance level). As illustrated, almost all statistically significant variables (Sig. <.05) refer to overall health and agedness factors, which may reflect a generalized vulnerability to disease among recently discharged patients—inpatients regularly lose their strength and develop new difficulties in doing their day-to-day activities (Gill, Allore, Gahbauer, & Murphy, 2010). Interestingly, ‘sequence’ turns out to be (positively) related to readmission risk, which highlights the fact that the chance of unnecessary returns to hospital is greater in patients with prior history of readmission.

In the baseline model, the c -statistics is .793; sample-level OOB error rates are 3.16%, 2.35%, and 8.05% for overall, No-readmission class, and PAR class, respectively; and there are large interactions [based on Breiman’s variable interaction model (Breiman, 2001)] between Agent Orange and Radiation, between Priority and LOS, and between Priority and Insurance, to name a few.

- **Model calibration**

We then calibrated the baseline model as follows: 1) we focused only on the 16 most important variables found in the baseline model; 2) we imputed missing values based on Breiman's replacement technique (Breiman, 2001); 3) we modified the optimal cut-off points with regards to maximizing the *c*-statistics (the new cutpoints are 69 years for age, 70 for CAN score, and 4.7 for Charlson index, while others remain unchanged); and 4) we altered the class weights to 1 on class 'No-readmission' and 8 on class 'PAR', to adjust for the imbalanced prediction errors in the classes. Then we rerun the model with 10,000 trees and 4 variables to try at each split. Depiction of variable importance for the calibrated model is shown in Table 9. Expectedly, the ranking of variables does not change but we achieved better results in terms of scores and significance levels. It is noticed that, though Mental disorder and Malignant neoplasm are only marginally significant, we decide to keep them in the final model since 1) they are both medically significant in contribution to the risk of readmission, and 2) they together contribute largely to the model discrimination ability.

In the calibrated model, the *c*-statistics jumps to .836; no serious interactions remain among variables; and the overall, No-readmission, and PAR error rates become 3.67%, 2.51%, and 2.64%, respectively. It is remarkable that the calibrated model considerably decreases PAR misclassification rate, but at the expense of increasing the overall error rate a little bit. We perceive that this tuning in class weights is really appealing for our situation because in readmission prediction models, the cost of false negatives (which correspond to readmitted patients incorrectly predicted as No-readmission) is usually much higher than the cost of false positives (which correspond to non-readmitted patients incorrectly predicted as PAR cases).

Since the PHSF method takes an ensemble approach of trees, as we mentioned earlier, we can obtain an unbiased estimate of PAR probability for each patient. Therefore, it is possible to further check the model calibration by evaluating predicted and actual PAR rates at different risk deciles. These results appear in Table 10 and Figure 4.

We note that, both on average and over the whole range of predictions, the predicted probability of readmission matches up well with the actual probabilities. Average predicted readmission (not shown here) also monotonically increases with growing risk, ranging from 8.79% in the lowest decile to 43.75% in the highest, a range of 34.96% in total. For the 12% of readmissions that happens between deciles four and five, the PHSF model under-predicts by roughly 8.5%. It also over-predicts by about 4%–14% for the small number of readmissions (21%) which occur in deciles 6–10.

- **Model validation**

Here, we used the calibrated model and studied its internal validity (also called reproducibility), based on the same population underlying the sample. To this end, since the PHSF does perform bootstrapping internally, we slightly modified the split-sample technique for our purposes: we randomly partitioned the sample into 50% training and 50% testing sets and redid this 7 times. For each partition we ran the PHRF algorithm and obtained the c -statistics. The average c -statistics for the seven runs of training sets reached .839 and for the test sets, it was .821. Hence, there exists an “optimism” of .018 in the mean AU-ROCs for the training and testing splits, and as a result, the internally-validated (or optimism-corrected) c -statistics is estimated as .818.

To provide more robust evidence of validity, we further conducted external (in fact: spatial) validation (also called generalizability) with a new sample of 478 patients admitted (with primary diagnosis of HF, AMI, PN, and COPD) in the months of August and September 2012. It is noted that we included the same patient factors studied in the new sample. The c -statistics in the external sample decreased to .809 (a decrease of .027) which is slightly more than results from internal validation (a decrease of .018). However, both internal and external validations confirm the superiority of our proposal over the current approaches in terms of discrimination power and stability. Nonetheless, we obtain greater c -statistics (at least .813) when the PHSF is applied separately on each condition. It should also be remarked that with the current sample data, the CMS endorsed model can only produce a c -statistics of about .63.

4.4.3 Numerical comparisons

In this section, we evaluate the proposed PHSF method with Logistic Regression (LR), Breiman’s Radom Forest (RF), Support Vector Machine (SVM), and Neural Network (NN) in terms of predictive measures introduced in Section 3.4 (plus mean squared error). The models are built and compared with the R version 3.0.2 (RDC Team, 2005) using packages `randomForest` (Liaw & Wiener, 2002), `e1071` (Dimitriadou, Hornik, Leisch, Meyer, & Weingessel, 2008), `glm2` (Marschner, 2011), and also MATLAB neural network toolbox (Demuth & Beale, 1993). It is worth mentioning that we used different kernels such as polynomial and radial basis function for the SVM method; and for the NN approach, we also tested for two and three layers with different numbers of sigmoid hidden neurons and linear output neurons. For the pure random forest method, we did the same calibration as with the PHSF, and for the logistic regression, we used generalized estimation equation to account for clustering at the patient level. The comparison results are summarized in Table 11 and Figure 5. As shown, the proposal works better than

other alternatives in all predictive criteria. The Breiman's random forest approach and SVM produce very close results in this sample but the NN approach seems unable to compete with other models having a modest discrimination of about 0.7. Not surprisingly, all models predict 'No-readmission' cases better than the PAR cases. It is of interest that SVM slightly outperforms the RF in terms of precision (.5% higher) and true negative rate (.17% higher). Furthermore, in the overall spectrum of false positive rates, the proposal assigns a higher probability of readmission for a patient with PAR compared to a 'No-readmission' patient, about 83.6% of the times. Looking at different ROC stairs graphs, we can infer that, with a false positive rate between .09 to .25, our PHSF is placed higher than others, but it falls behind the SVM and NN in case of very small rates of false positive. In higher false positive rates, we observe that RF and SVM are very similar in discrimination ability and they work as well as PHSF beyond .7 false positive rate. However, logistic regression turns out to fall short at a type I error rate of .8 to .9.

5 Conclusion

Concentration on reducing unnecessary readmission has never been higher, especially with the CMS augmenting the rates of penalties and introducing new waves of diseases that will be under scrutiny during next years. In response to this policy shift, hospitals and clinicians are become more interested in analytics ways to identify patients at elevated risk of avoidable readmission, since such tools can ultimately be used to guide more appropriate discharge planning and efficient resource utilization. Although a variety of approaches have been proposed to identify patients with higher risk, their potentials have been limited mainly because they do not incorporate timing of readmission in their prediction and/or they are not accurate enough (Kansagara et al., 2011).

In this study, we make several contributions to readmission reduction studies. First, we address the problem of characterizing avoidable (or unnecessary) readmissions from all other types of outcomes. Our algorithm (PAR) is based on administrative data and takes a more accurate look at preventability components of rehospitalization compared to existing methods. We also suggest using a more comprehensive risk adjustment tool (DCG/HCC) in counting avoidable readmissions, as well as getting help from other sources of information, like clinic visits between index admission and readmission, in assessing the avoidability of readmissions.

Second, we assert that the government-endorsed 30-day timeframe that is used to count readmissions is not "optimal" for chronic conditions such as COPD. Therefore, we develop a stochastic model based on Coxian phase-type distribution to analytically calculate the optimum cut-point that best stratifies among quickly-readmitted and slowly-readmitted COPD patients. We then adopt the new time window in the PAR algorithm to adjust for COPD readmissions.

Third, by combining algorithmic and data models, we put forward a hybrid predictive approach that exploits good aspects of classification and timing-based analytics models. We then demonstrate the superiority of our model over current solutions with respect to various accuracy criteria. Further, to confirm that the high discrimination ability of our proposal is irrespective to overfitting, we perform internal and external validation practices. Also, unlike some studies in the literature, we do not limit our work to a specific disease (Hernandez et al., 2010) or within a specific hospital (Smith, Huster, Fitzgerald, & Martin, 1996), but instead we aggregate data from four different VA facilities containing inpatients diagnosed with four different conditions.

Even though our results introduce new aspects of readmission studies, one should pay attention to some limitations in interpreting and generalizing them. First, the data used in the study is from one region (Veteran Integrated Service Network 11, Veterans In Partnership) in the State of Michigan, with a veteran population that is mostly male and veteran, and a government-funded care delivery system; hence the results may not be identical in other health care systems. Second, the study is limited to administrative data (that are regularly available to all health plans) and it does not have laboratory test results and vital signs such as hemoglobin or serum level at discharge, which may affect the risk of unnecessary readmission.

In future work, we plan to use our proposal to compare and profile the hospitals on their readmission rates using proper risk adjustment for case mix and service mix. The approach currently employed by the CMS (and the VHA) is to calculate a ratio of observed to expected outcomes for a given hospital, and evaluate it across the normal range of all other hospitals given the same mix. Methods in this context are primarily based on models in which the hospital effects on outcome are taken as random. Nonetheless, they have been recently argued because 1) they often produce biased estimates of outcomes at the provider level; and 2) they cannot prevent confounding issues when the patient characteristics are correlated with facility effects (Kalbfleisch & Wolfe, 2013).

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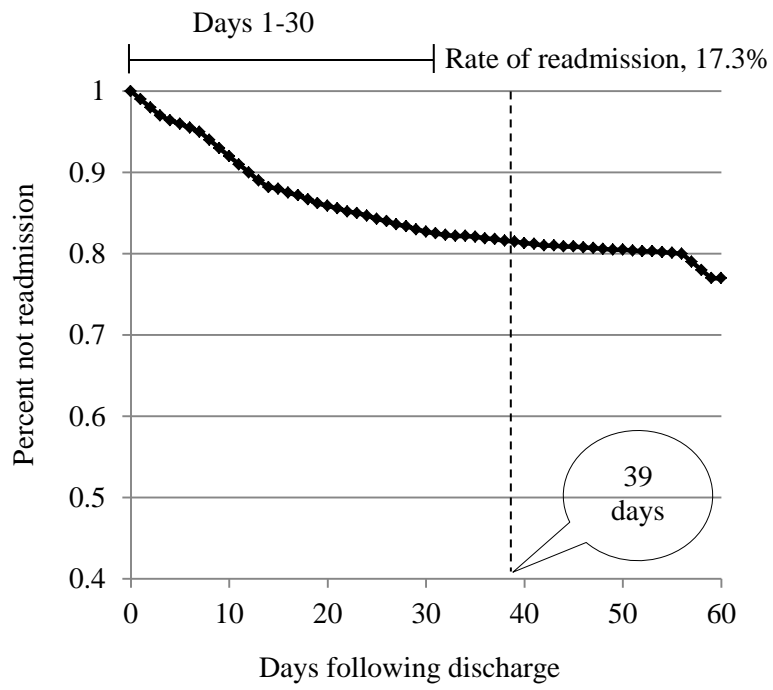


Figure 1 Percentage of COPD patients not readmitted

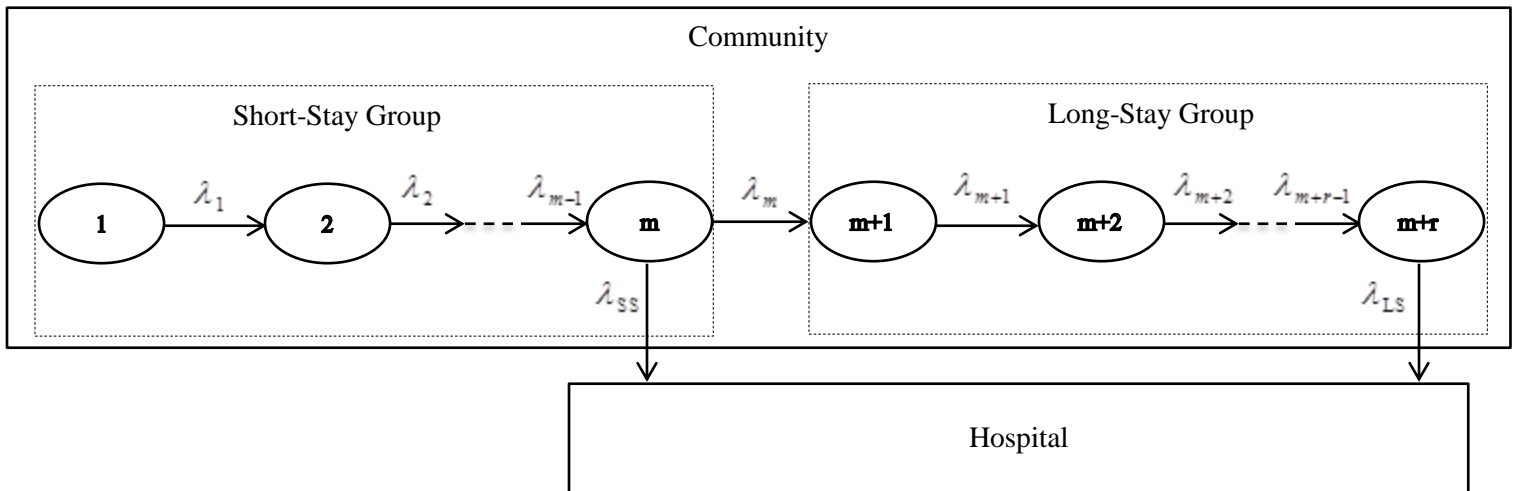


Figure 2 Markov model for flow of COPD patients after discharge

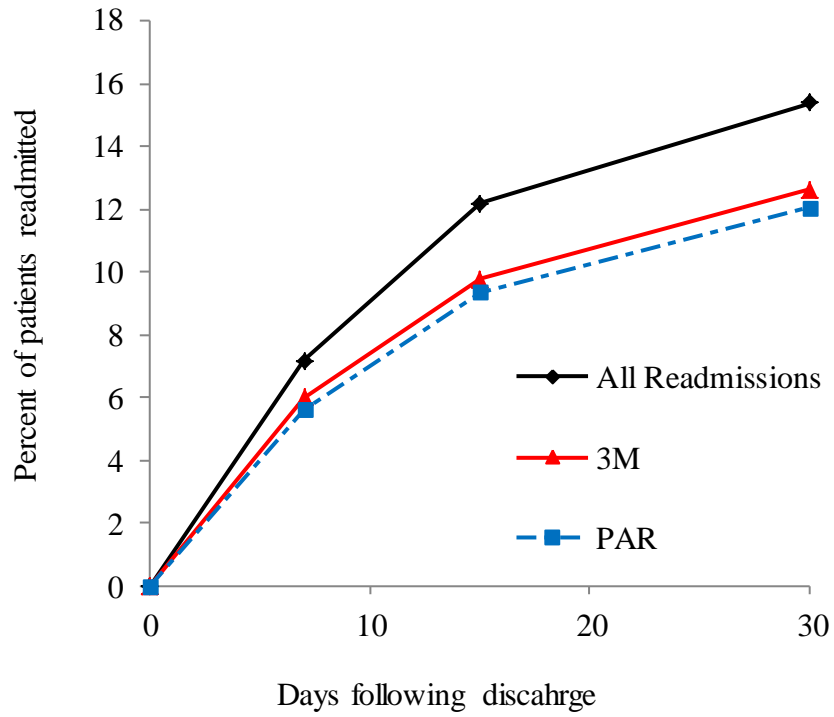


Figure 3 Percent of readmission over time

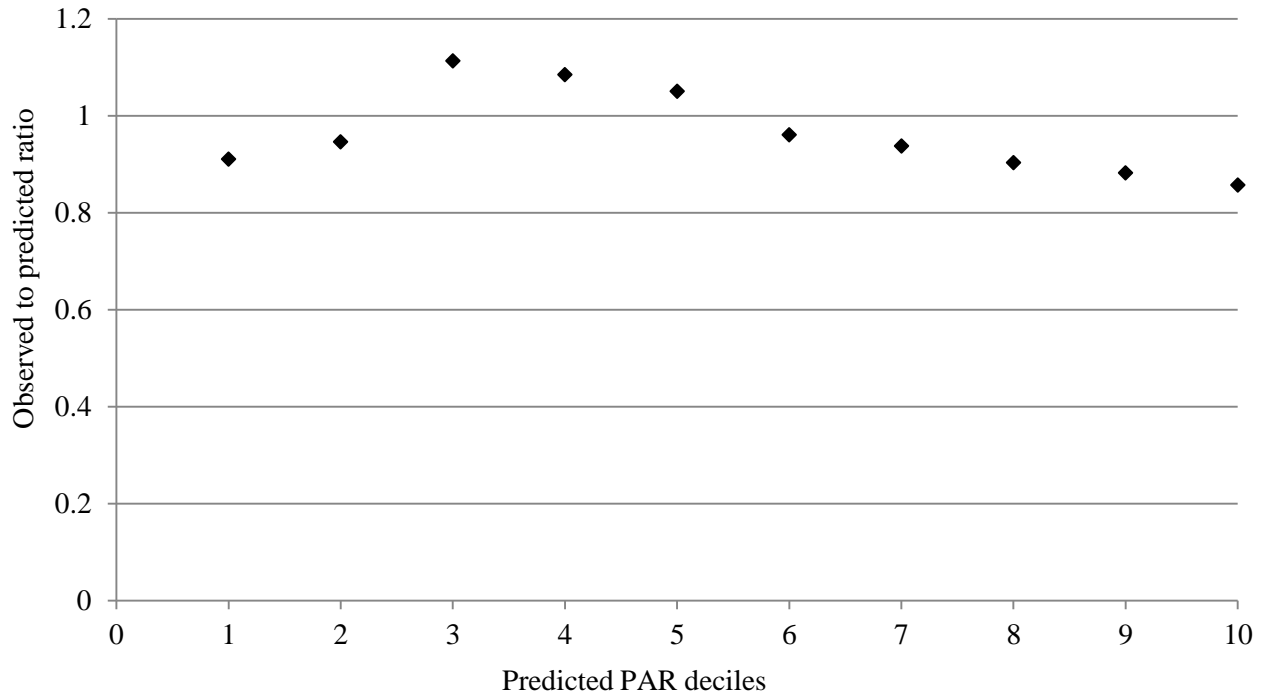


Figure 4 Calibration curve for the PHSF model

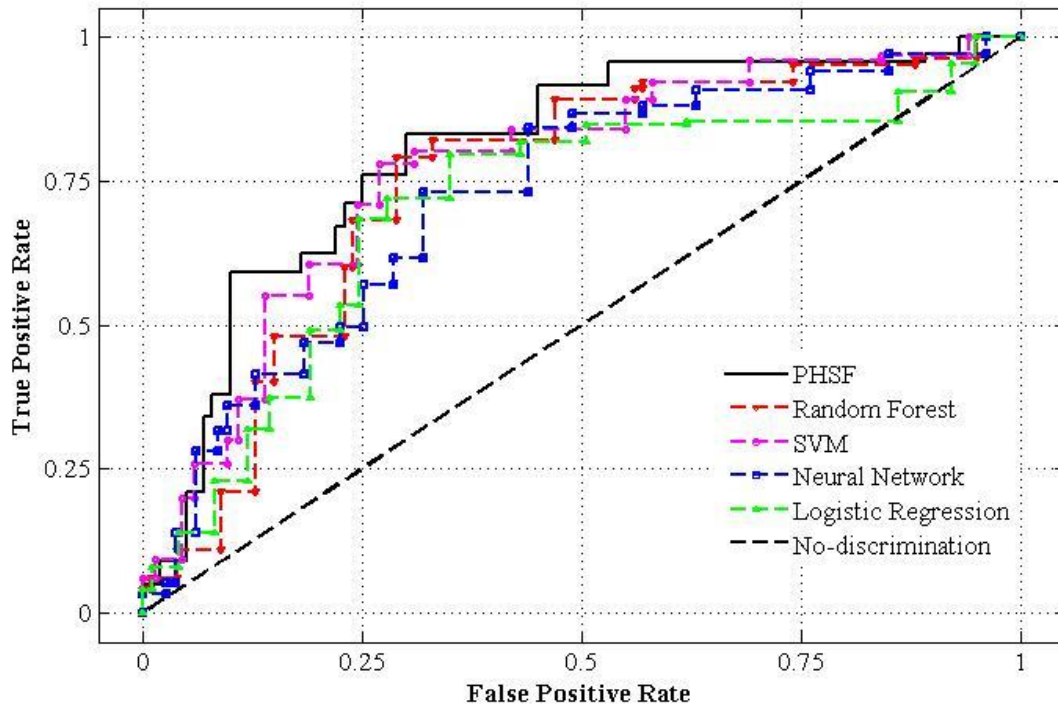


Figure 5 ROC curves for different predictive models

Table 1 Distribution of avoidable readmissions over time

Cohort	Days following discharge			
	0–7	0–15	0–21	0–30
Heart Failure	28.6%	57.9%	81.7%	100%
Acute Myocardial Infarction	33.4%	54.6%	86.4%	100%
Pneumonia	31.1%	60.1%	83.3%	100%
COPD	21.1%	42.6%	58.5%	78.3%

Table 2 Top readmission diagnoses for patients hospitalized with HF, AMI, PN, and COPD

Rank	HF cohort		AMI cohort		PN cohort		COPD cohort	
	Diagnosis	Percent of PAR	Diagnosis	Percent of PAR	Diagnosis	Percent of PAR	Diagnosis	Percent of PAR
1	Heart failure	39.6%	Heart failure	28.3%	COPD	21.4%	COPD	32.5%
2	Renal failure	9.3%	Coronary artery disease	13.7%	Pneumonia	15.3%	Bronchitis	16.3%
3	Arrhythmias	6.7%	Pneumonia	8.6%	Heart failure	10.6%	Cardio-Respiratory Failure and Shock	7.6%
4	Cardio-respiratory failure and shock	4.1%	Septicemia/ Shock	5.5%	Cardio-respiratory failure and shock	4.4%	Pneumonia	5.3%
5	Pneumonia	3.5%	Renal failure	3.3%	Renal failure	3.9%	Hypertension	3.4%

Table 3 Percentile distribution of the COPD time to readmission

Percentile	0	10	25	50	75	90	100
Day	1	7	19	36	88	174	283

Table 4 Results of various Coxian PH fits

Fit	r	AIC	BIC
$m=1$	3	35443.4	35443.02
$m=2$	2	35181.3	35180.90
$m=3$	1	34077.7	34077.32
$m=4$	2	33816.5	33818.28
$m=5$	2	33844.2	33851.54

Table 5 Estimated intensity rates for the COPD Coxian PH model

Parameter	Estimate	Standard Error
λ_1	0.04	0.008
λ_2	3.62	0.126
λ_3	5.17	0.233
λ_4	1.58	0.075
λ_5	0.96	0.039
λ_{SS}	1.26	0.097
λ_{LS}	0.83	0.004

Table 6 Baseline characteristics (mean (SD) for continuous variables; n(%) for categorical variables) and univariate analyses at the time of discharge

Characteristic	Heart Failure (n=1674)			Acute Myocardial Infarction (n=1417)		
	No Readmission (n=1447)	PAR (n=227)	P-Value	No Readmission (n=1211)	PAR (n=206)	P-Value
Age (years)	68.6 (5.2)	71.3 (3.2)	<.01	69.3 (5.6)	73.3 (3.7)	<.01
Sex, Male	1406 (97.2)	215 (96.9)	.04	1097 (90.6)	192 (93.2)	.07
Race						
Black	986 (68.1)	193 (85.0)] <.01	769 (63.5)	169 (82.0)] <.01
White	432 (29.8)	29 (12.8)		405 (29.8)	29 (14.1)	
Other	29 (2.1)	5 (2.2)		37 (3.1)	8 (3.9)	
Marital status						
Current spouse	839 (57.9)	137 (58.3)] .35	631 (52.1)	112 (54.4)] .42
Never married	307 (21.2)	52 (21.4)		320 (26.4)	58 (26.7)	
Previously married	301 (20.9)	38 (20.3)		260 (21.5)	36 (18.9)	
Primary insurance						
Medicare	732 (50.6)	126 (55.5)] .03	624 (51.5)	97 (47.1)] .07
Medicaid	249 (17.2)	27 (11.9)		226 (18.7)	32 (15.5)	
Private	107 (7.4)	25 (11.0)		103 (8.5)	28 (13.6)	
Not insured	359 (24.8)	49 (21.6)		258 (21.3)	49 (23.8)	
Length of stay (days)	5.2 (6.1)	6.2 (4.4)	.07	5.8 (5.8)	5.1 (6.8)	.11
Source of admission						
Direct from home	797 (55.1)	129 (56.8)] .31	623 (51.4)	107 (51.9)] .26
Outpatient clinic	392 (27.1)	63 (27.8)		392 (32.4)	67 (32.5)	
Transfer	17 (1.2)	3 (1.3)		23 (1.9)	4 (1.9)	
VA NHCUC	62 (4.3)	12 (5.3)		62 (5.1)	10 (4.9)	
VA Domiciliary	13 (0.9)	4 (1.8)		13 (1.1)	5 (2.4)	
Other	166 (11.5)	16 (7.0)		98 (8.1)	13 (6.3)	
Enrollment priority						
1	126 (8.7)	17 (7.5)] <.001	104 (8.6)	19 (9.2)] <.001
2	167 (11.5)	9 (4.0)		136 (11.2)	13 (6.3)	
3	293 (20.2)	38 (16.7)		239 (19.7)	41 (19.9)	
4	173 (12.0)	52 (22.9)		133 (11.0)	23 (11.2)	
5	316 (21.8)	66 (29.1)		331 (27.3)	56 (27.2)	
6	115 (7.9)	15 (6.6)		172 (14.2)	12 (5.8)	
7	103 (7.1)	19 (8.4)		26 (2.1)	15 (7.3)	
8	154 (10.6)	11 (4.8)		70 (5.8)	27 (13.1)	
Distance to hospital						
Near (<25m)	856 (59.2)	155 (68.3)] .02	781 (64.5)	151 (73.3)] .03
Middle ([25, 50]m)	549 (37.9)	69 (30.4)		406 (33.5)	53 (25.7)	
Far (>50m)	42 (2.9)	3 (1.3)		24 (2.0)	2 (1.0)	
Prisoner of war, Yes	17 (1.2)	8 (3.5)	<.01	11 (0.9)	6 (2.8)	.01

Table 6—Continued Baseline characteristics (mean (SD) for continuous variables; n(%) for categorical variables) and univariate analyses at the time of discharge

Characteristic	Heart Failure (n=1674)			Acute Myocardial Infarction (n=1417)		
	No Readmission (n=1447)	PAR (n=227)	<i>P</i> -Value	No Readmission (n=1211)	PAR (n=206)	<i>P</i> -Value
Radiation, Yes	11 (0.8)	5 (2.2)	.03	9 (0.7)	6 (2.9)	.02
Agent Orange, Yes	63 (4.4)	16 (7.0)	.02	42 (3.5)	13 (6.3)	.03
CAN score	67.4 (4.1)	71.7 (2.9)	<.01	64.5 (4.6)	68.6 (3.7)	.02
No. of past year hospitalization						
0	663 (45.8)	71 (31.3)] <.001	503 (41.5)	52 (25.2)] <.001
1-4	713 (49.3)	122 (53.7)		616 (50.9)	124 (60.2)	
>4	71 (4.9)	34 (15.0)		92 (7.6)	30 (14.6)	
Comorbidity						
CAD	486 (33.6)	94 (41.4)	.04	81 (6.7)	16 (7.8)	.53
Heart failure	—	—	—	346 (28.6)	73 (35.4)	.04
Vascular disease w/c	202 (14.0)	45 (19.8)	.02	306 (25.3)	67 (32.5)	.02
Cardiorespiratory	153 (10.6)	37 (16.3)	.01	134 (11.1)	14 (6.8)	.06
Pneumonia	97 (6.7)	19 (8.4)	.32	51 (4.2)	15 (7.3)	.05
Atrial fibrillation	403 (27.9)	77 (33.9)	.05	291 (24.0)	62 (30.1)	.04
Anemia	225 (15.5)	47 (20.7)	.05	378 (31.2)	81 (39.3)	.03
Diabetes	351 (24.3)	71 (31.3)	.02	159 (13.1)	37 (18.0)	.05
COPD	242 (16.7)	49 (21.6)	.05	63 (5.2)	17 (8.3)	.07
Chronic bronchitis	83 (5.7)	12 (5.3)	.66	17 (1.4)	6 (2.9)	.14
Malignant neoplasm	71 (4.9)	19 (8.4)	.03	25 (2.1)	12 (5.8)	<.01
Mental disorder	160 (11.1)	37 (16.3)	.01	102 (8.4)	31 (10.7)	<.01
Substance abuse	118 (8.2)	31 (13.7)	<.01	112 (9.2)	33 (16.0)	<.01

Table 7 Baseline characteristics (mean (SD) for continuous variables; n(%) for categorical variables) and univariate analyses at the time of discharge

Characteristic	Pneumonia (n=1306)			COPD (n=1198)		
	No Readmission (n=1117)	PAR (n=189)	P-Value	No Readmission (n=1025)	PAR (n=173)	P-Value
Age (years)	67.7 (4.9)	68.3 (2.8)	<.01	63.6 (4.5)	65.3 (2.6)	<.01
Sex, Male	1035 (92.7)	182 (96.3)	.07	966 (94.2)	169 (97.7)	.04
Race						
Black	731 (65.4)	153 (81.0)] <.01	597 (58.2)	126 (72.8)] <.01
White	335 (30.0)	25 (13.2)		390 (4.7)	42 (24.3)	
Other	51 (4.6)	11 (5.8)		38 (37.1)	5 (2.9)	
Marital status						
Current spouse	571 (51.1)	106 (56.1)] .27	579 (56.5)	103 (59.5)] .09
Never married	244 (21.8)	32 (16.9)		201 (19.6)	41 (23.7)	
Previously married	302 (27.1)	51 (27.0)		245 (23.9)	29 (16.8)	
Primary insurance						
Medicare	602 (53.9)	91 (48.1)] .06	535 (52.2)	103 (59.5)] .05
Medicaid	185 (16.6)	24 (12.7)		157 (15.3)	18 (10.4)	
Private	89 (8.0)	26 (13.8)		94 (9.2)	9 (5.2)	
Not insured	241 (21.6)	48 (25.4)		239 (23.3)	43 (24.9)	
Length of stay (days)	4.9 (5.4)	5.7 (4.2)	.03	3.7 (5.1)	4.3 (3.2)	.08
Source of admission						
Direct from home	651 (58.3)	114 (60.3)] .09	575 (56.1)	102 (59.0)] .36
Outpatient clinic	225 (20.1)	40 (21.2)		324 (31.6)	51 (29.5)	
Transfer	21 (1.9)	5 (2.6)		32 (3.1)	3 (1.7)	
VA NHCUC	59 (5.3)	14 (7.4)		62 (6.0)	9 (5.2)	
VA Domiciliary	16 (1.4)	5 (2.6)		14 (1.5)	1 (0.6)	
Other	145 (13.0)	11 (5.8)		18 (1.7)	7 (4.0)	
Enrollment priority						
1	74 (6.6)	22 (11.6)] <.001	119 (11.6)	26 (15.0)] .01
2	141 (12.6)	17 (9.0)		50 (4.9)	14 (8.1)	
3	219 (19.6)	35 (18.5)		182 (17.8)	23 (13.3)	
4	115 (10.3)	29 (15.3)		202 (19.7)	41 (23.7)	
5	341 (30.5)	36 (19.0)		341 (33.3)	51 (29.5)	
6	172 (15.4)	8 (4.2)		22 (2.1)	5 (2.9)	
7	37 (3.3)	14 (7.4)		26 (2.5)	7 (4.0)	
8	18 (1.6)	28 (7.4)		83 (8.1)	6 (3.5)	
Distance to hospital						
Near	692 (62.0)	127 (67.2)] .01	713 (69.6)	132 (76.3)] <.01
Middle	421 (37.7)	59 (31.2)		307 (30.0)	37 (21.4)	
Far	4 (0.4)	3 (1.6)		5 (0.5)	4 (2.3)	
Prisoner of war, Yes	23 (2.1)	11 (5.8)	<.01	27 (2.6)	14 (8.1)	<.01

Table 7—Continued Baseline characteristics (mean (SD) for continuous variables; n(%) for categorical variables) and univariate analyses at the time of discharge

Characteristic	Pneumonia (n=1306)			COPD (n=1198)		
	No Readmission (n=1117)	PAR (n=189)	<i>P</i> -Value	No Readmission (n=1025)	PAR (n=173)	<i>P</i> -Value
Radiation, Yes	10 (0.9)	8 (4.2)	<.01	13 (1.3)	9 (5.2)	<.01
Agent Orange, Yes	39 (3.5)	15 (7.9)	<.001	64 (6.2)	21 (12.1)	<.001
CAN score	68.3 (4.6)	69.1 (2.8)	<.01	70.4 (3.7)	72.7 (2.6)	<.01
No. of past year hospitalization						
0	485 (43.4)	56 (29.6)] <.01	526 (51.3)	33 (19.1)] <.001
1–4	593 (53.1)	114 (60.3)		447 (43.6)	117 (67.6)	
>4	39 (3.5)	19 (10.1)		52 (5.1)	23 (13.3)	
Comorbidity						
CAD	216 (19.3)	31 (16.4)	.3	137 (13.4)	25 (14.5)	.64
Heart failure	335 (27.7)	71 (34.5)	.03	119 (11.6)	17 (9.8)	.53
Vascular disease w/c	181 (16.2)	35 (18.5)	.4	82 (8.0)	20 (11.6)	.14
Cardiorespiratory	273 (24.4)	58 (30.7)	.05	94 (9.2)	10 (5.8)	.14
Pneumonia	—	—	—	355 (34.6)	72 (41.6)	.06
Atrial fibrillation	66 (5.7)	14 (7.4)	.3	35 (3.4)	8 (4.6)	.43
Anemia	33 (3.0)	10 (5.3)	.09	13 (1.3)	5 (2.9)	.11
Diabetes	132 (11.8)	35 (18.5)	.01	288 (28.1)	63 (36.4)	.02
COPD	339 (30.3)	69 (36.5)	.04	—	—	—
Chronic bronchitis	72 (6.4)	9 (4.8)	.4	402 (39.2)	86 (49.7)	<.01
Malignant neoplasm	31 (3.1)	10 (5.3)	.06	156 (15.2)	43 (24.9)	<.001
Mental disorder	106 (9.5)	27 (14.3)	.03	221 (21.6)	52 (30.1)	.01
Substance abuse	138 (12.4)	33 (17.5)	.04	269 (26.2)	60 (34.7)	.02

Table 8 Variable importance for the baseline PHSF model

Attribute	Raw score	Z-score	Sig.
CAN score	4.87	2.372	.009
Age	4.53	2.296	.011
Charlson CI	4.17	2.010	.022
No. of Past-year hospitalization	4.09	1.816	.035
Sequence	3.85	1.738	.041
LOS	3.79	1.658	.049
CAD	3.36	1.390	.082
Vascular disease w/c	3.41	1.381	.084
Admission source	3.21	1.303	.096
Atrial fibrillation	3.28	1.255	.105
Priority	2.88	1.068	.143
Agent Orange	2.52	.961	.168
Pneumonia	2.75	.930	.176
Sex	2.19	.869	.194
Mental disorder	2.66	.815	.207
Malignant neoplasm	2.53	.762	.223
Race	1.55	.653	.257
Radiation	1.43	.564	.286
Cardiorespiratory disease	1.71	.550	.291
Insurance	1.21	.483	.314
Heart failure	1.17	.466	.321
Diabetes	1.64	.454	.325
POW	.88	.330	.371
COPD	1.42	.323	.373
Marital status	.80	.283	.389
All others	.63	.197	.422

Table 9 Variable importance for the calibrated PHSF model

Attribute	Raw score	Z-score	Sig.
CAN score	7.88	3.582	<.0001
Age	7.32	2.874	.002
Charlson CI	7.06	2.398	.008
No. of Past-year hospitalization	7.18	2.324	.010
Sequence	6.72	2.077	.019
LOS	6.47	1.957	.025
CAD	6.24	1.898	.029
Vascular disease w/c	6.31	1.847	.032
Admission source	5.95	1.794	.036
Atrial fibrillation	6.03	1.736	.041
Priority	5.77	1.705	.044
Agent Orange	5.62	1.682	.046
Pneumonia	5.66	1.662	.048
Sex	5.24	1.656	.049
Mental disorder	5.39	1.632	.051
Malignant neoplasm	5.27	1.615	.053

Table 10 Calibration by risk decile for the PHSF model

Risk decile	Sample size	Predicted PAR	Observed PAR	O/P ratio
1	2286	201	183	0.910
2	1112	149	141	0.946
3	893	106	118	1.113
4	481	94	102	1.085
5	343	79	83	1.051
6	215	77	74	0.961
7	138	48	45	0.938
8	82	31	28	0.903
9	29	17	15	0.882
10	16	7	6	0.857

Table 11 Performance comparisons of our model over the selected methods

Method	Predictive accuracy measure							
	Sensitivity	Specificity	PPV	NPV	F-score	MCC	MSE	AUROC
Our proposal	91.95%	97.65%	86.61%	98.65%	.892	.874	.032	.836
Random Forest	88.43%	97.35%	84.70%	98.07%	.865	.843	.039	.802
SVM	86.16%	97.52%	85.20%	97.70%	.857	.833	.041	.775
Logistic Regression	83.40%	97.21%	83.19%	97.25%	.833	.805	.048	.721
Neural Network	82.39%	97.06%	82.28%	97.08%	.823	.794	.051	.704