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Research Article

Feedback Arc Number and Feedback Vertex Number of Cartesian Product of Directed Cycles

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For a digraph D, the feedback vertex number $\tau(D)$, (resp. the feedback arc number $\tau'(D)$) is the minimum number of vertices, (resp. arcs) whose removal leaves the resultant digraph free of directed cycles. In this note, we determine $\tau(D)$ and $\tau'(D)$ for the Cartesian product of directed cycles $D = \overrightarrow{C}_{n_1} \square \overrightarrow{C}_{n_2} \square \ldots \overrightarrow{C}_{n_k}$. Actually, it is shown that $\tau'(D) = n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$, and if $n_k \geq \ldots \geq n_1 \geq 3$ then $\tau(D) = n_2 \ldots n_k$.

1. Introduction

Let $G = (V, E)$ be an undirected graph. A set $S \subseteq V(G)$ is called a feedback vertex set of G if $G - S$ contains no cycle. The feedback vertex number of G, denoted by $\tau(G)$, is the cardinality of a minimum feedback vertex set of G. In general, it is NP-hard to determine the feedback vertex number of a graph G [1]. However, it becomes polynomial for specific families of graphs such as interval graphs [2], permutation graphs [3], graphs with maximum degree 3 [4], and k-trees. The readers are referred to [5, 6] for a review of some earlier results and open problems, and [7–9] for some recent results on the feedback vertex number of graphs. Some bounds or exact values are established for various families of graph, for instance, outerplanar graphs [10], grids and butterflies [11], cubic graphs [12, 13], bipartite graphs [14], generalized Petersen graphs [15], regular graphs [16, 17]. Bau et al. [18] investigated the feedback number of grid graphs.

Apart from its graph-theoretical importance, the feedback vertex problem has many applications, such as operating system [19, 20], artificial intelligence [21], synchronous distributed systems [22, 23], optical networks [24]. The feedback vertex set and the feedback vertex number are also known as decycling set and the decycling number, respectively, see [25].

In 2005, Pike and Zou [26] determined the feedback vertex number of the Cartesian products of two cycles as follows:

$$\tau(C_m \square C_n) = \begin{cases} \left\lceil \dfrac{3n}{2} \right\rceil, & \text{if} \quad m = 4 \\ \left\lceil \dfrac{3m}{2} \right\rceil, & \text{if} \quad n = 4 \\ \left\lceil \dfrac{mn+2}{3} \right\rceil, & \text{otherwise.} \end{cases} \tag{1}$$

Our main concern in this note is the directed version of the feedback vertex number. A directed graph D is said to be *acyclic* if it does not contain any directed cycle. A *feedback vertex set* in a digraph D is a set S of vertices such that $D - S$ is acyclic, and the *feedback vertex number* of D is the minimum size of such a set is denoted by $\tau(D)$. We denote by $\nu(D)$ the number of vertex-disjoint cycles of D. Clearly, $\tau(D) \geq \nu(D)$ for any digraph D. A *feedback arc set* of a digraph D is a set S of arcs such that $D - S$ is acyclic. The *feedback arc number* of D, denoted by $\tau'(D)$, is the cardinality of a minimum feedback arc set of D. We denote by $\nu'(D)$ the number of arc-disjoint cycles of D'. Clearly, $\tau'(D) \geq \nu'(D)$ for any digraph D.

Not much works were known for the feedback vertex number or the feedback arc number of directed graphs. Lien et al. [27] gave an upper bound for the feedback vertex number of generalized Kautz digraphs. Figueroa et al. [28] investigated the relation for the relationship between the minimum feedback arc set and the acyclic disconnection of a digraph. Even et al. [29] gave a $O(\log n \log\log n)$-approximation algorithm for the feedback vertex problem for a digraph of order n. For planar digraphs, the approximation ratio is not greater than 9/4

[30], and for tournament, it is 2.5 [31]. We refer to [32–34] for more results on feedback vertex set problems for tournaments and bipartite tournaments.

The Cartesian product $D_1 \square D_2 \square \ldots D_k$ of directed digraph $D_1, D_2, \ldots, D_k$ is the digraph with the vertex set $V(D_1) \times V(D_2) \times \ldots V(D_k)$, in which there is an arc directed from $(x_1, x_2, \ldots, x_k)$ to $(y_1, y_2, \ldots, y_k)$ if and only if there exists an integer $j \in \{1, \ldots, k\}$ such that $x_j y_j \in A(D_j)$ and $x_i = y_i$ for any other $i \neq j$. For any integer $n \geq 3$, $\overrightarrow{C_n}$ denotes the directed cycle of order n. Various kinds of properties of $\overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}$ are investigated. Trotter and Erdös [35] give a necessary and sufficient condition for $\overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}}$ being hamiltonian. Keating [36] gave a necessary and sufficient condition for $\overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}}$ being decomposed into directed Hamilton cycles. Recently, the previous result is extended by Bogdanowicz [37] with the decomposition into directed cycles of equal length. The domination number [38–41], respectively, the total domination number [42] of the Cartesian product of two directed cycles are investigated.

We shall determine the exact values of $\tau(D)$ and $\tau'(D)$ for the Cartesian product of directed cycles $D = \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}$.

2. Main Results

In this section, we denote $\overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}$ by $D(n_1, n_2, \ldots, n_k)$ or, simply by D. For convenience, label the vertices of D as $(x_1, x_2, \ldots, x_k)$, where $x_j \in \{0, 1, \ldots, n_j - 1\}$ for each $j \in \{1, 2, \ldots, k\}$. For an integer $i \in \{0, 1, \ldots n_k - 1\}$, let D_i be the subgraph of D induced by the set of vertices

$$\left\{(x_1, x_2, \ldots, x_{k-1}, i) : 0 \leq x_j \leq n_j - 1 \text{ for each } j \in \{1, \ldots, k-1\}\right\}. \quad (2)$$

It is clear that $D_i \cong \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_{k-1}}}$ for each i.

Theorem 1. *For any $k \geq 2$ integers $n_1, \ldots, n_k$ with $n_i \geq 3$ for each $i \in \{1, \ldots, k\}$,*

$$\tau'\left(\overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}\right) = n_1 n_2 \ldots n_k \sum_{i=1}^{k} \frac{1}{n_i}. \quad (3)$$

Proof. First, we show that $\tau'(D) \geq n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$ by showing that.

$$\nu'(D) \geq n_1 n_2 \ldots n_k \sum_{i=1}^{k} \frac{1}{n_i}. \quad (4)$$

We proceed with induction on k. Let $k = 2$. By our notation, $D_i \cong \overrightarrow{C_{n_1}}$ for each $i \in \{0, 1, \ldots, n_2 - 1\}$. Note that D_i and D_j are vertex-disjoint (and thus arc-disjoint). Moreover, since $D \backslash \cup_{i=0}^{n_2-1} A(D_i) \cong n_1 \overrightarrow{C_{n_1}}$, $\nu'(D) \geq n_1 + n_2$. Now assume that $k \geq 3$. Since $D_i \cong \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_{k-1}}}$ for each $i \in \{0, 1, \ldots n_k - 1\}$, by the induction hypothesis,

$$\nu'(D) \geq n_1 n_2 \ldots n_{k-1} \sum_{i=1}^{k} \frac{1}{n_i}. \quad (5)$$

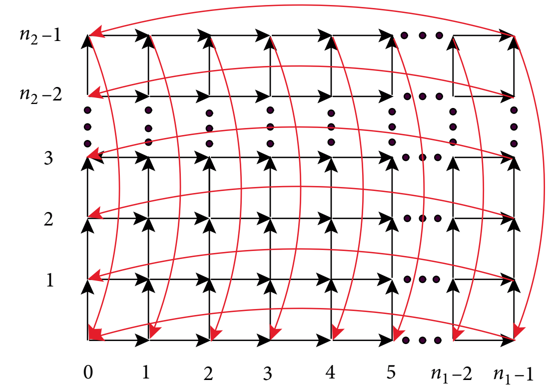

Figure 1: The feedback arc set A_2 of $D = \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}}$.

for each $i \in \{0, 1, \ldots n_k - 1\}$. After removing these $n_1 n_2 \ldots n_k \sum_{i=1}^{k-1} 1/n_i$ cycles from D, it results in exactly $n_1 n_2 \ldots n_{k-1}$ arc-disjoint directed cycles. This gives

$$\nu'(D) \geq n_1 n_2 \ldots n_{k-1} n_k \sum_{i=1}^{k-1} \frac{1}{n_i} + n_1 n_2 \ldots n_{k-1} = n_1 n_2 \ldots n_k \sum_{i=1}^{k} \frac{1}{n_i}. \quad (6)$$

Next we show that $\tau'(D) \leq n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$ by finding a feedback arc set of D with cardinality $n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$. Such a set feedback arc set A_k for $D(n_1, n_2, \ldots, n_k)$ is constructed recursively as follows. For convenience, let $A_1 := \{(n_1 - 1, 0)\}$. Note that A_1 is a feedback arc set of $\overrightarrow{C_{n_1}}$. For $k \geq 2$, $A_k = \{(x_1, \ldots, x_{k-1}, j)(y_1, \ldots, y_{k-1}, j) : (x_1, \ldots, x_{k-1})(y_1, \ldots, y_{k-1}) \in A_{k-1}, 0 \leq j \leq n_k - 1\} \cup \{(x_1, \ldots, x_{k-1}, n_k - 1)(x_1, \ldots, x_{k-1}, 0) : (x_1, \ldots, x_{k-1}) \in V(D_{k-1})\}$ By the above construction and the induction hypothesis,

$$\begin{aligned} |A_k| &= |A_{k-1}| n_k + n_1 n_2 \ldots n_{k-1} \\ &= n_1 n_2 \ldots n_{k-1} n_k \sum_{i=1}^{k-1} \frac{1}{n_i} + n_1 n_2 \ldots n_{k-1} \\ &= n_1 n_2 \ldots n_k \sum_{i=1}^{k} \frac{1}{n_i}. \end{aligned} \quad (7)$$

Moreover, since $D \backslash A_k \cong \overrightarrow{P_{n_1}} \square \overrightarrow{P_{n_2}} \square \cdots \overrightarrow{P_{n_k}}$ is acyclic, we conclude that A_k is a feedback arc set of D. This proves $\tau'(D) \leq n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$. □

For an illustration, for the case when $k = 2$, we have $D = \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}}$, and $A_2 = \{(n_1 - 1, 0)(0, 0), (n_1 - 1, 1)(0, 1), \ldots, (n_1 - 1, n_2 - 1)(n_2 - 1, 0)(0, n_2 - 1)(0, 0), (1, n_2 - 1)(1, 0), \ldots, (n_1 - 1, n_2 - 1)(n_1 - 1, 0)\}$, that is, the set of arcs colored in red as shown in Figure 1.

Theorem 2. *For any $k \geq 2$ integers $n_1, n_2, \ldots, n_k$ with $n_k \geq \ldots \geq n_1 \geq 3$,*

$$\tau\left(\overrightarrow{C_{n_1}} \to \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}\right) = n_2 n_3 \ldots n_k. \quad (8)$$

Proof. Let $D = \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}$.

First, we show that $\tau(D) \geq n_2 \ldots n_k$ by showing that

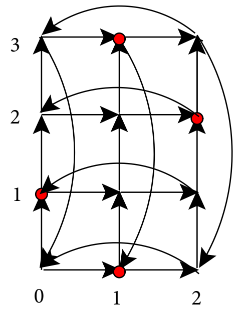

FIGURE 2: The feedback vertex set S_D of $D = \overrightarrow{C_3} \square \overrightarrow{C_4}$.

$$\nu(D) \geq n_2 \ldots n_k. \tag{9}$$

We proceed with induction on k. For the case when $k = 2$, $D_i \cong \overrightarrow{C_{n_1}}$ for each $i \in \{0, 1, \ldots, n_2 - 1\}$. It follows that D contains n_2 vertex-disjoint copies of $\overrightarrow{C_{n_1}}$, and thus $\nu(D) \geq n_2$. Now assume that $k \geq 3$. For every integer $i \in \{0, 1, \ldots n_k - 1\}$, $D_i \cong \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_{k-1}}}$, and hence, by the induction hypothesis,

$$\nu(D_i) \geq n_2 \ldots n_{k-1}. \tag{10}$$

Moreover, since D_i and D_j are vertex-disjoint for any $0 \leq i < j \leq n_k - 1$, we have

$$\nu(D) \geq n_2 \ldots n_{k-1} n_k. \tag{11}$$

As an example for the case when $n_1 = 3$ and $n_2 = 4$, we have $D = \overrightarrow{C_3} \square \overrightarrow{C_4}$ and $S_D = \{(1,0)(0,1)(2,2)(1,3)\}$, see Figure 2 for an illustration. Clearly, S_D is a feedback vertex set of D with $|S_2| = 4$.

For any $k \geq 2$, and $j \in \{0, 1, \ldots, n_k - 1\}$, let

$$S_k := \{(x_1, x_2, \ldots, x_{k-1}, x_k) : x_1 + x_2 + \ldots + x_k \equiv 1 \bmod n_1, 0 \leq x_i \leq n_i - 1\}. \tag{12}$$

Since for any given value of $(x_2, \ldots, x_{k-1}, x_k)$ with $0 \leq x_i \leq n_i - 1$, there exists unique value of x_1 with $0 \leq x_1 \leq n_1 - 1$ satisfying $x_1 + x_2 + \ldots + x_k \equiv 1 \bmod n_1$, implying that $|S_k| = n_k n_{k-1} \ldots n_2$.

To show that S_k is a feedback vertex set of D, we consider any directed cycle $\overrightarrow{C} = v_1 v_2 \ldots v_t v_1$ of D, where $v_i = (x_1^i, x_2^i, \ldots, x_k^i)$ for each $i \in \{1, \ldots, t\}$. Since for each i, $v_i v_{i+1} \in A(D)$, we have

$$x_1^{i+1} + x_2^{i+1} + \ldots + x_k^{i+1} \equiv x_1^i + x_2^i + \ldots + x_k^i + 1 \bmod n_1. \tag{13}$$

Moreover, since $t \geq n_1 = \min\{n_1, n_2, \ldots, n_k\}$, there exists an integer $j \in \{1, \ldots, t\}$ such that

$$x_1^j + x_2^j + \ldots + x_k^j \equiv 1 \bmod n_1, \tag{14}$$

implying that $v_j = (x_1^j, x_2^j, \ldots, x_k^j) \in S_k$, and thus S_k is a feedback vertex set of D. This proves

$$\tau(D) \leq n_2 n_3 \ldots n_k. \tag{15}$$

3. Conclusion

In this note, we determined the two important parameters $\tau(D)$ and $\tau'(D)$ for the Cartesian product of directed cycles $D = \overrightarrow{C_{n_1}} \square \overrightarrow{C_{n_2}} \square \ldots \overrightarrow{C_{n_k}}$. Actually, it is shown that $\tau'(D) = n_1 n_2 \ldots n_k \sum_{i=1}^{k} 1/n_i$ and if $n_k \geq \ldots \geq n_1 \geq 3$, then $\tau(D) = n_2 \ldots n_k \cdots$.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in conducting this research work and writing this paper.

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