

Adversarial Fault Tolerant Training for Deep Neural Networks

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Abstract—Deep Learning Accelerators and Neuromorphic circuits are prone to faults which manifest in the form of errors in Neural Networks. Fault Tolerance in Neural Networks is crucial, especially in real-time safety critical applications such as Autonomous Vehicles, which requires computation for long durations. Neural Networks with high regularisation exhibit superior fault tolerance, however, at the cost of classification accuracy. Practical applications require Neural Networks with superior fault tolerance as well as high classification accuracy. In the view of difference in functionality, a Neural Network is modelled as two separate networks, i.e, the *Feature Extractor* with unsupervised learning objective and the *Fully Connected Classifier* with a supervised learning objective. Traditional approaches of training the entire network using a single supervised learning objective is insufficient to achieve the objectives of the individual components optimally. In this work, a novel multi-criteria objective function, combining unsupervised training of the Feature Extractor and the supervised training of the Classifier Network has been proposed. The unsupervised training is modelled using two games solved simultaneously in the presence of adversary neural networks with conflicting objectives to the Feature Extractor. The first game minimises the loss in reconstructing the input image *given* the features of an image from the Extractor, in the presence of a generative decoder, such that the input and reconstructed image are indistinguishable. The second game solves a minimax constraint optimisation for smoothening the distribution of feature space to match a prior gaussian distribution, in the presence of a Discriminator network. The resultant Feature Extractor is strongly regularised and extracts robust features of the input image, which is combined with the Classifier Network for supervised fine-tuning. The proposed Adversarial Fault Tolerant Neural Network Training is scalable to large state of the art networks and is independent of the network architecture. The evaluation on benchmarking datasets: FashionMNIST and CIFAR10, indicates that the resultant networks have high classification accuracy with *superior* tolerance to stuck at “0” faults as compared to widely used regularisation functions.

Index Terms—Fault Tolerance, Neural Networks, Reliability, Adversarial Game, Regularization.

I. INTRODUCTION

The tremendous growth and adoption of advanced machine learning algorithms, such as Deep Learning, can be attributed to the improvements in algorithms, massive data for training the models and significant advances in the hardware design. Companies such as Google, Amazon, Microsoft and Apple have invested significantly in the development of Artificial Intelligence(AI) algorithms while hardware companies such

as NVIDIA, Intel and Xilinx have invested heavily in the development of efficient hardware for AI algorithms [32]. Research and development efforts in AI have led to growth of several applications in Natural Language Processing, Computer Vision, Speech Processing, Autonomous Vehicles and Cybersecurity [19]. Deep Learning algorithms are being explored for use in protection of national critical infrastructure, safety critical real-time applications and military such as flight control and radars. However, this tremendous interest in Deep Learning has also raised questions about the model security, data privacy, transparency and ethics of AI systems which are yet to be addressed.

Reliability and fault tolerance is another important property of AI systems deployed for critical applications. Neuromorphic computing chips and Neural Network accelerators are designed for energy efficient computation and fast inference. However, the underlying semiconductor devices and hardware are inherently unreliable due to process variations, thermal issues and leakages which affects the Neural Network computations on the circuit in the form of errors [36].

According to Neurobiological research, the human brain can tolerate small number of synapse or neurone faults [22][33]. This allows for replacing dead memory cells with new ones without loss of memory. Further, the distributed nature of the human memory cells allows for graceful degradation over the human lifetime. Notably, the mathematical abstraction of Human Brain, i.e, Artificial Neural Networks, has allowed some desirable properties like fault tolerance to be incorporated through over-parameterisation. However, this fault tolerance is very limited and cannot be generalised to all types of Neural Network models. This creates the need to incorporate fault tolerance and reliability fundamentally into the Artificial Neural Networks.

Practical Applications. Fault Tolerance is an important property for real time applications with high reliability requirements along with low power consumption and high performance. Faults could be catastrophic especially in critical-real time applications as it may not always be possible to retrain or reconfigure the model. A recent case is of Autonomous Vehicles like self driving cars and Unmanned Aerial Vehicles. These applications require uninterrupted computation for long duration of time where they use Deep Learning techniques for object recognition and detection in real time, for split second

decision making. Industrial Applications such as Oil and Gas industry and Nuclear Power Plants require high degree of reliability specified as part of the IEC Reliability specification [1]. For space applications, a satellite using Neural Networks for onboard sensor data processing requires the hardware and Neural Networks to be fault tolerant to be able to perform processing for significantly long time periods. Further, an aircraft flight control systems and onboard avionics rely heavily on fault tolerance through redundancy by replication of circuit components. Incorporating tolerance in the hardware architectures is crucial specifically for Brain Inspired computing hardware which are highly efficiency in terms of computation and energy consumption making them viable for defence and space applications [17][5]. In [3] and [7] the authors made investigations for getting a good modelling of the reliability of axonal transport. They deal with reliability schemes, in particular with hammock networks. Hence, deploying Neural Networks in the data processing pipelines for any practical application requires a rigorous evaluation of fault tolerance.

Key Challenges in Designing Fault Tolerant Neural Networks. Traditional reliability engineering techniques incorporate fault tolerance into hardware via explicit redundancy combined with voting strategies [21]. However, such approaches cannot be directly incorporated for Neural Network models. For instance, using *N-Modular Redundancy* results in additional implementation and computational cost making it infeasible for applications like Autonomous Vehicles with resource constraints. Such schemes need to address synchronicity of message exchanges between subset of Neural Networks. For Neural Networks, *augmenting redundancy* is achieved by adding additional nodes and synapses to distribute the computational load among more number of units [36]. However, additional redundancy results in large networks with too many hidden nodes and parameters, further requiring post-processing techniques like pruning to remove irrelevant nodes. Identifying critical neurons in every layer and distributing the load to other neurons is not feasible for large networks with millions of parameters [39]. Further, fault tolerance modelled as a *constrained optimisation* is expensive to solve. For instance, the fault tolerance as a min-max optimisation objective cannot be solved using gradient descent as the function is not differentiable. This requires approximating the objective function which results in partial fault tolerance [27][9].

Proposed Approach. The key idea in improving the fault tolerance *through regularisation* is to identify the individual Neural Network components with different functionality and train them separately with different objectives for best performance. The Deep Neural Network is divided into a Feature Extractor network with an unsupervised learning objective of identifying and extracting the dominant and robust features in the input image. On the other hand, the Classifier Network has a supervised objective to predict the image label given the extracted features. Previously proposed Fault Tolerant Neural Network algorithms, train the entire network using a single supervised learning objective which is insufficient to achieve the learning objectives of the individual components.

This further leads to a tradeoff between the model accuracy and the extent of generalisation which influence the fault tolerance. In this work, a novel multi-criteria Fault Tolerant training algorithm is proposed, comprising of an unsupervised objective function for the Feature Extractor and a supervised objective function for training the Classifier Network. The objective of training the Feature Extractor in unsupervised fashion is to achieve robust features with a smooth distribution which is modelled and solved as two strategic games. For robust features, the training is modelled as a game between the Feature Extractor and a Generative Model with conflicting objectives: the Feature Extractor model maps the input images to corresponding dominant features, while the Generative model reconstructs image *given* the extracted features. Both the networks are trained jointly, to minimise the reconstruction loss between the input and reconstructed images such that they are indistinguishable. For distributional smoothness, the Features extractor is trained adversarially with a minimax objective function in the presence of a Discriminator Network to match the feature space distribution with a target gaussian prior. This minimax objective acts as a Fault Tolerant constraint to *minimise the maximum deviation* between the feature space distribution and target prior distribution. The resultant network is attached to the Classifier Network and retrained by minimising supervised loss for object classification.

Main Contributions. In this work, it is shown that widely used regularisation functions such as *Tikhonov functions* have a *tradeoff between accuracy and generalisation*, i.e, for a highly generalised model (low generalisation error), the model has low inference accuracy (Section IV). This tradeoff makes standard regularisation functions unsuitable for incorporating fault tolerance into the network.

To achieve high generalisation, a novel framework for training the Neural Network is proposed *using a combination of unsupervised and supervised learning* in an adversarial setting which strongly regularises the model (Section V). The unsupervised training is modelled as a strategic game between two conflicting Neural Networks. This results in strongly regularised Neural Networks with *superior* tolerance to node and parameter (weights and filters) faults.

On comparing the proposed algorithm with other regularisation approaches, the resultant Neural Networks indicate lower generalisation error as well as a higher fault tolerance compared to traditional algorithms (Section VI). The evaluation is performed on networks for varying network complexities to indicate that *the training approach is scalable and independent of the model architecture*.

II. BACKGROUND

A. Machine Learning

Given the space of data instances X and space of corresponding ground truth labels Y , the goal of Machine Learning algorithms is to learn a classification function $f : X \rightarrow Y$ that accurately maps the data samples in X to its corresponding class in Y . This is modelled as an optimisation problem where the parameters are computed by minimising the loss

function $l(f(x), y)$ over each data instance (x, y) by capturing the error in model's prediction $f(x)$ and the ground truth label y . Instead of performing the optimisation problem on the entire data population $P(X, Y)$, the loss (L_D) is estimated over the training dataset $D \subset X \times Y$ where each data point (x, y) are sample i.i.d from D .

However, machine learning models tend to overfit on the training data, i.e, the accuracy on the training data is much higher than the accuracy on evaluation (previously unseen) data [4]. To ensure that the model does not overfit, a regularisation function $R(\theta)$ is added to the loss function making the final optimisation as below,

$$\min_f L_D + \lambda R(\theta) \quad (1)$$

The regularisation function penalises large parameter values and avoids the parameters from optimising specifically to fit the training dataset D . This ensures that the model performs well even on unseen data points sampled from $P(X, Y)$. The extent of regularisation is controlled by the regularisation hyperparameter λ which maintains a balance between the classification loss and the penalisation of large parameters.

Deep Learning. Deep Learning algorithms comprise of distributed network of computational units (neurons) connected amongst each other through edges or synapses. Each of the synapses is associated with a weight which indicates the importance of the input from the particular neuron for the computation in the next layer. Each neuron computes the weighted average of all the input synapses as an affine function, $a^l = W^l \times a^{l-1} + b^l$ where a^l is the activation (intermediate sum) of the l^{th} layer, W^l are weights learned for each of the synapses during training, a^{l-1} are the activation values from the previous layer fed as input and bias values are given by b^l . The affine computation is a matrix-vector multiplication between the weight matrix W and the input activation vector a . A non-linear activation function follows the matrix-vector computation which restricts the activation values from growing too large.

Convolutional Neural Networks use convolution and max-pool operations in addition to the above operations. Maxpool computes the maximum pixel value of the $k \times k$ region of preceding layer feature map where k is the kernel size. Convolutional operation, on the other hand, is a weighted sum operation which computes the multiplication of the parameters $W_{ij}^{(l)}$ of layer l and the input feature map a_{ij}^{l-1} and adds the output, i.e, $a_i^{(l)} = \sum_i \sum_j W_{ij}^{(l)} \times a_{ij}^{(l-1)}$.

B. Adversarial Networks

The idea of Generative Adversarial Networks was first given by Goodfellow *et al.* [15] where generative networks were modelled within a game theoretic framework. The goal of the generative model is to learn the distribution p_{model} of the underlying data p_{data} such that $p_{model} \sim p_{data}$. This is done by jointly solving a minimax optimisation between two networks with conflicting objectives, i.e, the generative model outputs data samples $z \sim p_{model}$ to maximise the Discriminator's network loss. While, the Discriminator distinguishes data

samples which are part of training data ($p(x \sim p_{data})$) and sampled by the Generator network ($z \sim p_{model}$). This can be formulated as a minimax optimisation between the Generator and Discriminator networks,

$$\min_G \max_D \mathbb{E}_{x \sim p_{data}} [\log(D(x))] + \mathbb{E}_{z \sim p_{model}} [1 - \log(D(G(x)))] \quad (2)$$

The training criteria in Equation 2 can be expanded as,

$$V(G, D) = \int_x p_{data}(x) \log(D(x)) dx + \int_z p_{model}(z) \log(1 - D(G(z))) dz \quad (3)$$

On solving this min-max optimisation problem as a game between the Generator and the Discriminator, over time, the Generator learns to mimic the target distribution p_{data} . The generator and discriminator functions can be any differentiable function with arbitrary complexity. In this work, however, both the functions are Deep Learning models. The advancements in adversarial networks allowed to design novel architectures and could be used to solve minimax optimisation problems as a game between two Neural Networks [26]. Adversarial training approaches can be extended to unsupervised learning paradigm where the labels corresponding to the inputs are not available [24]

C. Fault Tolerant Neural Network

The fault tolerance property of neural networks ensures that a neural network continues to operate even in the presence of node and synapse faults and degrades gracefully over time. According to the definition of ϵ -fault tolerance [27], a Neural Network \mathcal{N} performing computations $\mathcal{H}_{\mathcal{N}}$ is said to be fault tolerant if the computation $\mathcal{H}_{\mathcal{N}_{fault}}$ performed by a faulty neural network \mathcal{N}_{fault} is close to $\mathcal{H}_{\mathcal{N}}$, Formally,

$$\| \mathcal{H}_{\mathcal{N}}(\mathcal{X}) - \mathcal{H}_{\mathcal{N}_{fault}}(\mathcal{X}) \| \leq \epsilon \quad (4)$$

for $\epsilon > 0$ and input image \mathcal{X} is sampled from the training dataset D . For a Neural Network to be completely fault tolerant, the value of $\epsilon = 0$. However, this strict condition of complete fault tolerance can be relaxed by designing a Neural Network with graceful degradation with the condition for $\epsilon > 0$. The resultant model \mathcal{N} satisfying the above constraint is referred to as ϵ -Fault Tolerant.

Fault tolerance techniques can be broadly classified into active fault tolerance and passive fault tolerance based on the objectives (refer to [39] for a detailed survey). Active approaches explicitly and dynamically recognise and manage the system's redundant resources to compensate the fault as they appear by adaptation, relearning and self repair mechanisms. The design of active approaches however is complex as it includes the implementation of detection and localisation components within the system. Generally, a higher degree of fault tolerance can be achieved using passive techniques as active approaches cannot cover all the possible cases. In

passive techniques, intrinsic redundancy and fault masking is incorporated into the models before training to ensure correct operation in the presence of faults.

Fault Tolerance can be incorporated as a constraint during training using Quadratic programming [27], Genetic Algorithms [34] and minimax optimisation [9]. These approaches provide enhanced fault tolerance with theoretical guarantees but require significantly higher computation compared to simple regularisation.

Explicitly incorporating additional redundancy by adding nodes and synapses to share the computational load achieves tolerance to single node faults [12][6]. Additional redundancy like Triple Modular Redundancy (TMR) and node and weight replication provides partial fault tolerance with graceful degradation [28].

Several algorithms modify the training algorithm to generalise the model by adding noise to weights or injecting faults during training which acts as a regulariser. However, this regularisation is equivalent to mean square error plus Tikhonov function [25]. In fact, a theoretical analysis of most of the proposed fault tolerant training algorithms using regularisation are equivalent to either Tikhonov and Lasso functions [16]. Training a Neural Network with Lasso regularisation results in sparse parameters as compared to Tikhonov, which allows to prune certain nodes or weights, resulting in certain degree of fault tolerance [42]. The Kullback-Liebler Divergence can also be used as an objective function to improve the regularisation of RBF networks which is equivalent to the cross entropy loss used in the current state of the art DNN architecture training [20][41].

Hardware Fault Tolerance. Incorporating fault tolerance into the Neural Network algorithms is not sufficient and has to be accompanied with fault tolerant hardware design for Neural Networks. In hardware accelerators, single bit flip error (soft errors) are major source of unreliability which can cause application failure [21]. To address this, circuit replication is used but at the cost of a high overhead in energy and cost. Designing additional circuitry to detect anomalies in the computation helps to detect faults and their locations in the hardware. Further, design modifications at transistor level helps in mitigating the errors [37].

III. FAULT AND ERROR MODEL

The fault model, considered in this work, identifies the possible locations where faults can occur in the Neural Network hardware. Unlike fault models, error models consider the deviation occurring due to faults in the hardware. During the Neural Network development lifecycle, faults occur either in the training phase or in the inference (evaluation) phase. Faults during training may slow down the overall computation time but are less likely to effect the performance after network deployment. This work considers faults occurring after the deployment of Neural Networks which degrade the model performance over time.

Previous research considers the effects of single faults on simple Neural Network topologies with trivial learning tasks.

For the current state of the art Neural Networks, evaluating using single faults is ineffective due to the inherent over parameterisation from hyperparameter selection. To this extent, this work considers *multiple, concurrent faults* randomly occurring in the entire Neural Network hardware. Further, this work considers, for the first time, faults in Convolutional Neural Networks and thus extending the fault taxonomy to filter faults. The most commonly occurring faults in the hardware, manifest in the form of stuck at “0” faults in the parameters, i.e, weights in Fully Connected Layers and filters in Convolutional Layers. Due to large number of parameters and nodes in Deep Neural Networks, exhaustive testing of all possible single faults is prohibitive. For tractability, this work adopts the strategy of randomly testing for a fraction of faults and simulating the effect by measuring the corresponding error in Neural Network prediction. Instead of evaluating the effect of faults in hardware on the Neural Networks, the tolerance is measured by simulating the errors manifested in the Neural Networks due to faults. This yields fault tolerance estimates that are statistically very close to those obtained by exhaustive testing [28].

The faults in Neural Networks occurring in either the nodes or the synapses are assumed to be independent of previous faults. During the execution of Neural Networks, the major computation is the Multiply and Accumulate(MAC) operation between the weights of the layer and the input activation from the previous layer. Since, the weights or intermediate computation are reused for subsequent computations during the forward pass, Neural Network accelerators include additional scratchpad memory along with each computation circuit [35]. Inability to access the memory units due to hardware failures results in reading a “0” value for the corresponding node activations and parameters. These *parameter faults* occur as errors in the filters of convolutional layers and the synapses or edge connections between nodes in fully connected layers of the Neural Networks. On the other hand, *node faults* manifest as errors in the activations values in the intermediate layers. Another fault is when the circuit components are stuck at V_{max} on being connected directly to the supply voltage. However, this work only considers the more frequently occurring case of stuck at “0” faults in the nodes (activations) and the parameter (weights and filters) values.

To simulate the stuck at “0” faults in nodes and parameters in the network, binary masks are created which are multiplied with the weights and activations. The pytorch implementation used to generate the masks and simulate the faults is shown in Figure 1. The function *generate_mask* takes the percentage of total nodes/parameters to remove for each layer. The function *set_masks* multiplies each mask with the corresponding weights or activations in the network. In case of node faults, the binary mask is multiplied over the activation tensors during forward propagation in the Deep Neural Network.

In case of distributed learning, Byzantine participants output arbitrary values and the performance drop is more aggressive as compared to generic node or parameter faults. Further, Stochastic Gradient Descent(SGD) cannot tolerate even a

```

def generate_mask(model, pruning_perc):
    masks = []
    shape = []

    for p in model.parameters():
        if len(p.data.size()) == 4: #convolutional layer
            shape.append(p.size())

    for i in shape:
        tensor=torch.FloatTensor(i).uniform_()>(
            pruning_perc)
        masks.append(tensor.numpy())

    model.set_masks(masks)

def set_mask(model, mask):
    model.weight.data = model.weight.data * model.mask.
    data

```

Fig. 1: Code to Generate Fault Mask in Pytorch. To simulate stuck at “0” faults, a binary mask is created based on the percentage of total faults which is multiplied to the convolutional filter or fully connected layer weights. For Node Faults, the masks are multiplied with the intermediate layer activations during forward propagation.

single Byzantine participant who can force a parameter value to be arbitrarily large [8]. In such cases, it is important to design learning algorithms with Byzantine Fault Tolerance. In this work, however, the case of federated learning and design the networks for Byzantine Fault Tolerance is not considered. Rather, the focus is on designing Fault Tolerant Neural Networks for the general case of node and parameter faults stuck at “0”. In case of an adversarial setting, an attacker introduces faults and malicious modifications in the form of Hardware Trojans into the Integrated Circuit supply chain [2]. While faults have been studied as an attack on Neural Networks, this work focuses on fault tolerance in the absence of an adversary.

IV. REGULARISATION FOR FAULT TOLERANCE

Fault Tolerance is exhibited by models with high generalisation and regularisation functions play an important role in enhancing Fault Tolerance [10]. To achieve highly generalised model, regularisation functions penalise large parameter values and ensure that the models do not overfit. The key effect of regularisation is to uniformly distribute the information across all the nodes and synapses. This ensures that all the nodes are given equal importance during the computation such that in the presence of faults, other nodes or synapses can take over the computation without loss of accuracy. In case of unregularised networks, models tend to overfit and the parameters adapt themselves specifically to the training data. Overfitting model tends to perform well on the training dataset while the performance on the testing dataset is low [4]. Overfitting tunes the parameters such that some of the nodes or synapses are given more preference compared to others. Hence, the loss of the important nodes or synapses results in a significant drop in performance.

Effect of Regularisation on Weight Distribution. To evaluate the effect of regularisation on parameters, two

neural networks are trained with same architecture: one with regularisation while the other without regularisation. The information in the Neural Network in the form of parameter values follow a Gaussian distribution as seen in Figure 2. The parameters after gradient descent optimisation with regularisation have similar values, i.e, the parameter distribution has low variation. On the other hand, the parameters of un-regularised model after training are significantly more varied which is indicated by the distribution having a larger standard deviation. This validates the initial statement of equal weightage given to all nodes and uniform parameter distribution.

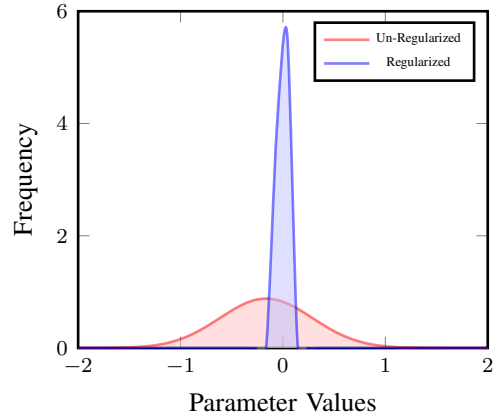


Fig. 2: Effect of Regularization on Parameter Distribution. The distribution of parameters for unregularized model has a higher standard deviation indicating more variance in the values. On regularization, the standard deviation of the distribution decreases significantly making all the parameters values close to the mean. Architecture: [784,512,512,10]

Quantifying Overfitting. Given a population of data and corresponding labels, $P(X, Y)$, the goal of training a machine learning model is to minimise the expected loss computed over the entire population.

$$E_{pop} = \mathbb{E}_{(x,y) \sim P(X,Y)} [l(f(x), y)] \quad (5)$$

However, this error cannot be exactly computed as it requires knowing the probability distribution of data $P(X, Y)$. Instead of estimating the loss for data points (x, y) sampled from $P(X, Y)$, the error is computed over a training dataset $D \subset P(X, Y)$ by sampling a few points i.i.d. The empirical minimisation loss over D is given by,

$$E_{train} = \frac{1}{|D|} \sum_{(x,y) \sim D} (l(f(x), y)) \quad (6)$$

The difference between the empirical loss over the entire population and the expected loss over the training data helps to identify the extent of overfitting. To compute the generalisation error, the population error (E_{pop}) is approximated by computing the expected error over the test data which the model has not seen during the training.

$$E_{test} = \frac{1}{|D_{test}|} \sum_{(x,y) \sim D_{test}} (l(f(x), y)) \quad (7)$$

For a large number of samples n , $\lim_{n \rightarrow \infty} E_{test} = E_{pop}$.

To compute the degree of overfitting, the difference between the training accuracy (R_{train}) and the testing accuracy (R_{test}) is measured corresponding to the respective training and testing error.

$$G_{error} = R_{train} - R_{test} \quad (8)$$

A higher generalisation error percent G_{error} indicates more overfitting and the model is prone to significant performance degradation in presence of faults. This measure of generalisation error is used throughout the paper for evaluating the proposed training algorithm on various neural network architectures.

State of the Art Regularisation. Current Deep Neural Network architectures rely heavily on either Lasso or Tikhonov Regularisation. While these regularisers generalise the model by clipping large parameter values, there exists a tradeoff between the test accuracy and the degree of generalisation. For instance, Tikhonov regularisation is extensively used for most of the neural networks but the resultant models are not maximally tolerant to faults due to this tradeoff. As seen Table I, despite having a model with high accuracy, the resulting generalisation error for the Neural Networks is high (overfitting), while, for strongly regularised models the test accuracy degrades. Finding the optimal values of the hyperparameters which results in good accuracy with lower generalisation error is a search problem, typically solved using Grid Search or Random Search algorithms. Given the input data samples from the training dataset $(x_i, y_i) \in D$, the classification loss for a given machine learning function $f()$ is computed as $L = \sum_i (y_i - f(x_i)) + \lambda R(\theta_j)$, i.e., the error in prediction of the machine learning model from the true label y for the input data point x . In order to penalise large values of parameters θ and ensuring that all the parameters are of the same scale, an additional term corresponding to the norm of the parameters is added to the loss function. In case

Regularisation Hyperparameter	Training Accuracy	Testing Accuracy	Generalization Error
0	98.60%	88.90%	9.70%
0.0001	97.84%	89.89%	7.95%
0.001	93.88%	88.86%	5.02%
0.1	66.57%	65.74%	0.83%

TABLE I: Tikhonov Regularisation. Simple Regularisation approaches like Tikhonov penalty results in a tradeoff between Model Test Accuracy and Generalization Error (measure of overfitting), i.e., as the Generalisation error decreases, the model performance decreases.

of Lasso regularisation, the resulting regularisation function is the absolute sum of the individual parameters θ ,

$$L = \sum_i (y_i - f(x_i)) + \lambda \sum_j |\theta_j| \quad (9)$$

where λ is the regularisation hyperparameter to control the scaling for penalising large parameters.

In case of Tikhonov Regularisation, the loss includes the sum of the square of the parameter values,

$$L = \sum_i (y_i - f(x_i)) + \lambda \sum_j |\theta_j|^2 \quad (10)$$

Tikhonov regularisation penalises large parameters more strongly as compared to Lasso. A combination of both Lasso and Tikhonov can be used to regularise the resultant Neural Networks. For example, Dey *et al.*[10] incorporate a combination of different parameter penalisation terms to regularise the models. However, all these approaches have a trade-off between the model generalisation and performance.

Fault Tolerance and Regularisation. The comparison of the model performance degradation to random node faults, filter faults and parameter faults on convolutional and deep neural network architectures is shown in Figure 3. The performance degrades gracefully for models with regularisation as compared to models without regularisation. In case of fault in parameters, a loss of 50% of the total nodes result in an

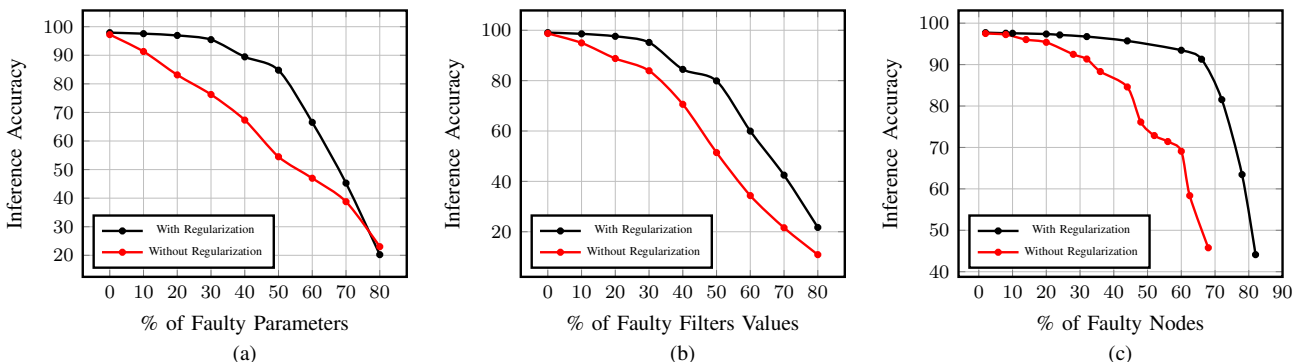


Fig. 3: Comparing the Tolerance of Regularised and Un-Regularised (overfitting) Models trained on Digit Recognition Data. (a) Degradation of Model Accuracy in presence of Weight Faults in Deep Neural Networks; (b) Performance Degradation in presence of Filter Faults in Convolutional Neural Networks and (c) Degradation of Performance due to Node Faults in forward propagation. The regularised model depicts a higher fault tolerance as compared to un-regularised model. Faults are injected by multiplying binary masks with the weights/node activations as shown in Figure 1

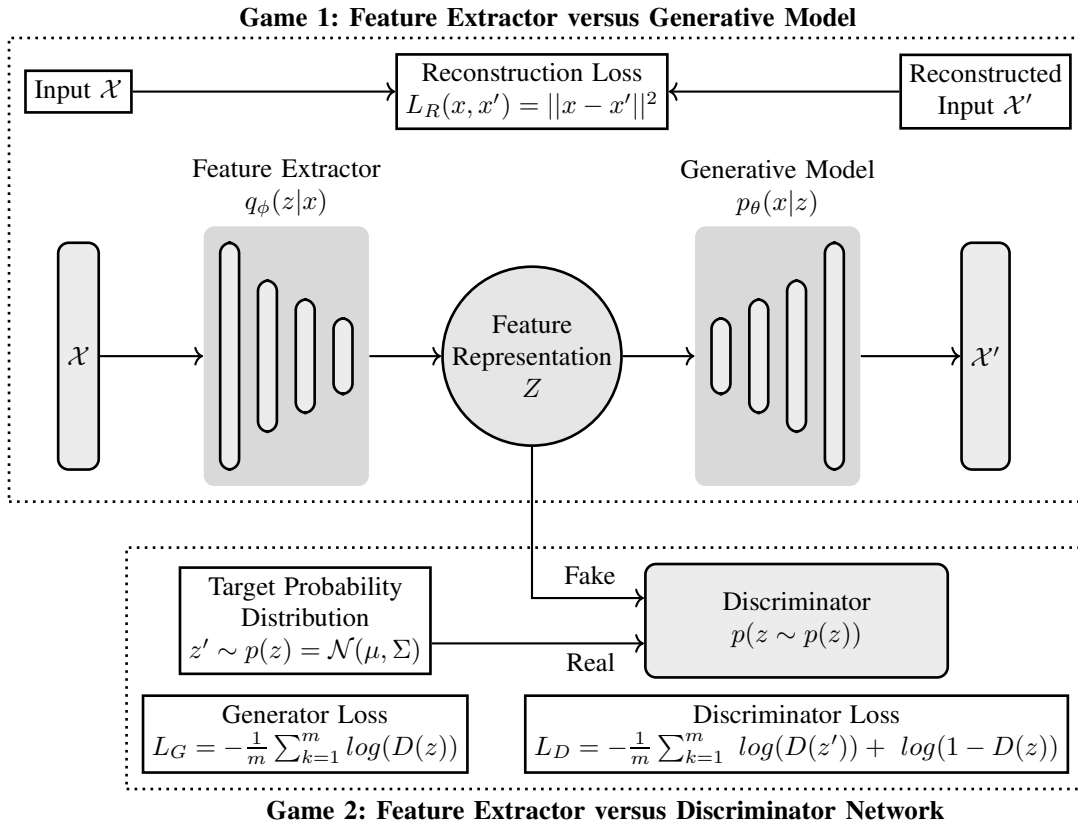


Fig. 4: Unsupervised Pre-Training for Regularising Feature Extractor. The joint optimisation is solved as games between different Neural Networks with conflicting objectives. In Game 1, the Feature Extractor is trained with a Generative Model p_θ to ensure the intermediate feature representation of the images is accurate and robust, by minimising the reconstruction loss. In Game 2, the Feature Extractor is adversarially trained by solving a min-max game with the Discriminator to ensure the distribution of feature space matches the target prior distribution. This adversarial loss along with the reconstruction loss results in a strongly regularized Feature Extractor.

accuracy of 54.49% for overfitting model compared to 84.78% in regularised models. In case of convolutional filter faults, a loss of 50% of the filter result in an accuracy of 51.43% for overfitting models compared to 79.90% in regularised models. For node faults, a 93.46% accuracy was observed for regularised models as compared to 69.08% accuracy of overfitting models when 60% of the nodes are not functioning. Despite a significant number of faults injected into the system, the performance of the generalised model is still high and operable compared to models without regularisation.

V. ADVERSARIAL FAULT TOLERANT TRAINING

Since, a strongly regularised model shows superior fault tolerance, the ultimate objective of the proposed algorithm is to attain superior fault tolerance through strong regularisation. However, simple regularisation functions are not sufficient for maximal fault tolerance and requires to address the problem from a different perspective.

Deep Neural Network is divided into a Feature Extractor and a Fully Connected Classifier Network based on the difference in functionality and learning objectives. The goal of the Feature Extractor is to identify and extract dominant features in a given input image which is passed to the Classifier

Network for predicting the final class of the object. Training the entire network using a single supervised objective function does not achieve the best classification accuracy with minimal overfitting. The different functionality of both the networks requires them to be trained separately with different objective functions to achieve the maximal performance *and* generalisation.

A. Unsupervised Learning

The Feature Extractor has an unsupervised learning objective of mapping the input images to feature space representation while the Classifier Network can be trained using traditional supervised learning algorithms. The unsupervised learning has two main objectives, *extract robust features* given the image and *distributional smoothness* to ensure that the extracted features follow a particular prior distribution. To achieve this, the Extractor is adversarially trained by solving two games in the presence of Neural Networks with conflicting objectives. Each game achieves a particular objective required for unsupervised learning which is discussed below.

Game 1: Feature Extractor versus Generative Network

The objective of this game is to train the Feature Extractor to identify and extract the dominant features in an image. The

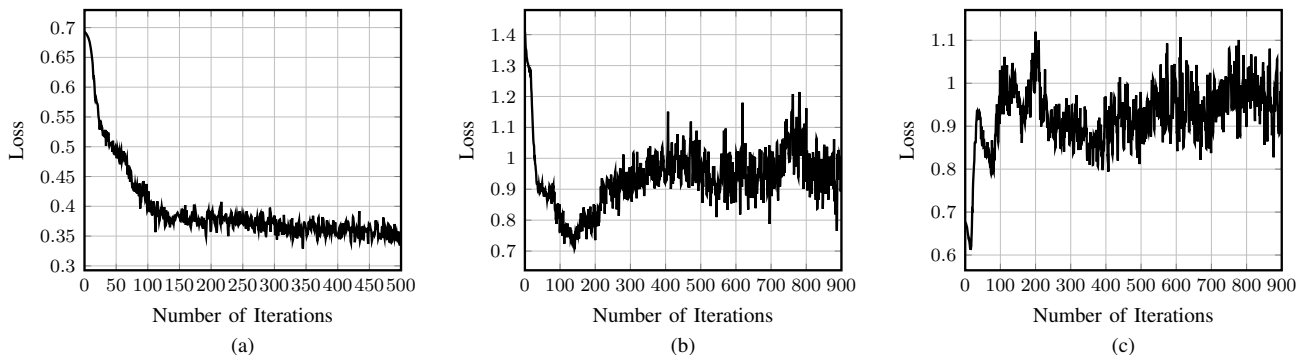


Fig. 5: Loss Functions. (a) Trajectory of the Reconstruction loss computed as the difference between the input image and the reconstructed image by the decoder as given in Equation 11; (b) Minimising the Loss of the Feature Extractor during the adversarial min-max optimisation as given in Equation 13; (c) Maximising the gain of the Discriminator model during the min-max optimisation as given in Equation 12.

game is played between the Feature Extractor Network (q_ϕ) and the Generative Network (p_θ) with conflicting objectives. The goal of the Feature Extractor is identify and extract the dominant features $q_\phi(x)$ given an input image (x). On the other hand, the goal of the Generative Network is to reconstruct the input image (x') given the features extracted from the actual image (x), given by $p_\theta(q_\phi(x))$. The two models are trained simultaneously to minimise the loss computed in the reconstruction process while ensuring a bottleneck condition where the feature dimensions are lower than the original image dimensions. The models train and update their parameters to choose the best possible strategy where the reconstructed image from the generated features is close to the original input image [24]. Both the networks together learn the identity function $p_\theta(q_\phi(x)) = x$ using the Feature Extractor as image encoder and Generative Network as feature decoder. The reconstruction loss minimised during the training is given by,

$$L_R(x, x') = \|x - x'\|^2 \quad (11)$$

The reconstruction loss is computed between x and x' and the gradient is backpropagated through p_θ and q_ϕ accordingly and their weights are updated. Ideally, after multiple epochs of training and parameter updates of both the adversary networks, the reconstructed image is indistinguishable for the original input image. This indicates that the Feature Extractor network is capable of generating dominant and robust image features which can be efficiently reconstructed.

Game 2: Feature Extractor versus Discriminator

The objective of the this game is distributional smoothening of the feature space by training the Extractor to output image features following a prior distribution. The optimisation is solved as a zero-sum non-cooperative (minimax) game between two Neural Networks where both the networks choose the optimal strategy to reduce the success of the other network objective. Since, both the Networks are playing the game against each other, a Nash Equilibrium exists where both the networks (players) take the optimal decision [15].

For distributional smoothness, the prior distribution is chosen as Gaussian to ensure that there are no gaps in the feature

space [24]. The Extractor Network is trained adversarially in the presence of a Discriminator Network to solve a minimax optimisation problem over the feature space distribution. The goal of the Discriminator Network is to distinguish between “fake” data points (z) generated from the Feature Extractor and “real” data points (z') sampled from prior distribution. While the Feature Extractor aims to *minimise* the difference between generated features and the target prior distribution such that the Discriminator Network error is *maximised*. Formally, a data instance z is sampled according to the Feature Extractor’s output $q_\phi(z|x)$ and z' is sampled from the true prior $p(z)$. The Discriminator receives z distributed as $q_\phi(z|x)$ and z' sampled from the true prior $p(z)$ and assigns a probability to each of coming from $p(z)$. The gain of the discriminator due to correctly distinguishing between the data points sampled from different distribution is computed as

$$G_D(z, z') = -\frac{1}{m} \sum_{k=1}^m \log(D(z')) + \log(1 - D(z)) \quad (12)$$

where m is the minibatch size, z is generated by the encoder and z' is a sample from the true prior.

For the Feature Extractor, the goal is to maximise the errors made by the Discriminator by generating samples similar to the prior distribution samples. In other words, the Extractor trains to minimise the maximum gain of the Discriminator Network. The loss for the Feature Extractor is computed as,

$$L_G(z) = -\frac{1}{m} \sum_{k=1}^m \log(D(z)) \quad (13)$$

The loss computed is back-propagated through the discriminator to update its weights followed by which the Feature Extractor updates its parameters. The Feature Extractor eventually generates samples $z \sim q_\phi(z|x)$ close to the target prior $p(z)$ such that the discriminator cannot distinguish between the two inputs (random guess strategy). Over time, the loss of the discriminator increases while the Feature Extractor minimises the maximum gain of the Discriminator. This min-max joint optimisation is shown in Equation 2 and the loss

plots indicating the trajectories of loss functions while playing the game over multiple epochs are shown in Figure 5.

The overall *adversarial loss* of the Feature Extractor used to update the parameters includes both the reconstruction loss and the loss given by the discriminator network (D) as shown below,

$$\min_{q_\phi} \left(L_R(x, x') + \max_D G_D(z, z') \right) \quad (14)$$

where the model is updated with the reconstruction loss first, followed by the update over the discriminator loss.

B. Supervised Fine-Tuning

The unsupervised training of the Feature Extractor acts as a strong regulariser [13]. The pre-trained Feature Extractor (q_ϕ), now, is attached to the Classifier Network (f_{class}) for supervised training. Formally, the classifier network maps the input latent space representation of image(z) to the corresponding class in Y , i.e., $f_{class} : q_\phi(z|x) \rightarrow Y$ for a given input x . While the parameters of the Classifier Network are trained for predicting the class from the features of an image, the parameters of the Feature Extractor are fine-tuned using the prediction error. The classifier and the Feature Extractor are retrained by minimising the *classification loss* computed using the ground truth labels y and the predicted labels $f(x)$ given by,

$$L_C(y, f(x)) = \|f(x) - y\|^2 \quad (15)$$

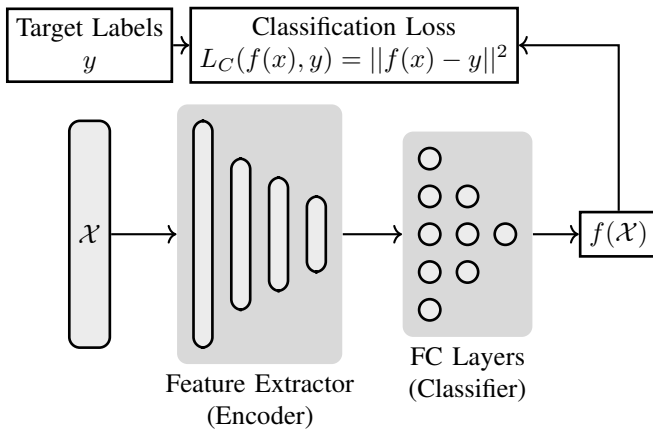


Fig. 6: Training Neural Network Classifier. The trained Feature Extractor is attached to a Fully Connected network which is trained by minimising the classification loss. The Fully Connected network maps the latent space output of the feature extractor corresponding to the input to a target label for classification.

Notably, the unsupervised pre-training of the Feature Extractor captures the dominant features of the input distribution and updates the parameters of the network accordingly. Among different regularisation functions, standard regularisation schemes like Tikhonov and Lasso functions are drastically less effective than the unsupervised pre-training strategy [13]. The effectiveness of the unsupervised learning strategy lies in efficiently learning the input distribution $P(X)$ in order to

improve the supervised classification $P(Y|X)$ for input space X and corresponding label space Y . Instead of randomly, initialising the weights of the vector in a classifier, pre-training the feature extractor to learn the input distribution followed by supervised fine-tuning of the resulting classifier strongly regularises the model [14].

C. Theoretical Analysis

In this section, the reconstruction loss minimisation with respect to the parameters of the both the Feature Extractor (q_ϕ) and the Generative Model (p_θ) is discussed. Further, the connection between Fault Tolerance and the minimax optimisation for training the Feature Extractor is explored. The optimal solution of the minimax game is shown along with the convergence criteria where both the Feature Extractor and the Discriminator Network choose the best optimal strategy to obtain their objective.

Reconstruction Loss Minimisation. The reconstruction loss (Game 1) computed between the original image and the reconstructed image by the Generative Network is used to update the parameters of the both the networks for improving the performance. Given the loss function, $L_R(x, x') = \|x - x'\|^2$, the gradients are computed first to update the Generative Model parameters θ as $\nabla_\theta \|x - x'\|^2$. The parameters θ are updated using Stochastic Gradient Descent to tune the parameters in the direction of minimum reconstruction loss. The parameters of the feature extractor are then updated similarly by computing the gradients of loss function with respect to the activations, $\nabla_\phi \|x - x'\|^2$. The parameters are iteratively updated till the minima of the loss function is achieved.

Fault Tolerance as Min-Max Constraint Optimisation. Minimax constraint during Neural Network helps to generalise the model by minimising the maximum deviation between the error computed over Fault-less Network $\mathcal{H}_N(\mathcal{X})$ and Faulty Network $\mathcal{H}_{N_{fault}}(\mathcal{X})$ [27][9]. The minimax constraint trains an ϵ -Fault Tolerant Network to minimize the threshold ϵ for maximum possible deviation, $|\mathcal{H}_N(\mathcal{X}) - \mathcal{H}_{N_{fault}}(\mathcal{X})| < \epsilon$ (Equation 4). In other words, the goal is to determine the minimum value of ϵ such that the resultant weights of \mathcal{N} maintains a high accuracy and additionally satisfies ϵ -fault tolerance.

Since, the minimax constraint optimisation is highly non-linear and non-differentiable, previous approaches approximate it using a sum of unconstrained least squares to solve the problem [9]. This approximation, however, results in partial fault tolerance and achieving fault tolerance in this approach is computationally more expensive. Quadratic programming algorithm for different values of ϵ can identify the minimum weights for which the objective is near zero [27]. However, this does not scale for the current state of the art neural network architectures with millions of parameters.

In this work, the minimax optimisation is efficiently solved and modelled as a game between two adversary Neural Networks with conflicting objectives. The game is played between the Discriminator Network and the Feature Extractor. The

Algorithm 1 Adversarial Fault Tolerant Training Algorithm for Neural Networks. The algorithm first optimises the reconstruction loss from Game 1 and updates the parameters ϕ of Feature Extractor (Equation 11). The Feature Extractor is then adversarially trained with the Discriminator to minimise the maximum deviation between the feature space and true prior distribution (Equation 14). The Feature Extractor is attached to the Fully Connected Network and retrained for minimising the classification loss (Equation 15).

- 1: **for** unsupervised training epochs **do**
 - 2: **(1) Train the Feature Extractor and decoder for reconstruction**
 - 3: Sample $\{x^{(i)}\}_{i=1}^m \sim \mathcal{P}(X, Y)$
 - 4: Compute $z^{(i)} = q_\phi(x^{(i)})$
 - 5: Compute $x'^{(i)} = p_\theta(x^{(i)}|z^{(i)})$
 - 6: Update the Feature Extractor and Decoder using *Stochastic Gradient Descent* over the parameters θ and ϕ
 - 7: Backpropagation loss, $\mathcal{L}_{\text{rec}} = \|x - x'\|^2$
 - 8: **(2) Train the Discriminator Model to Distinguish between True Prior and Generated Distribution (Keeping Feature Extractor Fixed)**
 - 9: **for** k steps **do**
 - 10: Sample data points from true prior distribution $z^{(i)} \sim p(z) = \mathcal{N}(\mu, \Sigma)$
 - 11: Compute $z^{(i)} = q_\phi(x^{(i)})$
 - 12: Update the Discriminator using *Stochastic Gradient Ascend*
 - 13: Backpropagation Loss: $\mathcal{L}_D(z, z') = -\frac{1}{m} \sum_{k=1}^m \log(D(z')) + \log(1 - D(z))$
 - 14: **end for**
 - 15: **(3) Update the parameters of the Feature Extractor after training the Discriminator**
 - 16: Update the Feature Extractor q_ϕ using *Stochastic Gradient Descent* over the parameters ϕ
 - 17: Backpropagation Loss: $-\frac{1}{m} \sum_{k=1}^m \log(D(z))$
 - 18: **end for**
 - 19: **(4) Attach Classifier to Feature Extractor and Retrain the Network for Classification**
 - 20: **for** supervised training epochs **do**
 - 21: Sample data points with corresponding labels $\{(x^{(i)}, y^{(i)})\}_{i=1}^m \sim D$.
 - 22: Update the Classifier Model using *Stochastic Gradient Descent* over its parameters
 - 23: Backpropagation Loss: $\mathcal{L}_C(y, f(x)) = \|f(x) - y\|^2$
 - 24: **end for**
-

Discriminator Network maximises the deviation between the feature space distribution (from faulty network) and the prior target distribution (from fault-less network). The goal of the Feature Extractor is to optimise the weights by minimising the maximum deviation computed by the Discriminator between the feature space and target prior distribution. This game theoretic solution is scalable to large Neural Networks and incorporates fault tolerance as a constraint during training.

Solution of the Min-Max Game. To solve the min-max optimisation, in each epoch of training, the Feature Extractor and Discriminator are alternatively trained to select the best strategy against the other player (Algorithm 1). First, the Discriminator is trained tries to maximise the distinction between the data points sampled from feature space distribution (attributed as “fake”) and sampled from chose prior distribution (attributed as “real”) (inner maximisation in Equation 14). The Feature Extractor on the other hand is trained to minimise the

maximum deviation given in Equation 13. Both the networks are jointly trained to attain the equilibrium (saddle) point using Stochastic Gradient Descent algorithm.

$$\begin{aligned}
\min_{q_\phi} \max_D V(q_\phi, D) &= \mathbb{E}_{x \sim p_z} [\log(D(x))] + \mathbb{E}_{x \sim q_\phi(z|x)} [1 - \log(D(G(x)))] \\
&= \int_x p_z(x) \log(D(x)) dx + p_{q_\phi}(x) \log(1 - D(x)) dx
\end{aligned} \tag{16}$$

On differentiating the function inside the integral and equating to zero, the following optimal value of the Discriminator is obtained given that the Feature Extractor ($q_\phi(\cdot)$) is fixed,

$$D_G^*(x) = \frac{p(z)}{p(z) + q_\phi(z|x)} \tag{17}$$

As an extension of Theorem 1 in [15], it can be shown that the min-max game has a global optimum for $p(z) = q_\phi(z|x)$, i.e., the prior distribution is same as the feature space distribution. Since, the global optima is achieved at $p(z) = q_\phi(z|x)$, at the point of global optimum, the Discriminator $D^*(x) = \frac{1}{2}$, i.e., the model cannot distinguish (random guess) between the data points sampled from the true prior distribution and feature space distribution. The proof of convergence to equilibrium of min-max game of Algorithm 1 follows directly from Proposition 2 in [15].

VI. EVALUATION

This section shows the evaluation of Deep Neural Networks trained using the proposed training algorithm for different classification tasks. The algorithm is implemented in Pytorch¹ deep learning framework. The evaluation is performed on Google Colaboratory GPU servers with a Tesla K80 having 2496 CUDA cores and 12GB memory.

Previous fault tolerant algorithms have explored simple Neural Network architectures to solve trivial learning problems like XOR computation or binary/ternary classification [29]. Unfortunately, these algorithms are not scalable for the current state of the art Deep Neural Networks with millions of parameters. Further, the classification tasks considered for evaluation are not sufficient for complex object recognition tasks required for current applications. However, in this work, a scalable and efficient training approach has been proposed for fault tolerant Neural Networks for state of the art image recognition tasks.

A. Datasets

The algorithm is evaluated on two major benchmarking datasets: FashionMNIST[43] and CIFAR10[18].

FashionMNIST. The dataset is similar to the MNIST dataset and consists of a training set of 60,000 examples and a test set of 10,000 examples. Each data sample is a 28×28 grayscale image associated with a label from 10 classes.

CIFAR10. It is a major image classification dataset for evaluating image recognition algorithms. The dataset contains 60,000 images, with 50,000 images for training and 10,000 images for testing where each data point is 32×32 coloured image. The images are clustered into 10 classes representing different objects.

B. Architectures

For experiments, four architectures with different network depth (number of layers) and complexity are considered to evaluate the scalability of the algorithm. For each dataset, two different architectures are chosen and trained till saturation.

For FashionMNIST dataset, a Fully Connected Deep Neural Network (*Architecture 1*) and a Convolutional Neural Network Architecture *Architecture 2* is chosen for evaluation. The first architecture comprises of three hidden layers of sizes [512, 1024, 512] with ReLU activation while the second architecture includes two convolutional layers of 20 and 50 filters with

5×5 filters and two maxpool layers of kernel size 2×2 and stride of 2. The Classifier Network comprises of a hidden layer with 512 nodes followed by the output layer of 10 nodes corresponding to the number of classes in the dataset. The Generative Network for both the architectures is a Fully Connected network of size [512, 512, 784].

For CIFAR10 dataset, *Architecture 1* includes seven convolutional layers with the number of filters as [64, 128, 256, 512, 256, 128, 128] with LeakyReLU activation with scaling factor of 0.1. On the other hand, *Architecture 2* is a smaller network with three convolutional layers with number of filters as [64, 128, 128], kernel size of 3×3 and stride of 2. Both the networks use a Classifier Network with 1024 nodes and output layer of 10 nodes corresponding to the classes. The Generative Network for both the architectures has four convolutional layers, each followed by an upsampling layer with a bilinear scaling factor of 2.

Discriminator Network. The Discriminator architecture is common to both the datasets and includes a simple multilayer perceptron with two hidden layers of 512 nodes each followed by the single output neurone for binary classification of “Fake” vs “Real” data distributions. The Discriminator architecture is trained using a learning rate of $5e - 5$ along with Dropout regularisation. CIFAR10 is adversarially trained for 10k iterations while MNIST was trained for 2k epochs adversarially in the presence of the discriminator network.

C. Adversarial Framework

The effectiveness of the proposed training to improve the generalisation of Neural Network classifiers is evaluated and addressed in this section. The results for the experiments to compare the generalisation performance for the Neural Networks using the proposed approach with state of the art regularisers is shown in Table II. The extent of overfitting in the model is measured using the generalisation error which is the difference between the training accuracy and the testing accuracy.

For FashionMNIST dataset, unregularised *Architecture 1* has about 1.82x lower generalisation as compared to model trained using the proposed algorithm, while *Architecture 2* shows 1.9x higher generalisation. In case of CIFAR10 architectures, *Architecture 1* shows a higher generalisation of about 2.7x using the proposed approach while about 3x superior generalisation in *Architecture 2* compared to the corresponding unregularised model. Further, the results show comparison of model generalisation resulting from the proposed Fault Tolerant Training Algorithm with commonly used regularisation functions: Lasso and Tikhonov. The accuracies indicated in Table II are the best model performance obtained after tuning the hyperparameter values using Grid Search technique. In case of Tikhonov, the proposed approach has a lower generalisation error by 3.11% for FashionMNIST, while a 2.88% lower generalisation in case of CIFAR10 dataset. On the other hand, for Lasso, the Adversarial training results in 3.93% lower generalisation error in FashionMNIST. For *Architecture 1* of CIFAR10, the generalisation error of Lasso to be lower

¹<https://pytorch.org/>

	FashionMNIST			CIFAR10		
	Training Accuracy	Testing Accuracy	Generalization Error	Training Accuracy	Testing Accuracy	Generalization Error
No Regularization	99.51%	89.42%	10.09%	96.35%	86.11%	10.24%
Lasso	98.42%	88.96%	9.46%	79.95%	77.78%	2.17%
Tikhonov	97.16%	88.52%	8.64%	93.87%	87.21%	6.66%
Proposed Approach	95.78%	90.25%	5.53%	91.36%	87.58%	3.78%
No Regularization	99.60%	90.94%	8.66%	85.16%	81.71%	3.45%
Lasso	96.98%	91.19%	5.79%	83.90%	81.68%	2.22%
Tikhonov	95.74%	90.61%	5.13%	84.83%	81.71%	3.12%
Proposed Approach	96.00%	91.55%	4.55%	82.00%	80.87%	1.13%

TABLE II: Performance and Generalization of Fault Tolerant Adversarial Classifier. As compared to unregularised models and models trained using Tikhonov and Lasso functions, the proposed training approach results in lower generalisation error while resulting in higher test accuracy. Generalisation error is used as a measure of fault tolerance which indicates the degree of regularisation. Highlighted cells indicate Architecture 1 while the cells below correspond to Architecture 2.

than the Adversarial training but at the cost of $\sim 10\%$ drop in the classification accuracy.

To summarise, the proposed Adversarial Fault Tolerant Training approach clearly results in strongly generalised Neural Network classifier compared to Neural Networks trained using Lasso and Tikhonov regularisation. Further, the proposed training methodology results in best possible classification accuracy and generalisation error, observed as tradeoff in Tikhonov and Lasso regularisation.

D. Performance in the Presence of Faults

After training the model using the proposed approach, the performance of the model is measured to evaluate the resilience to faults after deployment. The metric used to determine the performance is the test accuracy on data samples that the Neural Network has not previously seen before. The evaluation considers weight faults for Deep Neural Networks with only Fully Connected layers, filter faults for Convolutional Neural Networks and node faults for the Fully Connected Layers. This work provides a practical approach to evaluate the performance of models in the presence of multiple concurrent faults instead of analysing the performance in the presence of single faults which is not applicable to large scale Neural Networks. Figure 7 depicts the performance degradation of the Neural Network accuracy on injecting parameter and node faults. Neural Networks trained using the proposed training

algorithm results in higher fault tolerance compared to models trained using Lasso and Tikhonov.

In case of node faults, for a 68% faults in nodes in the hidden layers, the model performance is 84.60% for proposed algorithm, compared to 78.51% (Tikhonov) and 51.09% (Lasso). Further, for the proposed approach, the performance drop is just 4.8% compared to 9.92% (Tikhonov) and 38.29% (Lasso). In case of weight faults, the accuracy is 84.36% for proposed approach compared to 81.66% (Tikhonov) and 61.42% (Lasso) for a 60% parameter faults. For filter faults, the Neural Networks can tolerate upto 40% filter faults with 12.58% accuracy drop (proposed) compared to 24.98% (Tikhonov) and 30.44% accuracy (Lasso).

This indicates the superior fault tolerance of the Neural Networks trained using the proposed approach. While the evaluation is done for sample Neural Networks, the performance degradation and corresponding tolerance to faults is specific to the model topology.

E. Training Complexity and Computation Overhead

Space Complexity of Fully Connected Layer. To understand the overall space complexity of the model, individual layers, i.e, convolutional and Fully Connected layer are first addressed. In case of Fully Connected Layer, each node in the l^{th} layer is connected to each node in the $(l + 1)^{th}$ layer.

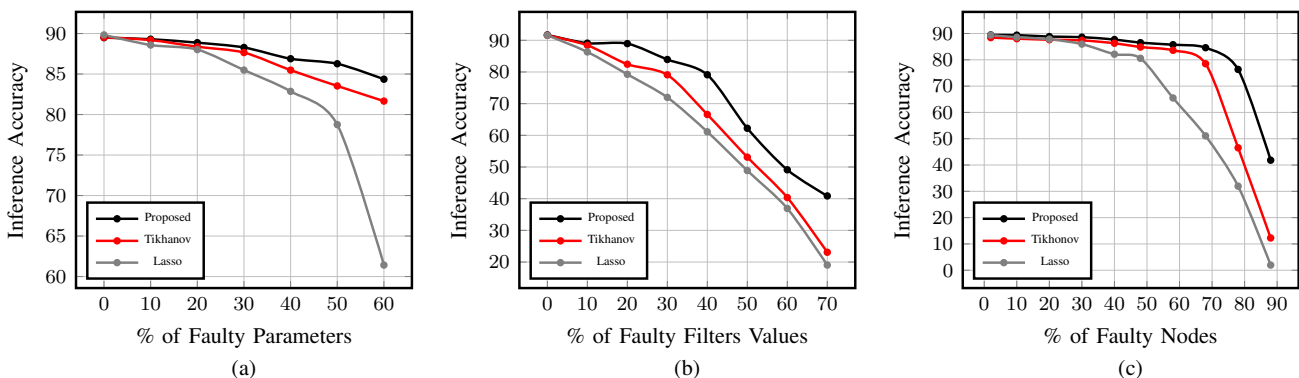


Fig. 7: Comparison of Fault Tolerance in Different Regularisation Functions. Performance degradation of the Neural Network models on injecting (a) Weight Faults (b) Filter Faults and (c) Node Faults, trained using proposed technique, Tikhonov and Lasso regularisation. The higher fault tolerance of the proposed approach compared to Tikhonov and Lasso is indicated by the gap in the curves.

Hence, the parameters stored in the form of matrix will have $|\mathbf{w}|$ elements given by,

$$|\mathbf{w}| = nk + k = (n + 1)k \quad (18)$$

where n is the number of input nodes and k is the number of output nodes. The additional k elements are to accommodate the bias values in the layer. Assuming each parameter is represented by p bits in the hardware, the total size of storing the parameters of Fully Connected layers is

$$|\mathbf{w}| = (n + 1)kp \text{ bytes} \quad (19)$$

During the operation, we have to consider an additional matrix to store the gradients computed during backpropagation. Here, the number of elements for $|\nabla w|$, is the same as the number of elements in $|\mathbf{w}|$. The total space during both forward and backward computation of an single Fully Connected layer is given by,

$$|\mathbf{w}| + |\nabla \mathbf{w}| + 2n + 2k \quad (20)$$

where the $2n$ and $2k$ are to store the intermediate computation of both outputs of the Neural Network layer and the activation non-linearity.

Computational Complexity of Fully Connected Layer.

The dotproduct between the weight matrix and the input activation vector requires n multiplications followed by (n) additions (including bias addition) *per output node* where n is the number input nodes. Hence, the total number of operations is given by $2nk$ where k is the number of output nodes. The number of floating point operations that a machine can perform in parallel in one second is used as a measure to monitor the computation performance.

Space Complexity of Convolution Layer. Given a kernel of width w , height h and c channels, the total number of elements in the matrix per kernel is given by,

$$|\mathbf{w}| = cwh \quad (21)$$

For k kernels, the total memory required to store the weights is,

$$|\mathbf{w}| = cwhk + k = (cwh + 1)k \quad (22)$$

which includes k bias connections and k total output channels for the forward propagation. Hence, for an input image of size $M \times N$, the total space complexity including the backward propagation is given by,

$$cMN + k(cwh + 1) + 2k(M - w + 1)(N - h + 1) \quad (23)$$

where $2k(M - w + 1)(N - h + 1)$ includes the intermediate output storage in both forward and backward propagation, assuming a stride of 1.

Computational Complexity for Convolutional Layer.

During convolution operation, the dotproduct between the kernel matrix with the image is computed. The total number of multiplications is given by $cwh(M - w + 1)(N - h + 1)$ and corresponding $cwh(M - w + 1)(N - h + 1)$ additions including the bias values, per kernel. Hence, the total number of operations for k kernels is given by, $2cwh(M - w + 1)(N - h + 1)k$.

Based on the total number of layers in the Neural Network, the overall computational and space complexity can be computed by adding the individual layer complexities discussed above. Unlike other training algorithms which train the Neural Network using a single training objective, the proposed Fault Tolerant training uses two training stages, an unsupervised training for Feature Extractor and supervised training of combined Neural Network. While the supervised training technique is same as the traditional approaches, the unsupervised training component incurs a significant computational overhead due to the game theoretic optimisation. As compared to traditional training algorithms which uses a single Neural Network for training, the proposed algorithm requires four networks in total, where three of them are trained simultaneously. This results in higher computation and space complexity during the training phase in addition to the complexity of single network training.

F. Drawbacks

Training the Feature Extractor network by solving the reconstruction game and the minimax game takes large number of iterations. Further, the minimax optimisation is highly sensitive to selection of hyperparameters and an incorrect selection of hyperparameters results in non-convergence of parameters which oscillate. In case the Discriminator network is able to perform very well in distinguishing the “fake” and “real” data samples, the Feature Extractor faces vanishing gradient problem due to which its parameters are not updated and the model does not learn. While solving the minimax problem as a game between two networks, the Feature Extractor could fail to generalise and starts to produce only finite number of features for different samples (mode collapse) [15]. These are commonly faced problems in adversarial training between two Neural Networks and stable training of both the networks is an active research problem [38].

VII. DISCUSSION

Robustness versus Fault Tolerance. Reliability or resilience addresses the performance of system in presence of faults, while robustness addresses the correct operation of the system despite noise or perturbation in the inputs. In Neural Networks, robustness has been studied as a security issue against perturbed inputs called adversarial examples which fool the model into misclassifying the input image [11]. However, *robustness against adversarial examples does not necessarily mean fault tolerance*. For instance, the current state of the art defence, Projected Gradient Descent (PGD), trains the model to minimise the maximise possible loss incurred from an adversarial example [23]. This ensures that the network performs well against the strongest adversary. However, there exists a tradeoff between generalisation error and robustness to adversarial examples [40] which degrades the overall fault tolerance of the Neural Networks. The empirical validation of the hypothesis is kept as future work.

Membership Privacy and Fault Tolerance. The goal of Membership Privacy in Machine Learning systems is to en-

sure indistinguishability between the output predictions of data instances belonging to the training dataset and non-training(evaluation) dataset [26]. Parameters of overfitting models are tuned to perform very well on the training data. This can be exploited to identify whether a given data instance is part of the training data of the model or not [31][30]. This is a serious privacy issue specially in the case where the training data includes sensitive personal information about users like medical and financial data. Notably, *overfitting is a common enemy for privacy of training data and fault tolerant Neural Networks*. Both membership privacy and fault tolerance have the same goals: improve generalisation of models by minimising overfitting. This can allow us to combine fault tolerance objective with indistinguishability for Membership Privacy. The analysis and joint optimisation for privacy and fault tolerance is kept for future research.

For deploying Neural Networks for practical applications, it is important to combine privacy, security (robustness) and fault tolerance within a single unified framework. This paper calls for further research in designing and evaluating such a unified framework for real world deployment of Neural Networks.

VIII. CONCLUSIONS

Incorporating fault tolerance through regularisation in Neural Networks using simple functions is insufficient due to the trade-off between the model classification accuracy and the extent of generalisation. In this work, a novel training technique combining both unsupervised and supervised learning is proposed, to improve the generalisation and attain superior fault tolerance. The proposed work identifies two different components of the Neural Networks, Feature Extractor and Classifier Network, based on the difference in functionality and learning objective. Instead of training the entire network using a single supervised learning objective, the Feature Extractor is trained using unsupervised learning paradigm which is modelled as two simultaneous games in the presence of adversary networks with conflicting objectives to the Extractor. This strategic training strongly regularises the Feature Extractor which is attached to the Classifier Network for supervised tasks. The resultant Neural Networks depict superior fault tolerance compared to widely used state of the art regularisers, Tikhonov and Lasso. The algorithm is evaluated extensively on two benchmarking datasets, FashionMNIST and CIFAR10, with different architectures of varying complexities. The Fault Tolerant Training Algorithm is scalable, efficient and extensible to practical applications requiring deployment of Deep Neural Networks.

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