SIGNAL PARAMETERS ESTIMATION USING TIME-FREQUENCY REPRESENTATION FOR LASER DOPPLER ANEMOMETRY

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ABSTRACT

This paper describes a processing method to estimate parameters of chirp signals for Laser Doppler Anemometry (LDA). The Doppler frequency as well as additional useful parameters are considered here. These parameters are the burst width and the frequency rate. Several estimators based on the spectrogram are proposed. Cramer-Rao bounds are given and performance of the estimators compared to the state of the art using Monte-Carlo simulations for synthesized LDA signals. The characteristics of these signals are provided by a flight test campaign. The proposed estimation procedure takes into account the requirements for a real-time application.

1. INTRODUCTION

Laser Doppler Anemometry is increasingly used in speed estimation systems. When crossing the laser beam, each particle, naturally present in the atmosphere, generates a burst signal which is a chirp with a Gaussian shape time-varying amplitude. The frequency varies with time and the central frequency corresponds to the Doppler frequency. It provides information on the particle speed. The burst width is the crossing time of the particle in the laser beam and the frequency rate represents the frequency speed of change.

The problem of parameter estimation of LDA signals has received a great deal of attention [1], [2], [3]. It has been shown that estimators of the Doppler frequency reach the Cramer-Rao Bounds (CRB) for a Signal to Noise Ratio (SNR) over 4 dB. Estimators of the burst width using a Kalman filter [1] or a wavelets transform [2] have been studied, but they do not reach the CRB. Estimators of the frequency rate using nonlinear least-squares (NLS) approaches have been proposed [4], [5]. It has been proven that they are close to the Cramer-Rao bounds for SNR above 5 dB. Nevertheless, these methods are time consuming and cannot be used in a real-time application. In [4], an additional method using the NLS approach with the high order ambiguity function (HAF) is proposed. It reduces the computational cost but it has lower performances.

The proposed approach consists in estimating all these parameters with one method, whose characteristics are accuracy and ease of on-line implementation. The spectrogram (square module of the Short-Time Fourier Transform) has these characteristics, due to the speed of the Fast Fourier Transform (FFT) and its robustness to noise for spectral line analysis. Moreover, it was successfully used in a previous flight test campaign for an LDA application [6].

This paper is organized as follows: in section 2, the signal model is presented. The time-frequency representation is presented in section 3. The proposed methods of estimation are described in section 4, the CRB are calculated in section 5 and the results of numerical simulations are presented in section 6 to illustrate the performance of our estimators compared to those proposed in [2] and [4].

2. SIGNAL MODEL

The backscattered signal is a linear chirp, whose expression is: $s(t) = x(t) + w(t), 0 \le t \le T$

$$x(t) = A_0 \exp\left(-\frac{8(t-t_0)^2}{D^2}\right)\cos\left(2\pi\left(f_D t + \frac{\beta}{2}(t-t_0)^2\right)\right) \quad (1)$$

A_0 is the signal intensity and t_0 is the time instant when the particle crosses the laser beam axis. The Doppler frequency f_D carries the speed information. The burst width D corresponds to the crossing time of the particle in the laser beam. β is the frequency rate.

w is a Gaussian white noise, its power spectral density is σ_w^2.

3. TIME-FREQUENCY REPRESENTATION

The time-frequency representations are commonly used for non-stationary signals analysis in real-time applications. The spectrogram is computationally efficient and robust to noise for spectral line analysis. Its main drawback, for the present problem, is a poor time-frequency concentration which leads to a bad localization of chirps. The proposed estimators are designed to compensate for this, by using center of mass computations and least squares approaches.

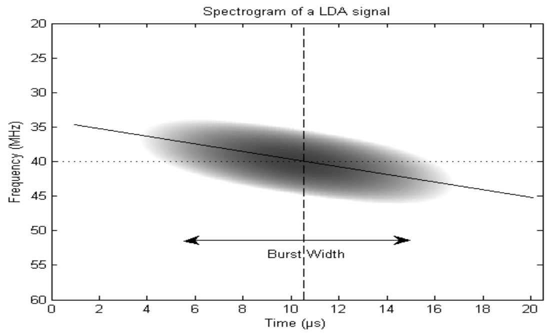


Figure 1: Spectrogram of an LDA signal: f_D is the dotted line, t_0 the dashed line and β the slope of the solid line.

The spectrogram of the analytic signal associated to x (Eq. 1) with the window $h(t) = \frac{1}{\sqrt{2\pi}\lambda}\exp\left(-\frac{t^2}{2\lambda^2}\right)$ is [7]:

$$S(t,f) = S_0 \exp\left(-\frac{A(t-t_0)^2 + B(f-f_0)^2 - C(t-t_0)(f-f_0)}{Y}\right) \quad (2)$$

with:

- $A = \frac{1}{\lambda^2}\left(\frac{256}{D^4} + \frac{16}{D^2\lambda^2} + 4\pi^2\beta^2\right)$
- $B = 4\pi^2\left(\frac{16}{D^2} + \frac{1}{\lambda^2}\right)$
- $C = \frac{8\pi^2\beta}{\lambda^2}$
- $Y = \left(\frac{16}{D^2} + \frac{1}{\lambda^2}\right)^2 + 4\pi^2\beta^2$

Figure 1 illustrates the spectrogram of a LDA signal and the parameters of interest.

4. ESTIMATION PROCEDURE

The proposed estimators are based on the spectrogram. The first step consists in detecting the signal and grouping points representing it in the spectrogram. Then, the points are used to estimate the three parameters of interest. The Doppler frequency is the frequency center of mass. The burst width is proportional to the standard deviation in time of the spectrogram amplitude, which is a Gaussian process. The burst width estimator is biased and a method is proposed to compensate for it. The frequency rate is estimated using a weighted least squares approach, assuming that it does not vary with time.

4.1. Detection

In the spectrogram, a signal is composed of all connected points whose amplitude is greater than a given threshold. This threshold has been chosen to allow a low false alarm rate and a high probability of detecting a signal. A false alarm occurs when a point due to noise is greater than the threshold. It has been determined experimentally that 8 dB over the noise power spectral density is the optimal value for the threshold.

4.2. Estimation of the Doppler frequency

The Doppler frequency is the barycenter of all the points representing the signal in the spectrogram.

$$f_D = \frac{\int\int f S(t,f)dtdf}{\int\int S(t,f)dtdf}$$

4.3. Estimation of the burst width

The duration D corresponds to the crossing time of the particle in the laser beam. At the extremities of the laser beam, the energy density is e^{-2} times lower than on the axis. Therefore, the particle goes out of the laser beam when the amplitude is $A(t) = A_0\exp(-2)$.

The spectrogram of the signal has a Gaussian shape time-varying amplitude (Eq .2), and its variance is given by:

$$\sigma^2 = \frac{\iint (t-t_0)^2 S(t,f)dtdf}{\iint S(t,f)dtdf}$$

Let us introduce from the spectrogram (Eq. 2) $p(t)$ which can be seen, for short windows, as an approximation of the instantaneous power multiplied by the energy of the window: $p(t) = \int S(t,f)df = p_0\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right)$, with $\sigma^2 = D^2/32 + \lambda^2/2$

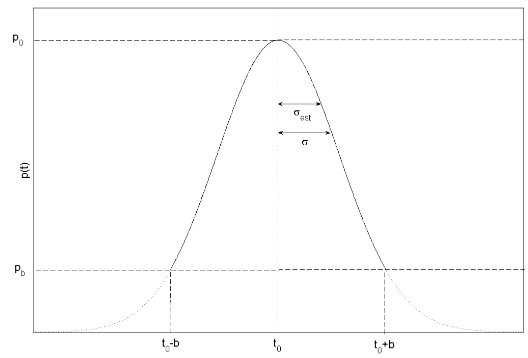


Figure 2 : Instantaneous power. It can be approximated for $t \in [t_0 - b\ t_0 + b]$ (solid line).

σ is estimated using the points of the time frequency representation whose amplitude is higher than the threshold. These points are between $t_0 - b$ and $t_0 + b$ (Fig. 2). The estimation of σ, $\hat{\sigma}$, does not take into account all the points representing the signal, and is lower than σ. Using the spectrogram, the estimated variance is:

$$\hat{\sigma}^2 = \frac{\int_{t_0-b}^{t_0+b}(t-t_0)^2 p(t)dt}{\int_{t_0-b}^{t_0+b} p(t)dt} = \sigma^2 - \sqrt{\frac{2}{\pi}}\frac{b\sigma\exp\left(-\frac{b^2}{2\sigma^2}\right)}{\mathrm{erf}\left(\frac{b}{\sqrt{2}\sigma}\right)}$$

with $\frac{b}{\sigma} = \sqrt{2\ln\left(\frac{p_0}{p_b}\right)}$ and $p_b = \frac{p(t_0-b)+p(t_0+b)}{2}$

A correction factor is proposed to compensate for the bias:

$$C_f = \frac{\sigma^2}{\hat{\sigma}^2} = \left(1 - \sqrt{\frac{2}{\pi}}\frac{b}{\sigma}\frac{\exp\left(-\frac{b^2}{2\sigma^2}\right)}{\mathrm{erf}\left(\frac{b}{\sqrt{2}\sigma}\right)}\right)^{-1} = f\left(\frac{b}{\sigma}\right) \quad (3)$$

The estimation of D is:

$$\widehat{D} = 4\sqrt{2\hat{\sigma}^2 C_f - \lambda^2}$$

4.4. Estimation of the frequency rate

For LDA signals, the instantaneous frequency is $f_{inst}(t) = f_D + \beta(t - t_0)$. It can be approximated by $\frac{\int f S(t,f) df}{\int S(t,f) df}$ for short windows ($\lambda \ll D/4$). Simulations show that estimating the instantaneous frequency as a center of mass instead of the spectrogram maxima reduces the effect of the discretization and is more robust to the noise for short windows. The frequency rate is estimated using a weighted least squares approach according to:

$$\hat{\beta} = \frac{\int_{t_0-b}^{t_0+b} (f_{inst}(t) - f_D)(t - t_0) p(t) dt}{\int_{t_0-b}^{t_0+b} (t - t_0)^2 p(t) dt}$$

5. CRAMER-RAO BOUNDS

The vector of unknown parameters is therefore $\theta = [f_D \quad D \quad \beta]^T$. It is estimated from the noisy LDA signal $s(t) = x(t) + w(t), 0 \leq t \leq T$. x is a chirp signal (Eq. 1) and w is supposed to be a Gaussian white noise with a power spectral density σ_w^2. The joint probability of having s for a given θ is:

$$p(s|\theta) = \frac{\exp\left(-\frac{1}{2\sigma_w^2} \int_0^T \left(s(t) - x(t)\right)^2 dt\right)}{\sqrt{2\pi}\sigma_w}$$

The Fisher information matrix F is composed of elements:

$$F_{j,k} = \frac{1}{\sigma_w^2} \int_0^T \frac{\partial x(t)}{\partial \theta_j} \frac{\partial x(t)}{\partial \theta_k} dt$$

The CRB are the diagonal elements of the inverse of the Fisher information matrix. The expressions of the CRB for the parameters of interest are:

$$\mathrm{CRB}(D) = \frac{16 D \sigma_w^2}{\sqrt{\pi} A_0^2}$$

$$\mathrm{CRB}(\beta) = \frac{8\sigma_w^2}{3\pi^{\frac{5}{2}} A_0^2 (D/4)^5}$$

$$\mathrm{CRB}(f_D) = \frac{\left(\frac{1}{\pi^2 (D/4)^3} + \beta^2 D\right) \sigma_w^2}{\sqrt{\pi} A_0^2}$$

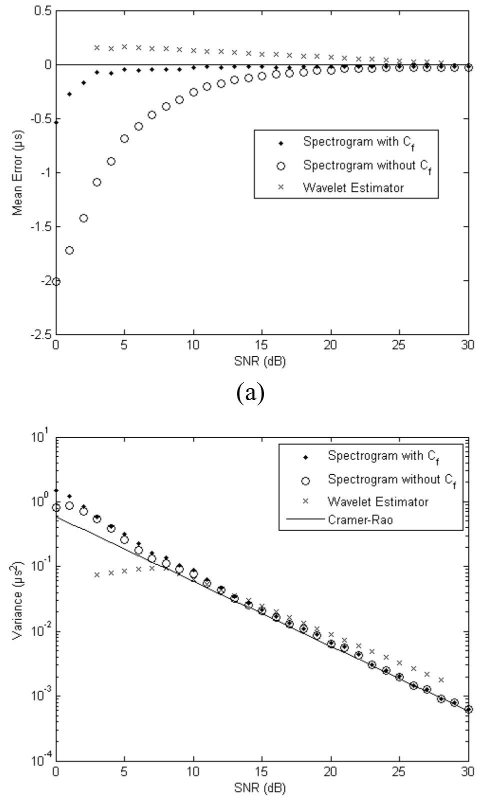
6. PERFORMANCES

Monte-Carlo simulations of an LDA signal are carried out and the performances of the proposed burst width and Doppler frequency estimators are compared with the wavelet estimator [2]. The proposed frequency rate performance estimator is compared with the NLS estimators [4]. Then, estimations of the three parameters for typical LDA signals are computed and the proposed spectrogram estimators are compared with the CRB. The problem of estimating the three parameters with the same method was never considered, to the best of the authors' knowledge.

As in [6], the spectrogram used in the proposed estimators has been computed with N-points ($N = 512$) windowed FFT (with $h(t)$). The overlap of the windows is 96.88 % instead of 75 % in [6]. The parameters estimation requires $\mathcal{O}\left(N_s N \log_2(N)\right)$ operations where N_s is the number of spectrum containing the signal ($N_s \ll N$) for spectrogram computation and $\mathcal{O}\left(N_p\right)$ operations where N_P is the number of points of the signal on the spectrogram ($N_p \approx N$).

6.1. Burst width and Doppler frequency estimation

The wavelet estimator is evaluated on an LDA signal with a Gaussian shape time varying amplitude and a constant Doppler frequency. In [2], the wavelet estimator reaches the CRB for the Doppler frequency estimation for an SNR higher than 4 dB. For the burst width, the estimator is biased. In the present method, the burst width and the Doppler frequency estimations are computed with the same signals as those described in [2] for the LDA case. These parameters are f_D = 0.986 MHz and $D/4$ = 2.6 µs. The estimators' performances are compared in figures 3 and 4.

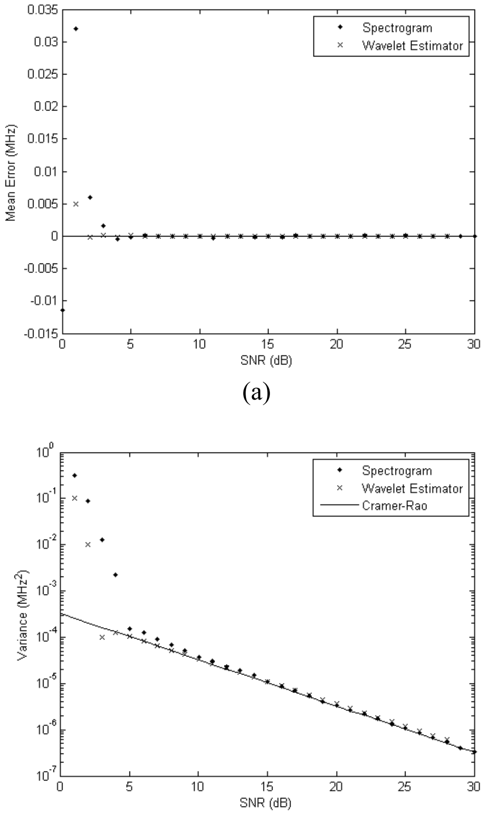


(a) (b)

Figure 3 : Mean error (a) and variance (b) of the burst width using the spectrogram and the wavelet estimators

Figure 3 shows the burst width estimation performance. The proposed method seems to outperform the wavelet estimator. It reduces the bias as well as the variance. The proposed correction factor (Eq. 3) decreases the bias, especially for low SNR. The proposed estimator with the correction factor is bias free for an SNR over 7 dB. Moreover, its variance is close to the CRB for an SNR higher than 11 dB.

The results of the Doppler frequency estimation (Fig. 4) show that our estimator is bias free for an SNR over 5 dB. The variance is close to the CRB for an SNR over 9 dB.



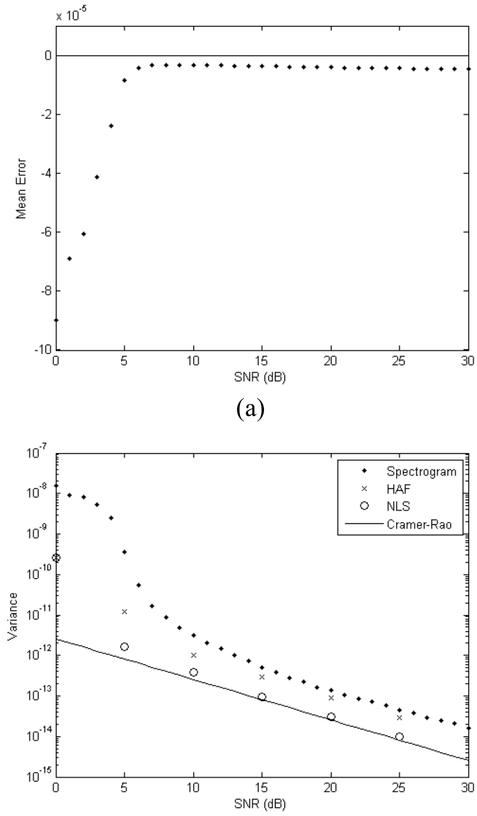
(a)

(b)

Figure 4 : Mean error (a) and variance (b) of the Doppler frequency using the spectrogram and the wavelet estimators

6.2. Estimation of the frequency rate

In [4], two frequency rate estimators are proposed for chirp signals with time-varying amplitude. The NLS estimator relies on nonlinear least-squares estimation of the chirp parameters and, according to the author; it is not a viable method for real-time applications. In order to reduce the computational cost of this estimator, the author describes a second method in the same article, which combines the high order ambiguity function (HAF) and the NLS approach. Performances of these estimators are presented in [4] with the following parameters: $\beta = 3.10^{-4}$ and $f_D = 0.18$. The proposed estimators are computed with the same signals. The results are presented Fig. 5 and the NLS estimator shows the best performance (it reaches the CRB for an SNR greater than 5 dB). The HAF estimator has lower performance (its variance is about 3.6 times higher than the CRB) but its cost is lower. The proposed estimator has an error lower than 1 % for an SNR over 7 dB. The variance is five times greater than the CRB for an SNR over 13 dB. It has a lower accuracy and computational cost than the NLS estimator. The performance and the cost of the proposed estimator are close to the HAF estimator. Moreover, the performance of the spectrogram estimator depends on the frequency rate value. For LDA signals, the variance of our estimator is three times higher than the CRB (Fig. 6c) and, in this case, it outperforms the HAF estimator.



(a)

(b)

Figure 5 : Mean error (a) and variance (b) of the frequency rate using the spectrogram and NLS estimators

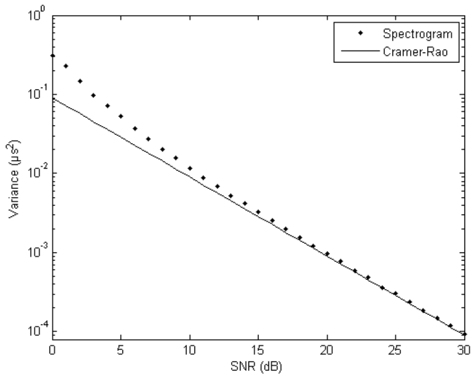
6.3. Estimation of typical LDA signal's parameters

The proposed estimators are evaluated with synthetized LDA signals using typical parameters encountered during the flight test campaign made by Thales [6]. The parameters are f_D = 40 MHz, D = 2 µs and β = 0.5 MHz/µs. The sampling frequency is 200 MHz and the SNR is defined as in [2], $\text{SNR} = A_0^2/\sigma_w^2$. The performances of spectrogram estimators are compared to the CRB.

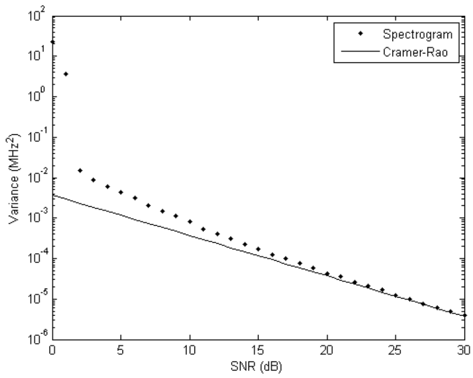
The burst width estimator is not biased for an SNR over 7 dB and the variance (Fig. 6a) is close to the CRB for an SNR over 11 dB.

The Doppler frequency estimator has no bias for an SNR over 2 dB. Its variance reach the CRB over 11 dB (Fig. 6b).

The frequency rate estimator is bias free for an SNR over 7 dB. For an SNR greater than 12 dB, the variance slightly decreases, from 3 to 1.5 times the CRB (Fig. 6c).



(a)



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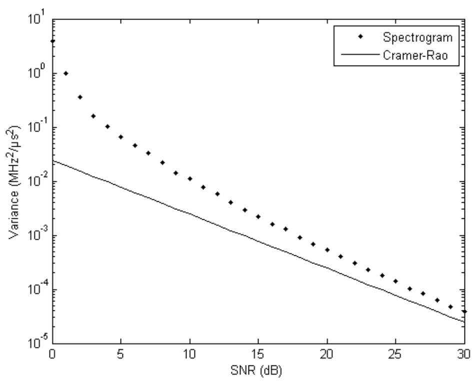

(c)

Figure 6 : Variance of the estimation of the burst width (a), the Doppler frequency (b) and the frequency rate (c)

The burst width and the Doppler frequency estimators have similar performance for signals described in [2] and for typical LDA signals. The performance of the frequency rate estimator is improved for typical LDA signals compared with those used in [4]: the estimator has no bias and the variance is closer to the CRB.

7. CONCLUSION

In this paper, the problem of estimating the parameters of LDA signals is addressed. Estimators based on a time-frequency representation are proposed and Cramer-Rao bounds are given. It is proven that they nearly reach the CRB for the Doppler frequency and the burst width. The frequency rate is also estimated without significant bias (the variance is similar to the one obtained in [4]). Moreover, the proposed estimators can be used in a real-time process. Spectrogram refinement methods, like spectrogram reassignment [8], will be studied in future works, mainly to improve the frequency rate estimation.

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