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Research Article

Robust Power Allocation for Cooperative Localization in Jammed Wireless Sensor Networks

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In this paper, we propose robust power allocation strategies to improve the localization performance in cooperative wireless sensor localization systems when suffering interference of jammer nodes. In wireless sensor localization systems, transmitting power strategies will affect the localization accuracy and determine the lifetime of wireless sensor networks. At the same time, the power allocation problem will be evolution to a new challenge when there are jammed nodes. So in this paper, we first present the optimization framework in jammed cooperative localization systems. Moreover, the imperfect parameter estimations of agent and jammer nodes are considered to develop robust power allocation strategies. In particular, this problem can be transformed into second-order cone programs (SOCPs) to obtain the end solution. Numerical results show the proposed power allocation strategies can achieve better performance than uniform power allocation and the robust schemes can ensure lower localization error than nonrobust power control when systems are subject to uncertainty.

1. Introduction

HIGH precious localization information is essential in many location-based applications and services, such as intelligent robot, logistics tracking, equipment management, and so on [1]. Traditional localization techniques, e.g., the global positioning system (GPS), may not provide satisficed localization accuracy in some harsh environments [2]. So the wireless sensor localization systems are motivated to provide necessary supplements.

In a wireless sensor localization system, there are always three types of nodes, i.e., agent nodes which are devices with unknown positions, anchor nodes which are infrastructures with known positions, and jammer nodes which are designedly or unintentional distributed in some places. Conventionally, the agent nodes can infer their positions by range measurements from anchor to agent nodes. Besides, the cooperation between agent nodes can improve localization accuracy through information sharing and additional measurements between agent nodes [3]. However, the jammer nodes will bring interference to degrade the localization performance of agent nodes. In other words, the localization accuracy of agent nodes is depended on the network topology and the measurement errors [4]. The measurement errors are related to the transmit power, signal waveform, channel condition, and interference condition. Consequently, power allocation strategies are critical to reduce the localization error and improve the lifetime of wireless sensor networks.

Existing studies have been worked on power allocation problems. In study [5], the author established an optimization framework to allocate robust power for anchor nodes and designed a distributed power allocation algorithm via conic programming. In [6], the power allocation strategies in both active and passive localization networks were researched. For network navigation, literature [7] developed efficient navigation algorithms to obtain optimized energy allocation strategies. Then for the cooperative localization, literature [4] built a general framework for wide-band cooperative localization networks and established the fundamental limits. In [7], the author proposed a distributed robust power allocation algorithm by infrastructure and cooperation phases. In [8], the power management problem was solved by game approach under the knowledge of local and global information. In [9], a hierarchical game was developed to obtain optimal power allocation strategies for different kinds of nodes simultaneously. What is more, when the jammer nodes are

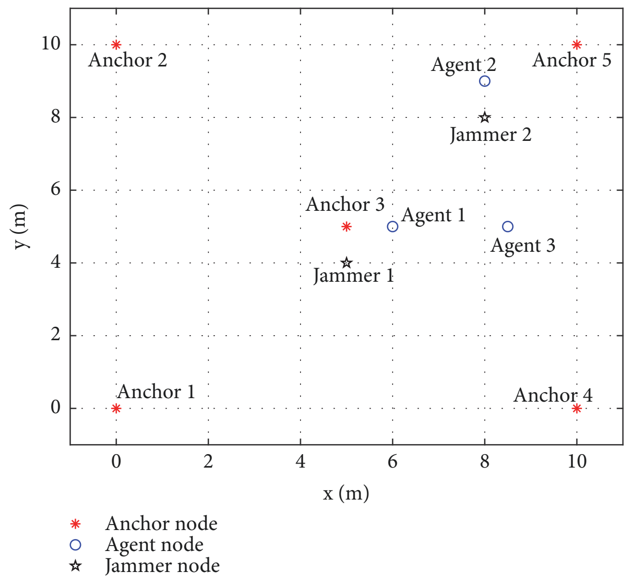

FIGURE 1: Illustration of a jammed wireless sensor localization system.

considered, two schemes were proposed to optimize power management for jammer nodes in [10, 11]. However, existing works in [10, 11] focus on jamming techniques to degrade the localization performance. So the author in [12] proposed an optimal power allocation approach based on semidefinite programs (SDP) to minimize the maximum Cramer-Rao lower bound (CRLB) or average CRLB in jammed wireless sensor localization systems.

While for above researches, there are some new challenges to be considered. First, the authors did not consider the effect of jammer nodes in [5–9]. If there are jammer nodes in localization systems, their positions and transmit power will affect the power allocation strategies of different nodes. Second, in [10, 11], the authors have investigated the analogous power allocation problems, but they focused on jamming techniques. The antijamming techniques through optimizing the power allocation strategies of anchor and agent nodes are still challenging tasks. Moreover, thought the jammer nodes were introduced, the cooperative technique and the parameter uncertainty did not consider in study [12]. For cooperative localization, it will be more complicated due to additional measurements. At the same time, the parameter uncertainty of different nodes should be tackled to guarantee the localization requirement. So the main contributions of this paper can be concluded as follows:

(i) We propose optimal power allocation strategies for cooperation in jammed wireless sensor localization system, aiming to guarantee the localization requirement.

(ii) We develop a robust optimization method to combat the uncertainty parameters of agent nodes and jammer nodes.

(iii) We exploit that the problem can be transformed into second-order cone programs (SOCPs) due to the functional properties of squared position error bound (SPEB) when considering the cooperative agent nodes as pseudo anchor nodes.

The rest of this paper is organized as follows. In Section 2, the system model is described, and the problem to optimize is formulated. Section 3 studies the uncertainty model and robust formulation. The robust power allocation strategies are presented in Section 4. The simulation results are presented in Section 5. Conclusions are given in Section 6.

Notation. $\mathbf{1_n}$ denotes a column vector with all 1's. The operation $\otimes$ denotes the Kronecker product. $\mathbf{e_k}$ denotes a unit vector and the k-th element is 1 while the others are 0's. $\|\cdot\|$ represents the Euclidean norm of its argument.

2. System Models and Problem Formulation

2.1. System Model. For a two-dimensional jammed wireless sensor localization system, there are three types of nodes illustrated in Figure 1. This network includes N_a agent nodes, N_b anchor nodes, and N_j jammer nodes (the jammer nodes may designedly or unintentionally distribute in some interested areas and they will degrade the localization accuracy of agent nodes in this network. If the jammer nodes are unintentional introduced into localization systems, they may be caused by different equipment. Then if the jammer nodes are designed, they may be employed by enemies), denoted by $\mathcal{N}_a = \{1, 2, \ldots, N_a\}$, $\mathcal{N}_b = \{1, 2, \ldots, N_b\}$, and $\mathcal{N}_j = \{1, 2, \ldots, N_j\}$, respectively. The position of node k is denoted by $\mathbf{p}_k$ for $k \in \mathcal{N}_a \cup \mathcal{N}_b \cup \mathcal{N}_j$. For arbitrary nodes k and j, the distance and angel from k to j are denoted by φ_{kj} and d_{kj}, respectively. In addition, it is assumed that jammer nodes transmit zero-mean Gaussian noise [10].

In this paper, each agent node can cooperate with its neighbors to increase localization accuracy. So each agent node not only receives signals from anchor nodes but also from neighbor agent nodes. The connectivity sets can be denoted as $\mathscr{A}_k = \{\{j \in \mathscr{N}_a \cup \mathscr{N}_b\} \mid$ *node j is connect with agent k*$\}$ for $k \in \mathscr{N}_a$. Then the received waveform at agent node k from node j can be represented as [10]

$$r_{kj}(t) = \sum_{i=1}^{L_{kj}} \sqrt{x_j}\alpha_{kj}^i s\left(t - \tau_{kj}^i\right) + \sum_{\ell=1}^{N_j} \gamma_{k\ell}\sqrt{P_\ell^J}\nu_{k\ell}(t) + n_{kj}(t), \tag{1}$$

where $t \in [0, T_{obs}]$ and T_{obs} represents the observation interval, x_j denotes the transmit power of node j, $s(t)$ represents known transmit signal waveform with Fourier transform $S(f)$, α_{kj}^i and τ_{kj}^i denote the amplitude and time delay of the k-th path, and L_{kj} denotes the number of multipath between agent node i and agent node j. Here the influence of jammer nodes is considered as jamming noise with a transmit power P_ℓ^J, $P_\ell^J \nu_{k\ell}(t)$ is assumed as independent zero-mean white Gaussian random Process, and $\gamma_{k\ell}$ denotes the channel coefficient between agent node k and the ℓ-th jammer node. Moreover, the different noises are modeled as independent zero-mean white Gaussian processes with the spectral density level $N_0/2$ and that of $\nu_{k\ell}(t)$ are equal to one. Then the time delay τ_{kj}^i can be expressed by

$$\tau_{kj}^i = \frac{\left\|\mathbf{p}_j - \mathbf{p}_k\right\| + b_{kj}^i}{c}, \tag{2}$$

where $\mathbf{p}_j$ and $\mathbf{p}_k$ represent the different positions of nodes, c is the propagation speed of signal, and b_{kj}^i is a range bias for non-line-of-sight (NLOS), denoting those NLOS anchor nodes as set $\mathscr{A}_k^{NL}$. If the i-th path is line-of-sight (LOS), such that $b_{kj}^i = 0$, denote those nodes as set $\mathscr{A}_k^L$. Here we consider the cooperative agent node j as a pseudo anchor node when it provides localization information to agent node k.

2.2. Performance Metric. For agent node k, the unknown parameters can be introduced as

$$\theta_k \triangleq \left[\mathbf{p}_k^T \kappa_{k1}^T \cdots \kappa_{k|\mathscr{A}_k|}^T\right], \tag{3}$$

where $\mathscr{A}_i = \mathscr{A}^{NL} \cup \mathscr{A}^L$ and $|\mathscr{A}_i|$ is the number of elements in $\mathscr{A}_i$; the vector of multipath parameters κ can be associated with $r_{kj}(t)$ [4], expressed as

$$\kappa_{kj} = \begin{cases} \left[\alpha_{kj}^1\ \alpha_{kj}^2\ b_{kj}^2\ \cdots\ \alpha_{kj}^{L_{kj}}\ b_{kj}^{L_{kj}}\right]^T, & j \in \mathscr{A}^L, \\ \left[\alpha_{kj}^1\ b_{kj}^1\ \alpha_{kj}^2\ b_{kj}^2\ \cdots\ \alpha_{kj}^{L_{kj}}\ b_{kj}^{L_{kj}}\right]^T, & j \in \mathscr{A}^{NL}. \end{cases} \tag{4}$$

Then let $\widetilde{\theta}$ denote the estimation of unknown parameter vector θ; the mean squared error (MSE) matrix for $\widetilde{\theta}$ will satisfy the following inequality [4]:

$$\mathbb{E}\left\{\left(\widetilde{\theta} - \theta\right)\left(\widetilde{\theta} - \theta\right)^T\right\} \succeq \mathbf{J}_\theta^{-1}, \tag{5}$$

where $\mathbf{J}_\theta$ is the Fisher information matrix (FIM). Let $\widetilde{\mathbf{p}}_k$ be the unbiased position estimation of individual agent node i, then (4) implies that

$$\mathbb{E}\left\{\left(\widetilde{\mathbf{p}}_k - \mathbf{p}_k\right)\left(\widetilde{\mathbf{p}}_k - \mathbf{p}_k\right)^T\right\} \succeq \left[\mathbf{J}_\theta^{-1}\right]_{2\times 2,k}, \tag{6}$$

For cooperative localization in jammed wireless sensor systems, the mean squared error (MSE) of position estimation for agent k is satisfied the following inequality [4]:

$$E\left\{\left\|\widehat{\mathbf{p}}_k - \mathbf{p}_k\right\|^2\right\} \geq \operatorname{tr}\left\{\mathbf{J}_k^{-1}\left(\mathbf{p}_k\right)\right\}, \tag{7}$$

where $\mathbf{J}_k(\mathbf{p}_k)$ denotes the Fisher information matrix (FIM). $\operatorname{tr}[\cdot]$ denotes the trace operator and $\mathscr{P}(\mathbf{p}_k) = \operatorname{tr}\{\mathbf{J}_k^{-1}(\mathbf{p}_k)\}$ is the individual squared position error bound (SPEB) for agent node k, which provides a lower bounded for unbiased estimate.

In addition, the network EFIM $\mathbf{J}(\mathbf{p})$ is given in [7]

$$\mathbf{J}(\mathbf{p}) = \sum_{k\in\mathscr{N}_a} \sum_{j\in\mathscr{N}_a\cup\mathscr{N}_b\setminus\{k\}} x_j\xi_{kj}\mathrm{u}_{kj}\mathrm{u}_{kj}^T, \tag{8}$$

with

$$x_j\xi_{kj} = \frac{4\pi^2W^2}{c^2}\left(1 - \chi_{kj}\right) SINR_{kj}^1, \tag{9}$$

$$\mathrm{u}_{kj} = \begin{cases} \mathrm{e}_k \otimes \left[\cos\varphi_{kj}\ \ \sin\varphi_{kj}\right]^T & \text{if } j \in \mathscr{N}_b, \\ \left(\mathrm{e}_k - \mathrm{e}_j\right) \otimes \left[\cos\varphi_{kj}\ \ \sin\varphi_{kj}\right]^T & \text{if } j \in \mathscr{N}_a, \end{cases} \tag{10}$$

where x_j is the transmit power of node j and ξ_{kj} is called equivalent ranging coefficient (ERC), W denotes the effective bandwidth of node j, $\chi_{kj} \in [0,1]$ is the path-overlap coefficient characterizing the effect of multipath propagation for localization, and $\mathbf{e}_k$ and $\mathbf{e}_j$ are N_a-dimensional vectors [7]. Moreover, the $SINR_{kj}^l$ denotes the energy ratio between the l-th component and the noise (here we only consider the line-of-sight connection between nodes, so $l = 1$.), given by

$$SINR_{kj}^l = \frac{\left|\alpha_{kj}^l\right|^2 \int_{-\infty}^{+\infty} \left|S(f)\right|^2 df}{N_0/2 + \sum_{n=1}^{N_j} \left|\gamma_{kn}\right|^2 P_n^J}, \tag{11}$$

in which α_{kj}^l is the amplitude of the l-th path, $S(f)$ is the Fourier transform of transmit signal waveform $s(t)$, γ_{kn} is the channel coefficient between agent node k and jammer node n, and P_n^J is the transmit power of jammer node n. Without loss of generality, the ERC can be transformed as [10]

$$\xi_{kj} = \frac{\zeta_{kj}/d_{kj}^\rho}{\left(N_0/2 + \sum_{n=1}^{N_j} \zeta_{kn}\left(P_n^J/d_{kn}^\rho\right)\right)}, \tag{12}$$

where ζ_{kj} is a positive coefficient determined by the channel properties and effective bandwidth; ρ is the pathloss coefficient during transmission. Here we simply recognize the cooperative agent nodes as pseudo anchor nodes. So the

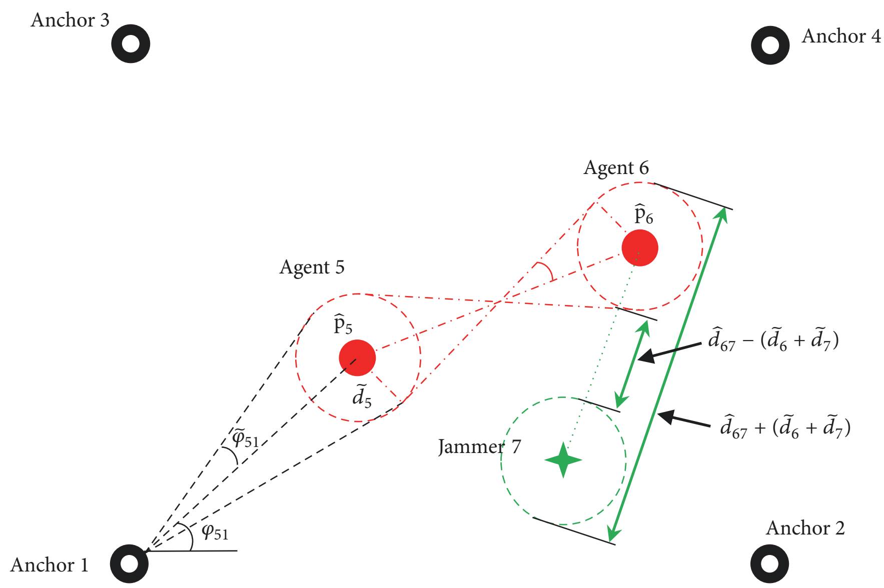

FIGURE 2: An example of uncertainty model for cooperation in a jammed wireless sensor localization system.

individual SPEB can be also obtained by (2). If the agent nodes k and j are not connected, the parameter can be set as $\xi_{kj} = \xi_{jk} = 0$.

In this paper, it is assumed that jammer nodes transmit zero-mean Gaussian noise. In practice, this assumption may be inappropriate for some situations. However, this assumption is used in our work for following purposes: First, to best of our knowledge, it is the first time to develop an antijamming approach through optimizing power allocation strategies for anchor and agent nodes. Those initial results can provide a fundamental feasibility for further studies on this problem. Second, the prior information of jammer nodes may not be reached for some situations, so we simplify the transmission of each jammer node as zero-mean Gaussian noise, which is commonly employed in [10, 11].

2.3. Power Allocation Formulation. The SPEB is adopted to be the performance matrix, so it is reasonable to minimize the total transmit power when each agent node requires the localization accuracy [6]. Thus, we can formulate the power allocation problem as

$$\begin{aligned} P_A : \min_{\{\mathbf{x}_a, \mathbf{x}_b\}} \quad & \mathbf{1}^T \mathbf{x}_a + \mathbf{1}^T \mathbf{x}_b \\ s.t. \quad & \mathscr{P}(p_k; \mathbf{x}_a, \mathbf{x}_b) \leq \tau_k, \quad \forall k \in \mathcal{N}_a \\ & c_q(\mathbf{x}_a, \mathbf{x}_b) \leq 0, \quad q = 1, 2, \ldots, Q \end{aligned} \tag{13}$$

where $\mathbf{x_a}$ and $\mathbf{x_b}$ are transmit power strategies of agent nodes and anchor nodes, τ_k is the localization accuracy requirement for agent node k, and $\{c_q(\cdot)\}$ represents the linear constraints of each power allocation strategy, such as the individual power constraints $0 \leq x_k \leq x_k^{\max}, \forall k \in \mathcal{N}_a \cup \mathcal{N}_b$. Note that the optimization objective and the optimization variables in our paper are different from the references [10, 11]. Moreover, it is impracticable to obtain the solution for the proposed problem through the approaches in [10, 11].

3. Uncertainty Model and Robust Formulation

Due to the imperfect estimates of network parameters, the robust formulation is necessary for the proposed power allocation problem. Figure 2 shows an example of uncertainty model. For any agent node k, its position can be defined in an area with center $\mathbf{p}_k$ and radius $\tilde{d}_k$. Let $\hat{\varphi}_{kj}$, $\hat{\xi}_{kj}$, and $\hat{d}_{kj}$ be the nominal value of angle φ_{kj}, channel coefficient ξ_{kj}, and distance d_{kj}. For any agent node k, the actual network parameters with anchor nodes j can be represented by

$$\varphi_{kj} \in \left[\hat{\varphi}_{kj} - \tilde{\varphi}_{kj}, \hat{\varphi}_{kj} + \tilde{\varphi}_{kj}\right] =: \mathscr{F}_{kj}, \tag{14}$$

$$d_{kj} \in \left[\hat{d}_{kj} - \tilde{d}_k, \hat{d}_{kj} + \tilde{d}_k\right] =: \mathscr{D}_{kj}, \tag{15}$$

Form Figure 2, we can find the angular uncertainty $\tilde{\varphi}_{kj}$ fits $\sin \tilde{\varphi}_{kj} = \tilde{d}_k / \hat{d}_{kj}$ (it is assumed that the radius is larger than the minimum distance between nodes.). Similarly, the actual network parameters for anchor nodes k with agent node i can be also represented by

$$\varphi_{ki} \in \left[\hat{\varphi}_{ki} - \tilde{\varphi}_{ki}, \hat{\varphi}_{ki} + \tilde{\varphi}_{ki}\right] =: \mathscr{F}_{ki}, \tag{16}$$

$$d_{ki} \in \left[\hat{d}_{ki} - \tilde{d}_k - \tilde{d}_i, \hat{d}_{ki} + \tilde{d}_k + \tilde{d}_i\right] =: \mathscr{D}_{ki}. \tag{17}$$

Here we can find from Figure 3 that the angular uncertainty $\tilde{\varphi}_{ki}$ fits $\sin \tilde{\varphi}_{ki} = (\tilde{d}_k + \tilde{d}_i)/\hat{d}_{ki}$. Then for the jammer node n, the angular uncertainty will not affect the result, so the network parameters are given by

$$d_{kn} \in \left[\hat{d}_{kn} - \tilde{d}_k - \tilde{d}_n, \hat{d}_{kn} + \tilde{d}_k + \tilde{d}_n\right] =: \mathscr{D}_{kn}. \tag{18}$$

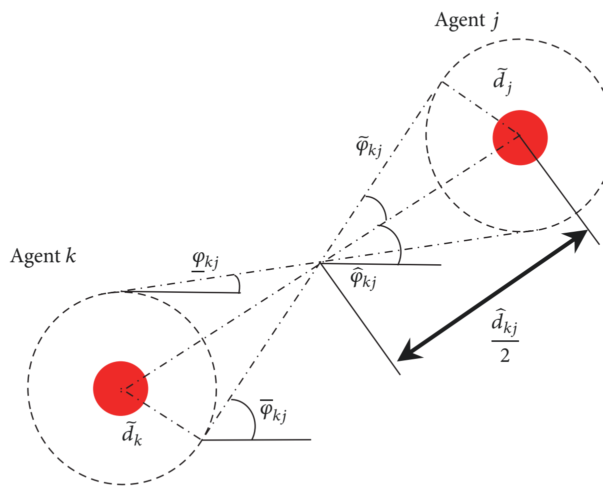

FIGURE 3: A detailed example of uncertainty model for two cooperative nodes.

From (6), we can find that the channel coefficient ξ_{kj} is a monotonically nonincreasing function of d_{kj}, $j \in \mathscr{A}_k$ and a monotonically increasing function of d_{kn}, $n \in \mathscr{N}_j$. Thus, the ERC ξ_{kj} is bounded by

$$\underline{\xi}_{kj} = \frac{\zeta_{kj}/\overline{d}_{kj}^{\rho}}{N_0/2 + \sum_{n=1}^{N_j} \zeta_{kn} \left(P_n^J/\underline{d}_{kn}^{\rho}\right)}, \tag{19}$$

$$\overline{\xi}_{kj} = \frac{\zeta_{kj}/\underline{d}_{kj}^{\rho}}{N_0/2 + \sum_{n=1}^{N_j} \zeta_{kn} \left(P_n^J/\overline{d}_{kn}^{\rho}\right)}, \tag{20}$$

$$\xi_{kj} \in \left[\underline{\xi}_{kj}, \overline{\xi}_{kj}\right] =: \mathscr{C}_{kj}. \tag{21}$$

In summary, we have the set of actual network parameters as

$$\left\{\varphi_{kj}, \xi_{kj}\right\}_{k\in\mathscr{N}_a, j\in\mathscr{A}_k} \in \prod_{k\in\mathscr{N}_a, j\in\mathscr{A}_k} \mathscr{F}_{kj} \times \mathscr{C}_{kj}. \tag{22}$$

To ensure the localization accuracy, the worst-case should be considered with parameter uncertainty. Then the worst-case SPBE is given by

$$\mathscr{P}_R\left(\mathbf{p}_k\right) = \max_{\{\{\varphi_{kj},\xi_{kj}\}\in\mathscr{F}_{kj}\times\mathscr{C}_{kj}\}} \operatorname{tr}\left\{\mathbf{J}_k^{-1}\left(\mathbf{p}_k\right)\right\}. \tag{23}$$

4. Robust Power Allocation Strategies

To solve the proposed problem, we introduce following proposition.

Proposition 1. *The SPEB of each agent node is convex about* $\mathbf{x} = [x_1, x_2, \ldots, x_{N_a+N_b}]$. *At the same time, it can be formulated as*

$$\mathscr{P}\left(\mathbf{p}_k; \mathbf{x}\right) = \frac{4 \cdot \mathbf{1}^T \mathbf{R}_k \mathbf{x}}{\mathbf{x}^T \mathbf{R}_k^T \mathbf{\Lambda}_k \mathbf{R}_k \mathbf{x}}, \tag{24}$$

where $\mathbf{R}_k = \operatorname{diag}\{\xi_{k1}, \xi_{k2}, \ldots, \xi_{k(N_a+N_b)}\}$ *and* $\mathbf{\Lambda}_k$ *is the symmetric matrix to reflect the topology, given by*

$$\mathbf{\Lambda}_k = \mathbf{1}\mathbf{1}^T - \mathbf{c}\left(2\varphi_k\right)\mathbf{c}\left(2\varphi_k\right) - \mathbf{s}\left(2\varphi_k\right)\mathbf{s}\left(2\varphi_k\right), \tag{25}$$

where

$$\varphi_k = \left[\varphi_{k1} \;\; \varphi_{k2} \;\; \cdots \;\; \varphi_{k(N_a+N_b)}\right]^T, \tag{26}$$

$$\mathbf{c}\left(2\varphi_k\right) = \left[\cos\left(2\varphi_{k1}\right) \;\; \cos\left(2\varphi_{k2}\right) \;\; \cdots \;\; \cos\left(2\varphi_{k(N_a+N_b)}\right)\right]^T \tag{27}$$

$$\mathbf{s}\left(2\varphi_k\right) = \left[\sin\left(2\varphi_{k1}\right) \;\; \sin\left(2\varphi_{k2}\right) \;\; \cdots \;\; \sin\left(2\varphi_{k(N_a+N_b)}\right)\right]^T. \tag{28}$$

Proposition 2. *According to the result in Proposition 1, the proposed power allocation problem can be transformed into the SOCP, given as*

$$\begin{aligned} P_A^{SOCP}: \quad & \mathbf{1}^T\mathbf{x}_a + \mathbf{1}^T\mathbf{x}_b \\ \text{s.t.} \quad & \left\|\mathbf{A}^T\mathbf{R}_k\mathbf{x} + \mathbf{b}_k\right\| \le \mathbf{1}^T\mathbf{R}_k\mathbf{x} - 2\cdot\tau_k^{-1}, \\ & \qquad \forall k \in \mathscr{N}_a \\ & c_l\left(\mathbf{x}_a, \mathbf{x}_b\right) \le 0, \quad l = 1, 2, \ldots, L \end{aligned} \tag{29}$$

where $\mathbf{A}_k = [\mathbf{c}(2\varphi_k) \;\; \mathbf{s}(2\varphi_k) \;\; \mathbf{0}]^T$ *and* $\mathbf{b}_k = [0 \;\; 0 \;\; 2\tau_k^{-1}]^T$. *Note that the proofs of Propositions 1 and 2 are similar to Appendix B and proposition 3 in [6], so here we omit the details for brevity. The next task is to address the uncertainty parameters about angel and ERC.*

Form Proposition 1, we can conclude that the SPEB is a monotonically nonincreasing function of ERC. So it will be the worst-case for SPEB when the ERC $\xi_{kj} = \underline{\xi}_{kj}$ for all $j \in \mathscr{A}_k$. In other words, when $\underline{\mathbf{R}}_k = \operatorname{diag}\{\underline{\xi}_{k1}, \underline{\xi}_{k2}, \ldots, \underline{\xi}_{k(N_a+N_b)}\}$, the maximization of SPEB over ERC ξ can be reached.

Moreover, because the SPEB over angel φ_{kj} is not an explicit expression, it will be more complicated to address the angular uncertainty.

From (18), we can find that only the denominator includes the angel φ_{kj}. Then the angular uncertainty problem can be transformed to find the lower bound of the denominator.

Proposition 3. *In the (18), the denominator can be expressed by*

$$\mathbf{x}^T\mathbf{R}_k^T\boldsymbol{\Lambda}_k\mathbf{R}_k\mathbf{x} = \left(\mathbf{1}^T\mathbf{R}_k\mathbf{x}\right)^2 - \left\|\left[\mathbf{c}_k \quad \mathbf{s}_k\right]^T\mathbf{R}_k\mathbf{x}\right\|^2. \tag{30}$$

Then let

$$\sin\breve{\varphi}_{kj} = \max_{|\varepsilon|\le\widetilde{\varphi}_{kj}}\left|2\sin\left(2\widehat{\varphi}_{kj}+\varepsilon\right)\sin\varepsilon\right|, \tag{31}$$

$$\cos\breve{\varphi}_{kj} = \max_{|\varepsilon|\le\widetilde{\varphi}_{kj}}\left|2\cos\left(2\widehat{\varphi}_{kj}+\varepsilon\right)\cos\varepsilon\right|, \tag{32}$$

and

$$\begin{aligned}&\breve{\mathbf{c}}\left(\varphi_k\right)\\&=\left[\cos\left(\breve{\varphi}_{k1}\right)\ \cos\left(\breve{\varphi}_{k2}\right)\ \cdots\ \cos\left(\breve{\varphi}_{k(N_a+N_b)}\right)\right]^T,\end{aligned} \tag{33}$$

$$\breve{\mathbf{s}}\left(\varphi_k\right) = \left[\sin\left(\breve{\varphi}_{k1}\right)\ \sin\left(\breve{\varphi}_{k2}\right)\ \cdots\ \sin\left(\breve{\varphi}_{k(N_a+N_b)}\right)\right]^T, \tag{34}$$

We can get

$$\mathscr{P}_R\left(\mathbf{p}_k,\mathbf{x}\right) \le \mathscr{P}_U\left(\mathbf{p}_k,\mathbf{x}\right), \tag{35}$$

$$\begin{aligned}&\mathscr{P}_U\left(\mathbf{p}_k,\mathbf{x}\right)\\&= \max_{e_1,e_2=\pm1}\frac{4\cdot 1^T\mathbf{R}_k\mathbf{x}}{\left(\mathbf{1}^T\mathbf{R}_k\mathbf{x}\right)^2-\left\|\left(\left[\widehat{\mathbf{c}}_k\ \ \widehat{\mathbf{s}}_k\ \ \mathbf{0}\right]^T+\left[e_1\breve{\mathbf{c}}_k\ \ e_2\breve{\mathbf{s}}_k\ \ \mathbf{0}\right]^T\right)\mathbf{R}_k\mathbf{x}\right\|^2}.\end{aligned} \tag{36}$$

Proof. One has

$$\mathbf{c}_k{}^T\mathbf{R}_k\mathbf{x} = \widehat{\mathbf{c}}_k^T\mathbf{R}_k\mathbf{x} + \left(\mathbf{c}_k-\widehat{\mathbf{c}}_k\right)^T\mathbf{R}_k\mathbf{x}, \tag{37}$$

and

$$\begin{aligned}&\max_{\varphi_{kj}\in\mathscr{F}_{kj}}\left|\left(\mathbf{c}_k-\widehat{\mathbf{c}}_k\right)^T\mathbf{R}_k\mathbf{x}\right|\\&=\max_{\max|\varepsilon|\le\widetilde{\varphi}_{kj}}\left|\sum\left\{\left\{\cos\left(2\widehat{\varphi}_{kj}\right)-\cos\left[2\left(\widehat{\varphi}_{kj}+\varepsilon\right)\right]\right\}\right.\right.\\&\left.\left.\cdot\,\xi_{kj}x_j\right\}\right|\\&\le\sum\left(\xi_{kj}x_j\right)\max_{\max|\varepsilon|\le\widetilde{\varphi}_{kj}}\left|\left\{\cos\left(2\widehat{\varphi}_{kj}\right)-\cos\left[2\left(\widehat{\varphi}_{kj}+\varepsilon\right)\right]\right\}\right|\\&=\sum\left(\xi_{kj}x_j\right)\sin\breve{\varphi}_{kj}=\breve{\mathbf{s}}\left(\varphi_k\right)^T\\&\cdot\,\mathbf{R}_k\mathbf{x}.\end{aligned} \tag{38}$$

So

$$\begin{aligned}\max_{\varphi_{kj}\in\mathscr{F}_{kj}}\left|\mathbf{c}_k{}^T\mathbf{R}_k\mathbf{x}\right| &\le \left|\widehat{\mathbf{c}}_k^T\mathbf{R}_k\mathbf{x}\right| + \max_{\varphi_{kj}\in\mathscr{F}_{kj}}\left|\left(\mathbf{c}_k-\widehat{\mathbf{c}}_k\right)^T\mathbf{R}_k\mathbf{x}\right|\\&\le\left|\widehat{\mathbf{c}}_k^T\mathbf{R}_k\mathbf{x}\right|+\breve{\mathbf{s}}\left(\varphi_k\right)^T\mathbf{R}_k\mathbf{x}\\&\le\max_{e_1=\pm1}\left|\left[\widehat{\mathbf{c}}_k+e_1\breve{\mathbf{s}}\left(\varphi_k\right)\right]^T\mathbf{R}_k\mathbf{x}\right|.\end{aligned} \tag{39}$$

Consequently, we have

$$\max_{\varphi_{kj}\in\mathscr{F}_{kj}}\left|\mathbf{s}_k{}^T\mathbf{R}_k\mathbf{x}\right| \le \max_{e_1=\pm1}\left|\left[\widehat{\mathbf{c}}_k+e_1\breve{\mathbf{c}}\left(\varphi_k\right)\right]^T\mathbf{R}_k\mathbf{x}\right|. \tag{40}$$

Thus,

$$\begin{aligned}&\max_{\varphi_{kj}\in\mathscr{F}_{kj}}\left\{\left(\mathbf{c}_k{}^T\mathbf{R}_k\mathbf{x}\right)^2+\left(\mathbf{s}_k{}^T\mathbf{R}_k\mathbf{x}\right)^2\right\}\\&\le\max_{e_1,e_2=\pm1}\left\{\left\{\left[\widehat{\mathbf{c}}_k+e_1\breve{\mathbf{c}}\left(\varphi_k\right)\right]^T\mathbf{R}_k\mathbf{x}\right\}^2\right.\\&\left.+\left\{\left[\widehat{\mathbf{s}}_k+e_2\breve{\mathbf{s}}\left(\varphi_k\right)\right]^T\mathbf{R}_k\mathbf{x}\right\}^2\right\}.\end{aligned} \tag{41}$$

Therefore, Proposition 3 is proved. □

In summary, combining the uncertainty parameters about angel and ERC, the upper bound for the worst-case SPEB can be expressed by

$$\begin{aligned}&\mathscr{P}'_U\left(\mathbf{p}_k,\mathbf{x}\right)\\&=\max_{e_1,e_2=\pm1}\frac{4\cdot 1^T{}_k\mathbf{x}}{\left(\mathbf{1}^T\underline{\mathbf{R}}_k\mathbf{x}\right)^2-\left\|\left(\left[\widehat{\mathbf{c}}_k\ \ \widehat{\mathbf{s}}_k\ \ \mathbf{0}\right]^T+\left[e_1\breve{\mathbf{c}}_k\ \ e_2\breve{\mathbf{s}}_k\ \ \mathbf{0}\right]^T\right)_k\mathbf{x}\right\|^2}.\end{aligned} \tag{42}$$

Then the constraint in (29) can be relaxed and the proposed problem becomes

$$\begin{aligned}P_{R-A}^{SOCP}:\ &\min_{\{x_a,x_b\}}\quad \mathbf{1}^T\mathbf{x}_a+\mathbf{1}^T\mathbf{x}_b\\&s.t.\quad \left\|\overline{\mathbf{A}}_k\underline{\mathbf{R}}_k\mathbf{x}+\mathbf{b}_k\right\|\le\mathbf{1}^T{}_k\mathbf{x}-2\cdot\tau_k^{-1},\\&\qquad\qquad\qquad\qquad\forall k\in\mathscr{N}_a\\&\qquad c_l\left(\mathbf{x}_a,\mathbf{x}_b\right)\le 0,\quad l=1,2,\ldots,L\end{aligned} \tag{43}$$

where

$$\begin{aligned}\overline{\mathbf{A}}_k&=\left[\left(\widehat{\mathbf{c}}\left(2\varphi_k\right)+e_1\breve{\mathbf{s}}\left(\varphi_k\right)\ \ \left(\widehat{\mathbf{s}}\left(2\varphi_k\right)+e_2\breve{\mathbf{c}}\left(\varphi_k\right)\right)\ \ \mathbf{0}\right]^T\\&\qquad\qquad e_1,e_2=\pm1\end{aligned} \tag{44}$$

and $\mathbf{b}_k = [0\ \ 0\ \ 2\tau_k^{-1}]^T$.

Successively, the power allocation strategies can be described in Algorithm 1.

5. Simulation Result

To evaluate the proposed robust power allocation method, the simulation scenario is illustrated in Figure 1. Here we compare the proposed algorithm with the uniform power management scheme and the nonrobust power allocation strategy. In this paper, the normalized power is considered as $x_k^{\max}=10, k\in\mathscr{N}_a\cup\mathscr{N}_b$ and $P_n^J=5,\forall n\cup\mathscr{N}_j$. The channel parameter is given as $\varsigma_{kj}=100N_0/2$ for different nodes and $N_0=2$. Moreover, the standard optimization solver CVX is used to address the proposed problem [13].

Figure 4 illustrates the average and worst SPEBs respect to different normalized total power. First, the cooperative

Input $\{\tilde{d}_k, x_k^{\max}\}, k \in \mathcal{N}_a \cup \mathcal{N}_b, \{\tilde{d}_n, P_n^J\}, n \in \mathcal{N}_j$.
Output $\{x_k\}, k \in \mathcal{N}_a \cup \mathcal{N}_b$.
(Step 1) Estimate the positions of each agent nodes with each anchor node power strategy $x_j = x_j^{\max}, j \in \mathcal{N}_b$.
(Step 2) For $k \in \mathcal{N}_a \cup \mathcal{N}_b$, solve the P_{R-A}^{SOCP} problem in (43).
(Step 3) Output $\{x_k\}, k \in \mathcal{N}_a \cup \mathcal{N}_b$.

Algorithm 1: Robust power allocation strategies via SOCP.

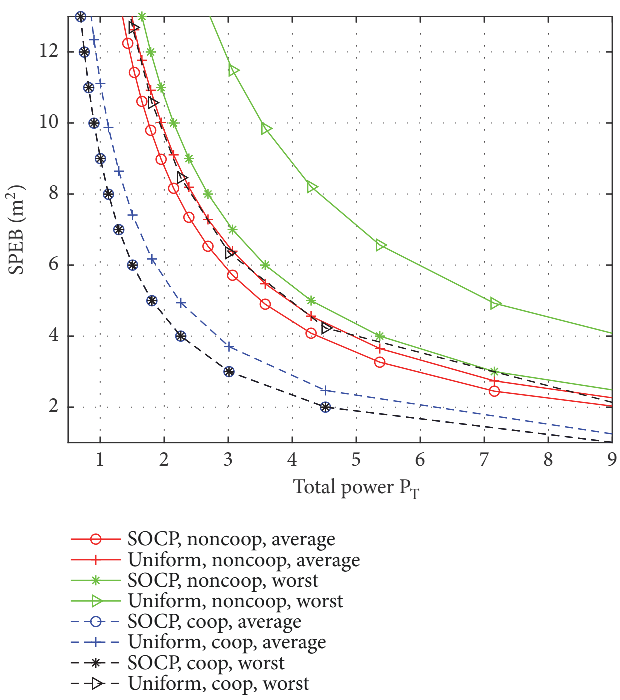

Figure 4: Different SPEBs with respect to total power consumption.

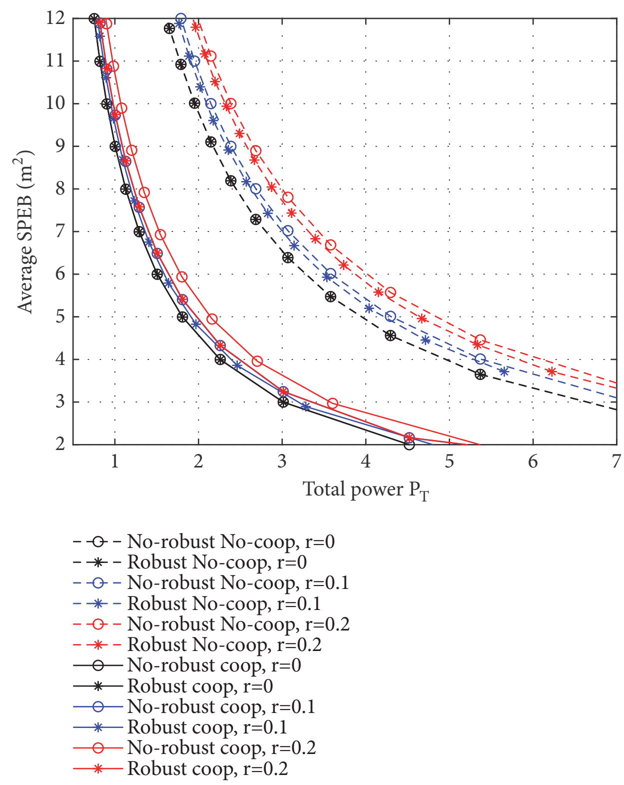

Figure 5: Different average SPEBs with respect to total power consumption.

localization can obtain lower SPEB than the noncooperative localization in all cases. Second, when consuming the same power, the proposed method via SOCP can reach a better performance than the uniform allocation strategy in both average SPEB and worst SPEB.

Figure 5 shows the average SPEB in different methods with respect to the total power consumption. In both noncooperative and cooperative localization systems, the robust power allocation strategies have better localization accuracy than the nonrobust approaches. When the uncertainty size $\tilde{d} = 0$, the problem P_{R-A}^{SOCP} will be equal to P_{A}^{SOCP} and they have the same performance for nonrobust and robust approaches.

For the same localization accuracy requirement $\tau_k = 4, \forall k \in \mathcal{N}_a$, the total power consumption and the worst-case SPEB with respect to the uncertainty size are demonstrated in Figures 6 and 7. It can be seen that the cooperative localization consumes less power to achieve the same localization of noncooperative localization. When the parameter uncertainty size increases, the power consumption will also increase to ensure the localization requirement in robust cases. At the same time, the worst-case SPEB will decrease due to the robust formulation. But in the nonrobust cases, the worst-case SPEB increase with the uncertainty size, significantly violating the localization accuracy requirement.

6. Conclusion

In this paper, we investigated the robust power allocation strategies for cooperation in jammed wireless sensor localization systems. First, the optimization framework is presented in jammed cooperative localization systems. Then, the robust power allocation strategies are developed to address the parameter uncertainty problem. Moreover, the problem can be transformed into SOCP and obtained the end solution

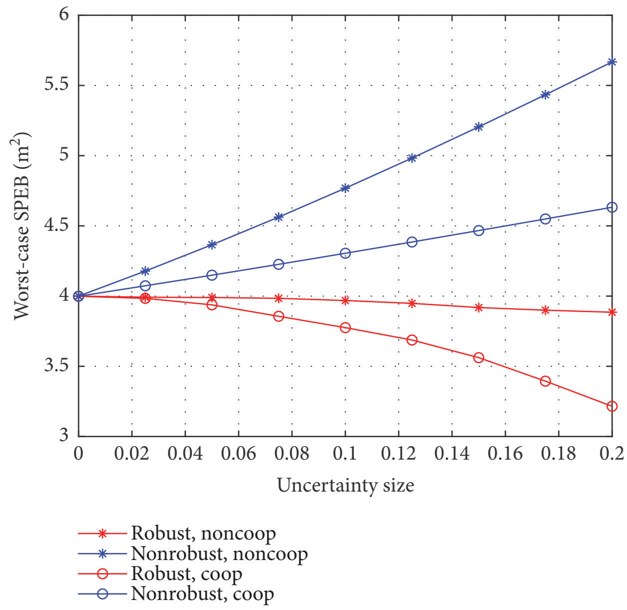

Figure 6: The total power consumption with respect to the uncertainty size.

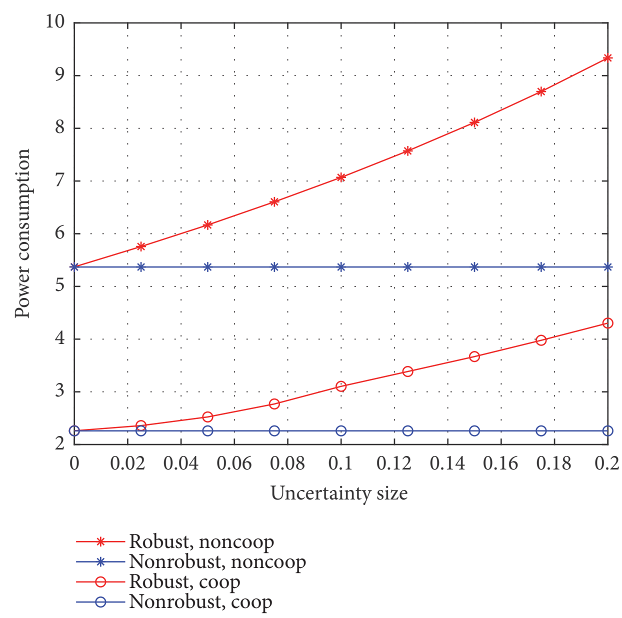

Figure 7: The worst-case SPEB with respect to the uncertainty size.

via conic programming. The simulation results demonstrated that the cooperative localization can reach better localization accuracy than noncooperative localization, the power allocation scheme via SOCP outperforms the uniform scheme, and the robust formulation approach outperforms the nonrobust approach.

Data Availability

The data in this paper is generated from the simulation in Matlab, and the detail simulation settings can refer to Section 5. Therefore, the data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] G. Han, J. Jiang, C. Zhang, T. Q. Duong, M. Guizani, and G. K. Karagiannidis, "A survey on mobile anchor node assisted localization in wireless sensor networks," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 3, pp. 2220–2243, 2016.

[2] M. Ke, Y. Wang, M. Li, F. Gao, and Z. Du, "Distributed power allocation for cooperative localization: A potential game approach," in *Proceedings of the 3rd IEEE International Conference on Computer and Communications, ICCC 2017*, pp. 616–621, Chengdu, China, December 2017.

[3] Y. Shen, H. Wymeersch, and M. Z. Win, "Fundamental limits of wideband localization—part II: cooperative networks," *IEEE Transactions on Information Theory*, vol. 56, no. 10, pp. 4981–5000, 2010.

[4] Y. Shen and M. Z. Win, "Fundamental limits of wideband localization—part I: a general framework," *IEEE Transactions on Information Theory*, vol. 56, no. 10, pp. 4956–4980, 2010.

[5] W. W.-L. Li, Y. Shen, Y. J. Zhang, and M. Z. Win, "Robust power allocation for energy-efficient location-aware networks," *IEEE/ACM Transactions on Networking*, vol. 21, no. 6, pp. 1918–1930, 2013.

[6] Y. Shen, W. Dai, and M. Z. Win, "Power optimization for network localization," *IEEE/ACM Transactions on Networking*, vol. 22, no. 4, pp. 1337–1350, 2014.

[7] W. Dai, Y. Shen, and M. Z. Win, "Distributed power allocation for cooperative wireless network localization," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 1, pp. 28–40, 2015.

[8] J. Chen, W. Dai, Y. Shen, V. Lau, and M. Z. Win, "Power management for cooperative localization: a game theoretical approach," *IEEE Transactions on Signal Processing*, vol. 64, no. 24, pp. 6517–6532, 2016.

[9] M. Ke, Y. Sun, M. Wang, S. Tian, and L. Lu, "Distributed power optimization for cooperative localization: a hierarchical game approach," *IEEE Wireless Communications Letters*, In press.

[10] S. Gezici, M. R. Gholami, S. Bayram, and M. Jansson, "Jamming of wireless localization systems," *IEEE Transactions on Communications*, vol. 64, no. 6, pp. 2660–2676, 2016.

[11] S. Bayram, M. F. Keskin, S. Gezici, and O. Arikan, "Optimal power allocation for jammer nodes in wireless localization systems," *IEEE Transactions on Signal Processing*, vol. 65, no. 24, pp. 6489–6504, 2017.

[12] M. Ke, G. Zhao, S. Tian, C. Wang, and Y. Liu, "Optimal power allocation for anchor nodes in jammed wireless localization systems," *IEEE Wireless Communications Letters*, In press.

[13] M. Grant and S. Boyd, "CVX: MATLAB software for disciplined convex programming," *Version 1.21*, 2010, http://cvxr.com/cvx.