## LOW COMPLEXITY ITERATIVE RECEIVERS FOR SPACE-TIME BLOCK CODED MC-CDMA DOWNLINK SYSTEMS

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## ABSTRACT

Multi-carrier code division multiple access (MC-CDMA) and space-time block coding (STBC) are two promising techniques to fulfill the high-speed and spectral efficiency requirements of fourth generation mobile communication systems. In this contribution we present an iterative receiver for the downlink of STBC MC-CDMA systems, that aims to mitigate the effects of multiple access interference (MAI) by jointly optimizing multiuser detection and channel decoding. Our proposal is a two-stage parallel interference canceller (PIC), where soft information in the form of loglikelihood ratios (LLRs) is exchanged between the multiuser detector (MUD) and individual channel decoders for successive refinement. In the first iteration a minimum mean square error (MMSE) equalizer, adapted to the equivalent channel as seen after space-time combining, is used for multiuser detection, whereas for the second and successive iterations a bank of MMSE filters adapted to each user and optimized for the quasi MAI free case is employed.

#### 1. INTRODUCTION

MC-CDMA, based on the concatenation of direct sequence CDMA and orthogonal frequency code division multiplexing (OFDM), is a serious candidate for the physical layer in the downlink of future broadband multiuser communication systems. OFDM multicarrier modulation provides robustness against multipath, avoiding intersymbol interference (ISI) and interblock interference (IBI) as long as carrier spacing and guard interval duration are appropriately chosen. The use of CDMA with orthogonal spreading codes provides multiple access flexibility as well as frequency diversity [1]. Unfortunately, the loss of orthogonality among users induced by the channel causes MAI, which limits the performance of MC-CDMA.

In order to combat the effects of fading and multipath, all diversity sources offered by the channel should be exploited. STBC, first introduced by Alamouti for two transmit antennas [2] and later generalized to a greater number of antennas by Tarokh *et al.* [3], is an efficient way to exploit spatial diversity. Even if the use of multiple antennas at the remote terminals might not be feasible due to size, weight, power or cost limitations, STBC can still provide diversity in the downlink for all the remote units by using multiple transmit antennas at the base station (BS).

Although initially designed for flat fading channels, STBC can be successfully employed in wideband frequency selective channels when combined with MC-CDMA, because the channel is split in as many flat fading subchannels as the number of carriers in the OFDM multiplex. Several authors have previously considered the combination of MC-CDMA and STBC [4, 5, 6], mostly employing single-user detection techniques. In order to minimize the effect of MAI and fully exploit the potential of this combination, multiuser detection techniques are required. The huge complexity of the optimal multiuser detector makes suboptimal approaches necessary. In [6] simple PIC receivers are proposed, but neither channel coding nor soft cancellation are considered. Channel coding and interleaving help to exploit time diversity, and given that most real systems will employ some form of channel coding, it makes sense to jointly consider multiuser detection and channel decoding in iterative structures, as it has been studied in great detail for single-antenna MC-CDMA systems [7, 8]. It has been shown that inclusion of channel decoding in the cancellation loop, together with the use of soft input-soft output (SISO) algorithms for detection and decoding, helps to mitigate the error propagation effect that could otherwise limit the performance of PIC receivers. Recently this approach has been extended to the uplink of MC-CDMA systems with Alamouti STBC [9]. In this paper we focus on downlink systems, where complexity issues are a major concern. We try to reduce complexity by using linear components and avoiding matrix inversions where possible. Moreover, by working with the equivalent channel the size of the matrices involved in all calculations is reduced.

The rest of this paper is organized as follows: in section 2 a general description of the system is given, together with a brief review of STBC as applied to MC-CDMA. Section 3 describes in detail the interference cancellation schemes. Simulation results are presented in section 4, and we finally draw our conclusions in section 5.

## 2. SYSTEM MODEL

We consider a STBC MC-CDMA system in the downlink with  $N_u$  active users, two antennas in the transmitter and q antennas in the receiver. The transmitter model is shown in Fig. 1, and the configuration of the proposed receiver for the first user in the system is shown in Fig. 2, being similar for the rest of the users.

#### 2.1 Transmitter Model

As shown in Fig. 1, the bit information sequences  $\{b^{(k)}\}$ for each user  $k, k = 1, \ldots, N_u$ , are encoded with standard convolutional encoders, and each encoded sequence is passed through a pseudo-random bit interleaver before Gray symbol mapping. At a given symbol period t, each modulated symbol  $x_t^{(k)}$  is multiplied by its corresponding spreading code  $c^{(k)}$ , and the spread symbols of all users are synchronously added to be simultaneously transmitted sharing the same bandwidth. The vector transmitted in the t-th OFDM symbol is given by  $y_t = C x_t$ , where the  $N_c \times N_u$  spreading matrix C has  $c^{(k)}$  as its k-th column, and  $x_t = [x_t^{(1)}, \ldots, x_t^{(N_u)}]^T$ . In our system the length of the spreading codes is equal to the number of carriers  $N_c$  in the OFDM multiplex.

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Figure 1: STBC MC-CDMA transmitter model (BS).



Figure 2: STBC MC-CDMA receiver model for the first user in the system.

Because of its unitary rate, we focus attention on the STBC proposed by Alamouti [2]. Two consecutive spread symbol vectors  $\boldsymbol{y}_1$  and  $\boldsymbol{y}_2$  are transmitted employing two antennas over two contiguous OFDM symbols: during the first symbol period,  $\boldsymbol{y}_1$  is transmitted from the first antenna, and simultaneously  $\boldsymbol{y}_2$  is transmitted from the second one. During the next symbol interval, the first antenna outputs  $-\boldsymbol{y}_2^*$  while the second antenna transmits  $\boldsymbol{y}_1^*$ , with  $[\cdot]^*$  standing for complex conjugate.

After space-time coding, OFDM modulation is performed. For each transmit antenna a  $N_c$ -point inverse fast Fourier transform (IFFT) operation is required, followed by appropriate guard interval insertion.

#### 2.2 Receiver Model

The proposed receiver, shown in Fig. 2, consists of four blocks: OFDM demodulators, the space-time combiner, the SISO MUD, and  $N_u$  individual SISO channel decoders. We remark that the space-time combiner is not included in the MAI cancellation loop, so that the iterative exchange of tentative soft decisions only involves the MUD and the channel decoders. This suboptimal approach still provides very good performance, and is essential in order to reduce complexity, since the MUD can be designed considering the equivalent channel as seen after space-time combining, for which we derive a model in the following paragraphs.

Let  $\mathbf{r}_{tj}$  denote the  $N_c \times 1$  vector obtained after OFDM demodulation of the *t*-th symbol received at the *j*-th antenna. Since the memory of the STBC does not extend from block to block, we define the stacked vector  $\mathbf{r}_j = [\mathbf{r}_{1j}^T \ \mathbf{r}_{2j}^T]^T$ . Our channel model assumes frequency non-selective fading per subcarrier, and we denote by  $\mathbf{H}_{ij}$  the  $N_c \times N_c$  diagonal matrix having as its *l*-th diagonal entry the complex frequency response of the channel from transmit antenna *i* to receive antenna *j*, at the frequency of the *l*-th carrier. We do not include a time index because the fading coefficients are assumed to remain constant within a space-time block. Under these hypothesis  $\mathbf{r}_j$  is given by  $\mathbf{r}_j = \mathbf{H}_j \mathbf{y} + \mathbf{n}_j$ , where

$$\boldsymbol{H}_{j} = \begin{bmatrix} \boldsymbol{H}_{1j} & \boldsymbol{H}_{2j} \\ \boldsymbol{H}_{2j}^{H} & -\boldsymbol{H}_{1j}^{H} \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_{1} \\ \boldsymbol{y}_{2} \end{bmatrix}, \quad \boldsymbol{n}_{j} = \begin{bmatrix} \boldsymbol{n}_{1j} \\ \boldsymbol{n}_{2j}^{*} \end{bmatrix} \quad (1)$$

Each vector  $\mathbf{n}_{tj}$  contains the additive white gaussian noise samples that affect  $\mathbf{r}_{tj}$ , with  $E\{\mathbf{n}_{tj}\} = \mathbf{0}$  and  $E\{\mathbf{n}_{tj}\mathbf{n}_{tj}^H\} = \sigma_n^2 \mathbf{I}$ . We denote by  $\mathbf{I}$  the  $N_c \times N_c$  identity matrix,  $[\cdot]^H$  stands for hermitian transposition, and  $E\{\cdot\}$  represents mathematical expectation.

The space-time combiner does not take into account the multiuser structure of the signals, but simply builds an estimate z' of y applying the maximum ratio receive combining (MRRC) rule, which aims to maximize the signal-to-noise ratio (SNR) per subcarrier in the combined signal:

$$\boldsymbol{z}' = \sum_{j=1}^{q} \boldsymbol{H}_{j}^{H} \boldsymbol{r}_{j} = \Big[\sum_{j=1}^{q} \boldsymbol{H}_{j}^{H} \boldsymbol{H}_{j}\Big] \boldsymbol{y} + \boldsymbol{\eta}'$$
(2)

Writing  $\mathbf{z}' = [\mathbf{z}_1^T \ \mathbf{z}_2^T]^T$ ,  $\boldsymbol{\eta}' = [\boldsymbol{\eta}_1^T \ \boldsymbol{\eta}_2^T]^T$ , the above equation can be split as

$$\boldsymbol{z}_t = \boldsymbol{\mathcal{H}} \boldsymbol{C} \boldsymbol{x}_t + \boldsymbol{\eta}_t, \quad t = 1, 2 \tag{3}$$

where  $\mathcal{H}$  is a diagonal matrix representing the equivalent channel after space-time combining, given by

$$\mathcal{H} = \sum_{i=1}^{2} \sum_{j=1}^{q} \boldsymbol{H}_{ij}^{H} \boldsymbol{H}_{ij}$$
(4)

Both equivalent noise vectors,  $\eta_1$  and  $\eta_2$ , have the same mean,  $E\{\eta_t\} = \mathbf{0}_{N_c \times 1}$ , and the same covariance matrix

$$E\{\boldsymbol{\eta}_t \boldsymbol{\eta}_t^H\} = \sigma_n^2 \boldsymbol{\mathcal{H}}, \quad t = 1, 2$$
(5)

Hence, the use of the MRRC combiner effectively eliminates the ISI artificially created by the space-time transmission, and provides a simple channel model, similar to that of a single antenna MC-CDMA system, summarized in equations (3), (4) and (5). Since these equations are equally valid for any of the symbols in a space-time block, the time subindex will be removed from  $\boldsymbol{z}_t$ ,  $\boldsymbol{x}_t$  and  $\boldsymbol{\eta}_t$  in the following sections for notational simplicity.

## 3. SOFT INTERFERENCE CANCELLATION SCHEMES

The signals output by the space-time combiner are first processed by the MUD, which consist of a linear equalizer, despreading, and a module that performs soft demapping, evaluating the LLRs about the coded bits of every user. After deinterleaving, the LLRs are passed to individual SISO channel decoders. The improved LLRs output by the decoders are interleaved again and soft-mapped into symbols, in order to estimate and cancel the MAI.

#### 3.1 Multiuser detection for the first iteration

The approach used for multiuser detection depends on the iteration index. For the first iteration, the  $N_u \times N_c$  equalization matrix  $\boldsymbol{Q}$  is chosen according to the multiuser MMSE (MU-MMSE) criterion, which aims to globally minimize the error between the equalized symbols of all users and the transmitted symbol vector  $\boldsymbol{x}$ ,  $E\{||\boldsymbol{Q}\boldsymbol{z} - \boldsymbol{x}||^2\}$ , and is hence able to cope with high MAI situations. Application of the orthogonality principle leads to

$$\boldsymbol{Q} = \boldsymbol{C}^{H} \left( \boldsymbol{\mathcal{H}} \boldsymbol{C} \boldsymbol{C}^{H} + \frac{1}{\gamma_{s}} \boldsymbol{I} \right)^{-1}$$
(6)

where  $\gamma_s = \sigma_x^2/\sigma_n^2$  is the SNR per symbol. We note that MU-MMSE requires the inversion of a  $N_c \times N_c$  matrix, whatever the number of active users. This contrasts with the case of single antenna MC-CDMA systems, where an equivalent expression for the MU-MMSE equalization matrix can be found which only requires the inversion of a  $N_u \times N_u$  matrix [10]. Instead of MU-MMSE, in the first iteration we may also employ per-carrier MMSE (PC-MMSE) equalization, with

$$\boldsymbol{Q} = \boldsymbol{C}^{H} \left( \boldsymbol{\mathcal{H}} + \frac{1}{\gamma_{c}} \boldsymbol{I} \right)^{-1}$$
(7)

where  $\gamma_c = \gamma_s \frac{N_u}{N_c}$  is the SNR per carrier. Inversion is not required, since  $\mathcal{H} + 1/\gamma_c I$  is a diagonal matrix, but the PC-MMSE criterion does not take into account the structure of MAI, hence sacrificing performance for the sake of a complexity reduction. In the full load case, when  $N_u = N_c$ , PC-MMSE and MU-MMSE are equivalent.

We note that in (6) and (7) despreading is included in the equalization matrix Q by means of the  $C^{H}$  factor, and that the equalized vector v = Qz can be decomposed as

$$\boldsymbol{v} = \operatorname{diag}(\operatorname{diag}(\boldsymbol{U})) \boldsymbol{x} + (\boldsymbol{U} - \operatorname{diag}(\operatorname{diag}(\boldsymbol{U}))) \boldsymbol{x} + \boldsymbol{Q}\boldsymbol{\eta} \quad (8)$$

where  $\boldsymbol{U} = \boldsymbol{Q} \boldsymbol{\mathcal{H}} \boldsymbol{C}$  and diag  $(\text{diag}(\boldsymbol{U}))$  is a diagonal matrix with its main diagonal entries equal to those of  $\boldsymbol{U}$ . Hence, the first term in the right hand side of (8) represents the desired signal, the second term represents MAI and the third term is noise. We finally note that if zero forcing (ZF) equalization were used, with  $\boldsymbol{Q} = \boldsymbol{C}^{H} \boldsymbol{\mathcal{H}}^{-1}$ , orthogonality would be restored and there would be no MAI, but noise enhancement would severely degrade performance in that case.

Based on the model given by (8), each equalized symbol  $v^{(k)}$  is processed by the soft demapper to obtain m LLRs, where  $m = \log_2 M$  and M is the size of the constellation employed. These *a posteriori* LLRs are the optimal reliability information that can be exploited by the channel decoders, and are given by

$$\lambda^{\text{MUD}}(\tilde{c}_{n}^{(k)}) = \log \frac{p\left(\tilde{c}_{n}^{(k)} = 1 \mid v^{(k)}\right)}{p\left(\tilde{c}_{n}^{(k)} = 0 \mid v^{(k)}\right)}$$
(9)

where we denote by  $\{\tilde{c}_1^{(k)}, \ldots, \tilde{c}_m^{(k)}\}$  the interleaved coded bits that are mapped to symbol  $x^{(k)}$ . We assume that the BS transmits with equal power towards all users, and that the number of interfering users is high enough, so that the MAI term affecting  $v^{(k)}$  can be considered Gaussian distributed, with zero mean and independent from the noise term. Thus, for evaluation of the conditional probabilities in (9) the total perturbation is modelled as a complex Gaussian random variable with variance  $\sigma_{MAI}^2 + \sigma_N^2$ , where  $\sigma_N^2$  is the variance of the equivalent noise term after equalization.

### 3.2 Channel Decoding

After deinterleaving, each of the sequences of LLRs  $\{\lambda^{\text{MUD}}(c_n^{(k)})\}$  is independently processed by a SISO channel decoder. By taking into account the information received during a whole frame and the constraints of the encoder trellis, the decoders produce improved LLRs  $\{\lambda^{\text{DEC}}(c_n^{(k)})\}$  about the coded bits. We use the logMAP algorithm for channel decoding, aided by a look-up table as described in [11].

# **3.3** MAI Regeneration and Cancellation. Multiuser detection for the second and successive iterations

The LLRs output by each channel decoder are interleaved again and converted to soft bits by application of the well known  $\tanh(\lambda/2)$  non-linearity [7]. These soft bits are grouped and mapped into soft symbols following a statistical approach, so that the reliability information is not lost, see for example [12, Ch. 17.5.2].

In the second and successive iterations, a bank of  $N_u$ MMSE filters is used for equalization. The input to the k-th filter is obtained after subtraction from z of the MAI caused by all users but the k-th. The MAI contribution is estimated by spreading and distorting through the equivalent channel  $\mathcal{H}$  the vector of soft symbols  $\hat{x}$  calculated in the previous iteration. According to (3), the input vector to the k-th filter is given by

$$\boldsymbol{z}^{(k)} = \boldsymbol{z} - \boldsymbol{\mathcal{H}} \left( \boldsymbol{C} \hat{\boldsymbol{x}} - \boldsymbol{c}^{(k)} \hat{\boldsymbol{x}}^{(k)} \right)$$
(10)

and the equalized output symbol is given by  $\boldsymbol{q}_k^H \boldsymbol{z}^{(k)}$ , where the coefficient vector  $\boldsymbol{q}_k$  is chosen so as to minimize  $E\{|\boldsymbol{q}_k^H \boldsymbol{z}^{(k)} - \boldsymbol{x}^{(k)}|^2\}$ . Assuming that MAI had been completely eliminated after the the first cancellation we obtain

$$\boldsymbol{q}_{k}^{H} = \boldsymbol{c}^{(k)H} \mathcal{H} \left( \mathcal{H} \boldsymbol{c}^{(k)} \boldsymbol{c}^{(k)H} \mathcal{H} + \frac{1}{\gamma_{s}} \mathcal{H} \right)^{-1}$$
(11)

In high-load scenarios, which are the target of our proposal, direct calculation of the coefficients of the  $N_u$  filters would require a huge computational effort. Fortunately, application of the Woodbury matrix lemma leads to an equivalent expression for  $\boldsymbol{q}_k$  that avoids matrix inversion:

$$\boldsymbol{q}_{k}^{H} = \gamma_{s} \boldsymbol{c}^{(k)}{}^{H} \boldsymbol{\mathcal{H}}^{1/2} \Big( \boldsymbol{I} - \frac{\gamma_{s}}{1 + \gamma_{s} \boldsymbol{u}^{(k)}{}^{H} \boldsymbol{u}^{(k)}} \boldsymbol{u}^{(k)} \boldsymbol{u}^{(k)}{}^{H} \Big) \boldsymbol{\mathcal{H}}^{-1/2}$$
(12)

where  $\boldsymbol{u}^{(k)} = \mathcal{H}^{1/2} \boldsymbol{c}^{(k)}$ . Since  $\mathcal{H}$  is a diagonal positive definite matrix, its square root is well defined and straight forward to evaluate.

The rest of the elements of the cancellation loop are identical to those used in the first iteration, except for the soft demapping module, that approximates the variance of the total perturbation affecting each equalized symbol by the variance of the noise term. In the last iteration only the branch of the desired user is active, and the channel decoder is modified to calculate LLRs about the data bits, not about the coded bits. Final decisions are based on the sign of those LLRs.

#### 4. SIMULATION RESULTS

In this section we discuss the performance of the the proposed receivers, evaluated by means of Monte Carlo simulation. Rate 1/2 convolutional encoders with octal generators  $(171, 133)_8$  and a frame length of 1000 uncoded bits are considered. Hadamard spreading codes are used, normalized so that  $C^H C = I$ , and the number of carriers is  $N_c = 64$ . It is assumed that the BS transmits 16-QAM symbols with equal power for all users, and that channel state information is estimated by the receiver without error.

Rayleigh distributed fading, independent from carrier to carrier, is assumed. To allow for optimal space-time combining, we further assume that fading is quasi static, so that, for a given carrier, it remains constant within a block of two consecutive OFDM symbols, but then changes independently from block to block. It is also assumed that the fading processes between any two pairs of transmit/receive antennas are uncorrelated. Under these hypothesis the optimal frequency, time and spatial diversity gains will be attained, thus allowing to evaluate an upper bound for the performance of the system.

In Fig. 3 results are presented in terms of bit error rate (BER) versus SNR per information bit  $(E_b/N_0)$ , whereas in Fig. 4 the value of  $E_b/N_0$  required for a BER =  $10^{-3}$  is plotted as a function of the system load,  $N_u/N_c$ . Close observation of this figures reveals that for intermediate and high values of  $E_b/N_0$ , and for all systems loads, MU-MMSE with two iterations performs very closely to the single user case. With one iteration, MU-MMSE clearly outperforms PC-MMSE, but the difference between them becomes very



Figure 3: BER vs.  $E_b/N_0$  after one iteration (solid lines) and two iterations (dashed lines). MU-MMSE is used for the first iteration except where shown. One receive antenna.

small when two iterations are considered, and is almost negligible with three iterations. As expected, performance of MU-MMSE and PC-MMSE systems with the same number of iterations coincide in the full load case. Fig. 3 also shows that, despite the absence of MAI, the results obtained with ZF equalization are poor.

The gain obtained when comparing systems with two and three iterations is much smaller than that between systems with one and two iterations, specially when MU-MMSE equalization is considered. This was expected, since for all iterations after the first we are assuming that  $\sigma_{MAI}^2 \approx 0$ . Besides, it should be noted that for two iterations, only one of the MMSE filters needs to be implemented, whereas the calculation of the coefficients of  $N_u$  filters is required if three iterations are to be done.

## 5. CONCLUSION

We presented two types of soft iterative receivers for Alamouti STBC MC-CDMA systems with convolutional channel coding in a synchronous downlink. Mitigation of the effect of MAI leads to increased spectral efficiency and a major performance improvement. PC-MMSE equalization for the first iteration, followed by MMSE filtering adapted to the quasi MAI-free case in the second one, represents a very good trade-off between complexity and performance, which comes close to that of a single user system for all loads. Complexity is kept at a reasonable level avoiding matrix inversions and thanks to equalization based on the equivalent channel after space-time combining. Moreover, the use of the equivalent channel allows for further improvement with slightly increased signal processing complexity by using multiple receive antennas, and makes possible the generalization of this receiving scheme to any other STBC based on orthogonal designs [3].

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Figure 4: Required  $E_b/N_0$  (dB) for a BER =  $10^{-3}$  as a function of the system load,  $N_u/N_c$ . One receive antenna.

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