FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 23, no. 1, April 2010, 87-96

Performance analysis of system with triple selection combining over correlated Weibull fading channel in the presence of cochannel interference

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Abstract: In this paper, closed-form expressions for probability density function (PDF) and cumulative distribution function (CDF) of the signal-to-interference ratio (SIR) at the output of the triple selection combining (SC) receiver over correlated Weibull fading channels in the presence of correlated cochannel interference (CCI) are derived. These expressions are used to study wireless system performance criteria, such as outage probability, average bit error probability and average output SIR.

Keywords: Cochannel interference, correlated fading, diversity, selection combining, Webull fading channels.

1 Introduction

DIVERSITY combining [1] is an efficient and widely employed technique in digital communication receivers for mitigating the multipath fading effects [2] and upgrading transmission reliability at relatively low cost. Space diversity techniques [3] combine input signals from multiple receive antennas, also called an antenna array, in some way to ameliorate system' s quality-of-service (QoS). The most popular linear diversity techniques are SC, equal-gain combining (EGC), and maximal-ratio combining (MRC) [4]. MRC is the optimal combining technique in the sense that it achieves the highest output signal-to-noise ratio (SNR) regardless of the fading statistics on the diversity branches. However, MRC requires the knowledge of the channel fading amplitudes and phases of each diversity branch which must be continuously estimated by the receiver. This estimations require

Manuscript received on August 15, 2009.

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separate receiver chain for each branch of the diversity system increasing its complexity. EGC provides performance comparable to MRC, but with simpler implementation complexity. EGC does not require the estimation of the channel fading amplitudes since it combines all of the branches with the same weighting factor. SC is the least complicated technique, since it is reposed on processing only one of the diversity branches. SC combiner chooses the branch with the highest SNR, or equivalently, with the strongest signal assuming equal noise power among the branches. In wireless communication systems the influence of the thermal noise is negligible as compared to the influence of the cochannel interference (CCI) so SC combiner processes the branch with the highest signal-to-interference ratio (SIR based selection diversity) [5].

Despite the fact that Weibul fading model confirms experimentally attained fading channel measurements, for both indoor [6], as well as for outdoor environments [7], it has not yet received as much attention as Rayleigh, Rice and Nakagami-m fading models. In paper [8], useful analytical expressions for the joint probability density function, cumulative density function, moment-generating function (MGF) and product moments for the bivariate and multivariate Weibull distribution have been derived and applied to analyze the performance of dual- and multibranch SC, EGC, and MRC receivers. In [9], performance of dual-branch SC receiver over correlated Weibull fading channels in the presence of correlated Weibull distributed CCI has been presented. Closed form expressions for the joint statistics of two correlated Weibull variates have been obtained in paper [10].Triple selection diversity system over correlated Nakagami-m fading channels has been envisaged in [11].

In this paper, triple SC receiver operating over correlated Weibull fading channels in the presence of correlated Weibull distributed CCI is considered. Expressions for PDF and CDF of SIR at the output of SC receiver are analytically derived. These expressions are applied to analyze the performance criteria of the triple SC receiver. Numerical results show the effect of fading severity and correlation coefficient on the combiners performance.

2 PDF and CDF of the output SIR at SC receiver

Since Weibull fading model exhibits an excellent fit to experimental fading channel measurements, especially in urban and nonhomogeneous environments, our efforts are concentrated on case in which both desired and interfering signal envelopes follow the correlated Weibull distribution with joint PDFs ([8] eq. (23) for L = 3):

$$p_{R_1R_2R_3}(R_1,R_2,R_3) = \frac{\beta_1\beta_2\beta_3}{\Omega_{d1}\Omega_{d2}\Omega_{d3}(1-\rho)^2} \times \exp\left\{-\frac{1}{1-\rho}\left[\frac{R_1^{\beta_1}}{\Omega_{d1}}+\frac{R_3^{\beta_3}}{\Omega_{d3}}+\frac{(1+\rho)R_2^{\beta_2}}{\Omega_{d2}}\right]\right\} \times \sum_{k_1,k_2=0}^{\infty}\left[\frac{\sqrt{\rho}}{\sqrt[3]{\Omega_{d1}\Omega_{d2}\Omega_{d3}}(1-\rho)}\right]^{2(k_1+k_2)} \times \frac{R_1^{(k_1+1)\beta_1-1}R_3^{(k_2+1)\beta_3-1}R_2^{(k_1+k_2+1)\beta_2-1}}{(k_1!k_2!)^2} \tag{1}$$

and

$$p_{r_1r_2r_3}(r_1,r_2,r_3) = \frac{\beta_1\beta_2\beta_3}{\Omega_{c1}\Omega_{c2}\Omega_{c3}(1-\rho)^2} \times \exp\left(-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_{c1}}+\frac{r_3^{\beta_3}}{\Omega_{c3}}+\frac{(1+\rho)r_2^{\beta_2}}{\Omega_{c2}}\right)\right) \times \sum_{l_1,l_2=0}^{\infty}\left[\frac{\sqrt{\rho}}{\sqrt[3]{\Omega_{c1}\Omega_{c2}\Omega_{c3}}(1-\rho)}\right]^{2(l_1+l_2)} \frac{r_1^{(l_1+1)\beta_1-1}r_3^{(l_2+1)\beta_3-1}r_2^{(l_1+l_2+1)\beta_2-1}}{(l_1!l_2!)^2} \tag{2}$$

respectively. Parameter β is Weibull fading parameter $(\beta > 0)$ and represents fading intensity measure, ρ is correlation coefficient, $\Omega_{di} = \overline{R_i^2}$ and $\Omega_{ci} = \overline{r_i^2}$ are the average powers of desired and interfering signal at *i*th branch, respectively. When value of parameter β increases, fading intensity decreases, while for $\beta = 2$ proposed Weibull distribution becomes Rayleigh distribution. Instantaneous value of SIR on the *i*th diversity branch of SC receiver can be defined as , $(i = 1,2,3)$. Joint PDF of these random variables can be obtained as

$$p_{\mu_1\mu_2\mu_3}(\mu_1,\mu_2,\mu_3) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} r_1r_2r_3\, p_{R_1R_2R_3}(\mu_1r_1,\mu_2r_2,\mu_3r_3) \times p_{r_1r_2r_3}(r_1,r_2,r_3)\,dr_1dr_2dr_3 \tag{3}$$

Substituting (1) and (2) in (3), and after triple integration using ([12], eq.(3.478/1)), joint PDF becomes

$$p_{\mu_1\mu_2\mu_3}(\mu_1,\mu_2,\mu_3) = (1-\rho)^2\beta_1\beta_2\beta_3 \times \sum_{k_1,k_2=0}^{\infty}\sum_{l_1,l_2=0}^{\infty} \frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)}(\Omega_{c1}\Omega_{c2}\Omega_{c3})^{1+\frac{2}{3}(l_1+l_2)}} \times \frac{\mu_1^{(k_1+1)\beta_1-1}\mu_1^{(k_2+1)\beta_3-1}\mu_1^{(k_2+k_1+1)\beta_2-1}}{(k_1!l_1!k_2!l_2!)^2}\left(\frac{\mu_1^{\beta_1}}{\Omega_{d1}}+\frac{1}{\Omega_{c1}}\right)^{-(k_1+l_1+2)} \times \left(\frac{\mu_3^{\beta_3}}{\Omega_{d3}}+\frac{1}{\Omega_{c3}}\right)^{-(k_2+l_2+2)}\left(\frac{\mu_2^{\beta_2}}{\Omega_{d2}}+\frac{1}{\Omega_{c2}}\right)^{-(k_1+l_1+k_2+l_2+2)} \times \Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2) \tag{4}$$

For this case joint CDF can be written as

$$F_{\mu_1\mu_2\mu_3}(\mu_1,\mu_2,\mu_3) = \int_0^{\mu_1}\int_0^{\mu_2}\int_0^{\mu_3} p_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)dx_1dx_2dx_3 \tag{5}$$

Substituting (4) in (5) and after some manipulations, triple integral can be solved with the use of [12, eq.(3.194/1)] obtaining

$$\begin{aligned} F_{\mu_1\mu_2\mu_3}(\mu_1,\mu_2,\mu_3) &= (1-\rho)^2\sum_{k_1,k_2=0}^{\infty}\sum_{l_1,l_2=0}^{\infty}\frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}} \\ &\times\frac{\Omega_{c1}^{k_1+l_1+1-\frac{2}{3}(l_1+l_2)}\Omega_{c3}^{k_2+l_2+1-\frac{2}{3}(l_1+l_2)}\Omega_{c2}^{k_1+k_2+l_1+l_2+1-\frac{2}{3}(l_1+l_2)}}{(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)}} \\ &\times\Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2) \\ &\times\frac{\mu_1^{(k_1+1)\beta_1}\mu_2^{(k_1+k_2+1)\beta_2}\mu_3^{(k_2+1)\beta_3}}{(k_1+1)(k_1+k_2+1)(k_2+1)}\,{}_2F_1(k_1+l_1+2,k_1+1;k_1+2;-\frac{\Omega_{c_1}}{\Omega_{d1}}\mu_1^{\beta_1}) \\ &\times{}_2F_1(k_1+k_2+l_1+l_2+2,k_1+k_2+1;k_1+k_2+2;-\frac{\Omega_{c_2}}{\Omega_{d2}}\mu_2^{\beta_2}) \\ &\times{}_2F_1(k_2+l_2+2,k_2+1;k_2+2;-\frac{\Omega_{c_3}}{\Omega_{d3}}\mu_3^{\beta_3}) \end{aligned} \tag{6}$$

where ${}_2F_1(a,b;c;d)$ is hypergeometric function.

Let μ_{sc} denotes the instantaneous SIR at the output of SC combiner, i.e. $\mu_{sc} = \max\mu_1,\mu_2,\mu_3$. CDF of μ_{sc} could be derived from (6), by equating input instantaneous SIRs $\mu_1 = \mu_2 = \mu_3 = \mu$ resulting in

$$\begin{aligned} F_{\mu_{sc}}(\mu) &= (1-\rho)^2\sum_{k_1,k_2=0}^{\infty}\sum_{l_1,l_2=0}^{\infty}\frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}} \\ &\times\frac{\Omega_{c1}^{k_1+l_1+1-\frac{2}{3}(l_1+l_2)}\Omega_{c3}^{k_2+l_2+1-\frac{2}{3}(l_1+l_2)}\Omega_{c2}^{k_1+k_2+l_1+l_2+1-\frac{2}{3}(l_1+l_2)}}{(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)}}\frac{1}{(k_1!l_1!k_2!l_2!)^2} \\ &\times\Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2) \\ &\times\frac{\mu^{(k_1+1)\beta_1+(k_1+k_2+1)\beta_2+(k_2+1)\beta_3}}{(k_1+1)(k_1+k_2+1)(k_2+1)}\,{}_2F_1(k_1+l_1+2,k_1+1;k_1+2;-\frac{\Omega_{c_1}}{\Omega_{d1}}\mu^{\beta_1}) \\ &\times{}_2F_1(k_1+k_2+l_1+l_2+2,k_1+k_2+1;k_1+k_2+2;-\frac{\Omega_{c_2}}{\Omega_{d2}}\mu^{\beta_2}) \\ &\times{}_2F_1(k_2+l_2+2,k_2+1;k_2+2;-\frac{\Omega_{c_3}}{\Omega_{d3}}\mu^{\beta_3}) \end{aligned} \tag{7}$$

PDF of the output SIR can be obtained by definition

$$\begin{aligned} p_{\mu_{sc}}(\mu) = &\int_0^{\mu}\int_0^{\mu} p_{\mu_1\mu_2\mu_3}(\mu,\mu_2,\mu_3)d\mu_2d\mu_3 \\ &+\int_0^{\mu}\int_0^{\mu} p_{\mu_1\mu_2\mu_3}(\mu_1,\mu,\mu_3)d\mu_1d\mu_3 \\ &+\int_0^{\mu}\int_0^{\mu} p_{\mu_1\mu_2\mu_3}(\mu_1,\mu_2,\mu)d\mu_1d\mu_2 \end{aligned} \tag{8}$$

or by differentiating expression (7)

$$p_{\mu_{sc}}(\mu) = \frac{d}{d\mu}F_{\mu_{sc}}(\mu) \tag{9}$$

In both cases PDF of instantaneous SIR at the output of SC receiver is

$$
\begin{aligned}
p_{\mu_{sc}}(\mu) = {} & (1-\rho)^2 \sum_{k_1,k_2=0}^{\infty} \sum_{l_1,l_2=0}^{\infty} \frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}} \\
& \times \frac{\Omega_{c1}^{k_1+l_1+1-\frac{2}{3}(l_1+l_2)} \Omega_{c3}^{k_2+l_2+1-\frac{2}{3}(l_1+l_2)} \Omega_{c2}^{k_1+k_2+l_1+l_2+1-\frac{2}{3}(l_1+l_2)}}{(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)}} \frac{1}{(k_1!l_1!k_2!l_2!)^2} \\
& \times \Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2)\mu^{(k_1+1)\beta_1+(k_1+k_2+1)\beta_2+(k_2+1)\beta_3-1} \\
& \times \Bigg[\frac{\beta_1}{(k_2+1)(k_1+k_2+1)(1+\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1})^{k_1+l_1+2}} \, {}_2F_1(k_2+l_2+2, k_2+1; k_2+2; -\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3}) \\
& \times {}_2F_1(k_2+k_1+l_1+l_2+2, k_2+k_1+1; k_2+k_1+2; -\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_3}) \\
& + \frac{\beta_2}{(k_1+1)(k_2+1)(1+\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_2})^{k_1+k_2+l_1+l_2+2}} \, {}_2F_1(k_1+l_1+2, k_1+1; k_1+2; -\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1}) \\
& \times {}_2F_1(k_2+l_2+2, k_2+1; k_2+2; -\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3}) \\
& + \frac{\beta_3}{(k_1+1)(k_2+k_1+1)(1+\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3})^{k_1+l_2+2}} \, {}_2F_1(k_1+l_1+2, k_1+1; k_1+2; -\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1}) \\
& \times {}_2F_1(k_2+k_1+l_2+l_1+2, k_2+k_1+1; k_2+k_1+2; -\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_2}) \Bigg]
\end{aligned}
\tag{10}
$$

Fig.1 shows PDF of SIR at the output of triple SC for various values of correlation coefficient and average powers of desired and interfering signals.

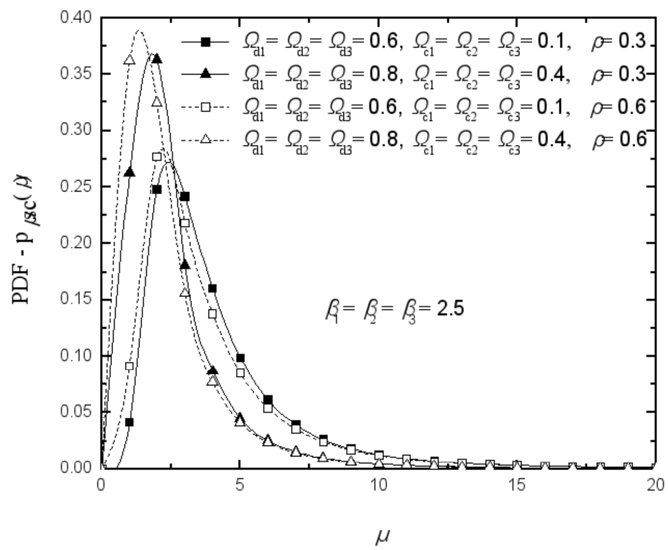

Fig. 1. Probability density function of instantaneous SIR at the output of triple SC receiver.

3 System performance measures

Analytically derived expressions for PDF and CDF can be applied to analyze the performance of triple SC receivers operating in correlated Weibull fading environment. Several performance quality indicators are considered in this section.

3.1 Outage probability

Outage probability is standard system performance measure of diversity system operating over fading channels. In the interference limited environment, outage probability is defined as the probability that the output SIR falls below a certain specified threshold μ_{th}. It can be obtained by replacing μ with μ_{th} in the CDF expression given by (7).

$$P_{OUT} = P_R(\mu_{sc} < \mu_{th}) = \int_0^{\mu_{th}} p_{\mu_{sc}}(\mu)d\mu = F_{\mu_{sc}}(\mu_{th}) \tag{11}$$

Numerical results show that outage probability converges for all values of system parametres. As table 1 indicates, the number of terms that are needed to be summed in expression for outage probability to achieve accuracy at the 4th digit strongly depends on the correlation coefficient. The number of terms increases as correlation coefficient increases.

Table 1. Terms need to be summed in expression for outage probability to achieve accuracy at the 4th digit

	Number of terms
$\rho = 0.2$	7
$\rho = 0.4$	14
$\rho = 0.6$	22

Based on (7) and (11), in Fig.2, outage probability for triple SC receiver is plotted as a function of the outage threshold μ_{th} for different system parameters. For lower values of outage threshold, outage probability decreases as Weibull fading parametres increase. When desired signal dominates (higher values of μ_{th}), increasing of Weibull fading parametres leads to deterioration of system performance. For a fixed values of Weibull fading parameters, better system performance are in the case for lower correlation coeficient.

3.2 Average bit error probability (ABEP)

Average bit error probability is another useful performance criterion characteristic of wireless communication systems. Conditional bit error probability (BEP) is

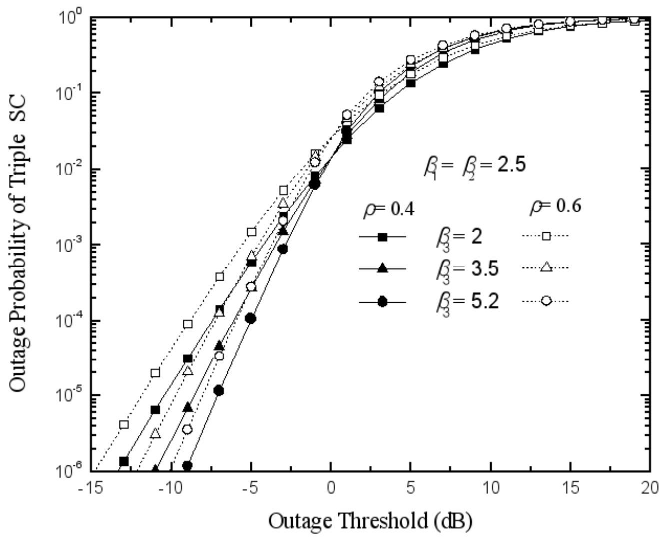

Fig. 2. Outage probability of triple SC as a function of the outage threshold.

a nonlinear function of the intstantaneous SIR and the nature of the nonlinearity is a function of the modulation/detection scheme employed by the system. The conditional BEP for a given SIR is

$$P_e(\mu) = \frac{1}{2} e^{-g\mu} \tag{12}$$

where g denotes modulation constant, i.e., g = 1 for BDPSK and g =1/2 for BFSK. ABEP at the SC output can be evaluated directly by averaging the conditional BEP over PDF of μ_{sc}.

$$P_e(\mu) = \int_0^\infty p_{\mu_{sc}}(\mu) P_e(\mu) d\mu \tag{13}$$

Based on (10), (12) and (13), the error performance of triple SC receiver can be obtained for several modulation schemes. Average BEP for triple and dual [9] SC receiver with BFSK and BDPSK signaling is plotted in Fig.3 as a function of the input average signal to average interference power ratio for several values of correlation coeficient. From Fig.3, the obtained performance evaluation results for balanced input SIRs ($\Omega_{d1}/\Omega_{c1} = \Omega_{d2}/\Omega_{c2} = \Omega_{d3}/\Omega_{c3} = S$) show that ABEP imroves with the decrease of correlation coefficient and increase of number of diversity branches, as expected. Moreover, system performances are better for BDPSK signaling. It is very interesting to observe that for lower values of S, ABEP performance of dual SC receiver with BDPSK signaling is better than ABEP performance of triple SC receiver with BFSK signaling.

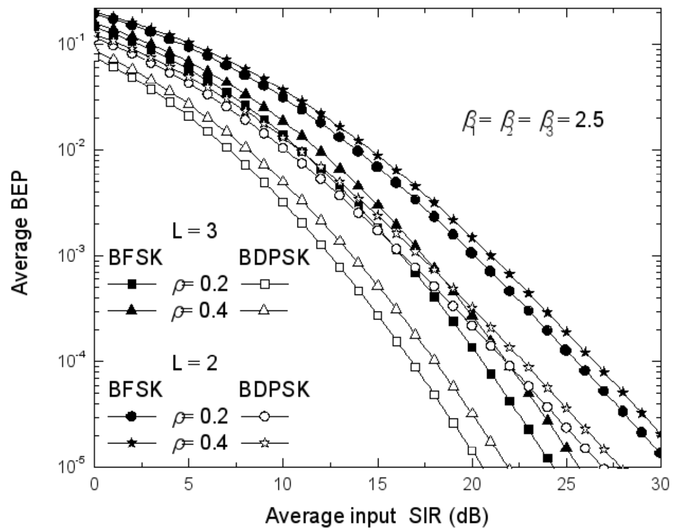

Fig. 3. Average BER of triple SC as a function of the average input SIR for equal branch average SIRs.

3.3 Average output SIR

Average output SIR is very useful parameter for describing wireless communication systems in the presence of CCI and it can be evaluated as

$$\overline{\mu_{sc}} = \int_0^{\infty} \mu p_{\mu_{sc}}(\mu) d\mu \tag{14}$$

Based on (10) and (14), Fig.4 demonstrates numericaly obtained results for average output SIR as a function of the correlation coefficient for diffrent system parameters. It is obviously that diversity gain decreases as correlation coefficient and/or fading parameters increase. Increasing of average input SIR leads to amelioration of system performance. This amelioration is more significant for lower values of fading parameters.

4 Conclusion

The performance of triple SC receiver operating over Weibull fading channels in the presence of correlated Weibull distributed CCI was studied. Closed-form expressions for PDF and CDF of the instantaneous SIR at the output of SC combiner were derived and applied to analyze system performance measures, such as outage probability, ABEP for several modulation schemes and average output SIR. Vari-

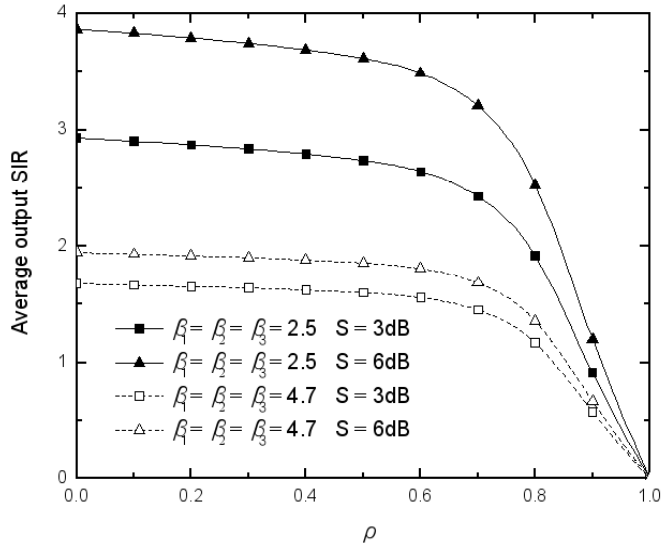

Fig. 4. Average output SIR as a function of the correlation coefficient.

ous numerical results of these performance measures are presented, describing their dependence on correlation coefficient, fading severity and modulation format.

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