

Spatial Discrete Soliton in Homogeneous Two Dimensional Waveguide Arrays with Kerr Medium

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Behaviors of discrete spatial solitons in nonlinear mediums have recently received a plenty of research interest. In this study, for the first time, the effects of diagonal neighbors on spatial discrete solitons in a two dimensional waveguide with Kerr medium are investigated. A numerical model for propagation of the Gaussian beam through a two dimensional waveguides array is proposed. The waveguide contains medium with Kerr nonlinearity considering coupling to vertical, horizontal and diagonal neighbor through light electric field. Different values of intensity, nonlinear coefficient Kerr and Gaussian beam width of incident Gaussian beam are examined and finally suitable parameters for providing central spatial solutions are obtained.

Key words: Waveguide Arrays, Discrete Diffraction, Anisotropic Medium, Discrete Soliton.

Behaviors of discrete spatial solitons in nonlinear mediums have recently received a plenty of research interest. Such solitons are localized modes of nonlinear lattices that are formed when discrete nonlinearity balances diffraction. Discrete solitons have been shown to exist in a wide range of physical systems¹⁻³. An array of coupled optical waveguides is a setting representing a convenient laboratory for experimental observations. Discrete solitons in an optical waveguide array were theoretically predicted by Christodoulides and Joseph in 1988⁴. Indeed, Christodoulides and Joseph were the first who predicted discrete solitons in waveguide arrays¹. They showed that a nonlinear array of coupled waveguides can exhibit discrete self-focusing that in the continuum

approximation obeys the so-called nonlinear Schrodinger equation. In their study, the arrays are considered as discrete solitons with alternating refraction index which are weakly coupled in a nonlinear medium. Later on, many theoretical studies of discrete solitons in a waveguide array reported switching, steering, and other collision properties of these solitons^{1,3,5}. Optical energy is transferred from one waveguide to neighboring ones through electric field leakage^{2,6}. In the discrete self-focusing coherent optical field propagating along a nonlinear array obeys a nonlinear difference-differential equation that resembles that of Davydov in biophysics^{7, 8}. Propagation of the light also depends on the phase difference between neighboring waveguides. On the other hand, the waveguides can be considered as linear or matrix in one-dimensional or two-dimensional states; respectively. Generally, light is investigated by the use of nonlinear Schrödinger equation in

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discrete systems where the light is propagated along the direction of waveguides⁹.

Fleischer *et al* (2003) were the first who discovered the optical discrete solitons in two-dimensional arrays⁹. In his study, two dimensional array was produced by light induction into a photorefractive crystal⁹. Various theoretical and modeling studies assessing the complicated nature of two dimensional systems have shown their innovative and unique capabilities^{3, 5, 9, 10}. As a results, several other applications and potential developments of these systems have been proposed^{3, 5, 9, 10}. Schematic diagram of the two dimensional waveguide is presented in figure 1¹¹.

The Model Presentation

System discussed is a two dimensional array of the same N×M waveguide that are equally placed at the distance of each other (Fig.1).

Where, C₁ is the coupling between vertical and horizontal neighboring waveguides and C₂ is diagonal coupling constant. It is also assumed that the field of each waveguide (E_{m,n}) only affects its neighboring waveguide. By modeling of two dimensional coupled oscillators [12], the Nonlinear Schrödinger equation and two dimensional discrete alternating coupling can be expressed for such a system as follows [12]:

$$\begin{aligned} \frac{idE_n}{dz} + i\beta E_{nm} + C_1[E_{m+1,n}(z) + E_{m-1,n}(z) + E_{m,n+1}(z) + E_{m,n-1}(z)] \\ + C_2[E_{m+1,n+1}(z) + E_{m-1,n-1}(z) + E_{m+1,n-1}(z) + E_{m-1,n+1}(z)] + \\ \gamma |E_{nm}|^2 E_{nm} = 0 \end{aligned} \dots (1)$$

where, β is the light propagation constant inside waveguide and third expression represents the Kerr nonlinear effect in which γ is a constant parameter that corresponds with nonlinear Kerr constant. Constant parameters corresponding to experimental values in all calculations are as follows: C₁=109 and C_d = $\frac{C_1}{\sqrt{2}}$; wavelength, λ=633nm, distance between waveguides, a=8.5μm; and propagation distance, z=10 cm has been selected from waveguide length.

Simulation

To solve equation (2) and simulating the behaviors of light in passing through waveguides, the central waveguide arrays are modeled using a Gaussian wave with the profile intensity as equation (2):

$$A = A_0 \exp\left(\frac{-x^2}{w^2}\right) \dots(2)$$

The *relaxation method* is used in this model. In this method, when equation (2) is converted to N×M coupled equation, having initial field values, Runge-Kutta method is used to obtain two dimensional central solitons that are created more easily in comparison with two dimensional solitons at corners or edges of waveguides¹³.

The effect of input intensity parameter

As a result of the waves coupling, the optical energies are distributed among waveguides immediately after its entrance. This phenomenon is called “discrete diffraction”¹⁴. Now, if the effect of diffraction balances nonlinear self-focusing factor, spatial solitons are generated. However, this requires the proper selection of input beam. Calculations showed that for the nonlinear factor of γ=2, peak amplitude of input Gaussian wave for solitons propagation, the exact amount of A₀=7.4038 at the width of w₀=4, two dimensional central spatial solitons can be observed. Wave output and output profile are presented in figure 2. In figure 3a, output profile (shown by dotted line for A₀=7,2) is plotted for three different A₀ values implying that diffraction has dominated nonlinearity and consequently solitons have not been obtained. The output profile is also plotted for A₀=7,6 indicating that if the input amplitude exceeds a given value, it will lose its balance. Figure 3b shows peak intensity changes and propagation distance per different amplitudes. It also implies

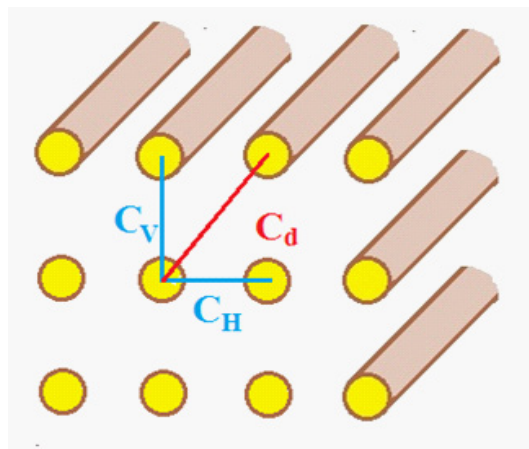


Fig. 1. Schematic diagram of two dimensional waveguide arrays

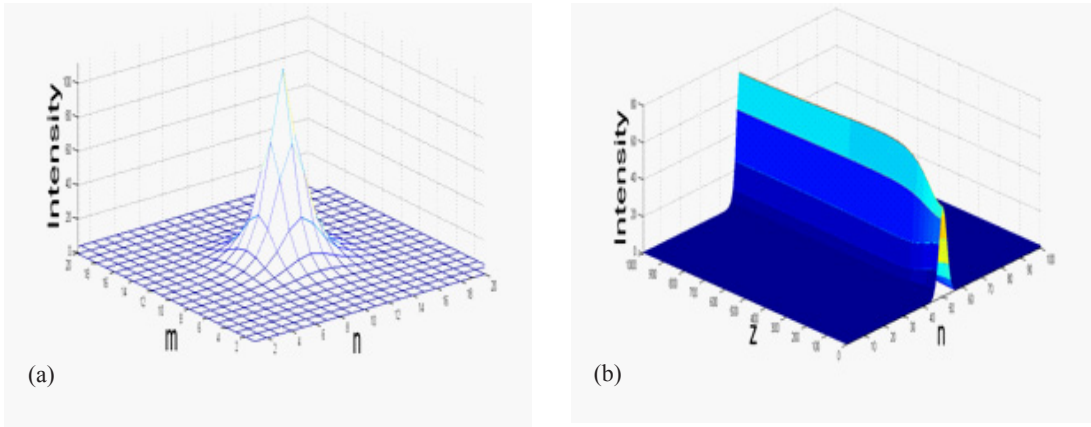


Fig. 2(a). Gaussian wave output; (b) changes of wave profile during propagation

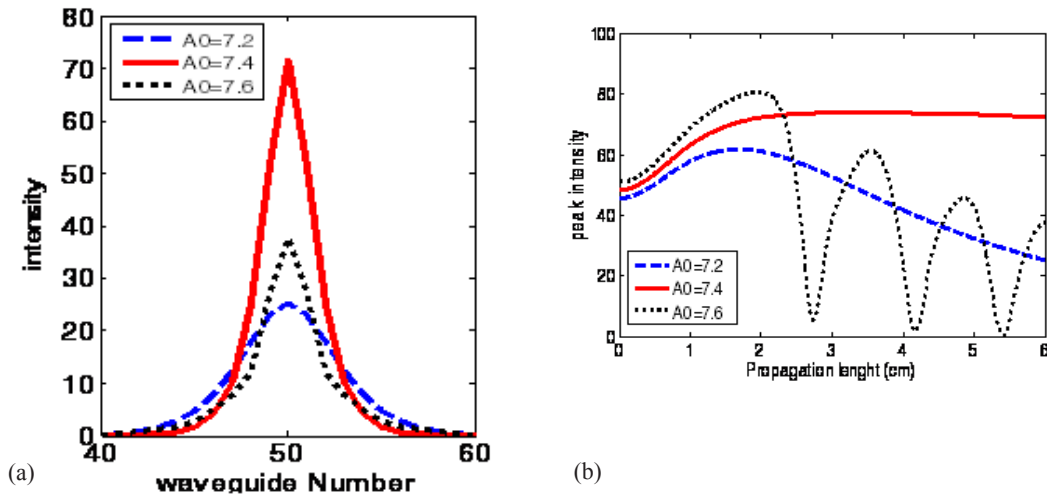


Fig. 3(a). Output profile, (b) changes of peak intensity and propagation distance (z) per different values of input amplitudes

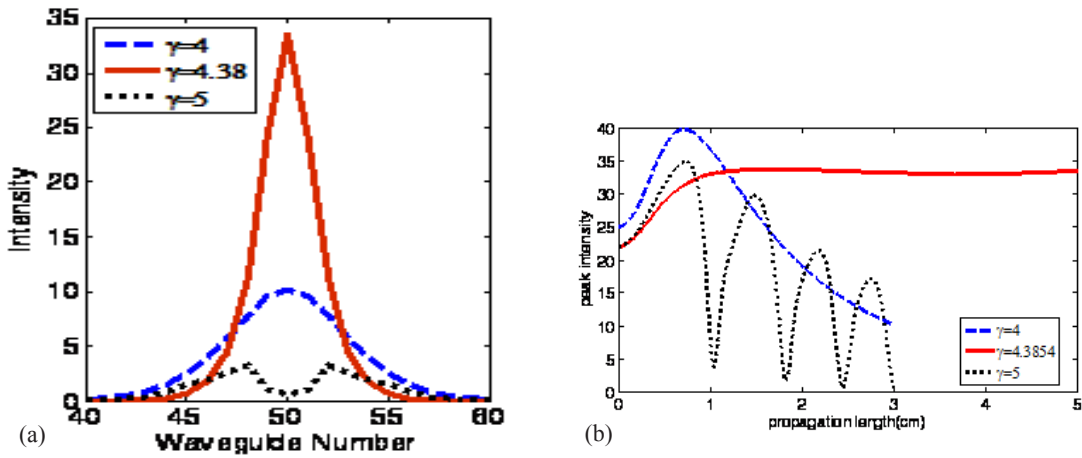


Fig. 4. (a) Output profile, (b) changes of peak intensity and propagation distance (z) per different values of nonlinear parameter (γ)

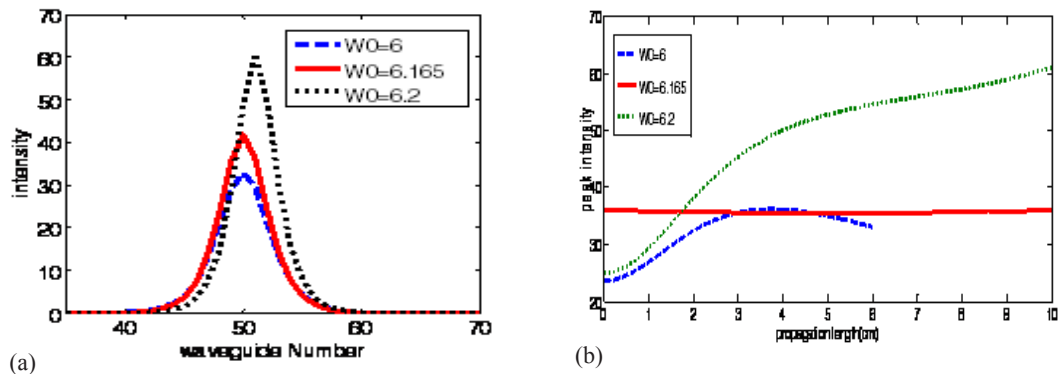


Fig. 5. (a) output profile, (b) changes of peak intensity with propagation distance (Z) per different values of Gaussian wave width

that in $A_0=7, 2$ the wave doesn't maintain its shape during propagation and gradually declines. In $A_0=7, 6$, with the increase in intensity, nonlinearity and diffraction contrast and expand as declining peaks. So that only in the certain value of $A_0=7, 4038$ diffraction and nonlinearity balance and travel their direction in a relatively straight line and maintain their shape and soliton state.

Investigating the effect of nonlinear Kerr Factor (γ)

This section deals with the investigation of nonlinear Kerr factor. To specify this factor, other parameters should be maintained constant. According to calculations, input intensity value was hold constant at $A_0=5$ and width of Gaussian wave at $w_0=4$. Then, γ was changed and it was observed that at the exact value of $\gamma=4,3854$, it can be considered as value of stable soliton. In figures 4a and b, output profile and changes of peak intensity are plotted per propagation distance for different values of nonlinear Kerr. A diagram is plotted as well for two values of less (dotted line) and more than soliton values. In figure 4a, the diagram has flattened; and in figure 4b, it declines during propagation and fully disappears at last. This is because of the fact that diffraction dominates nonlinearity. In figure 4a, the diagram loses its balance and in figure 4b, it is expanded as peaks having maximum and minimum points. In this diagram, maximum points are the result of nonlinear Kerr, and minimum points are the product of diffraction effect that finally has expanded and declined as peaks having maximum and minimum points and plotted only for the given value of $\gamma=4,3854$ as a straight line. Figure 4b shows a

Gaussian diagram that has maintained its shape during propagation and in figure 4b propagates in a relatively straight line along propagation distance indicating the balance between nonlinearity and diffraction. The diagram is the result of two spatial solitons.

Investigating the effect of Gaussian wave width (W_0)

In this section, the effect of Gaussian wave width on wave propagation is investigated. According to the equation (2), change in Gaussian wave width triggers the change in transverse distribution and consequently in wave's behavior. Calculations shows that if the wave enters the system with the constant value of $A_0=5$ and $\gamma=2$, by choosing the value of $w_0=6.165$, soliton will be obtained. Simulation results are presented in figure 5a and b. In figure 5b, output profile, and in figure 5b, changes of peak intensity per propagation distance are presented. The figure indicates that there is a decline in dotted $w_0=6.165$ till $z=6\text{cm}$; and in dotted $w_0=6.2$, self-focusing nonlinearity has dominated and with the increase in wave's concentration, peak intensity has ascended and only at the exact value of dotted $W_0=6.165$, it propagates as a straight line from the beginning to the end and has created a full balance between diffraction and nonlinearity; as a result, spatial soliton has been created.

CONCLUSION

In this study, for the first time, the effects of diagonal neighbors on spatial discrete soliton in a two dimensional waveguide with Kerr medium

are investigated.

Our findings showed that the spatial soliton can be created in two dimensional waveguide arrays by changing each of the parameters separately for the input beam that has been considered as Gaussian wave. In the predetermined values for nonlinear self-focusing Kerr (n_2), intensity (A_0), and Gaussian wave width (w_0) parameters, the model can obtain two dimensional discrete optical stable spatial soliton. For the values less than soliton, diffraction will dominate, whereas for the values more than soliton, the concentration of input Gaussian wave will increase.

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