Max-Planck-Institut
für Mathematik
in den Naturwissenschaften
Leipzig

Uncertainty relation based on Wigner-Yanase-Dyson skew information with quantum memory

by

Jun Li and Shao-Ming Fei

Preprint no.: 52 2018

Article

Uncertainty relation based on Wigner-Yanase-Dyson skew information with quantum memory

Jun Li [1] and Shao-Ming Fei [1,2*]

[1] School of Mathematical Sciences, Capital Normal University, Beijing 100048, China; lijunnl123@163.com
[2] Max-Planck-Institute for Mathematics in the Sciences, 04103 Leipzig, Germany;
* Correspondence: feishm@cnu.edu.com;

Version February 11, 2018 submitted to MDPI

Abstract: We present uncertainty relations based on Wigner-Yanase-Dyson skew information with quantum memory. Uncertainty inequalities both in product and summation forms are derived. It is shown that the lower bounds contain two terms: one characterizes the degree of compatibility of two measurements, one is the quantum correlation between the measured system and the quantum memory. Detailed examples are given for product, separable and entangled states.

Keywords: Uncertainty relation; Wigner-Yanase-Dyson skew information; Quantum memory

1. Introduction

Uncertainty principle is an essential feature of quantum mechanics, characterizing the experimental measurement incompatibility of non-commuting quantum mechanical observables in the preparation of quantum states. Heisenberg first introduced the variance-based uncertainty [1]. Later, Robertson [2] proposed the well-known formula of uncertainty relation, $V(\rho,R)V(\rho,S) \geq \frac{1}{4}|Tr\rho[R,S]|^2$, for arbitrary observables R and S, where $[R,S] = RS - SR$ and $V(\rho,R)$ is the standard deviation of R. Schrödinger gave a further improved uncertainty relation [3]

$$V(\rho,R)V(\rho,S) \geq \frac{1}{4}|\langle[R,S]\rangle|^2 + |\frac{1}{2}\langle\{R,S\}\rangle - \langle R\rangle\langle S\rangle|^2,$$

where $\langle R\rangle = Tr(\rho R)$ and $\{R,S\} = RS + SR$ is the anti-commutator. Since then many kinds of uncertainty relations have been presented [4,5,6,7,8,9,10,11]. In addition to the uncertainty of standard deviation, entropy can be used to quantify uncertainties [12]. The first entropic uncertainty relation was given by Deutsch [13] and then improved by Maassen and Uffink [14],

$$H(R) + H(S) \geq \log_2 \frac{1}{c},$$

where $R = \{|u_j\rangle\}$ and $S = \{|v_k\rangle\}$ are two orthonormal bases on d-dimensional Hilbert space H, and $H(R) = -\Sigma_j p_j log p_j$ ($H(S) = -\Sigma_k q_k log q_k$) is the Shannon entropy of the probability distribution $p_j = \langle u_j|\rho|u_j\rangle$ ($q_k = \langle v_k|\rho|v_k\rangle$) for state ρ of H. The number c is the largest overlap among all $c_{jk} = |\langle u_j|v_k\rangle|^2$ between the projective measurements R and S. Berta et al [15] bridged the gap between cryptographic scenarios and the uncertainty principle, and derived this landmark uncertainty relation for measurements R and S in the presence of quantum memory B:

$$H(R|B) + H(R|B) \geq \log_2 \frac{1}{c} + H(A|B),$$

where $H(R|B) = H(\rho_{RB}) - H(\rho_B)$ is the conditional entropy with $\rho_{RB} = \Sigma_j(|u_j\rangle\langle u_j| \bigotimes I)\rho_{AB}(|u_j\rangle\langle u_j| \bigotimes I)$ (similarly for $H(S|B)$), d is the dimension of the subsystem

A. The term $H(A|B) = H(\rho_{AB}) - H(\rho_B)$ appearing on the right-hand side is related to the entanglement between the measured particle A and the quantum memory B. The bound of Berta et al has been further improved [16,17,18]. Moreover, there are also some uncertainty relations given by the generalized entropies, such as the Rényi entropy [19,20,21] and the Tsallis entropy [22,23,24], and even more general entropies such that the (h, Φ) entropies [25]. These uncertainty relations not only manifest the physical implications of the quantum world, but also play roles in entanglement detection [26,27], quantum spin squeezing [28,29] and quantum metrology [30,31].

In [32], an uncertainty relation based on Wigner-Yanase skew information $I(\rho, H)$ has been obtained with quantum memory, where $I(\rho, H) = \frac{1}{2}Tr[(i[\sqrt{\rho}, H])^2] = Tr(\rho H^2) - Tr(\sqrt{\rho}H\sqrt{\rho}H)$ quantifies the degree of non-commutativity between a quantum state ρ and an observable H, which is reduced to the variance $V(\rho, H)$ when ρ is a pure state. In fact, the Wigner-Yanase skew information $I(\rho, H)$ is generalized to Wigner-Yanase-Dyson skew information $I_\alpha(\rho, H)$, $\alpha \in [0, 1]$, see [33],

$$\begin{aligned} I_\alpha(\rho, H) &= \frac{1}{2}Tr[(i[\rho^\alpha, H])(i[\rho^{1-\alpha}, H])] \\ &= Tr(\rho H^2) - Tr(\rho^\alpha H \rho^{1-\alpha} H), \qquad \alpha \in [0, 1]. \end{aligned} \tag{1}$$

Here the Wigner-Yanase-Dyson skew information $I_\alpha(\rho, H)$ reduces to the Wigner-Yanase skew information $I(\rho, H)$ when $\alpha = \frac{1}{2}$. And the Wigner-Yanase-Dyson skew information $I_\alpha(\rho, H)$ reduces to the standard deviation $V(\rho, H)$ when ρ is a pure state.

The convexity of $I_\alpha(\rho, H)$ with respect to ρ has been proven by Lieb in [34]. In [35] Kenjiro introduced another quantity,

$$\begin{aligned} J_\alpha(\rho, H) &= \frac{1}{2}Tr[(\{\rho^\alpha, H_0\})(\{\rho^{1-\alpha}, H_0\})] \\ &= Tr(\rho H_0^2) + Tr(\rho^\alpha H_0 \rho^{1-\alpha} H_0), \qquad \alpha \in [0, 1], \end{aligned} \tag{2}$$

where $H_0 = H - Tr(\rho H)I$ with I the identity operator.

For a quantum state ρ and observables R, S and $0 \leq \alpha \leq 1$, the following inequality holds [35],

$$U_\alpha(\rho, R)U_\alpha(\rho, S) \geq \alpha(1-\alpha)|Tr\rho[R, S]|^2, \tag{3}$$

where $U_\alpha(\rho, R) = \sqrt{I_\alpha(\rho, R)J_\alpha(\rho, R)}$ can be regarded as a kind of measure for quantum uncertainty, in the sense given by [35]. For a pure state, a standard deviation based relation is recovered from (3). When $\alpha = \frac{1}{2}$, it is reduced to the result of [36].

Inspired by the works [32] and [35], in this paper we study the uncertainty relations based on Wigner-Yanase-Dyson skew information in the presence of quantum memory, which generalize the results in [32] to the case of Wigner-Yanase-Dyson skew information, and the results in [35] to the case with the presence of quantum memory. We present uncertainty inequalities both in product and summation forms, and show that the lower bounds contain two terms: one concerns the compatibility of two measurement observables, one concerns the quantum correlations between the measured system and the quantum memory. We compare the lower bounds for product states, separable and entangled states by detailed examples.

2. Results

Let $\phi_k = |\phi_k\rangle\langle\phi_k|$ and $\psi_k = |\psi_k\rangle\langle\psi_k|$ be the rank one spectral projectors of two non-degenerate observables R and S with the eigenvectors $|\phi_k\rangle$ and $|\psi_k\rangle$, respectively. Similar to [32], we define $UN_\alpha(\rho, \phi) = \sum_k U_\alpha(\rho, \phi_k) = \sum_k \sqrt{I_\alpha(\rho, \phi_k)J_\alpha(\rho, \phi_k)}$ as the uncertainty of ρ associated to the projective measurement $\{\phi_k\}$, and $U_\alpha(\rho, \psi)$ to $\{\psi_k\}$.

Let ρ_{AB} be a bipartite state on $H_A \otimes H_B$, where H_A and H_B denote the Hilbert space of subsystem A and B respectively. Let V be any orthogonal basis space on H_A and $|\phi_k\rangle$ be an orthogonal basis of H_A. We define a quantum correlation of ρ_{AB} as:

$$\tilde{D}_\alpha(\rho_{AB}) = \min_V \sum_k [I_\alpha(\rho_{AB}, \phi_k \otimes I_B) - I_\alpha(\rho_A, \phi_k)] \tag{4}$$

where the minimum is taken over all the orthogonal basis on H_A, $\rho_A = Tr_B \rho_{AB}$.

For any bipartite state ρ_{AB} and any observable X_A on H_A, we have $I_\alpha(\rho_{AB}, X_A \otimes I_B) \geq I_\alpha(\rho_A, X_A)$, which follows form the Corollary 1.3 in [34] and the Lemma 2 in [37]. Therefore, $\tilde{D}_\alpha(\rho_{AB}) \geq 0$. Furthermore, $\tilde{D}_\alpha(\rho_{AB}) = 0$ when ρ_{AB} is a classical-quantum correlated state, which follows from the proof in Theorem 1 of [38]. $\tilde{D}_\alpha(\rho_{AB})$ has a measurement on subsystem A, which gives an explicit physical meaning: it is the minimal difference of incompatibility of the projective measurement on the bipartite state ρ_{AB} and on the local reduced state ρ_A. $\tilde{D}_\alpha(\rho_{AB})$ quantifies the quantum correlations between the subsystems A and B. We have

Theorem 1. *Let ρ_{AB} be a bipartite quantum state on $H_A \otimes H_B$, $\{\phi_k\}$ and $\{\psi_k\}$ be two sets of rank one projective measurements on H_A. Then*

$$UN_\alpha(\rho_{AB}, \phi \otimes I) UN_\alpha(\rho_{AB}, \psi \otimes I) \geq \sum_k L^2_{\alpha,\rho_A}(\phi_k, \psi_k) + \tilde{D}^2_\alpha(\rho_{AB}), \tag{5}$$

where $L_{\alpha,\rho_A}(\phi_k, \psi_k) = \alpha(1-\alpha)\frac{|Tr\rho_A[\phi_k,\psi_k]|^2}{\sqrt{J_\alpha(\rho_A,\phi_k)\cdot J_\alpha(\rho_A,\psi_k)}}$.

Proof of Theorem 1. By definition, we have

$$\begin{aligned}
&UN_\alpha(\rho_{AB}, \phi \otimes I) UN_\alpha(\rho_{AB}, \psi \otimes I) \\
&= \sum_k \sqrt{I_\alpha(\rho_{AB}, \phi_k \otimes I) \cdot J_\alpha(\rho_{AB}, \phi_k \otimes I)} \cdot \sum_k \sqrt{I_\alpha(\rho_{AB}, \psi_k \otimes I) \cdot J_\alpha(\rho_{AB}, \psi_k \otimes I)} \\
&\geq \sum_k I_\alpha(\rho_{AB}, \phi_k \otimes I) \cdot \sum_k I_\alpha(\rho_{AB}, \psi_k \otimes I) \\
&= [\sum_k (I_\alpha(\rho_{AB}, \phi_k \otimes I) - I_\alpha(\rho_A, \phi_k)) + \sum_k I_\alpha(\rho_A, \phi_k)] \\
&\cdot [\sum_k (I_\alpha(\rho_{AB}, \psi_k \otimes I) - I_\alpha(\rho_A, \psi_k)) + \sum_k I_\alpha(\rho_A, \psi_k)] \\
&\geq [\tilde{D}_\alpha(\rho_{AB}) + \sum_k I_\alpha(\rho_A, \phi_k)] \cdot [\tilde{D}_\alpha(\rho_{AB}) + \sum_k I_\alpha(\rho_A, \psi_k)] \\
&\geq \tilde{D}^2_\alpha(\rho_{AB}) + \sum_k I_\alpha(\rho_A, \phi_k) I_\alpha(\rho_A, \psi_k) \\
&\geq \tilde{D}^2_\alpha(\rho_{AB}) + \sum_k \frac{\alpha^2(1-\alpha)^2 |Tr\rho_A[\phi_k, \psi_k]|^4}{J_\alpha(\rho_A, \phi_k) J_\alpha(\rho_A, \psi_k)} \\
&\triangleq \tilde{D}^2_\alpha(\rho_{AB}) + \sum_k L^2_{\alpha,\rho_A}(\phi_k, \psi_k),
\end{aligned} \tag{6}$$

where the first inequality is due to $J_\alpha(\rho, H) \geq I_\alpha(\rho, H)$ [35], the last inequality follows form (3). □

Theorem 1 gives a product form of uncertainty relation. Comparing the results (3) without quantum memory with (5) with quantum memory, one finds that if the observables A and B satisfy $[A, B] = 0$, the bound is trivial in (3), while in (5), even if the projective measurements ϕ_k and ψ_k satisfy $[\phi_k, \psi_k] = 0$, i.e. $L_{\alpha,\rho_A}(\phi_k, \psi_k) = 0$, but $\tilde{D}_\alpha(\rho_{AB})$ may still be not trivial due to correlations between the system and the quantum memory.

Corresponding to the product form of uncertainty relation, we can also derive the sum form of uncertainty relation:

Theorem 2. *Let ρ_{AB} be a quantum state on $H_A \otimes H_B$, $\{\phi_k\}$ and $\{\psi_k\}$ be two sets of rank one projective measurements on H_A. Then*

$$UN_\alpha(\rho_{AB}, \phi \otimes I) + UN_\alpha(\rho_{AB}, \psi \otimes I) \geq 2\sum_k L_{\alpha,\rho_A}(\phi_k, \psi_k) + 2\tilde{D}_\alpha(\rho_{AB}), \tag{7}$$

Proof of Theorem 2. By definition and taking into account the fact that $J_\alpha(\rho, H) \geq I_\alpha(\rho, H)$ [35], we have

$$\begin{aligned}
&UN_\alpha(\rho_{AB}, \phi \otimes I) + UN_\alpha(\rho_{AB}, \psi \otimes I) \\
&= \sum_k \sqrt{I_\alpha(\rho_{AB}, \phi_k \otimes I) \cdot J_\alpha(\rho_{AB}, \phi_k \otimes I)} + \sum_k \sqrt{I_\alpha(\rho_{AB}, \psi_k \otimes I) \cdot J_\alpha(\rho_{AB}, \psi_k \otimes I)} \\
&\geq \sum_k I_\alpha(\rho_{AB}, \phi_k \otimes I) + \sum_k I_\alpha(\rho_{AB}, \psi_k \otimes I).
\end{aligned}$$

While

$$\begin{aligned}
&\sum_k I_\alpha(\rho_{AB}, \phi_k \otimes I) + \sum_k I_\alpha(\rho_{AB}, \psi_k \otimes I) \\
&= \sum_k I_\alpha(\rho_A, \phi_k) + \sum_k I_\alpha(\rho_A, \psi_k) + \sum_k [I_\alpha(\rho_{AB}, \phi_k \otimes I) - I_\alpha(\rho_A, \phi_k)] \\
&+ \sum_k [I_\alpha(\rho_{AB}, \psi_k \otimes I) - I_\alpha(\rho_A, \psi_k)] \\
&\geq \sum_k I_\alpha(\rho_A, \phi_k) + \sum_k I_\alpha(\rho_A, \psi_k) + 2\tilde{D}_\alpha(\rho_{AB}),
\end{aligned}$$

where the inequality follow from (4). By using the inequality $a + b \geq 2\sqrt{ab}$ for positive $a = I_\alpha(\rho_A, \phi_k)$ and $b = I_\alpha(\rho_A, \psi_k)$, we further obtain

$$\begin{aligned}
&UN_\alpha(\rho_{AB}, \phi \otimes I) + UN_\alpha(\rho_{AB}, \psi \otimes I) \\
&\geq 2\sum_k \sqrt{I_\alpha(\rho_A, \phi_k) \cdot I_\alpha(\rho_A, \psi_k)} + 2\tilde{D}_\alpha(\rho_{AB}) \\
&\geq 2\sum_k \alpha(1-\alpha)\frac{|Tr\rho_A[\phi_k, \psi_k]|^2}{\sqrt{J_\alpha(\rho_A, \phi_k) \cdot J_\alpha(\rho_A, \psi_k)}} + 2\tilde{D}_\alpha(\rho_{AB}) \\
&\triangleq 2\sum_k L_{\alpha,\rho_A}(\phi_k, \psi_k) + 2\tilde{D}_\alpha(\rho_{AB}),
\end{aligned} \tag{8}$$

where the second inequality follows from (3). □

We note that (7) reduces to an inequality which agrees with the result of [32] when $\alpha = \frac{1}{2}$. Theorem 2 is a generalization of the Theorem in [32].

From Theorem 1 and 2, we obtain uncertainty relations in the form of product and sum of skew information, which is different from the uncertainty of [39], which only deals with single partite state. However, we treat the bipartite case with a quantum memory B. It is shown that the lower bound contains two terms: one is the quantum correlation $\tilde{D}_\alpha(\rho_{AB})$, the other is $\sum_k L_{\alpha,\rho_A}(\phi_k, \psi_k)$ which characterizes the degree of compatibility of two measurements, just like the meaning of $\log_2 \frac{1}{c}$ in the entropy uncertainty relation [15].

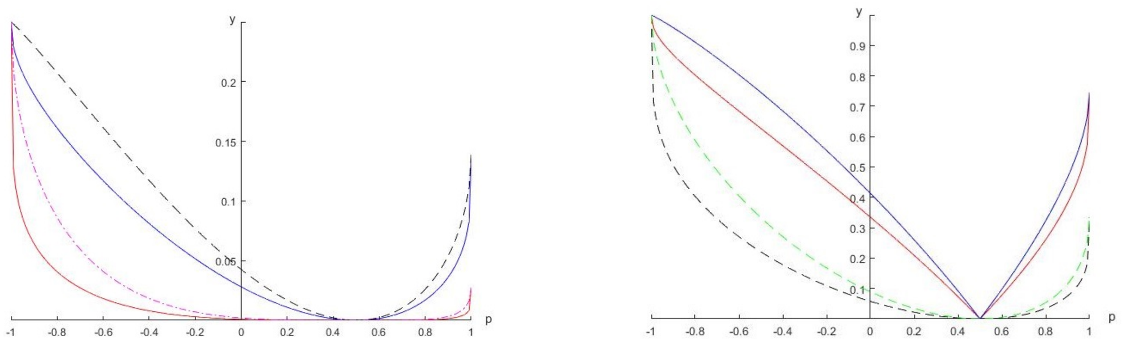
Example 1. *We consider the 2-qubit Werner state* $\rho = \frac{2-p}{6} I + \frac{2p-1}{6} V$*, where* $p \in [-1, 1]$ *and* $V = \sum_{kl} |kl\rangle\langle lk|$*. Let the Pauli matrices* σ_x *and* σ_z *be the two observables, and* $\{|\psi_k\rangle\}$ *and* $\{|\varphi_k\rangle\}$ *be the eigenvectors of* σ_x *and* σ_z *respectively, which satisfy* $|\langle\psi_i|\varphi_j\rangle|^2 = \frac{1}{2}$*,* $i, j = 1, 2$*. For all k, we have* $Tr\rho_A[\psi_k, \varphi_k] = 0$*, i.e.* $L_{\alpha,\rho_A}(\psi_k, \varphi_k) = 0$*. The values of the left hand side and the right hand side of (5) are given by*

$$4\left(\frac{2-p}{12} - \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{24}\right) \times \left(\frac{4+p}{12} + \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{24}\right)$$

and

$$\left(\frac{2-p}{6} - \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{12}\right)^2$$

respectively, see Figure. 1(a) for the uncertainty relations with different values of α*.*



(a) (b)

Figure 1. The axis y is the uncertainty and its lower bounds. (a) Blue (red) solid line for the value of the left (right) hand side of (5) with $\alpha = 0.2$; black dotted (red dotdashed) line represents the value of the left (right) hand side of (5) with $\alpha = 0.5$. (b) Red solid (black dotted) line represents the value of the left (right) hand side of (7) with $\alpha = 0.2$; blue solid (green dotted) line represents the value of the left (right) hand side of (7) with $\alpha = 0.5$, which corresponds to Fig. 1 in [32].

Similarly, we can get the values of the left and the right hand sides of (7),

$$4\sqrt{\left(\frac{2-p}{12} - \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{24}\right)} \times \sqrt{\left(\frac{4+p}{12} + \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{24}\right)}$$

and

$$\frac{2-p}{3} - \frac{(3-3p)^{\alpha}(1+p)^{1-\alpha} + (1+p)^{\alpha}(3-3p)^{1-\alpha}}{6},$$

respectively, see Figure. 1(b).

Here we see explicitly that, just like the Shannon entropy, Rényi entropy, Tsallis entropy, (h, Φ) entropies and Wigner-Yanase skew information, the Wigner-Yanase-Dyson skew information characterizes a special kind of information of a system or measurement outcomes, which needs to satisfy certain restrictions for given measurements and correlations between the system and the memory. Different parameter α gives rise to different kind of information. From Figure 1 we see that for given state and measurements, the differences between the left and the right hand sides of the inequalities (5) or (7) varies with the parameter α. Moreover, the degree of compatibility of the two measurements, $L_{\alpha,\rho_A}(\phi_k, \psi_k)$, vanishes for $\alpha = 0$ or 1, which is a fact in accordance with (3), the case without quantum memory. For $p = 1/2$, the state ρ is maximally mixed. In this case, both sides of the inequalities (5) and (7) vanishes for any α.

Example 2. *Consider a separable bipartite state, $\rho^{AB} = \frac{1}{2}[|+\rangle\langle+| \otimes |0\rangle\langle 0| + |-\rangle\langle-| \otimes |1\rangle\langle 1|]$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.*

We still choose σ_x and σ_z to be the two observables. By calculation we get: For product states $|+\rangle\langle+| \otimes |0\rangle\langle 0|$ and $|-\rangle\langle-| \otimes |1\rangle\langle 1|$, both left and right hand sides of (5) are 0, and the right hand side of (7) is 0. And for the separable bipartite state ρ^{AB}, the left hand and the right hand sides of (5) are $\frac{1}{2}$ and 0, respectively. Both left and right hand sides of (7) are 0.

Example 3. *For the Werner state $\rho_w^{AB} = (1-p)\frac{I}{4} + p|\varphi\rangle\langle\varphi|$, where $|\varphi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is the Bell state, $p \in [0, 1]$, and the state is separable when $p \leq \frac{1}{3}$.*

We have the values of the left and right hand sides of (5), respectively,

$$4\left(\frac{1+p}{8} - \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{16}\right)$$
$$\times\left(\frac{3-p}{8} + \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{16}\right)$$

and

$$4\left(\frac{1+p}{8} - \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{16}\right)^2,$$

see Figure. 2(a) for a comparison with different values of α.

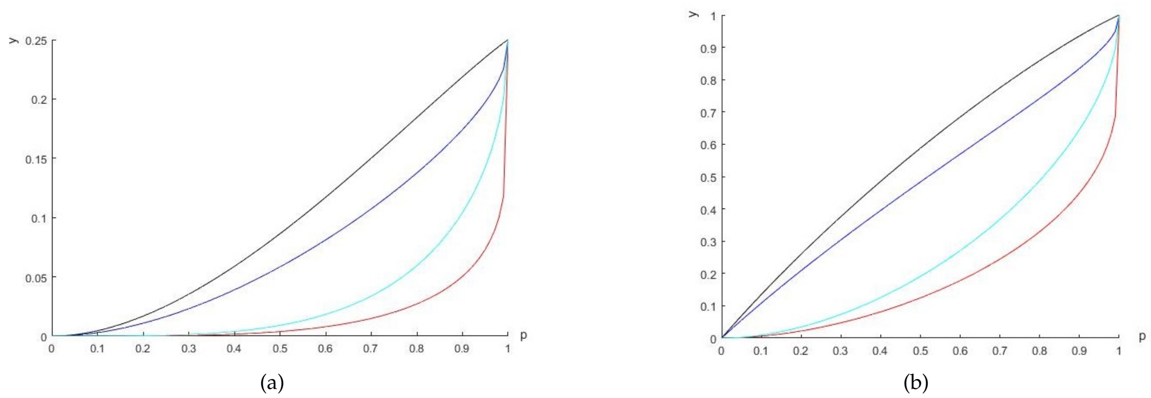


Figure 2. The axis y is the uncertainty and the lower bounds. (a) blue (red) solid line is the value of the left (right) hand side of (5) for $\alpha = 0.2$; black (blue-green) solid line represents the value of the left (right) hand side of (5) for $\alpha = 0.5$. (b) the blue (red) solid line represents value of the left (right) hand side of (7) for $\alpha = 0.2$; black (blue-green) solid line represents the value of the left (right) hand side of (7) for $\alpha = 0.5$.

We can also get the values of the left and right hand sides of (7),

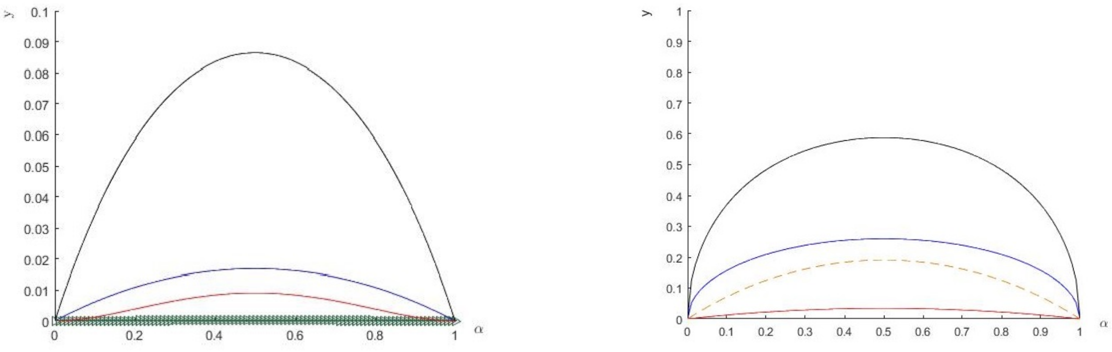
$$4\sqrt{\frac{1+p}{8} - \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{16}}$$
$$\times\sqrt{\frac{3-p}{8} + \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{16}}$$

and

$$\frac{1+p}{2} - \frac{(1-p)^{\alpha}(1+3p)^{1-\alpha} + (1-p)^{1-\alpha}(1+3p)^{\alpha}}{4}$$

respectively, see Figure. 2(b).

Moreover, when ρ_w^{AB} is separable, namely, $p \leq \frac{1}{3}$, the differences between the left and the right sides of the inequalities is smaller than that of the entangled states. Figure. 3 shows the differences for different values of p.



(a) (b)

Figure 3. The axis y is the uncertainty and its lower bound. (a) $p = 0.2$ (ρ_w^{AB} is a separable state), blue solid line represents the value of the left hand side of (5), the line (very near the x-axis) marked by triangles represents the corresponding lower bound; $p = 0.5$ (ρ_w^{AB} is an entangled state), the black (red) solid line represents the value of the left (right) hand side of (5). (b) blue (red) solid line represents the value of the left (right) hand side of (7) for $p = 0.2$; black solid (red dashed) line represents the value of the left (right) hand side of (7) for $p = 0.5$.

3. Conclusion

We have investigated the uncertainty relations both in product and summation forms in terms of the Wigner-Yanase-Dyson skew information with a quantum memory. It has been shown that the lower bounds contain two terms: one is the quantum correlation $\tilde{D}_\alpha(\rho_{AB})$, the other is $\sum_k L_{\alpha,\rho_A}(\phi_k, \psi_k)$ which characterizes the degree of compatibility of two measurements. By detailed examples we have compared the lower bounds for product states, separable and entangled states.

Acknowledgments: This work is supported by the NSF of China under Grant No. 11675113.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] W. Heisenberg, Z. Phys. **43**, 172-198 (1927).
[2] H. P. Robertson, Science, **73**, 65-98 (1929).
[3] E. Schrödinger, Berl. Ber. **19**, 296-303 (1930).
[4] P. Busch, P. Lahti, and R. F. Werner, Phys. Rev. Lett. **111**, 160405 (2013).
[5] P. Busch, P. Lahti, and R. F. Werner, Phys. Rev. A **89**, 012129 (2014).
[6] G. Sulyok, S. Sponar, B. Demirel, F. Buscemi, M. J. W. Hall, M. Ozawa, and Y. Hasegawa, Phys. Rev. Lett. **115**, 030401 (2015).
[7] W. Ma, Z. Ma, H. Wang, Y. Liu, Z. Chen, F. Kong, Z. Li, M. Shi, F. Shi, S.-M. Fei, and J. Du, Phys. Rev. Lett. **116**, 160405 (2016).
[8] Z. Puchała, Ł. Rudnicki, and K. Zyczkowski, J. Phys. A: Math. Theor. **46**, 272002 (2013).
[9] S. Friedland, V. Gheorghiu, and G. Gour, Phys. Rev. Lett. **111**, 230401 (2013).
[10] L. Maccone and A. K. Pati, Phys. Rev. Lett. **113**, 260401 (2014).

[11] W.C. Ma, B. Chen, Y. Liu, M.Q. Wang, X.Y. Ye, F. Kong, F.Z. Shi, S.M. Fei, and J.F. Du, Phys. Rev. Lett. **118**, 180402 (2017).
[12] P. J. Coles, M. Berta, M. Tomamichel, and S. Wehner, Rev. Mod. Phys. **89**, 015002 (2017).
[13] D. Deutsch, Phys. Rev. Lett. **50**, 631 (1983).
[14] H. Maassen and J. B. M. Uffink, Phys. Rev. Lett. **60**, 1103 (1988).
[15] M. Berta, M. Christandl, R. Colbeck, J. Renes, and R. Renner, Nat. Phys. **6**, 695 (2010).
[16] P. J. Coles and M. Piani, Phys. Rev. A **89**, 022112 (2014).
[17] Ł. Rudnicki, Z. Puchała, and K. Życzkowski, Phys. Rev. A **89**, 052115 (2014).
[18] Y. Xiao, N. Jing, S. M. Fei, and X. Li-Jost, J. Phys. A: Math. Theor. **49**, 49LT01 (2016).
[19] E. Romera and L. S. F. De, Phys. Rev. A **78**, 013837 (2008).
[20] I. Bialynicki-Birula, Phys. Rev. A **74**, 052101 (2012).
[21] J. Zhang, Y. Zhang, and C. S. Yu, Quantum Inf. Process, **14**: 2239-2253 (2015).
[22] A. E. Rastegin, Quantum Inf. Process, **12**: 2947-2963 (2013).
[23] Alexey and Rastegin, Commun. Theor. Phys. **63**, 687-694 (2015).
[24] D. Kurzyk, Łukasz Pawela, and Z. Puchała, arXiv: 1707. 09278 (2017).
[25] S. Zozor, G. M. Bosyk, and M. Portesi, J. Phys. A: Math. Theor. **47**, 495302 (2014).
[26] R. Prevedel, D. R. Hamel, and R. Colbeck, Nat. Phys. **7**, 757-761 (2011).
[27] H.F. Hofmann, and S. Takeuchi, Phys. Rev. A **68**, 032103 (2003).
[28] D.F. Walls, and P. Zoller, Phys. Rev. Lett. **47**, 709-711 (1981).
[29] M. Jian, X. Wang, and C.P. Sun, Physics Reports, **509**, 89-165 (2011).
[30] V. Giovannetti, S. Lloyd, and L. Maccone, Science, **306**, 1330 (2004).
[31] V. Giovannetti, S. Lloyd, and L. Maccone, Nature Photonics, **96**, 222-229 (2011).
[32] Z.H. Ma, Z.H. Chen, and S.M. Fei, Sci. China Phys. Mech. & Astron. **60**, 010321 (2017).
[33] E.P. Wigner, and M.M. Yanase, Proceedings of the National Academy of Sciences of the United States of America, **49**, 910-918 (1963).
[34] E.H. Lieb, Advances in Mathematics, **11**, 267-288 (1973).
[35] K. Yanagi, J. Math. Anal. Appl. **365**, 12-18 (2010).
[36] S. Luo, Phys. Rev. A **72**, 042110 (2005).
[37] S. Luo, Journal of Statistical Physics, **128**, 1177-1188 (2007).
[38] S. Luo, S. Fu, and C.H. Oh, Phys. Rev. A **85**, 32117 (2012).
[39] B. Chen and S. M. Fei, Sci. Rep. **5**, 14238 (2015).