Distributed Double-Differential Modulation for Cooperative Communications under CFO

Alfonso Cano†, Eduardo Morgado‡, Antonio Caamaño‡, Fco. Javier Ramos‡
†Dept. of ECE, University of Minnesota, MN, USA
‡Dept. of TSC, ETSIT-URJC, Fuenlabrada, Madrid, Spain
Email: {alfonso}@umn.edu, {eduardo.morgado, antonio.caamano, javier.ramos}@urjc.es

***Abstract*— When a terminal is recruited to cooperate with other neighboring terminals, its channel state and carrier frequency offset (CFO) may be unknown to the destination. Under these circumstances, this paper considers the use of distributed double-differential (DD) modulation, which simplifies receiver implementation because it by-passes channel and CFO estimation. Two double-differential codecs are proposed transmitting: i) across orthogonal channels using time-division multiplexing, achieving rate and error performance similar to that of co-located multi-antenna DD systems; or ii) simultaneously, benefiting from the distinct CFOs across terminals and bypassing the need of ordering protocols. Both (i)-(ii) approaches are considered in adaptive- and selective-relaying cooperation protocols demonstrating that maximum spatial diversity is achievable. Simulations corroborate the theoretical error performance claims.**

I. Introduction

User cooperation considers multiple terminals share data packets to form a distributed multi-antenna system [1], [2]. The objective is to enable spatial diversity along with resilience against shadowing and coverage enhancement. Thus, cooperation is particulary attractive for multi-access and adhoc networks. However, cooperative communications face several operation challenges from scheduling and signal processing perspectives that differentiate them from point-to-point or co-located multi-antenna links. As an example, whenever a terminal is recruited to cooperate with other neighboring terminals, efficient protocols for transmission ordering and orthogonal channels are required in many cases [1], [2]; moreover accurate per-terminal channel knowledge and synchronization are to be obtained [3]. Typically, these problems are overcome with signaling to estimate these parameters, which becomes particulary costly both from terminal and network perspectives, specially if many low-cost terminals are cooperating [1].

The present work considers the problem of designing cooperation protocols which do not require knowledge of the channel nor carrier frequency offset (CFO) at the destination. The presence of CFO between transmitter and receiver can be caused by: i) relative movement between transmitter and receiver, which causes a Doppler shifts [4]; and ii) drifts in transmitter and receiver oscillators [5]. When considered in distributed scenarios, point-to-point techniques that mitigate CFO, such as those based on harmonic recovering, may not be effective when simultaneous transmissions are considered [6]. Even with orthogonal channels, these techniques need to be re-stated whenever a new user joins cooperation. Instead of retrieved, the CFO can be by-passed using double-differential (DD) modulations as described in [7], [5]. The DD modulator at the transmitter defines a recursion such that detection at the receiver can be accomplished only using previously received symbols. DD demodulation features low complexity and thus motivates its consideration in distributed systems.

Using multi-antenna DD coding ideas in [5], we design a distributed DD modulator which cooperatively transmit the signals: i) through orthogonal channels using time-division multiplexing (TDM); or ii) simultaneously. The TDM approach is described in Section III and achieves rate and error performance similar to that of co-located multi-antenna systems, and can be understood as its natural distributed implementation [5]. Simultaneous transmissions are described in Section IV and assume the same coded block is transmitted by all terminals, thus not requiring terminals to be ordered. In Section V we analyze the performance of both transmission models and generalize them to either Selective Relaying (SR) or Link-Adaptive Regeneration (LAR) [2], [8]. In Section VI simulated results compare these strategies.

Notation: Upper (lower) bold face letters will be used for matrices (column vectors); calligraphic letters will be used for sets; $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^{\mathcal{H}}$ are the transpose, conjugate and transpose conjugate (hermitian) of a vector; $[\cdot]_k$ is the kth entry of a vector; $\otimes$ denotes Kroneker product; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{1}_N$ ($\mathbf{1}_{N\times M}$) is the $N \times 1$ ($N \times M$) all-one vector; $\mathbf{0}_N$ ($\mathbf{0}_{N\times M}$) is the $N \times 1$ ($N \times M$) all-zero vector; $\text{diag}(\mathbf{X}_1, \ldots, \mathbf{X}_N)$ is a matrix with matrices $\mathbf{X}_1, \ldots, \mathbf{X}_N$ in its diagonal; $\mathbf{D}_\mathbf{x}$ is a diagonal matrix with the elements of vector $\mathbf{x}$ in its diagonal; $\|\cdot\|$ is the Frobenius norm; $\mathbf{F}_N$ is the $N \times N$ Fast Fourier Transform (FFT) matrix with $[\mathbf{F}_N]_{k+1,n+1} := N^{-1/2}\exp(2\pi nk/N)$; and $\mathcal{CN}(\mu, \sigma^2)$ denotes the complex Gaussian distribution with mean μ and variance σ^2.

II. Channel Model

We consider a set of R terminals $\{T_r\}_{r=1}^R$ that collaborate to send a block of symbols $\mathbf{s}$ to an access point or destination (D). Vector $\mathbf{s}$ can be made available to $\{T_r\}_{r=1}^R$ through a previous *broadcasting phase*. For simplicity in explanation, we assume no transmission error occurred during the broadcasting phase and so $\mathbf{s}$ is the same in all terminals. Section V will

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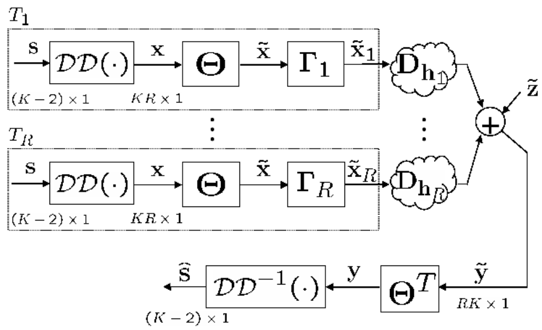

Fig. 1. System model

discuss system performance if one assumes $\mathbf{s}$ can be wrongly estimated at $\{T_r\}_{r=1}^R$.

We consider that terminals $\{T_r\}_{r=1}^R$ suffer independent pairwise channel fades and CFO with respect to (wrt) that of the destination D. The equivalent discrete-time low-pass channel of link $T_r - D$ at the nth sampling instant is modelled as:

$$h_r(n) = e^{j\omega_r n} h_r \quad r \in [1, R], n \in [0, N-1] \tag{1}$$

and h_r and ω_r are, respectively, the channel fading coefficient and the normalized CFO for link $T_r - D$. In (1) we assume operation under the following assumptions:

A1. *The fading channel h_r between T_r and D is quasi-static, flat over N symbols and can be modelled as a complex zero-mean Gaussian random variable; i.e., $h_r \sim \mathcal{CN}(0, \sigma_{h_r}^2 \bar{\gamma})$*

A2. *The normalized CFO ω_r between T_r and D is assumed to remain constant along the duration of the transmission and is $\omega_r = 2\pi T_s f_r$, where f_r is the physical frequency offset in Herzs, $f_r \in (-1/2T_s, 1/2T_s]$, and T_s is the sampling period in seconds.*

Given these assumptions, we are challenged to design a modulation strategy at $\{T_r\}_{r=1}^R$ such that detection at the destination can be accomplished without knowledge of either h_r or ω_r $\forall r \in [1, R]$. We are also interested in exploiting the maximum degrees of freedom the independent fades enable; i.e. our modulation scheme should be also designed so as to achieve the maximum spatial diversity enabled by the distributed set-up.

III. TDM TRANSMISSIONS

The baseband-sampled-equivalent system model is depicted in Fig. 1. We will start from the outer to the inner encoders at the transmitter side, and proceed through the channel to the inner and outer decoders at the receiver.

A block $\mathbf{s}$ of $K-2$ information symbols arrives to the DD modulator. Each element of this block belongs to a set $\mathcal{A}_s$ with cardinality $|\mathcal{A}_s| = M$ and thus transports $\log_2(M)$ information bits. Terminal T_r maps each element of $\mathbf{s}$, say the kth, to an unitary diagonal matrix $\mathbf{D}_{\mathbf{v}_k}$ size $R \times R$ picked from a constellation $\mathcal{V}$ with cardinality $|\mathcal{V}| = |\mathcal{A}_s| = M$. Matrix $\mathbf{D}_{\mathbf{v}_k}$ is used to yield double-differentially encoded blocks $\mathbf{x}_k$ according to the two following recursions:

$$\mathbf{x}_k = \begin{cases} \mathbf{D}_{\mathbf{g}_k}\mathbf{x}_{k-1}, & k = 3, \ldots, K \\ \mathbf{1}_R, & k = 1, 2 \end{cases} \tag{2}$$

with

$$\mathbf{g}_k = \begin{cases} \mathbf{D}_{\mathbf{v}_k}\mathbf{g}_{k-1}, & k = 3, \ldots, K \\ \mathbf{1}_R, & k = 2 \end{cases} . \tag{3}$$

Design conditions for $\mathbf{D}_{\mathbf{v}_k}$ will be detailed in Section V, devoted to system performance. For now, we only constrain $\mathbf{D}_{\mathbf{v}_k}$ to be diagonal with unitary entries. Symbols $\mathbf{x}_k$ are concatenated to build a block $\mathbf{x} := [\mathbf{x}_1^T, \ldots, \mathbf{x}_K^T]^T$ size $KR \times 1$ that is block-interleaved using a matrix $\mathbf{\Theta}$ to form $\tilde{\mathbf{x}} := \mathbf{\Theta}\mathbf{x}$. The $KR \times KR$ matrix $\mathbf{\Theta}$ is defined so that $[\tilde{\mathbf{x}}]_{(r-1)K+k} = [\mathbf{x}]_{(k-1)R+r}$ and can be compactly written as

$$\mathbf{\Theta} := [\mathbf{I}_R \otimes \mathbf{e}_1, \ldots, \mathbf{I}_R \otimes \mathbf{e}_K] \tag{4}$$

with $\mathbf{e}_k$ the kth column in $\mathbf{I}_K$. The interleaved block $\tilde{\mathbf{x}}$ is parsed through a $KR \times KR$ multiplexing matrix

$$\mathbf{\Gamma}_r := \text{diag}(\mathbf{0}_{(r-1)K\times(r-1)K}, \mathbf{I}_K, \mathbf{0}_{(R-r)K\times(R-r)K}) \tag{5}$$

forming $\tilde{\mathbf{x}}_r := \mathbf{\Gamma}_r\mathbf{x}$. Note that $\mathbf{\Gamma}_r$ depends on r and so does $\tilde{\mathbf{x}}_r$. Blocks $\tilde{\mathbf{x}}_r$ are transmitted through the channel. The received signal, which we name $\mathbf{y}$, is the noisy superposition of R signals from $\{T_r\}_{r=1}^R$. The overall input/output (I/O) can be thus written as (c.f. Fig. 1):

$$\tilde{\mathbf{y}} = \sum_{r=1}^{R} \mathbf{D}_{\mathbf{h}_r}\tilde{\mathbf{x}}_r + \tilde{\mathbf{z}} \tag{6}$$

where $\mathbf{D}_{\mathbf{h}_r} := \text{diag}([h_r(1), \ldots, h_r(KR)])$, with $h_r(n)$ as defined in (1). The noise term $\tilde{\mathbf{z}}_r = [z_r(1), \ldots, z_r(KR)]^T$ has entries $z_r(n)$ modelled to be average white gaussian noise (AWGN) $z_r(n) \sim \mathcal{CN}(0, 1)$. With reference to Fig. 1, we parse $\tilde{\mathbf{y}}$ through the de-interleaving matrix $\mathbf{\Theta}^T$ to form $\mathbf{y} := \mathbf{\Theta}^T\tilde{\mathbf{y}}$. Using the definitions of $\mathbf{\Theta}$ and $\mathbf{\Gamma}_r$ in (4) and (5), respectively, we can write $\mathbf{y}$ as

$$\mathbf{y} := \sum_{r=1}^{R} \mathbf{\Theta}^T\mathbf{D}_{\mathbf{h}_r}\mathbf{\Gamma}_r\mathbf{\Theta}\mathbf{x} + \mathbf{\Theta}^T\tilde{\mathbf{z}} = \mathbf{D}_{\mathbf{h}}\mathbf{x} + \mathbf{z} \tag{7}$$

with $\mathbf{z} := \mathbf{\Theta}^T\tilde{\mathbf{z}}$, still AWGN (permutations do not affect its distribution), and $\mathbf{D}_{\mathbf{h}} := \text{diag}(\mathbf{D}_{\mathbf{h}_1}, \ldots, \mathbf{D}_{\mathbf{h}_K})$ a $KR \times KR$ diagonal matrix with elements $[\mathbf{D}_{\mathbf{h}_k}]_{r,r} := e^{j\omega_r((r-1)K+k)}h_r$. We partition $\mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_K^T]^T$ in K sub-blocks size $R \times 1$ each. Considering (7), and recalling that $\mathbf{x} := [\mathbf{x}_1^T, \ldots, \mathbf{x}_K^T]^T$, one can write each sub-block $\mathbf{y}_k$ as a function of $\mathbf{x}_k$ as:

$$\mathbf{y}_k = \mathbf{D}_h\mathbf{D}_\omega\mathbf{D}_{\omega_k}\mathbf{x}_k + \mathbf{z}_r \tag{8}$$

where we conveniently rewrote $\mathbf{D}_{\mathbf{h}_k}$ using $[\mathbf{D}_h]_{r,r} := h_r$, $[\mathbf{D}_\omega]_{r,r} := e^{j\omega_r((r-1)K)}$ and $[\mathbf{D}_{\omega_k}]_{r,r} := e^{j\omega_r k}$. As mentioned in [5] in the context of double-differential designs for multi-antenna systems, the maximum-likelihood (ML) decoder for $\mathbf{D}_{\mathbf{v}_k}$ in (8) may depend on the frequency offsets. To avoid this problem, we search for an *heuristic* suboptimal and simple detector that decode $\mathbf{D}_{\mathbf{v}_k}$ by-passing CFO knowledge, and

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whose performance is surprisingly close to that of the ML detector [7]. For that mean, and observing the construction of $\mathbf{D}_{\mathbf{x}_k}$ in (2)-(3), we can conveniently rewrite three consecutive transmitted blocks $\mathbf{y}_k$, $\mathbf{y}_{k-1}$, $\mathbf{y}_{k-2}$ as

$$\begin{aligned}\mathbf{y}_k &= \mathbf{D}_h\mathbf{D}_\omega\mathbf{D}_{\omega_{k-1}}\mathbf{D}_{\omega_1}\mathbf{D}^2_{\mathbf{g}_{k-1}}\mathbf{D}_{\mathbf{v}_k}\mathbf{x}_{k-2}+\mathbf{z}_k\\ \mathbf{y}_{k-1} &= \mathbf{D}_h\mathbf{D}_\omega\mathbf{D}_{\omega_{k-1}}\mathbf{D}_{\mathbf{g}_{k-1}}\mathbf{x}_{k-2}+\mathbf{z}_{k-1}\\ \mathbf{y}_{k-2} &= \mathbf{D}_h\mathbf{D}_\omega\mathbf{D}_{\omega_{k-1}}\mathbf{D}_{\omega_{-1}}\mathbf{x}_{k-2}+\mathbf{z}_{k-2}. \end{aligned} \quad (9)$$

It becomes evident that $\mathbf{D}_{\mathbf{y}_k}\mathbf{y}^*_{k-1} = \mathbf{D}_\mathbf{v}\mathbf{D}_{\mathbf{y}_{k-1}}\mathbf{y}^*_{k-2} + \mathbf{z}'_k$, where

$$\mathbf{z}'_k = \mathbf{D}_{\mathbf{z}_k}\mathbf{y}^*_{k-1} + \mathbf{D}^*_{\mathbf{z}_{k-1}}\mathbf{y}_k + \mathbf{D}^*_{\mathbf{z}_{k-1}}\mathbf{z}_k - \mathbf{D}_\mathbf{v}\Big[\mathbf{D}_{\mathbf{z}_{k-1}}\mathbf{y}^*_{k-2} + \mathbf{D}^*_{\mathbf{z}_{k-2}}\mathbf{y}_{k-1} + \mathbf{D}^*_{\mathbf{z}_{k-2}}\mathbf{z}_{k-1}\Big]. \quad (10)$$

We discard high-order noise terms and approximate $\mathbf{z}'_k$ to be gaussian with covariance matrix $\mathbf{\Sigma}_k := E[\mathbf{z}'_k\mathbf{z}'^{\mathcal{H}}_k] = \Big(\mathbf{D}_{\mathbf{y}_k}\mathbf{D}^*_{\mathbf{y}_k} + 2\mathbf{D}_{\mathbf{y}_{k-1}}\mathbf{D}^*_{\mathbf{y}_{k-1}} + \mathbf{D}_{\mathbf{y}_{k-2}}\mathbf{D}^*_{\mathbf{y}_{k-2}}\Big)$. This gaussian approximation, also considered in [5], enables us to write the following *heuristic* detector for $\mathbf{D}_{\mathbf{v}_k}$ from (9), given by:

$$\hat{\mathbf{D}}_{\mathbf{v}_k} = \arg\min_{\mathbf{D}_\mathbf{v}\in\mathcal{V}}\Big\{\|\mathbf{\Sigma}_k^{-1/2}\left(\mathbf{D}_{\mathbf{y}_k}\mathbf{y}^*_{k-1} - \mathbf{D}_\mathbf{v}\mathbf{D}_{\mathbf{y}_{k-1}}\mathbf{y}^*_{k-2}\right)\|^2\Big\}. \quad (11)$$

De-mapping $\hat{\mathbf{D}}_{\mathbf{v}_k}$ we get an estimation for the k-th entry in $\mathbf{s}$ at the destination. Bounds for the error probability of the detector in (11) will be carried in Section V. We anticipate that a judicious construction of constellation $\mathcal{V}$ suffices to enable spatial diversity with this detector, by-passing at the same time channel fades and CFOs. Before that, let us first remark that the proposed strategy separates transmissions in time slots to avoid inter-terminal interference, dictated by $\mathbf{\Gamma}_r$. This transmission model, however, requires a previous source discovery and ordering protocol so that each terminal knows its corresponding r, and might be inefficient when sources are frequently recruted/discarded to join/stop collaboration. Next, we demonstrate that, indeed, one can design an ordering-unaware system with simultaneous transmissions between terminals that alleviates the alluded drawbacks.

IV. Simultaneous transmissions

With reference again to Fig. 1, we differentially encode symbols as in (2)-(3) and interleave them using $\mathbf{\Theta}$ as defined in (4). Now, we re-define the multiplexing matrix $\mathbf{\Gamma}_r$ to be $\mathbf{\Gamma}_r := \mathbf{I}_{KR}$. Consequently, we can drop the *terminal-dependent* subscript r for $\tilde{\mathbf{x}}_r$ and write the received signal at the destination as

$$\tilde{\mathbf{y}} = \sum_{r=1}^{R}\mathbf{D}_{\mathbf{h}_r}\tilde{\mathbf{x}} + \tilde{\mathbf{z}}. \quad (12)$$

We de-interleave the received signal and partition it again in K sub-blocks size $R\times 1$ each. The per sub-block I/O now becomes

$$\mathbf{y}_k = \mathbf{D}_{\mathbf{h}_k}\mathbf{x}_k + \mathbf{z}_k \quad (13)$$

and $[\mathbf{D}_{\mathbf{h}_k}]_{r,r} := \sum_{r=1}^{R} e^{j\omega_r((r-1)K+k)}h_r$ is now a superposition of sinusoids. Note that as opposed to the I/O relationship in (8), this sub-block I/O relationship entails amplitude (as opposed to only phase) variation with index k. We can rewrite (13) as

$$\mathbf{y}_k = \mathbf{D}_{\mathbf{x}_k}\tilde{\mathbf{F}}_k\mathbf{h} + \mathbf{z}_k \quad (14)$$

where $[\tilde{\mathbf{F}}_k]_{q,r} := e^{j\omega_r((q-1)K+k)}$. Note that fading coefficients $\mathbf{h} := [h_1,\ldots,h_R]^T$ are independent of k, whereas matrix $\tilde{\mathbf{F}}_k$ renders its k-fluctuations. Ignoring noise terms for clarity, we can use $\mathbf{y}_{k-1}$ and $\mathbf{y}_{k-2}$ to write

$$\mathbf{y}_k = \mathbf{D}_{\mathbf{x}_k}\tilde{\mathbf{F}}_k\tilde{\mathbf{F}}^{\mathcal{H}}_{k-1}\mathbf{D}^*_{\mathbf{x}_{k-1}}\mathbf{y}_{k-1} \quad (15)$$

$$\mathbf{y}_{k-1} = \mathbf{D}_{\mathbf{x}_{k-1}}\tilde{\mathbf{F}}_{k-1}\tilde{\mathbf{F}}^{\mathcal{H}}_{k-2}\mathbf{D}^*_{\mathbf{x}_{k-2}}\mathbf{y}_{k-2}. \quad (16)$$

Notice that $\tilde{\mathbf{F}}_k\tilde{\mathbf{F}}^{\mathcal{H}}_{k-1} = \tilde{\mathbf{F}}_{k-1}\tilde{\mathbf{F}}^{\mathcal{H}}_{k-2} := \mathbf{\Upsilon}$ is independent of k and Toeplitz. In fact, for large N, we can approximate $\mathbf{\Upsilon}$ to be circulant; thus, we can write it as

$$\mathbf{\Upsilon} \approx \mathbf{F}\mathbf{D}_\omega\mathbf{F}^{\mathcal{H}} \quad (17)$$

with the diagonal matrix $\mathbf{D}_\omega = \text{diag}([e^{j\omega_1},\ldots,e^{j\omega_R}])$. To get some insight as to how this circulant approximation for matrix $\mathbf{\Upsilon}$ is possible, notice that if the offset frequencies ω_r coincide with Fourier bases (i.e. $\omega_r = 2\pi f_r/KR$ for some discrete frequencies $f_r \in [1, KR]$), the approximation in (17) would become an equality. Thus, if we increase KR, we decrease $2\pi/KR$ and get a closer approximation to ω_r. On the other hand, we recall that there are practical limits for K (since R is fixed) that are related to the processing delay introduced by the interleaver $\mathbf{\Theta}$, which would accordingly increase. This trade-off will be further discussed through simulations.

Equation (17) is instrumental to build the detector for $\mathbf{D}_{\mathbf{v}_k}$. With that objective in mind, we first obtain matrix $\mathbf{D}_\omega$ from $\mathbf{y}_{k-1}$ and $\mathbf{y}_{k-2}$ in (16) as $\mathbf{D}_\omega = \text{diag}(\mathbf{F}^{\mathcal{H}}\mathbf{D}^*_{\mathbf{x}_{k-1}}\mathbf{y}_{k-1})\text{diag}^{-1}(\mathbf{F}^{\mathcal{H}}\mathbf{D}^*_{\mathbf{x}_{k-2}}\mathbf{y}_{k-2})$. Substituting $\mathbf{D}_\omega$ into (15), and including the noise term, we arrive to the following channel- and CFO-independent I/O relationship

$$\mathbf{y}_k = \mathbf{D}_{\mathbf{v}_k}\mathbf{D}_{\mathbf{g}_{k-1}}\mathbf{D}_{\mathbf{x}_{k-1}}\mathbf{F}\mathbf{D}_\omega\mathbf{F}^{\mathcal{H}}\mathbf{D}^*_{\mathbf{x}_{k-1}}\mathbf{y}_{k-1} + \mathbf{z}'_k. \quad (18)$$

After some standard manipulations, and discarding high-order noise terms, we can approximate $\mathbf{z}'_k$ to be gaussian with covariance matrix $\mathbf{\Sigma}_k := E[\mathbf{z}'_k\mathbf{z}'^{\mathcal{H}}_k] = 3\mathbf{I}_R$. Thus, one can build the following detector

$$\hat{\mathbf{D}}_{\mathbf{v}_k} = \arg\min_{\mathbf{D}_\mathbf{v}\in\mathcal{V}}\Big\{\|\mathbf{y}_k - \mathbf{D}_{\mathbf{v}_k}\hat{\mathbf{D}}_{\mathbf{g}_{k-1}}\hat{\mathbf{D}}_{\mathbf{x}_{k-1}}\mathbf{F}\hat{\mathbf{D}}_\omega\mathbf{F}^{\mathcal{H}}\hat{\mathbf{D}}^*_{\mathbf{x}_{k-1}}\mathbf{y}_{k-1}\|^2\Big\} \quad (19)$$

with $\hat{\mathbf{D}}_\omega := \text{diag}(\mathbf{F}^{\mathcal{H}}\hat{\mathbf{D}}^*_{\mathbf{x}_{k-1}}\mathbf{y}_{k-1})\text{diag}^{-1}(\mathbf{F}^{\mathcal{H}}\hat{\mathbf{D}}^*_{\mathbf{x}_{k-2}}\mathbf{y}_{k-2})$. Note that compared to (11), the detector (19) depends not only on on the previously received signals $\mathbf{y}_{k-1}$ and $\mathbf{y}_{k-2}$ but also on the estimation of the previously-received symbols $\hat{\mathbf{D}}_{x_{k-1}}$ and $\hat{\mathbf{D}}_{x_{k-2}}$. This dependence can cause error propagation, and its effect on error performance will be assessed through simulations in Section VI. If needed, there are means of mitigating such error propagation, with different complexities, by using, e.g., Multiple-Symbol-Detection as in [9], or the Viterbi Algorithm as in [4].

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V. Performance Analysis

In this section, we want to select the constellation $\mathcal{V}$ to provide performance guarantees, at least for high signal-to-noise ratios (SNRs). We start by analyzing the performance of detectors (11) and (19). Then, we move to cooperation protocols which consider estimation errors for $\mathbf{s}$ across $\{T_r\}_{r=1}^R$, and outline its error performance.

A. Error-free cooperation

We resort on the Pairwise Error Probability (PEP), defined as the probability of mistaking $\mathbf{D_v}$ for another $\tilde{\mathbf{D}}_\mathbf{v}$ in the constellation $\mathcal{V}$. Under the AWGN approximation, the channel-conditioned PEP can be bounded as [5]

$$\Pr(\mathbf{D_v} \to \tilde{\mathbf{D}}_\mathbf{v}|h) \leq \exp\left(-\frac{d_v^2}{4}\right) \tag{20}$$

and d_v^2 is the distance between two distinct constellation codewords. For the detector (11), d_v^2 takes the form

$$d_v^2 = \|(\mathbf{D_v} - \tilde{\mathbf{D}}_\mathbf{v})\mathbf{\Sigma}^{-1/2}\mathbf{D}_{\mathbf{y}_{k-1}}\mathbf{D}_{\mathbf{y}_{k-2}}^{\mathcal{H}}\|^2. \tag{21}$$

Under high-SNR analysis, $\mathbf{D}_{\mathbf{y}_{k-1}} \approx \mathbf{D}_{\mathbf{x}_{k-1}}\mathbf{D}_{\omega_{k-1}}\mathbf{D}_h$ and likewise $\mathbf{D}_{\mathbf{y}_{k-2}}$. Thus, after some manipulations, we can rewrite the distance (21) as

$$d_v^2 = \|(\mathbf{D_v} - \tilde{\mathbf{D}}_\mathbf{v})(\mathbf{D_h}\mathbf{D_h^{\mathcal{H}}})^{1/2}\|^2. \tag{22}$$

From here, the next steps are common to those for the performance analysis of coherent space-time systems. We wrap these steps up indicating that if $\mathcal{V}$ is designed to guarantee that $[\mathbf{D_v}]_{r,r} - [\tilde{\mathbf{D}}_\mathbf{v}]_{r,r} \neq 0 \ \forall r$ and $\forall \mathbf{D_v}, \tilde{\mathbf{D}}_\mathbf{v}$ in $\mathcal{V}$, $d_v^2 = \sum_{r=1}^R \delta_r^2 |h_r|^2$, for some non-zero constellation-dependent constant δ_r^2. Upon recalling that $\mathbf{h}_r \sim \mathcal{CN}(0, \sigma_r^2\bar{\gamma})$, the expectation of the PEP in (20) can be recast into an expression of the form

$$\Pr(\mathbf{D_v} \to \tilde{\mathbf{D}}_\mathbf{v}) \leq (G_c\bar{\gamma})^{-R} \tag{23}$$

where coefficient $G_c = G_c(\sigma_1^2, \ldots, \sigma_R^2)$ absorbs relative distances between terminals along with constellation distances δ_r^2, and $\bar{\gamma}$ will be our (average) signal-to-noise ratio (SNR).

For the case when simultaneous transmissions are scheduled, d_v^2 in (20) is written now as (c.f. (19))

$$d_v^2 = \|(\mathbf{D_v} - \tilde{\mathbf{D}}_\mathbf{v})\hat{\mathbf{D}}_{\mathbf{g}_{k-1}}^2\hat{\mathbf{D}}_{\mathbf{x}_{k-2}}\mathbf{F}\hat{\mathbf{D}}_\omega\mathbf{F}^{\mathcal{H}}\hat{\mathbf{D}}_{\mathbf{x}_{k-1}}^*\mathbf{y}_{k-1}\|^2. \tag{24}$$

assuming again that at high SNR $\mathbf{y}_{k-1} \approx \mathbf{D}_{\mathbf{x}_{k-1}}\tilde{\mathbf{F}}_{k-1}\mathbf{h}$, no error propagation occurs when estimating $\hat{\mathbf{D}}_{\mathbf{x}_{k-1}}$ and $\hat{\mathbf{D}}_{\mathbf{x}_{k-2}}$, and $\hat{\mathbf{D}}_\omega \approx \mathrm{diag}([e^{j\omega_1}, \ldots, e^{j\omega_R}])$, we can rewrite d_v^2 as

$$d_v^2 = \|(\mathbf{D_v} - \tilde{\mathbf{D}}_\mathbf{v})\mathbf{D}_{\mathbf{g}_{k-1}}^2\mathbf{D}_{\mathbf{x}_{k-2}}\mathbf{F}\hat{\mathbf{D}}_\omega\mathbf{F}^{\mathcal{H}}\tilde{\mathbf{F}}_{k-1}\mathbf{h}\|^2. \tag{25}$$

Being matrices $(\mathbf{D_v} - \tilde{\mathbf{D}}_\mathbf{v})$ and $\mathbf{D}_{\mathbf{g}_{k-1}}^2\mathbf{D}_{\mathbf{x}_{k-2}}\mathbf{F}\hat{\mathbf{D}}_\omega\mathbf{F}^{\mathcal{H}}\tilde{\mathbf{F}}_{k-1}$ full rank, it is not difficult to follow standard steps to conclude that the average PEP achieves diversity R. Taking a closer look to $\tilde{\mathbf{F}}_k$, full rank (hence, diversity) R might not be guaranteed if frequencies overlap, then, $\tilde{\mathbf{F}}_{k-1}$ would have rank as high as the number of non-equal frequencies. Fortunately, the probability of these overlaps to happen decreases with the interleaver depth characterized by the product KR.

B. Selective and Adaptive Relaying Protocols

So far, we assumed symbols $\mathbf{s}$ are given to all terminals. No transmission/detection procedures were given to guarantee $\mathbf{s}$ was error-free received at the cooperating terminals. Nonetheless, it is possible to ensure diversity advantage of order R even with estimation errors at T_r. We consider two possible strategies: i) LAR [3], [8] and ii) SR [2], [1]. The proof for (i)-(ii) is shortly outlined next.

We define a set $\mathcal{E}$ of terminals which erroneously decode $\mathbf{s}$ and thus the complementary set $\bar{\mathcal{E}}$ indicates all terminals that successfully decoded $\mathbf{s}$. When either coherent, differential or DD transmissions are employed to interchange information between cooperating terminals, one can always bound the error detection probability for block $\mathbf{s}$ at T_r as $\Pr(r \in \mathcal{E}) \leq \exp(-\delta_{s,r}^2|h_{s,r}|^2)$, where $h_{s,r}$ is the instantaneous SNR from the source to T_r, and the constant $\delta_{s,r}^2$ depends on the constellation and demodulation employed. If we assume $h_{s,r}$ is independent from any other SNR in the set-up, the conditional probability of having the error event $\mathcal{E}$ is $\mathrm{P}_{e,h}(\mathcal{E}) \leq \exp(-\sum_{r\in\mathcal{E}}\delta_{s,r}^2|h_{s,r}|^2)$. In LAR, T_r transmits $\sqrt{\alpha_r}\tilde{\mathbf{x}}_r$ to the destination and $0 \leq \alpha_r \leq 1$ is a link-adaptive channel-independent coefficient defined as in [3]. Note that the construction in [3], although designed for single-differential modulations, is also valid here.

Being T_r in $\mathcal{E}$, the mapping to $\mathcal{V}$ in T_r entails in errors, and denoting $\hat{\mathbf{D}}_\mathbf{v}$ as the mapped matrix, the rank of $(\hat{\mathbf{D}}_\mathbf{v} - \tilde{\mathbf{D}}_\mathbf{v})$ reduces *at most* to $R - |\mathcal{E}|$. After some tedious manipulations, and dropping without loss of generality $\delta_{s,r}^2$ and δ_r^2, the conditional error-aware PEP can be written as

$$\begin{aligned}&\Pr(\mathbf{D_v} \to \tilde{\mathbf{D}}_\mathbf{v}|h)\\ &= \sum_{\mathcal{E}} \mathrm{P}_{e,h}(\mathcal{E})\Pr(\hat{\mathbf{D}}_\mathbf{v} \to \tilde{\mathbf{D}}_\mathbf{v}|h, \mathcal{E})\\ &\leq \sum_{\mathcal{E}} \exp\left(-\sum_{r\in\mathcal{E}}|h_{s,r}|^2\right)\exp\left(-\frac{\sum_{r\in\bar{\mathcal{E}}}\alpha_r|h_r|^2 - \sum_{r\in\mathcal{E}}\alpha_r|h_r|^2}{\sqrt{\sum_{r=1}^R\alpha_r|h_r|^2}}\right)\end{aligned} \tag{26}$$

The SR protocol can be seen as a special case of the LAR protocol in which $\alpha_r = 1$ whenever $T_r \in \mathcal{E}$ and $\alpha_r = 0$ whenever $T_r \in \bar{\mathcal{E}}$. In such case, the expectation of (26) over the channel becomes

$$\begin{aligned}E\left[\Pr(\mathbf{D_v} \to \tilde{\mathbf{D}}_\mathbf{v}|h)\right] &\leq \sum_{\mathcal{E}} E\left[\exp\left(-\sum_{r\in\mathcal{E}}|h_{s,r}|^2\right)\right]E\left[\exp\left(-\sum_{r\in\bar{\mathcal{E}}}|h_r|^2\right)\right]\\ &= \sum_{\mathcal{E}}(\bar{\gamma})^{-|\mathcal{E}|}(\bar{\gamma})^{-(R-|\mathcal{E}|)}\\ &= \bar{\gamma}^{-R}.\end{aligned} \tag{27}$$

A little more elaboration is needed to average (26) whenever α_r continuous values. In this general case, expectations are not separable as in (27) because α_r, as defined in [3], depends on $h_{s,r}$. Fortunately, the inequality (26) is similar to the one encountered in [8] in the context of distributed coherent coding, and demonstrates diversity R.

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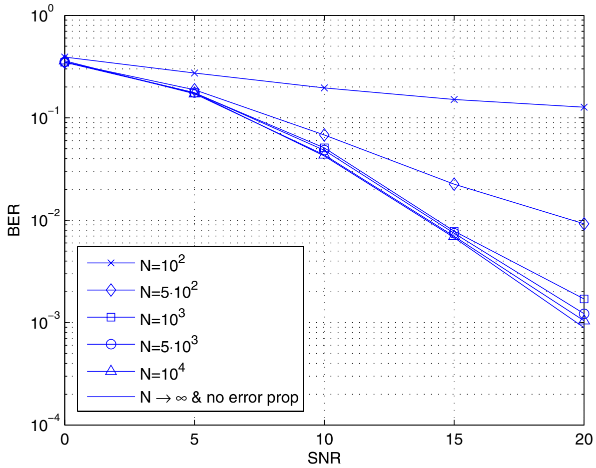

Fig. 2. Effect of error propagation and interleaver depth.

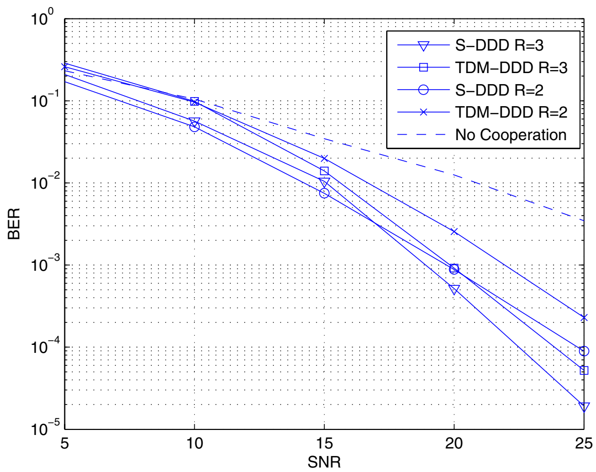

Fig. 3. TDM-DDD vs. S-DDD with no errors and $R = 2, 3$.

VI. Simulations

In this section, we present the results of simulating the bit-error-rate (BER) versus SNR of the distributed DD schemes in Sections III (TDM-DDD) and IV (S-DDD) under different assumptions. We choose diagonal matrices as proposed in [5] with $R = 1, 2$ and $|\mathcal{A}_s| = 2$. The channel and CFO are generated according to AS1-AS2, with $T_s = 0.0625\mu s$ and $f_c = 5.6GHz$ and $N = KR = 10^4$.

A. TDM vs. Simultaneous transmissions

First, we measure the effect of the approximation (17) for the system performance. Fig. 2 shows the BER-SNR curve for the ideal (infinite interleavers and no error propagation) versus the case when error propagation and and model mismatches are present. As shown, one can have negligible losses for practical N values. Fig. 2 compares TDM-DDD versus S-DDD for $R = 2$ and $R = 3$ and K=200 with no error propagation. As observed, both schemes achieve diversity of order R with coding gain losses that increase along with R. Note that the higher the coding gain, the higher SNRs are needed for diversity to 'kick-in' in practice [5].

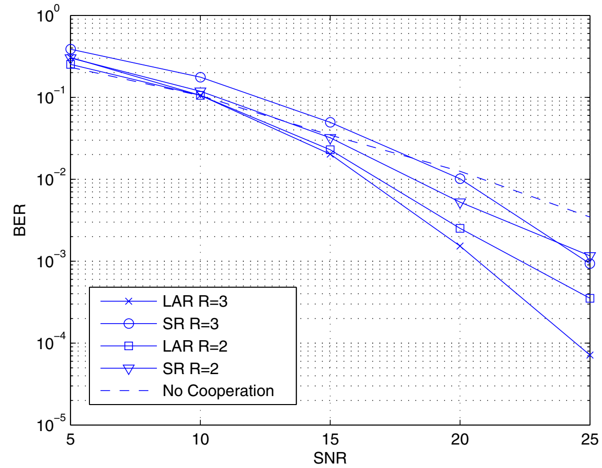

Fig. 4. TDM-DDD using SR and LAR protocols for $R = 2, 3$.

B. Errors in terminal-to-terminal channels

Fig. 4 shows now BER curves when LAR and SR protocols are considered. We assume the SNR between the source and $\{T_r\}_{r=1}^R$ is 3dB larger than that of $\{T_r\}_{r=1}^R$ to the destination. Compared to Fig. 3, errors in terminals unavoidably degrade system performance. The good news here are that diversity R is still achievable, as predicted in our theoretical performance analysis. We also remark that the LAR protocol attain larger coding gains because it considers 'soft' information at the receiver. SR, however, discards entire frames (here specially large for our interleaver).[1]

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