A Novel Differential Modulation Scheme Using Full-Rate STBC for 4×4 MIMO OFDM

Dhoni Putra Setiawan and Hua-An Zhao
Graduate School of Science and Technology, Kumamoto University, Japan
Email: {166d9304; cho}@cs.kumamoto-u.ac.jp

***Abstract*—Non-coherent communication which employs Differential Modulation (DM) technique is an interesting technique to combat the uncertainty of channel in the wireless communication system. This technique also provides a more spectral efficient system than coherent communication because it does not need the presence of signal overhead for channel estimation purpose which usually exists in the coherent transmission. To improve the performance of DM technique, differential unitary modulation technique which is combining DM and Space Time Block Code (STBC) is explored in this paper. In this paper, we modify a quasi-orthogonal STBC (QO-STBC) to be used as a unitary matrix generator, and then we introduce an element-wise calculation concept to minimize the system complexity. The experimental results show the proposed differential unitary modulation technique could be an excellent technique to overcome the uncertainty of channel in a wireless telecommunication system.**

***Index Terms*—channel uncertainty, complexity, differential modulation, non-coherent communication, QO-STBC**

I. Introduction

The uncertainty of channel is one of the main issues in the wireless telecommunication system. This uncertainty makes the Channel State Information (CSI), which is used to calculate the channel estimation, cannot be appropriately obtained. Without a proper channel estimation, the performance of the coherent transmission of wireless communication will decrease significantly. One of the example of the uncertainty of channel is when one of the receiver or transmitter or both of them are in high mobility condition, because in this condition the channel condition changes very fast. To ensure the quality of the received data, a non-coherent transmission which does not employ channel estimation is proposed. Non-coherent transmission does not need signal overhead for channel estimation purpose, which makes this technique more economical especially in fast varying channel [1]-[3].

Giving an example of this problem: an internet service provider (ISP) is using long term evolution (LTE) system with 2.6 GHz of the carrier frequency (F_c) to serve a passenger which is located inside a vehicle which is moving at a speed of 100 km/h (27.78 m/s). With this condition, we can calculate the maximum Doppler frequency (f_m):

$$f_m = \frac{vF_c}{c} = \frac{27.78 \times 2.6 \times 10^9}{3 \times 10^8} = 240.74 \text{ Hz}$$

By knowing the value of f_m, we can calculate the coherence time as follows:

$$Tc = \frac{0.423}{f_m} = \frac{0.423}{240.74} = 1.757 \text{ ms}$$

The symbol time (T_{sym}) of LTE is 66.67 µs which means the value of T_c/T_{sym} is 26.34. This condition means that the channel condition will change every 26.34 symbols. This value will decrease in accordance with the increase of the velocity and the use of higher carrier frequency. A shorter T_c/T_{sym} makes it harder to get the CSI accurately, while the increasing of the number of signal overhead for channel estimation purpose will make the system become uneconomical [1], [2].

In this paper, we analyze the performance of Differential Modulation (DM) as one of the non-coherent transmission systems. We try to combine the DM technique with orthogonal space time block code (OSTBC) and quasi-orthogonal space time block code (QO-STBC) for four antennas. There are several ways to combine the DM technique and space time block code (STBC). One of the most popular combinations between DM and STBC is the Differential Unitary Modulation (DUM) technique which is proposed in [3]. Tran *et al.* in [1], [2] proposed a modified concept of DUM so it can be implemented in multicarrier transmission. This modification is called Unitary Differential Space Time Frequency Modulation (UDSTFM). Both DUM and DUSTFM [1]-[4] use STBC Alamouti [5] to create the unitary matrix which is needed in the encoding system. The use of STBC Alamouti in this technique is the key to achieve full diversity and full orthogonality which is suitable for 2×2 Multiple-Input Multiple-Output (MIMO) system. differential modulation concept also has been analysed in several wireless communication systems such as multiband orthogonal frequency division multiplexing (MB-OFDM) [1], [2], LTE [6], multicarrier code division multiple access (MC-CDMA) [7], and also in the recently proposed modulations such as spatial modulation (SM) [8], [9] and the Subcarrier Index Modulation (SIM) [10].

Manuscript received January 21, 2019; revised April 8, 2019; accepted April 28, 2019.
Corresponding author: Dhoni Putra Setiawan (email: 166d9304@st.kumamoto-u.ac.jp).

doi: 10.18178/ijeetc.8.6.307-313

In this paper, we propose a modified QO-STBC for four transmitting antennas as a unitary matrix generator. By using QO-STBC we could improve the spectral efficiency of the system because there is no full rate OSTBC for four transmitting antennas. Several studies about using QO-STBC for differential modulation are proposed in [11], [12]. In this paper, the proposed QO-STBC is modified from the STBC scheme which was proposed by Tirkkonen in [13]. Although some researches have proposed differential modulation technique which uses quadrature amplitude modulation (QAM) and amplitude phase shift keying (APSK) modulation [14], [15], our proposed modification could be implemented only when the transceiver system uses phase shift keying (PSK) modulation as a symbol mapper. In the experimental process, the performance of our proposed scheme is compared with the differential modulation which uses OSTBC as a unitary generator. We also propose a low-complexity differential modulation which uses an element-wise calculation in order to solve the complexity problem of differential unitary modulation technique. This technique is a developed from the element-wise calculation technique which is proposed in [16]. Unlike the method which is proposed in [1], [2], which only proposes a low complexity technique for the decoding process, in this paper, we propose a technique which has low complexity either in the encoding process or decoding process. With low complexity encoding, we hope our proposed technique can be implemented not only for downlink communication but also uplink communication.

This paper is arranged this way: Section I: Introduction, Section II: Differential Unitary Modulation for 4×4 MIMO, Section III: Low Complexity Differential Modulation, Section IV: Complexity Analysis, Section V: Experimental Result, and Section VI: Conclusion.

II. Differential Unitary Modulation for 4×4 MIMO

LC Tran *et al*. proposed a DM for multicarrier transmission for 2×2 MIMO which is called unitary differential space time frequency modulation (UDSTFM) in [1], [2]. This technique is developed from the DM technique which was proposed by Ganesan *et al*. in [3]. These papers [1]-[4] used a similar unitary matrix which was derived from STBC Alamouti [5]. In [3], Ganesan *et al*. also proposed a DM technique for 4×4 MIMO. This proposed technique [3] uses an orthogonal STBC (OSTBC) for 4 antennas which has a rate of 3/4 as the unitary matrix.

In this paper, we introduce a new unitary matrix for 4 transmitting antennas which has 4/4 rate. The purpose of this new unitary matrix is to create a transmission which has higher spectral efficiency than using OSTBC. To create a new unitary matrix, we modify the QO-STBC which was proposed in [13] so it can be used as a unitary matrix. Let us consider that $\mathbf{U}$ is the encoding of STBC which is proposed by Tirkkonen in [13].

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \tag{1}$$

where

$$\mathbf{A} = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} c & d \\ -d^* & c^* \end{bmatrix} \tag{2}$$

Therefore, $\mathbf{U}$ can be written as the following:

$$\mathbf{U} = \begin{bmatrix} a & b & c & d \\ -b^* & a^* & -d^* & c^* \\ c & d & a & b \\ -d^* & c^* & -b^* & a^* \end{bmatrix}$$

However, unlike OSTBC, $\mathbf{U}$ is not an orthogonal matrix, and if this matrix is not an orthogonal matrix, it is impossible for us to create a unitary matrix from $\mathbf{U}$. Therefore, to make $\mathbf{U}$ into an orthogonal matrix, we have to make some modifications.

To make modifications, first we have to consider the definition of unitary matrix. A unitary matrix is a square matrix which the conjugate transpose of this matrix has the same value with the inverse of this matrix $\mathbf{X}^H = \mathbf{X}^{-1}$. Therefore, if we multiply a unitary matrix with its conjugate transpose, the result should be an identity matrix. However, in our case, if we multiply $\mathbf{U}$ with its conjugate transpose the result is as the following:

$$\mathbf{U}\mathbf{U}^H = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} \tag{3}$$

where $\alpha = aa^* + bb^* + cc^* + dd^*$ and $\beta = ac^* + ca^* + bd^* + db^*$

In order to make $\mathbf{U}$ into a unitary matrix, we have to modify this matrix so the value of α become 1 and the value of β become 0. Because in our proposed scheme we use PSK modulation as symbol mapper, making α become 1 is an easy task because in the PSK modulation the absolute value of its symbols will be always 1. However, the problem is how to make the value of β become 0, so $\mathbf{U}$ can be transformed into a unitary matrix.

The simplest way to make β become 0 is to consider that β will be 0 if the following equations are valid:

$$ac^* + ca^* = 0$$
$$bd^* + db^* = 0$$

To make this condition happens we propose some modification rules:

a. The modulation (symbol mapper) which is used in this system has to be PSK modulation.
b. Replace the symbols a, b, c, d with an addition or a subtraction between two symbols, example: $a = s_1 + s_2$, $b = s_3 + s_4$.
c. The multiplication between a and c, also between b and d, must produce $x^2 - y^2$. Example: because $a = s_1 + s_2$, then c should be $c = s_1 - s_2$, so the multiplication between a and c will produce ${s_1}^2 - {s_2}^2$. With the same concept, the value of d will be $d = s_3 - s_4$.

Proof:

$$ac^* + ca^* = 0$$

To prove the truth of our modification concept, let us substitute a and c respectively with $s_1 + s_2$ and $s_1 - s_2$

(the same with the examples above), where s_1 and s_2 are the outputs of PSK modulator.

$$(s_1+s_2)(s_1-s_2)^*+(s_1-s_2)(s_1+s_2)^*=0$$
$$s_1s_1^*-s_2s_2^*+s_1s_1^*-s_2s_2^*=0$$
$$1-1+1-1=0$$
$$0=0$$

Because the absolute value of every PSK symbol is 1, then the multiplication between each symbol with its conjugate will always be 1. Therefore, the equation in the left side will result in 0, and with this, this equation is proven. This proof is also appropriate for equation $bd^*+db^*=0$ which has exactly the same feature with c^*+ca^*.

Therefore, we will get this kind of equation as the new form of matrix **U**:

$$\mathbf{U}=\begin{bmatrix} s_1+s_2 & s_3+s_4 & s_1-s_2 & s_3-s_4 \\ -s_3^*-s_4^* & s_1^*+s_2^* & s_4^*-s_3^* & s_1^*-s_2^* \\ s_1-s_2 & s_3-s_4 & s_1+s_2 & s_3+s_4 \\ s_4^*-s_3^* & s_1^*-s_2^* & -s_3^*-s_4^* & s_1^*+s_2^* \end{bmatrix} \quad (4)$$

The multiplication between **U** and its conjugate transpose is:

$$\mathbf{U}\mathbf{U}^H=\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

We can see, that **U** already become an orthogonal matrix, although it still has not become a unitary matrix. The last step to make **U** to become a unitary matrix is to make the value of α become 1 by divide the matrix with $1/\sqrt{8}$. Therefore, the last form of matrix **U** as a unitary matrix is as the following:

$$\mathbf{U}=1/\sqrt{8}\begin{bmatrix} s_1+s_2 & s_3+s_4 & s_1-s_2 & s_3-s_4 \\ -s_3^*-s_4^* & s_1^*+s_2^* & s_4^*-s_3^* & s_1^*-s_2^* \\ s_1-s_2 & s_3-s_4 & s_1+s_2 & s_3+s_4 \\ s_4^*-s_3^* & s_1^*-s_2^* & -s_3^*-s_4^* & s_1^*+s_2^* \end{bmatrix} \quad (5)$$

This matrix is only one of the variations which could be designed from our proposed modification rules. We can produce the other matrices which have the same properties as this matrix as long as we follow the set of rules which is presented above.

III. Low Complexity Differential Modulation

A. Encoding

To minimize the complexity of differential unitary modulation, LC Tran in [1], [2] proposes a low complexity decoding process by decodes it two symbols by two symbols instead of jointly decodes it in a set of subcarriers as in their proposed encoding process. We show how our proposed algorithm is presented by (6) if we use the same encoding technique which is introduced by LC Tran in [1], [2]. The block diagram of our proposed low complexity differential modulation system is shown in Fig. 1.

The notation "di" represents matrix diagonalization process, $\mathbf{U}x_t$ represents the space time frequency coding (STFC) matrix and $\mathbf{s}_{t,m}$ is a column matrix size (N_{subc} ×1), the components of $\mathbf{s}_{t,m}$ are the outputs of PSK modulator, $\mathbf{s}_{t,m}=[s_{t,m,1}\, s_{t,m,2}\, \cdots\, s_{t,m,N_{subc}}]$.

Although this technique allows all symbols within one subcarrier to be encoded together, the diagonalization process adds a lot of zeros in this matrix in order to make this matrix into a square matrix. Because of this, the multiplication between two unitary matrices will also have to do multiplication between zeros of both unitary matrices, which does not give any gain to the system performance but increase the system complexity.

To solve this problem, in this paper we replace the multiplication between matrices into an element-wise multiplication process. With the element-wise multiplication, we do not have to add any zeros in our encoding and decoding process. The differential modulation using element-wise multiplication process for 2×2 MIMO has been proposed analyzed in the [16], while in this paper, we propose differential modulation using element-wise multiplication for 4×4 MIMO. We use the unitary matrix which is introduced in the previous section for the base of our proposed element-wise multiplication process. First, we consider (7) is the proposed STFC encoding, where $\mathbf{s}_{t,m}$ is a column matrix size (N_{subc} ×1), the components of $\mathbf{s}_{t,m}$ are the outputs of PSK modulator, $\mathbf{s}_{t,m}=[s_{t,m,1}\, s_{t,m,2}\, \cdots\, s_{t,m,N_{subc}}]$. We choose to call this matrix an STFC matrix rather than an STBC matrix because each element of this matrix consists of multiple symbols (size (N_{subc} ×1)), which after the encoding process they will be transmitted through different subcarriers.

$$\mathbf{U}x_t=1/\sqrt{8}\begin{bmatrix} di(\mathbf{s}_{t,1}+\mathbf{s}_{t,2}) & di(\mathbf{s}_{t,3}+\mathbf{s}_{t,4}) & di(\mathbf{s}_{t,1}-\mathbf{s}_{t,2}) & di(\mathbf{s}_{t,3}-\mathbf{s}_{t,4}) \\ di(-\mathbf{s}_{t,3}^*-\mathbf{s}_{t,4}^*) & di(\mathbf{s}_{t,1}^*+s_{t,2}^*) & di(\mathbf{s}_{t,4}^*-\mathbf{s}_{t,3}^*) & di(\mathbf{s}_{t,1}^*-\mathbf{s}_{t,2}^*) \\ di(\mathbf{s}_{t,1}-s_{t,2}) & di(\mathbf{s}_{t,3}-s_{t,4}) & di(\mathbf{s}_{t,1}+\mathbf{s}_{t,2}) & di(\mathbf{s}_{t,3}+\mathbf{s}_{t,4}) \\ di(\mathbf{s}_{t,4}^*-\mathbf{s}_{t,3}^*) & di(\mathbf{s}_{t,1}^*-\mathbf{s}_{t,2}^*) & di(-\mathbf{s}_{t,3}^*-\mathbf{s}_{t,4}^*) & di(\mathbf{s}_{t,1}^*+\mathbf{s}_{t,2}^*) \end{bmatrix} \quad (6)$$

$$\mathbf{U}_t=1/\sqrt{8}\begin{bmatrix} \mathbf{s}_{t,1}+\mathbf{s}_{t,2} & \mathbf{s}_{t,3}+\mathbf{s}_{t,4} & \mathbf{s}_{t,1}-\mathbf{s}_{t,2} & \mathbf{s}_{t,3}-\mathbf{s}_{t,4} \\ -\mathbf{s}_{t,3}^*-\mathbf{s}_{t,4}^* & \mathbf{s}_{t,1}^*+\mathbf{s}_{t,2}^* & \mathbf{s}_{t,4}^*-\mathbf{s}_{t,3}^* & \mathbf{s}_{t,1}^*-\mathbf{s}_{t,2}^* \\ \mathbf{s}_{t,1}-\mathbf{s}_{t,2} & \mathbf{s}_{t,3}-\mathbf{s}_{t,4} & \mathbf{s}_{t,1}+\mathbf{s}_{t,2} & s_{t,3}+s_{t,4} \\ \mathbf{s}_{t,4}^*-\mathbf{s}_{t,3}^* & \mathbf{s}_{t,1}^*-\mathbf{s}_{t,2}^* & -\mathbf{s}_{t,3}^*-\mathbf{s}_{t,4}^* & \mathbf{s}_{t,1}^*+\mathbf{s}_{t,2}^* \end{bmatrix} \quad (7)$$

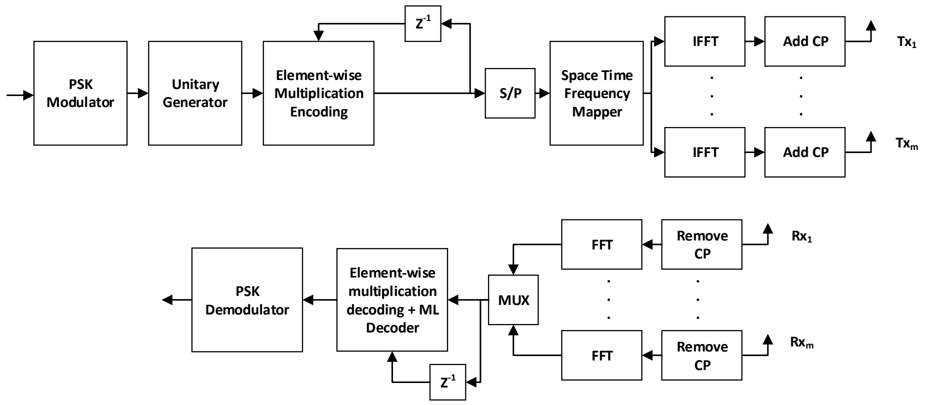

Fig. 1. Transceiver diagram of low-complexity differential modulation

By using a similar concept with the concept which proposed in [16], we change the multiplication between two unitary matrices into an element-wise multiplication process. The element-wise multiplication process is shown in (7). In this paper, we consider "$\circ$" as the notation of element-wise multiplication.

$$\mathbf{X}a_t = \mathbf{X}a_{t-1} \circ \mathbf{E}a_t + \mathbf{X}b_{t-1} \circ \mathbf{E}b_t + \mathbf{X}c_{t-1} \circ \mathbf{E}c_t + \mathbf{X}d_{t-1} \circ \mathbf{E}d_t \quad (8)$$

We consider $\mathbf{X}a_t$ is the result of the proposed element wise multiplication, except in the first transmission, where $\mathbf{X}a_1 = \mathbf{U}_1$. For understanding better our decoding concept, Xa_t can be explored as the following equation:

$$\mathbf{X}a_t = \begin{bmatrix} Xa_{t(1,1)} & Xa_{t(1,2)} & Xa_{t(1,3)} & Xa_{t(1,4)} \\ Xa_{t(2,1)} & Xa_{t(2,2)} & Xa_{t(2,3)} & Xa_{t(2,4)} \\ Xa_{t(3,1)} & Xa_{t(3,2)} & Xa_{t(3,3)} & Xa_{t(3,4)} \\ Xa_{t(4,1)} & Xa_{t(4,2)} & Xa_{t(4,3)} & Xa_{t(4,4)} \end{bmatrix} \quad (9)$$

where $Xa_{t(p,q)}$ represents the elements of matrix $\mathbf{X}a_t$, p represents the number of rows, and q represents the number of columns.

After getting $\mathbf{X}a_t$ we can generate $\mathbf{X}b_t$, $\mathbf{X}c_t$, and $\mathbf{X}d_t$ in the following way:

$$\mathbf{X}b_t = \begin{bmatrix} Xa_{t(1,2)} & Xa_{t(1,1)} & Xa_{t(1,4)} & Xa_{t(1,3)} \\ Xa_{t(2,2)} & Xa_{t(2,1)} & Xa_{t(2,4)} & Xa_{t(2,3)} \\ Xa_{t(3,2)} & Xa_{t(3,1)} & Xa_{t(3,4)} & Xa_{t(3,3)} \\ Xa_{t(4,2)} & Xa_{t(4,1)} & Xa_{t(4,4)} & Xa_{t(4,3)} \end{bmatrix} \quad (10)$$

$$\mathbf{X}c_t = \begin{bmatrix} Xa_{t(1,3)} & Xa_{t(1,4)} & Xa_{t(1,1)} & Xa_{t(1,2)} \\ Xa_{t(2,3)} & Xa_{t(2,4)} & Xa_{t(2,1)} & Xa_{t(2,2)} \\ Xa_{t(3,3)} & Xa_{t(3,4)} & Xa_{t(3,1)} & Xa_{t(3,2)} \\ Xa_{t(4,3)} & Xa_{t(4,4)} & Xa_{t(4,1)} & Xa_{t(4,2)} \end{bmatrix} \quad (11)$$

$$\mathbf{X}d_t = \begin{bmatrix} Xa_{t(1,4)} & Xa_{t(1,3)} & Xa_{t(1,2)} & Xa_{t(1,1)} \\ Xa_{t(2,4)} & Xa_{t(2,3)} & Xa_{t(2,2)} & Xa_{t(2,1)} \\ Xa_{t(3,4)} & Xa_{t(3,3)} & Xa_{t(3,2)} & Xa_{t(3,1)} \\ Xa_{t(4,4)} & Xa_{t(4,3)} & Xa_{t(4,2)} & Xa_{t(4,1)} \end{bmatrix} \quad (12)$$

Matrices $\mathbf{E}a_t$, $\mathbf{E}b_t$, $\mathbf{E}c_t$, and $\mathbf{E}d_t$ are derived from the components of STFC matrix $\mathbf{U}_t$ as the following:

$$\mathbf{E}a_t = \begin{bmatrix} U_{t(1,1)} & U_{t(2,2)} & U_{t(3,3)} & U_{t(4,4)} \\ U_{t(1,1)} & U_{t(2,2)} & U_{t(3,3)} & U_{t(4,4)} \\ U_{t(1,1)} & U_{t(2,2)} & U_{t(3,3)} & U_{t(4,4)} \\ U_{t(1,1)} & U_{t(2,2)} & U_{t(3,3)} & U_{t(4,4)} \end{bmatrix} \quad (13)$$

$$\mathbf{E}b_t = \begin{bmatrix} U_{t(2,1)} & U_{t(1,2)} & U_{t(4,3)} & U_{t(3,4)} \\ U_{t(2,1)} & U_{t(1,2)} & U_{t(4,3)} & U_{t(3,4)} \\ U_{t(2,1)} & U_{t(1,2)} & U_{t(4,3)} & U_{t(3,4)} \\ U_{t(2,1)} & U_{t(1,2)} & U_{t(4,3)} & U_{t(3,4)} \end{bmatrix} \quad (14)$$

$$\mathbf{E}c_t = \begin{bmatrix} U_{t(3,1)} & U_{t(4,2)} & U_{t(1,3)} & U_{t(2,4)} \\ U_{t(3,1)} & U_{t(4,2)} & U_{t(1,3)} & U_{t(2,4)} \\ U_{t(3,1)} & U_{t(4,2)} & U_{t(1,3)} & U_{t(2,4)} \\ U_{t(3,1)} & U_{t(4,2)} & U_{t(1,3)} & U_{t(2,4)} \end{bmatrix} \quad (15)$$

$$\mathbf{E}d_t = \begin{bmatrix} U_{t(4,1)} & U_{t(3,2)} & U_{t(2,3)} & U_{t(1,4)} \\ U_{t(4,1)} & U_{t(3,2)} & U_{t(2,3)} & U_{t(1,4)} \\ U_{t(4,1)} & U_{t(3,2)} & U_{t(2,3)} & U_{t(1,4)} \\ U_{t(4,1)} & U_{t(3,2)} & U_{t(2,3)} & U_{t(1,4)} \end{bmatrix} \quad (16)$$

The result of this element-wise multiplication, $\mathbf{X}a_t$, is the transmitted STFC symbol before IFFT process in the transmitter. This element-wise multiplication gives the exact same transmitted symbols as multiplication between $\mathbf{U}x_t$ and $\mathbf{U}x_{t-1}$. Because the elements of $\mathbf{X}a_t$ always have constant correlation each other, the transmitted matrix $\mathbf{X}a_t$ can be written as the following:

$$\mathbf{X}a_t = \begin{bmatrix} a_t & b_t & c_t & d_t \\ -b_t^* & a_t^* & -d_t^* & c_t^* \\ c_t & d_t & a_t & b_t \\ -d_t^* & c_t^* & -b_t^* & a_t^* \end{bmatrix} \quad (17)$$

B. Decoding

The received symbols after FFT process in the receiver can be described into this following equation:

$$\mathbf{Y}a_t = \mathbf{X}a_t \circ \mathbf{H}_t + \mathbf{N} \quad (18)$$

where $\mathbf{X}a_t$ is the output of element-wise multiplication (8) as well as the transmitted symbols before IFFT process in the transmitter, $\mathbf{H}_t$ represents the channel coefficient, and N represents noise.

$$\mathbf{H}_t = \begin{bmatrix} h_{t(1,1)} & h_{t(1,2)} & h_{t(1,3)} & h_{t(1,4)} \\ h_{t(2,1)} & h_{t(2,2)} & h_{t(2,3)} & h_{t(2,4)} \\ h_{t(3,1)} & h_{t(3,2)} & h_{t(3,3)} & h_{t(3,4)} \\ h_{t(4,1)} & h_{t(4,2)} & h_{t(4,3)} & h_{t(4,4)} \end{bmatrix} \quad (19)$$

The received STFC symbol $\mathbf{Y}a_t$ can be written as the form of (20).

$$\mathbf{Y}a_t = \begin{bmatrix} (a_t.h_{t(1,1)}+n) & (b_t.h_{t(1,2)}+n) & (c_t.h_{t(1,3)}+n) & (d_t.h_{t(1,4)}+n) \\ (-b_t^*.h_{t(2,1)}+n) & (a_t^*.h_{t(2,2)}+n) & (-d_t^*.h_{t(2,3)}+n) & (c_t^*.h_{t(2,4)}+n) \\ (c_t.h_{t(3,1)}+n) & (d_t.h_{t(3,2)}+n) & (a_t.h_{t(3,3)}+n) & (b_t.h_{t(3,4)}+n) \\ (-d_t^*.h_{t(4,1)}+n) & (c_t^*.h_{t(4,2)}+n) & (-b_t^*.h_{t(4,3)}+n) & (a_t^*.h_{t(4,4)}+n) \end{bmatrix} \quad (20)$$

For understanding better our decoding process, $\mathbf{Y}a_t$ can be written as the following:

$$\mathbf{Y}a_t = \begin{bmatrix} Ya_{t(1,1)} & Ya_{t(1,2)} & Ya_{t(1,3)} & Ya_{t(1,4)} \\ Ya_{t(2,1)} & Ya_{t(2,2)} & Ya_{t(2,3)} & Ya_{t(2,4)} \\ Ya_{t(3,1)} & Ya_{t(3,2)} & Y_{at(3,3)} & Ya_{t(3,4)} \\ Ya_{t(4,1)} & Ya_{t(4,2)} & Ya_{t(4,3)} & Ya_{t(4,4)} \end{bmatrix} \quad (21)$$

In this decoding process, $\mathbf{Y}a_t$ is not only a received STFC symbol but also functioned as one of decoding matrices which are needed to decode the received symbols. The decoding matrices $\mathbf{Y}b_t$, $\mathbf{Y}c_t$, and $\mathbf{Y}d_t$ which are derived from Ya_t are written as the following:

$$\mathbf{Y}b_t = \begin{bmatrix} Ya_{t(2,1)} & Ya_{t(2,2)} & Ya_{t(2,3)} & Ya_{t(2,4)} \\ Ya_{t(1,1)} & Ya_{t(1,2)} & Ya_{t(1,3)} & Ya_{t(1,4)} \\ Ya_{t(4,1)} & Ya_{t(4,2)} & Ya_{t(4,3)} & Ya_{t(4,4)} \\ Ya_{t(3,1)} & Ya_{t(3,2)} & Ya_{t(3,3)} & Ya_{t(3,4)} \end{bmatrix} \quad (22)$$

$$\mathbf{Y}c_t = \begin{bmatrix} Ya_{t(3,1)} & Ya_{t(3,2)} & Ya_{t(3,3)} & Ya_{t(3,4)} \\ Ya_{t(4,1)} & Ya_{t(4,2)} & Ya_{t(4,3)} & Ya_{t(4,4)} \\ Ya_{t(1,1)} & Ya_{t(1,2)} & Ya_{t(1,3)} & Ya_{t(1,4)} \\ Ya_{t(2,1)} & Ya_{t(2,2)} & Ya_{t(2,3)} & Ya_{t(2,4)} \end{bmatrix} \quad (23)$$

$$\mathbf{Y}d_t = \begin{bmatrix} Ya_{t(4,1)} & Ya_{t(4,2)} & Ya_{t(4,3)} & Ya_{t(4,4)} \\ Ya_{t(3,1)} & Ya_{t(3,2)} & Ya_{t(3,3)} & Ya_{t(3,4)} \\ Ya_{t(2,1)} & Ya_{t(2,2)} & Ya_{t(2,3)} & Ya_{t(2,4)} \\ Ya_{t(1,1)} & Ya_{t(1,2)} & Ya_{t(1,3)} & Ya_{t(1,4)} \end{bmatrix} \quad (24)$$

Aside from the matrices which are mentioned above, to decode the received symbols, these following matrices are required for the decoding process:

$$\mathbf{D}a_t = \begin{bmatrix} Ya^*_{t(1,1)} & Ya^*_{t(1,1)} & Ya^*_{t(1,1)} & Ya^*_{t(1,1)} \\ Ya^*_{t(2,2)} & Ya^*_{t(2,2)} & Ya^*_{t(2,2)} & Ya^*_{t(2,2)} \\ Ya^*_{t(3,3)} & Ya^*_{t(3,3)} & Ya^*_{t(3,3)} & Ya^*_{t(3,3)} \\ Ya^*_{t(4,4)} & Ya^*_{t(4,4)} & Ya^*_{t(4,4)} & Ya^*_{t(4,4)} \end{bmatrix} \quad (25)$$

$$\mathbf{D}b_t = \begin{bmatrix} Ya^*_{t(2,1)} & Ya^*_{t(2,1)} & Ya^*_{t(2,1)} & Ya^*_{t(2,1)} \\ Ya^*_{t(1,2)} & Ya^*_{t(1,2)} & Ya^*_{t(1,2)} & Ya^*_{t(1,2)} \\ Ya^*_{t(4,3)} & Ya^*_{t(4,3)} & Ya^*_{t(4,3)} & Ya^*_{t(4,3)} \\ Ya^*_{t(3,4)} & Ya^*_{t(3,4)} & Ya^*_{t(3,4)} & Ya^*_{t(3,4)} \end{bmatrix} \quad (26)$$

$$\mathbf{D}c_t = \begin{bmatrix} Ya^*_{t(3,1)} & Ya^*_{t(3,1)} & Ya^*_{t(3,1)} & Ya^*_{t(3,1)} \\ Ya^*_{t(4,2)} & Ya^*_{t(4,2)} & Ya^*_{t(4,2)} & Ya^*_{t(4,2)} \\ Ya^*_{t(1,3)} & Ya^*_{t(1,3)} & Ya^*_{t(1,3)} & Ya^*_{t(1,3)} \\ Ya^*_{t(2,4)} & Ya^*_{t(2,4)} & Ya^*_{t(2,4)} & Ya^*_{t(2,4)} \end{bmatrix} \quad (27)$$

$$\mathbf{D}d_t = \begin{bmatrix} Ya^*_{t(4,1)} & Ya^*_{t(4,1)} & Ya^*_{t(4,1)} & Ya^*_{t(4,1)} \\ Ya^*_{t(3,2)} & Ya^*_{t(3,2)} & Ya^*_{t(3,2)} & Ya^*_{t(3,2)} \\ Ya^*_{t(2,3)} & Ya^*_{t(2,3)} & Ya^*_{t(2,3)} & Ya^*_{t(2,3)} \\ Ya^*_{t(1,4)} & Ya^*_{t(1,4)} & Ya^*_{t(1,4)} & Ya^*_{t(1,4)} \end{bmatrix} \quad (28)$$

where $Ya^*_{t(\mathrm{p,q})}$ is the conjugate of $Ya_{t(p,q)}$.

Finally, the element-wise decoding can be written as follows:

$$\widehat{\mathbf{U}}_t = \mathbf{Y}a_t \circ \mathbf{D}a_{t-1} + \mathbf{Y}b_t \circ \mathbf{D}b_{t-1} + \mathbf{Y}c_t \circ \mathbf{D}c_{t-1} + \mathbf{Y}d_t \circ \mathbf{D}d_{t-1} \quad (29)$$

where, $\widehat{\mathbf{U}}_t$ is the received form of the matrix U_t which has been affected by channel and noise. In order to understanding better our decoding process, matrix $\widehat{\mathbf{U}}_t$ can be written as the following:

$$\widehat{\mathbf{U}}_t = \begin{bmatrix} \hat{r}_{t,1}+\hat{r}_{t,2} & \hat{r}_{t,3}+\hat{r}_{t,4} & \hat{r}_{t,1}-\hat{r}_{t,2} & \hat{r}_{t,3}-\hat{r}_{t,4} \\ -\hat{r}^*_{t,3}-\hat{r}^*_{t,4} & \hat{r}^*_{t,1}+\hat{r}^*_{t,2} & \hat{r}^*_{t,4}-\hat{r}^*_{t,3} & \hat{r}^*_{t,1}-\hat{r}^*_{t,2} \\ \hat{r}_{t,1}-\hat{r}_{t,2} & \hat{r}_{t,3}-\hat{r}_{t,4} & \hat{r}_{t,1}+\hat{r}_{t,2} & \hat{r}_{t,3}+\hat{r}_{t,4} \\ \hat{r}^*_{t,4}-\hat{r}^*_{t,3} & \hat{r}^*_{t,1}-\hat{r}^*_{t,2} & -\hat{r}^*_{t,3}-r^*_{t,4} & \hat{r}^*_{t,1}+\hat{r}^*_{t,2} \end{bmatrix} \quad (30)$$

where $\hat{r}_{t,m}$ represents the received symbol of $\mathbf{s}_{t,m}$ which has been affected by channel and noise. Because matrix $\widehat{\mathbf{U}}_t$ is a received STFC matrix, which each matrix element is received through a different channel coefficient and noise, we can take advantages of this diversity in order to get more accurate symbols ($\hat{r}_{t,m}$) by using these simple equations:

$$\hat{r}_{t,1} = 1/8(\widehat{U}_{t(1,1)} + \widehat{U}_{t(1,3)} + \widehat{U}^*_{t(2,2)} + \widehat{U}^*_{t(2,4)} + \widehat{U}_{t(3,1)} + \widehat{U}_{t(3,3)} + \widehat{U}^*_{t(4,2)} + \widehat{U}^*_{t(4,4)}) \quad (31)$$

$$\hat{r}_{t,2} = 1/8(\widehat{U}_{t(1,1)} - \widehat{U}_{t(1,3)} + \widehat{U}^*_{t(2,2)} - \widehat{U}^*_{t(2,4)} + \widehat{U}_{t(3,1)} - \widehat{U}_{t(3,3)} + \widehat{U}^*_{t(4,2)} - \widehat{U}^*_{t(4,4)}) \quad (32)$$

$$\hat{r}_{t,3} = 1/8(\widehat{U}_{t(1,2)} + \widehat{U}_{t(1,4)} - \widehat{U}^*_{t(2,1)} - \widehat{U}^*_{t(2,3)} + \widehat{U}_{t(3,2)} + \widehat{U}_{t(3,4)} - \widehat{U}^*_{t(4,1)} - \widehat{U}^*_{t(4,3)}) \quad (33)$$

$$\hat{r}_{t,3} = 1/8(\widehat{U}_{t(1,2)} - \widehat{U}_{t(1,4)} - \widehat{U}^*_{t(2,1)} + \widehat{U}^*_{t(2,3)} - \widehat{U}_{t(3,2)} + \widehat{U}_{t(3,4)} + \widehat{U}^*_{t(4,1)} - \widehat{U}^*_{t(4,3)}) \quad (34)$$

And last, the maximum likelihood decoding to decode the transmitted symbols can be written as the following:

$$\hat{s}_{t,m,k} = \arg min_{s_{t,m,k}\in\mathbb{C}} \{ (\Re|\hat{r}_{t,m,k} - s_{t,m,k}|)^2 + (\Im|\hat{r}_{t,m,k} - s_{t,m,k}|)^2 \} \quad (35)$$

IV. Complexity Analysis

A. OSTBC vs. Proposed QO-STBC

We can see the comparison of complexity between OSTBC [3] and QO-STBC in two different points of view. First, if we see it as a single matrix, then obviously the complexity of the proposed QO-STBC is higher than OSTBC because in our proposed QO-STBC we have to do extra additions or subtractions process. However, if we see it as a part differential modulation system the complexity of our proposed system which uses QO-STBC is lower than if we use OSTBC. The reason is if we use OSTBC we can only put $3N_{subc}$ symbols in one STFC Matrix, while if QO-STBC is used, we can put $4N_{subc}$ symbols in one STFC Matrix. This condition makes differential modulation using QO-STBC has fewer STFC Matrix than if we use OSTBC, and because of this, the number of multiplications which has to be done by the proposed system also fewer than if we use OSTBC. Because multiplication process is more computationally demanding than addition and subtraction, the computational time of QO-STBC will be shorter than OSTBC.

B. Unitary Model vs. Element-Wise Model

To compare the complexity between Unitary differential modulation and our proposed Element-Wise differential modulation, we compare the number of multiplications which has to be done by both systems. unitary differential modulation model for multicarrier transmission which uses a unitary matrix with diagonalization process as shown in (6) has $(4N_{subc})^3$ multiplications for one single differential symbol calculation, while our proposed system only has $64N_{subc}$ multiplications. The number of multiplications which has to be done if we use unitary differential modulation will increase exponentially in accordance with the number of subcarriers, while the complexity of our proposed system will increase arithmetically. Therefore, compared to the unitary model, our proposed scheme has a great improvement in term of computational complexity.

V. Experimental Results

In the experimental process, we compare our proposed system with UDSTFM for 2×2 MIMO which is proposed in [1], [2] and differential modulation using OSTBC. The parameters of our simulation are shown in Table I.

TABLE I. Simulation Parameters

Parameters	Value
Subcarrier	256
Subcarrier Spacing	15 kHz
Cyclic Prefix	25%
Convolutional Encoder's rate	1/2
Convolutional Decoder	Viterbi (hard)
Modulation	OFDM
Channel Model	Rayleigh fading channel
Carrier	2.6 MHz
Vehicle Speed	100 km/h
Simulation Technique	Monte Carlo
Communication Direction	Downlink

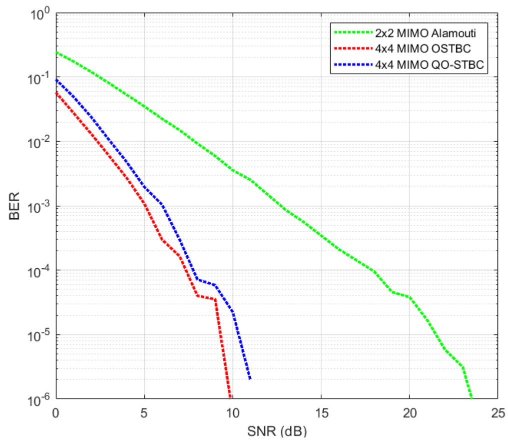

Fig. 2. Comparison between differential modulation using Alamouti, OSTBC and QO-STBC using BPSK

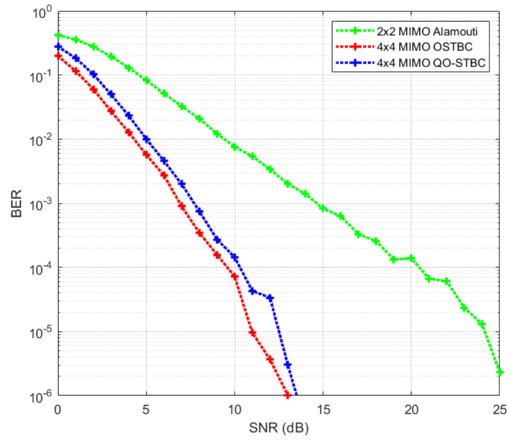

Fig. 3. Comparison between differential modulation using Alamouti, OSTBC and QO-STBC using QPSK

From Fig. 2 and Fig. 3, we can conclude that our proposed differential modulation using QO-STBC almost matches the performance of differential modulation using OSTBC. The difference of our proposed system and the differential modulation using OSTBC system is around 1 dB, which means that our proposed system needs 1 dB more power in order to achieve similar Bit Error Rate (BER) with differential modulation using OSTBC system.

VI. Conclusion

In this paper we propose a low complexity differential modulation for 4×4 MIMO OFDM system. In order to improve the spectral efficiency, we use a modified QO-STBC to become an STFC Matrix rather than OSTBC. The proposed QO-STBC is suitable only if PSK modulation is used in the transceiver system. The low complexity differential modulation concept is achieved by using element-wise multiplication concept rather than using unitary differential modulation concept. The experimental results confirm our proposed system could match the performance of differential modulation using OSTBC, while in the same time our proposed system gives better spectral efficiency and also lower computational complexity.

References

[1] L. C. Tran and A. Mertins, “Differential space-time-frequency codes for MB-OFDM UWB for dual carrier modulation,” presented at 2009 Int. Conf. on Communications, June, 2009.

[2] L. C. Tran, A. Mertins, and T. A. Wysocki, “Unitary differential space-time-frequency codes for MB-OFDM UWB wireless communications,” *IEEE Trans. on Wireless Communications*, vol. 12, no. 2, pp. 862-876, January, 2013.

[3] G. Ganesan and P. Stoica, “Differential modulation using STBC block code,” *IEEE Signal Processing Letters*, vol. 9, no. 2, pp. 57-60, August, 2002.

[4] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, October, 1998.

[5] F. Alsifiany, A. Ikhlef, and J. Chambers, “On differential modulation in downlink multiuser MIMO systems,” presented at 2017 25th European Signal Processing Conference, October, 2017.

[6] M. Hofer, M. Simko, S. Schwarz, and M. Rupp, “Performance evaluation of differential modulation in LTE downlink,” in *Proc. 20th Int. Conf. on Systems, Signals and Image Processing (IWSSIP)*, October, 2013, pp. 179-182.

[7] D. P. Setiawan, R. P. Astuti, and Sugihartono, "MIMO MC-CDMA with differential unitary space-time frequency modulation in high mobility scenario," presented at 2016 Asia Pacific Conference on Wireless and Mobile (APWiMob), January 2017.

[8] N. Ishikawa and S. Sugiura, "Rectangular differential spatial modulation for open-loop noncoherent massive MIMO downlink," *IEEE Trans. on Wireless Communications*, vol. 16, no. 3, pp. 1908-1920, March 2017.

[9] J. Liu, L. Dan, P. Yang, F. Yu, and Y. Xiao, "High-rate APSK-aided spatial modulation: Design method and performance analysis," *IEEE Communications Letters*, vol. 21, no. 1, pp. 168-171, January 2017.

[10] S. Alahunibat, R Mesleh, and E Basar, "Differential subcarrier index modulation," *IEEE Trans. on Vehicular Technology*, vol. 67, no. 8, pp. 7429-7436, August 2018.

[11] Y. Zhu and H. Jafarkhani, "Differential modulation based on quasi-orthogonal codes," *IEEE Trans. on Wireless Communications*, vol. 4, no. 6, pp. 3018-3030, November 2005.

[12] F. Alsifiany, A. Ilkhef, and J Chambers, "Exploiting high rate differential algebraic space-time block code in downlink multiuser MIMO systems," *IET Communications*, vol. 12, no. 17, pp. 2198-2205, October 2018.

[13] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. on Information Theory*, vol. 48, no. 2, pp. 1122–1126, Feb. 2002.

[14] D. Kong, X. G. Xia, and T. Jiang, "A differential QAM detection in uplink massive MIMO systems," *IEEE Trans. on Wireless Communications*, vol. 15, no. 9, pp. 6371-6383, June 2016.

[15] R. Y. Wei and X. J. Wang. "Differential 16-QAM and 16-APSK for uplink massive MIMO system," *IEEE Wireless Communications Letters*, vol. 7, no. 2, pp. 170-173, April 2018.

[16] D. P. Setiawan and H. A. Zhao, "Low-complexity differential modulation for high mobility MIMO-OFDM," *IEICE Commun. Express*, vol. 8, no. 1, pp. 1-6, January 2019.

Dhoni Putra Setiawan received Bachelor degree in Telecommunication Engineering from Telkom Institute of Technology in 2013 and Master degree in Electrical and Telecommunication Engineering from Telkom University in 2016. While he was working toward his Master degree, he received JASSO scholarship and became an exchange student from October 2015 until March 2016 in Kumamoto University, Japan. Since October 2016, he has been working on his Ph.D. Degree in the Fundamental Information Technique Laboratory, Graduate School of Science and Technology, Kumamoto University as an awardee of MEXT scholarship. His current research topics are about MIMO, relay communication, 4G, and advanced 4G.

Hua-An Zhao received B. S. and M. S. degrees in Electrical Engineering from Anhui University, China in 1982 and 1986, respectively. He received the Ph.D. degree in Computer Science from Hiroshima University, Japan in 1993. During 1993-2006, he joined the Faculty of Engineering, Kyushu Kyoritsu University. From 2007, he is a professor in Kumamoto University. His current research interests are wireless communications, graph theory and its applications, signal processing and VLSI layout design.