A New Fuzzy Bandwidth Carrier Recovery System in GPS for Robust Phase Tracking

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Abstract—Fast carrier recovery loops are essential in high-dynamic global positioning system receivers. A new digital phase-locked loop (DPLL) system incorporating a fuzzy bandwidth controller (FBC) is presented in this letter. The FBC provides a time-varying bandwidth and adjusts the loop filter coefficients to improve DPLL tracking capability. When the phase error or phase difference error becomes large in the presence of significant accelerations, the loop bandwidth increases adaptively and performs rapid locking. Simulation results show that our receiver does achieve a superior performance over conventional tracking loops in terms of shorter settling time and wider frequency ramp range, while preventing the occurrence of cycle slipping.

Index Terms—Carrier phase tracking, cycle slipping, digital phase-locked loop (DPLL), fuzzy bandwidth controller (FBC), global positioning system (GPS) receiver.

I. INTRODUCTION

CARRIER PHASE observable is widely used in global positioning systems (GPS) to obtain a centimeter-level positioning accuracy with applications of aircraft precision approaches, missile navigation, and etc. Unfortunately, high dynamics and shadowing may lead to cycle slipping that seriously degrades the ranging accuracy and requires a time-consuming ambiguity search algorithm for rinitialization. Previously, a fuzzy estimation filter [1] has been proposed for missile navigation. However, the model needed an *a priori* known specific trajectory to train the fuzzy phase-locked loop (PLL) and thus was unsuitable for practical receiver. Lee *et al.* [7] presented an adaptive PLL capable of varying the loop bandwidth by increasing the amount of charge-pump current. An alternative scheme operating by the control on the reference frequency and frequency division ratio was proposed for frequency synthesizer [8]. Thus, these adaptive bandwidth methods are only applicable for analog PLL architectures. In carrier loop design, a wider bandwidth is needed for rapid acquisition, whereas a smaller bandwidth is preferred for noise performance [2]. To enhance both carrier tracking and phase estimation accuracy simultaneously in GPS receivers, the digital bandwidth-adjustment scheme is indispensable to overcome this difficulty.

In this letter, we propose a new carrier recovery system composed of a third-order digital phase-locked loop (DPLL) and a fuzzy bandwidth controller (FBC) to achieve robust carrier phase tracking. The FBC provides a means of converting a control strategy comprised of a set of linguistic rules into a bandwidth-control operation and requires only moderate computational complexity. Under realistic mobile conditions, the carrier phase signal is modeled as a sum of three components, these being 1) phase offset, 2) frequency offset, and 3) frequency ramp offset. When the phase error or phase difference error increases due to large dynamics, the FBC provides an adaptive bandwidth to the DPLL and results in fast acquisition. If both absolute phase and phase difference errors are small, the loop bandwidth will reduce and thus improves the positioning accuracy. This new adaptive-bandwidth method has been shown to shorten the pull-in process and widen the locking range in comparison with conventional loops under a variety of dynamic environments.

II. MOBILE CARRIER PHASE MODEL

A detailed block diagram of the proposed GPS carrier loop is illustrated in Fig. 1(a). The received sample observation can be represented as

$$r_i = \sqrt{2P} C[iT_s - \tau] \cos(2\pi f_{\mathrm{IF}} iT_s + \theta_i) + n_i \qquad (1)$$

where P is the average power of the received signal, $C[\bullet]$ is the coarse/acquisition (C/A) code sequence, T_s is the sample period, τ is the code transmission delay time, ω_{IF} is the receiver intermediate radian frequency, and θ_i is the unknown carrier phase to be estimated. The noise component n_i is modeled as sample of a zero-mean white Gaussian noise with two-sided power spectral density of $N_0/2$ W/Hz, and the jamming source is not considered in this letter. The mathematical model for the carrier phase θ_i can be represented as

$$\theta_i = \theta_0 + 2\pi f_0(iT_s) + \pi \lambda_0 (iT_s)^2 \qquad (2)$$

where θ_0 (radians) is a constant phase offset, f_0 (hertz) is the amount of frequency offset due to the Doppler effect, and λ_0 (hertz per second) is the frequency ramp offset of a high-dynamic user. Since the frequency ramp caused by the satellite motion is usually small, we may consider only the motion of vehicles [4]. It is assumed that the user has an acceleration of a_c (m/s^2), and the equivalent Doppler frequency ramp becomes

$$\lambda_0 = \frac{a_c f_c}{c} \quad (\mathrm{Hz/s}) \qquad (3)$$

where c is the speed of light and f_c is the GPS L1 carrier frequency ($= 1575.42$ MHz). For a highly dynamic condition with an acceleration of 10 G ($G =$ gravitational acceleration with a value of 9.8m/s^2), the frequency ramp λ_0 will be 515 Hz/s.
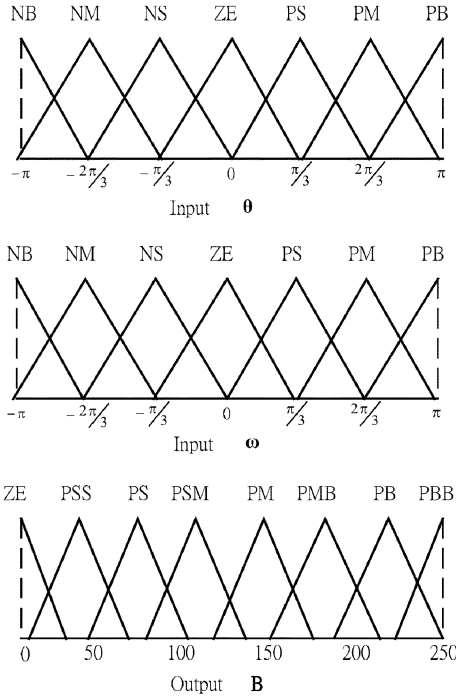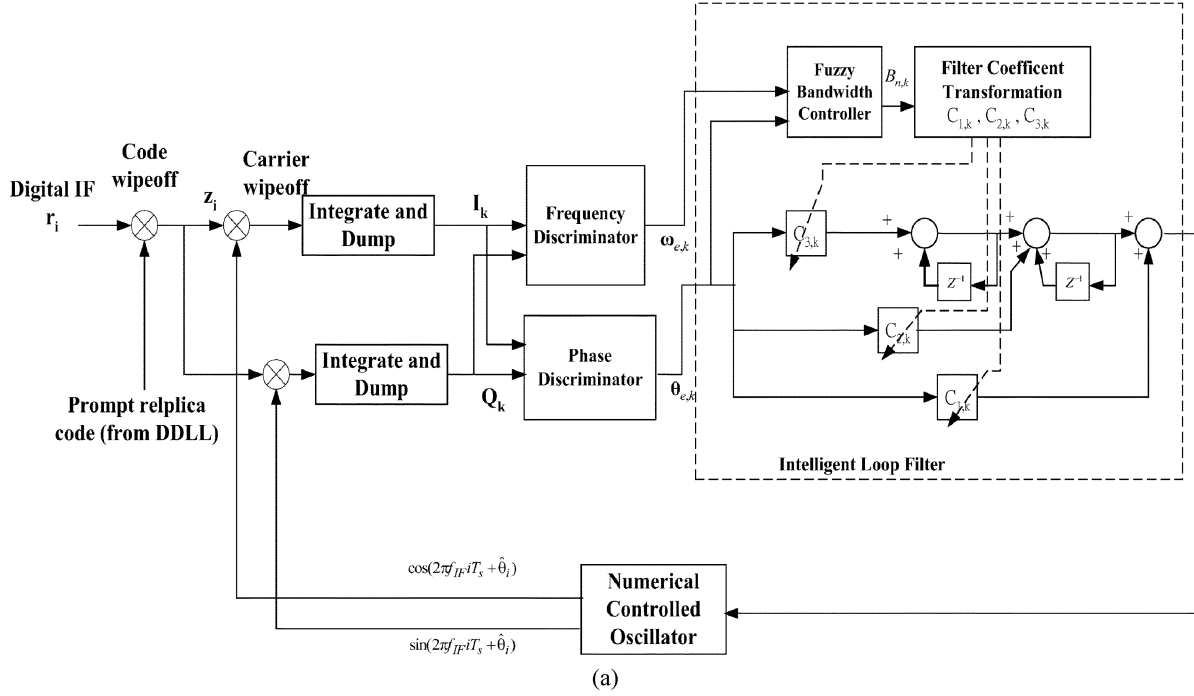
Fig. 1.   (a) Proposed fuzzy bandwidth carrier tracking loop. (b) Membership functions and rule table.

III. NEW FUZZY BANDWIDTH CARRIER TRACKING LOOP

A. Fuzzy Bandwidth Controller (FBC)

The integration and dump filters sum up their input data in I- and Q-arms, and the outputs of the digital accumulators [6] become

$$I_k = \sqrt{2P}M \sin c \left(\frac{\Delta\omega T}{2} \right) R(\tau - \hat{\tau})$$

$$\cdot \cos \left[\theta(kT) - \hat{\theta}(kT) \right] + n_{I,k} \qquad (4.1)$$

$$Q_k = \sqrt{2P}M \sin c \left(\frac{\Delta\omega T}{2} \right) R(\tau - \hat{\tau})$$

$$\cdot \sin \left[\theta(kT) - \hat{\theta}(kT) \right] + n_{Q,k} \qquad (4.2)$$

where $R(\bullet)$ is the autocorrelation function of C/A codes, T is the loop update time, $\Delta\omega = [\theta(kT) - \theta(kT - T)]/T$, and $M = T/T_s$ is the number of samples summed together to update the loop. $\theta(kT)$ and $\hat{\theta}(kT)$ are the phases of the received signal and local carrier signal. $n_{I,k}$ and $n_{Q,k}$ are I- and Q-phase noise components due to noise n_i. The phase error $\theta_{e,k}$ and phase

TABLE I
SETTLING TIMES FOR EACH DPLL SCHEME IN DOPPLER FREQUENCY RAMP ENVIRONMENTS

Gravitational acceleration (G)	1	2	3	4	5	6	7	8	9	10
Frequency ramp offset (Hz/s)	51.5	102.9	154.4	205.9	257.3	308.8	360.2	411.7	463.2	515
Types of digital carrier loop	Settling time (ms)									
Fuzzy bandwidth carrier loop	18	18	19	20	20	22	23	25	26	28
Conventional 2$^{\text{nd}}$-order loop [3]	228	230	233	LL	LL	LL	LL	LL	LL	LL
Conventional 3$^{\text{rd}}$-order loop [3]	715	721	725	982	LL	LL	LL	LL	LL	LL
Conventional 4$^{\text{th}}$-order loop [3]	4341	4344	LL	LL	LL	LL	LL	LL	LL	LL

LL: loss of lock.

difference error $\omega_{e,k}$ are chosen as input variables of the FBC and given by

$$\theta_{e,k} = \arctan\left[\frac{Q_k}{I_k}\right] \tag{5.1}$$

$$\omega_{e,k} = \arctan\left[\frac{(I_{k-1}Q_k - I_kQ_{k-1})}{(I_{k-1}I_k - Q_kQ_{k-1})}\right]. \tag{5.2}$$

The output variable of the FBC, $B_{n,k}$, is the desired loop bandwidth to vary the loop filter parameters accordingly. We choose triangular-shaped membership functions (MFs) for the input and output variables [shown in Fig. 1(b)]. The inference mechanism based on the Mamdani algorithm [5] is utilized. These control rules have the general form

$$\text{IF } \theta_e \text{ is } S_1^i \text{ AND } \omega_e \text{ is } S_2^i, \quad \text{THEN } B \text{ is } S_3^i \tag{6}$$

where θ_e, ω_e, and B are the linguistic variables, and S_1^i, S_2^i and S_3^i are the corresponding linguistic labels characterized by MFs. The fuzzy output $\bar{B}$ can be determined using the center-average defuzzification [5] formulated as follows:

$$\bar{B} = \frac{\sum_{i=1}^{n} B_i u(B_i)}{\sum_{i=1}^{n} u(B_i)} \tag{7}$$

where n is the number of fuzzy output sets, B_i is the numerical value of i output MF, and $u(B_i)$ represents its membership value at the ith quantization level.

B. Filter Coefficient Transformation

The closed-loop transfer function of a third-order PLL [3] can be represented as

$$H(s) = \frac{b\omega_n s^2 + a\omega_n^2 s + \omega_n^3}{s^3 + b\omega_n s^2 + a\omega_n^2 s + \omega_n^3}. \tag{8}$$

The single-sided noise bandwidth B_n for this loop is

$$B_n = \int_0^\infty |H(j2\pi f)|^2 \, df = M\omega_n \text{ (Hz)} \tag{9}$$

where $M = (ab^2 + a^2 - b)/(4ab - 4)$ is the ratio of the natural frequency ω_n to the loop bandwidth B_n. Since most GPS carrier loops are implemented as a digital structure, the bilinear transformation is used to map the continuous-time system into a discrete-time one. We consider the second-order loop filter with a transfer function

$$L(z) = C_1 + \frac{C_2}{1 - z^{-1}} + \frac{C_3}{(1 - z^{-1})^2} \tag{10}$$

where C_1, C_2, and C_3 are the digital filter coefficients. Inserting (10) into the carrier loop system, the discrete-time transfer function becomes as shown in (11) at the bottom of page, where C_d is the gain of phase discriminator, and C_n is the gain of the numerically controlled oscillator. The loop filter parameters in terms of loop bandwidth B_n can be derived as

$$C_1 = \frac{8b\omega_n T + 2\omega_n^3 T^3}{C_d C_n \left[8 + 4b\omega_n T + 2a\omega_n^2 T^2 + \omega_n^3 T^3\right]}$$
$$= \frac{8bM^{-1}B_n T + 2M^{-3}B_n^3 T^3}{C_d C_n \left[8 + 4bM^{-1}B_n T + 2aM^{-2}B_n^2 T^2 + M^{-3}B_n^3 T^3\right]} \tag{12.1}$$

$$C_2 = \frac{8a\omega_n^2 T^2 - 4\omega_n^3 T^3}{C_d C_n \left[8 + 4b\omega_n T + 2a\omega_n^2 T^2 + \omega_n^3 T^3\right]}$$
$$= \frac{8a\omega_n^2 T^2 - 4\omega_n^3 T^3}{C_d C_n \left[8 + 4bM^{-1}B_n T + 2aM^{-2}B_n^2 T^2 + M^{-3}B_n^3 T^3\right]} \tag{12.2}$$

$$C_3 = \frac{8\omega_n^3 T^3}{C_d C_n \left[8 + 4b\omega_n T + 2a\omega_n^2 T^2 + \omega_n^3 T^3\right]}$$
$$= \frac{8M^{-3}B_n^3 T^3}{C_d C_n \left[8 + 4bM^{-1}B_n T + 2aM^{-2}B_n^2 T^2 + M^{-3}B_n^3 T^3\right]}. \tag{12.3}$$

$$H(z) = \frac{C_d C_n z^{-1} \left[(C_1 + C_2 + C_3) - (2C_1 + C_2)z^{-1} + C_1 z^{-2}\right]}{1 + \left[C_d C_n(C_1 + C_2 + C_3) - 3\right]z^{-1} + \left[3 - C_d C_n(2C_1 + C_2)\right]z^{-2} + \left[C_d C_n C_1 - 1\right]z^{-3}} \tag{11}$$

IV. SIMULATION RESULTS

Computer simulations have been carried out to evaluate the performance of proposed scheme applied in high-dynamic receivers. The relevant parameters are: the IF carrier $f_{\text{IF}} = 1.25$ MHz, IF sampling frequency $f_s = 5$ MHz, and loop update time $T = 1$ ms. The value of the third-order filter coefficients a and b are chosen as 1.1 and 2.4 [3], respectively. The new fuzzy bandwidth DPLL is compared to the conventional second-, third-, and fourth-order loops with $B_n = 10$ Hz [3]. The desired dynamic limitations are set as a lock range of 100 Hz and a frequency ramp range of 515 Hz/s. The complete rule table for the FBC is shown in Fig. 1(b). Seven linguistic terms are used for each input variables, and 49 rules are developed in this controller. The dynamic tests are conducted at a lowest value of CNR 35 dB-Hz [2], and each result is obtained through 2000 independent Monte Carlo simulations.

Table I compares the simulated performances of our fuzzy loop with those of the conventional methods under the acceleration conditions. The constant-bandwidth loops can only operate in smaller frequency ramp ranges (near 220 Hz/s). On the other hand, the FBC provides an adaptive loop bandwidth to the DPLL, and the corresponding tracking capability can be significantly extended to 515 Hz/s with a shorter settling time of 28 ms (5% frequency ramp error specification).

Table II shows the acquisition limitation comparisons between each DPLL scheme. If no fuzzy algorithm is used, loss of locking will occur in conventional loops when the frequency offset is larger than 30 Hz. It is observed that the fuzzy bandwidth scheme can work very well to achieve wider lock range and pull-in range. The significant improvement by using our DPLL is that a robust transient behavior can be performed in kinematic environments.

V. CONCLUSION

This letter has presented a new architecture of an intelligent GPS receiver suitable for use under various dynamic conditions. Based on the bandwidth-adjustment criterion, the proposed car-

TABLE II
ACQUISITION LIMITATIONS FOR EACH DPLL SCHEME

Types of digital carrier loop	Lock range (Hz)	Pull-in range (Hz)	Freq. ramp range (Hz/s)
Fuzzy bandwidth carrier loop	100.2 Hz	134.6 Hz	515 Hz/s (10 G)
Conventional 2nd-order loop [3]	20.2 Hz	70.2 Hz	164.8 Hz/s (3.2 G)
Conventional 3rd-order loop [3]	25.6 Hz	95.3 Hz	221.5 Hz/s (4.3 G)
Conventional 4th-order loop [3]	16.4 Hz	17.8 Hz	118.5 Hz/s (2.3 G)

rier loop does achieve three better acquisition performances: 1) larger frequency ramp range (515 Hz/s); 2) wider lock range (100.2 Hz) and pull-in range (134.6 Hz); and 3) shorter settling time for the L1 carrier signal. These results demonstrate that our receiver is capable of faster acquisition speed and broader frequency ramp range compared to those of the conventional constant-bandwidth loops in a variety of mobile circumstances.

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