Technology Adoption in an OLG model with forward-looking agents[*]

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Abstract

We investigate the effects of forward looking behaviour in technology adoption. The setup is an overlapping generation model where agents choose between two alternative networks taking in consideration the installed base as well as the *expected* base. In other words users not only keep into account the actions of previous players, but also form expectations over future generations' choices before deciding which technology to purchase. The latter element is the distinctive feature of our approach. Contrary to the existing literature, we consider users that receive technology benefits over their whole life-time, rather than only upon purchasing one technology. As multiplicity of equilibrium is to be expected we introduce stochastic payoffs such that agents coordinate their expectations on a unique outcome.

We consider both the case of incompatible and compatible technology within our OLG setup, and show that technologies cannot lock-in, that is no technology can emerge as dominant in the long run. This realistic conclusion differs from standard results in the existing literature.

1 Introduction

Many industries exhibit increasing returns in consumption, i.e., the benefits that consumers obtain from the goods are increasing in the number of consumers who have already consumed the good or that will consume the good in the future. The presence of this type of increasing returns, also known as network effects can be explained through different channels.[1] Network effects are due to consumers valuing the services which are attached to the good and whose supply is only active once a considerable number of consumers joins the market. Or they are a result of the interaction that agents obtain with peers which also have the same good or use the same technology.

Increasing returns to consumption implies that once a network good becomes highly adopted in the market it becomes so valuable for consumers that rival networks are led off the market. This result has been put forth by Arthur (1983, 1989) and has been observed in several industries. Known examples are the QWERTY keyboard standard which has become dominant with respect to AZERTY; the VHS video system which became dominant versus BETAMAX and more recently the BluRay DVD system versus the HD DVD.

Despite this evidence, other network industries remained competitive, even after years of apparent domination by one standard. It is even possible to observe the inversion of the consumption trends, contradicting the result that a dominant technology eventually drives the rivals off the market. For example, in the console war, that has been fought by Nintendo and Sony

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[1]See Rohlfs (1974) for a seminal contribution.

Playstation since the 1990's, although Nintendo became a secondary competitor ever since Playstation 1 was introduced, with less available games, and less perceived quality, it was able to invert this trend after the introduction on the Wii console. A second example is the industry of personal computers. Despite market share dominance by Microsoft based PCs, Macintosh persisted in the market, being able to conquer market share in recent years. More so if we account for the combined market share of Macintosh PC and IPad. [2][3]

The possibility of inversion in consumption trends implies that consumers choosing a network good, should not only regard the installed base but also forecast the future adoption of the good.

From a theoretical point of view, there are strategic complementarities in the adoption of network goods. Models with this type of complementarities often possess multiple self fulfilling equilibria. Once a consumer expects certain behavior from other consumers, he will mimic this behavior, enhancing further the benefits for others of complying with the same choice. Obviously, different expectations of behavior lead to different equilibria. This feature weakens the predictive power of the model and precludes the design of policies. It is necessary, therefore, to use an equilibrium selection method to identify possible outcomes of the technology adoption game.

The properties of equilibria in games with strategic complementarities or *supermodular games* have been studied under different lights.[4] Milgrom and

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[2]Note that oscillating adoption trends are also observed in fashion goods such as clothes, bags and shoes, see Garcia and Resende (2010).

[3]See Bresnahan and Greenstein (1999) for an empirical study on Mac Vs PC adoption.

[4]See Amir (2005) for a survey.

Roberts (1990) showed that supermodular games always have Pure Strategy Nash Equilibria which are Pareto ranked. More recently, Reny (2011) and Van Zandt and Vives (2007) showed that this result can be extended to Monotone Bayesian Games. The theory of *global games,* first developed by Carlsson and Van Damme (1993) for binary games with strategic complementarities, provides conditions under which the equilibrium is unique. Their approach is based on the existence of payoff uncertainty and heterogeneous information among agents. Frankel Burdzy and Pauzner () extended this result to encompass more general classes of games. Regarding Dynamic games of strategic complementarities, Heidues and Melissas (2006) introduce cohort effects in a dynamic global game, that under certain conditions do not lead to dynamic increasing differences and hence multiple equilibria still arise. Angeletos et al. (2007) consider a dynamic global game of regime change where decisions can be taken in various periods in time and agents learn. They show that, multiple equilibria can arise under conditions for which it would not arise in the static game. This occurs because of the interaction between learning about previous actions and new information which arrives through time. From these articles we can conclude that uniqueness of equilibria in a game with strategic complements becomes less obvious once dynamics is introduced.

In the present paper we consider a dynamic game of technology adoption and characterize the unique equilibrium. We model the dynamics as an overlapping generations game, where each agent lives for two periods, interacting with the predecessor(s) and the sucessor(s). Consumers opt, in the first period for one of two available technologies. After the adoption

decision, consumers receive payoffs in all periods of their permanence in the network, in which case expectations over future agents' choices need to be factored in. We consider that agents enjoy the technology in two different ways: through a stand alone value that is independent of the other agent's actions and through a network effect that depends on whether there are other agents adopting the same technology. We assume that the stand alone relative values of the competing technologies are stochastic. The consumer knows the relative stand alone value of the technology at the time of the adoption, but he can only form expectations over the future relative stand alone value of the technologies. This feature of the model can arise for different reasons. First we can think that consumers' preferences are subject to stochastic shocks unknown at the time of the purchase.[5] Second we can think that technologies are subject to fashion effects and that this may drive consumer preferences. Finally, we can think that the willingness to pay of consumers varies according to the development stage of the technologies, as specified in Adner (2004) and Adner and Levinthal (2001).[6]

The main result of our paper is that a unique equilibrium exists in which individuals adopt technologies via switching strategies that depend on predecessors' choices and on the observed relative stand alone value. Van Zandt and Vives (2007) show that for Bayesian games of strategic complementar-

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[5]Bass (1974) presents a general theory of preference shocks and brand switching.

[6]According to Adner (2004) and Adner and Levinthal (2001), the marginal utility of technological improvements for consumers is not constant in every step of the technology evolution. It assumes the form of an S-curve: agents have a very high willingness to pay for the first improvements, but this willingness decreases after a certain threshold. If we think about two technologies whose values evolve through time, it is possible that at a certain point consumers have higher willingness to pay for one technology, and at another point they prefer the other technology.

ities, there exists a greatest and least Pareto ranked pure strategy Nash Equilibrium that are in monotone strategies. Their model is static in that agents do not have information on other agents' actions upon deciding. Our setup, nevertheless, is a dynamic bayesian game of strategic complementarities and hence a further complementarity assumption is needed such that coordination always brings positive rewards, independent on whether it is coordinating in the high or low action. With this further assumption, also our game has greatest and least pure strategy bayes Nash equilibrium which is in monotone strategies. Finally, using an argument of iterative dominance, we can further show that the maximal and minimal equilibrium threshold strategies coincide, and hence there will be a unique Bayes Nash equilibrium

A second result of our analysis is that technologies cannot lock in by historical events as would happen in the model by Arthur (1983). The absence of lock-in is in line with Liebowitz and Margolis (1994; 1995), who question the empirical relevance of lock-in patterns in technology adoption, arguing that lock-ins are extremely unlikely to occur. We provide a different rationale for the absence of lock-ins. Agents incorporate successors' choices and face technologies with stochastic stand alone values. Given that these values are independent of the number of adopters, network externalities are not the only driving force behind technology adoption. A lead in terms of installed base is not enough to attract all consumers. This occurs because agents are concerned with the relative stand alone value granted by the technology upon purchase, as well as with the value provided in subsequent periods.

Even though lock-in does not emerge in our setup, path dependence in technology choices is still present. Once the model parameters are laid out,

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the technology adopted by the predecessors affects the current user's decision. When this occurs the equilibrium path exhibits hysteresis and being the dominant technology for some periods, may guarantee further periods of dominance.

We characterize as well the unique equilibrium in technology adoption when a converting device is available and allows agents from different networks to interact. The conclusion on the absence of lock in (that is a direct consequence of the stochastic pattern of the technology value), does not change when converters are allowed. However converters contribute to mitigate hysteresis: agents have weaker incentives to coordinate their choices, and as a consequence technology adoption depends less on predecessors' decisions.

Since technologies do not lock in, a different measure of dominance should be studied. The first measure we propose is the limiting probability of technology adoption. This describes the likelihood that each technology is chosen in the long run, and provides a rough estimate of its expected demand schedule over long horizons. We show that technologies are more likely to be adopted in the long run if they 1) provide higher stand alone values, or 2) agents prefer bigger networks, or 3) they embed a converter device. In short run equilibrium a technology will be adopted for a certain number of consecutive periods, and then replaced. From the producer's point of view, it is therefore important to determine what is the expected time of adoption, our second measure for dominance. We find that the expected time of adoption decreases in the presence of converters due to the fact that compatible technologies reduce path dependence and one observes switching between

technologies more often.

1.1   Related Literature

Following seminal work by Rohlfs (1974), Farrell and Saloner (1985) and Katz and Shapiro (1986), the early literature adoption of network goods focused on relatively simple, static models.[7] Since then, the literature developed and recently some models have been put forth to analyse the dynamic implications of the technology adoption decision.

Ochs and Park (2010), take a similar standpoint from ours and highlight the importance of forward looking behavior in network formation. Their model differs from ours in that individual types (or technology values as in our setup) are uncorrelated, thus the global games equilibrium selection approach cannot be applied. In Ochs and Park (2010) it is possible to identify a unique symmetric perfect bayesian equilibrium since agents choose both when and whether to join a network.

We focus on adoption choices of two sponsored technologies, i.e., technologies which are non proprietary, or for which there is no strategic price interaction. Our analysis is, hence, directly related to Katz and Shapiro (1986), Arthur (1989) and Choi (1998). Some recent models focus on price competition between proprietary technologies, however, either forward looking behavior on the part of consumers is not considered or the multiplicity of equilibria resulting from factoring in the future behavior of consumers is disregarded. Also, pricing strategies are either assumed to be Markov or it

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[7]Farrell and Klemperer (2006) present an excellent survey of this literature. See also Economides (1996) and Garcia and Resende (2011).

is assumed that firms commit to a certain price level disregarding the fully dynamic features of the price interaction.

For instance, Fudenberg and Tirole (2000), Driskill (2007), Laussel and Resende (2009), Lee (2007) and Cabral (2010) study dynamic aspects of competition between proprietary networks. Fudenberg and Tirole (2000) establish a two-period model in which there are network effects in adoption and there is potential entry of a firm. Their main focus is on pricing to deter entry. Driskill (2007) presents a continuous-time overlapping-generations model where each cohort is heterogeneous in regards to the effect of a certain network good on individual's utility. Hence, each member of a cohort faces the problem of forecasting how many people in the future will purchase the good. It is then considered that firms may have two type of strategies: commitment to a price or Markov prices. Unlike ours, this paper does not deal with the issue of selecting among multiple equilibria in consumption choices. Laussel and Resende (2009) deals with the issue of firm competition in primary and aftermarkets, i.e., the competition in the market for the network good and for its complementary goods and services. While obtaining the equilibria, once again, the issue of multiple equilibria in consumption is dealt away by assuming that one of the firms locks-in. Markov perfect strategies in prices are then analysed. Cabral (2010) analyses the same problem as Laussel and Resende (2009), without restricting to linear Markov strategies in price competition. This gain in generality is at the cost of not obtaining a closed form solution for equilibrium prices and market shares. Cabral (2010) obtains numerically that price strategies in equilibrium are highly nonlinear and hence the analysis should not be restricted to linear Markov strategies.

Another series of papers attempts to address the dynamic competition between proprietary networks, namely Doganoglu (2003), Mitchell and Skrzypacz (2006), Markovich (2004), Markovich and Moenius (2009) and Doraszelski, Chen, and Harrington (2009), however, we mainly have that in this papers either consumers are not strategic as in Mitchell and Skrzypacz (2006) or that the utility of consumption of the network good only depends on the past market shares as in Doraszelski, Chen, and Harrington (2009).

Finally, our paper also contributes to the literature on persistence of firm dominance. Seminal papers are Gilbert and Newbery (1982), Reinganum (1983) and Arthur (1989). Other contributions to this literature are Budd, Harris and Vickers (1993), Cabral and Riordan (1994), Athey and Schmutzler (2001) and Cabral (2002). These papers provide conditions under which larger firms tend to become larger dominating the market. Our framework provides a rational for which firm dominance may be inverted and increasing dominance is not observed.

The remainder of the paper is organized as follows. The sequential move game with incompatible technologies is described in section 2. Section 3 characterizes the unique equilibrium and analyses the impact of the underlying parameters on the equilibrium outcome. The long run behavior of our adoption game is described in section 4. Section 5 focuses on the robustness of the equilibrium outcome to alternative specifications, as well as the introduction of partial converters. Finally section 6 concludes.

2　Theoretical model

We consider a sequence of users planning to adopt a technology within a discrete time and infinite horizon setting. At each time $n \in N$, player n enters the game and chooses among two competing technologies 1 and 0. Each player lives for two periods. In the first period, player n buys a single unit of one of the two technologies and commits to his choice in period $n+1$.[8] We denote player n's action set as $A_n = \{0, 1\}$ where $a_n = 1$ (resp. $a_n = 0$) corresponds to the choice of technology 1 (resp. 0). We restrict our analysis to unsponsored technologies resulting in their supply at a price equal to zero.[9]

2.1　Preferences over technologies

Player's valuation of a technology reflects two components, the stand alone and the network value. The former captures the utility of obtained if no other player adopts the same technology, while the latter is the benefit from interacting with other players. Players derive utility from adopting a given technology in both periods of their lives, and discount the payoffs received in the second period of life via a common discount factor $\beta \in (0, 1)$.

The stand alone payoff at time n depends on the contemporaneous technology value, x_n. We assume that the stand alone payoff is linear in x_n. Player n stand alone value is therefore a function of the current and future

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[8]Choice irreversibility is often assumed in the literature of (dynamic) network adoption. To our knowledge Farrell and Saloner (1985) are the only ones obtaining -rather than imposing- choice irreversibility. However they are admittedly unable to explain why this result arises (see Farrell and Saloner (1985), footnote 9 and Malin (2003))

[9]The term unsponsored technologies was first used by Arthur (1989) to refer to technologies that are non-appropriable. The absence of property rights leads to entry in the market until marginal cost pricing condition is met, i.e. in our case until prices are zero.

technology value, and his chosen technology a_n according to

$$\pi_S = \pi_S \left(a_n, x_n, x_{n+1} \right) = \begin{cases} x_n + \beta x_{n+1} & \text{if } a_n = 1 \\ - \left(x_n + \beta x_{n+1} \right) & \text{if } a_n = 0 \end{cases} \tag{1}$$

Network externalities arise when consumers interact with each other because they use the same technology. Since each player lives for two periods, at each time n there are two generations active in the market. Thus player n's network value depends on the technology chosen by his immediate predecessor $n - 1$ and by his immediate successor $n + 1$. We consider the network payoff at time n to be linear in the number of players that use the same technology at time n, and we assume that the two technologies are identical in terms of the network benefit they provide per unit of member in the base, $\nu > 0$. If player n and $n - 1$ adopt the same technology, they receive a network payoff of ν at time n. The same payoff is received at time $n + 1$ by player n and $n + 1$ if they coordinate on the same technology. Player n's network value is given by:

$$\pi_D = \pi_D \left(a_n, a_{n-1}, a_{n+1} \right) = \begin{cases} \nu \left(a_{n-1} + \beta a_{n+1} \right) & \text{if } a_n = 1 \\ \nu \left[1 - a_{n-1} + \beta \left(1 - a_{n+1} \right) \right] & \text{if } a_n = 0 \end{cases} \tag{2}$$

Player n overall utility is the sum of the stand alone and network values

$$u_n = u \left(a_n, a_{n-1}, a_{n+1}, x_n, x_{n+1} \right) = \pi_S \left(a_n, x_n, x_{n+1} \right) + \pi_V \left(a_n, a_{n-1}, a_{n+1} \right) \tag{3}$$

where the first component on the RHS corresponds to the sum of the current

and the (discounted) future stand alone payoffs, while the second component is the sum of the network payoffs derived by both the existing base and the (discounted) future base. Making use of (1) and (2), player n overall utility in (3) reads

$$u_n = \begin{cases} x_n + \beta x_{n+1} + \nu\left(a_{n-1} + \beta a_{n+1}\right) & \text{if } a_n = 1 \\ -x_n - \beta x_{n+1} + \nu\left[1 - a_{n-1} + \beta\left(1 - a_{n+1}\right)\right] & \text{if } a_n = 0 \end{cases} \qquad (4)$$

Finally, we denote by x_n the technology value at time n, and assume the evolution of technology values follows a Markov process

$$x_n = x_{n-1} + \sigma\varepsilon_n, \qquad (5)$$

where $\sigma > 0$ and ε_n are i.i.d. random shocks that can take two realizations, 1 or -1, with probability $p \in (0,1)$ and $q = 1 - p$, respectively. Since the time n stand alone payoff is linear in the current technology value, x_n can be equivalently thought of as a function of the the per-period relative value of technology 1 for a consumer that does not value interaction with his peers.

At time n, agent n is aware of the current technology valuation x_n and his predecessor's choice a_{n-1}. A strategy for player n is thus a function $s_n = s_n\left(a_{n-1}, x_n\right) : \{0,1\} \times \mathbb{R} \rightarrow \{0,1\}$. Player n chooses his action to maximize the expected payoff in (3). Letting $s_{n+1} = s_{n+1}\left(a_n, x_{n+1}\right)$ denote the strategy of player $(n+1)$, the $n-$th user solves the following:

$$\max_{a_n \in A_n} E\left[u_n | a_{n-1}, x_n; s_{n+1}\right]. \qquad (6)$$

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From (6) it emerges that, when choosing which technology to purchase, player n takes into account the effect of his action in determining the future size of the network, which in turn affects player $n + 1$ compatibility payoff. It is worth noting that the process (5) allows the technological valuation to be correlated through time, which turns out to be crucial in enabling player n to forecast the next generation's strategy after observing x_n. We allow user n to receive network benefits from the (expected) future base via (6), and thus we explicitly bring in a role for predicting future technology values.

3   Solving for an equilibrium

3.1   The benchmark case: no network externalities

If there are no network externalities, utility is simply given by the stand alone value and the maximization problem (6) therefore resumes to:

$$\max_{a_n \in A_n} E\left[\pi_S\left(a_n, x_n, x_{n+1}\right) | x_n\right]$$

In the absence of network benefits, player n is indifferent between the two technologies when the current technology value $x_n = \hat{x}$ solves

$$E\left[\pi_S\left(1, \hat{x}, x_{n+1}\right) | \hat{x}\right] = E\left[\pi_S\left(0, \hat{x}, x_{n+1}\right) | \hat{x}\right] \tag{7}$$

We refer to the technology value $\hat{x}$ as the pivotal point, that is the threshold above which the agent has a strict preference for technology 1. Making use of (1) and noting that for the technology process (5) we have $E\left(x_{n+1} | x_n\right) =$

$x_n + \sigma\left(2p - 1\right),$ condition (7) yields

$$\hat{x} = \frac{\beta\left(1 - 2p\right)}{1 + \beta}\sigma, \tag{8}$$

and the equilibrium strategy for player n is $a_n\left(x_n\right) = 1$ if $x_n \geq \hat{x}$ and $a_n\left(x_n\right) = 0$ if $x_n < \hat{x}$. When the disturbance term in (5) has zero mean, i.e. $p = 1/2$, the pivotal point is zero since the best forecast for the future technology value is the current value x_n. In this case player n does not expect the future technology value to deviate from the current one. As a consequence, if he prefers technology 1 to 0 at time n (which would occur when $x_n \geq 0$) then he keeps the same preference ordering between technologies at time $n + 1$. From (8) the pivotal point decreases in p. A larger p means that technology 1 is expected to provide large stand alone values in the future, $E\left(x_{n+1}\right) > x_n$, which creates an incentive for player n to choose technology 1, thus resulting in a lower value for the pivotal point. Moreover, $\hat{x}$ decreases in the discount factor for $p > 1/2$ and increases otherwise. This follows from the fact that technology 1 is more valuable (in the future) relative to technology 0 when $p > 1/2$. Since player n discounts the future expected stand alone payoff at rate β, then it will take a lower current technology value to make him adopt 1 at time n. Indeed, when the discount factor is close to zero, future payoffs becomes irrelevant and the pivotal point shrinks toward zero.

3.2 The effect of network externalities

We now analyze equilibrium strategies when ν is strictly positive. In order to solve for the equilibrium in the sequential move game outlined in section

2 We first state the following:

Lemma 1. Let $\sigma < \nu$. The space of technology values contains a region $[\bar{x}, +\infty)$ where technology 1 is dominant, and a region $(-\infty, \underline{x}]$ where technology 0 is dominant, i.e.

$$a_n\left(x_n\right) = \begin{cases} 1 & \text{if } x_n \geq \bar{x} = \hat{x} + \frac{\nu}{2} \\ 0 & \text{if } x_n < \underline{x} = \hat{x} - \frac{\nu}{2} \end{cases}$$

According to Lemma 1 there exist technology values for which the stand alone value offsets the benefits from coordinating on a network. This ensures that some players would choose a technology regardless of the network size: technology 1 is dominant whenever x_n is above the critical value $\bar{x}$, whereas technology 0 is dominant if the technology value falls below $\underline{x}$. In general, multiple equilibria would occur between $\underline{x}$ and $\bar{x}$, and the restriction $\sigma < \nu$ ensures that this region is non-empty. This region is symmetric around $\hat{x}$, i.e. the pivotal point for dominant actions in the absence of network benefits. Note that the gap $\bar{x} - \underline{x}$ depends positively on the network benefits. When players attach a large value to joining a network, a high stand alone value is needed in order to adopt a given technology regardless of other players' adoptions. As a result both $\bar{x}$ and $\underline{x}$ would move away from $\hat{x}$ the larger is ν. Lemma 1 plays a key role in applying an iterated elimination argument, and thus solving for the unique equilibrium. Proposition 2 constitutes our main result.

Proposition 1. The game has a unique equilibrium, in which for all n

$$s\left(a_{n-1}, x_n\right) = \begin{cases} 1 \text{ if } x_n \geq x^*\left(a_{n-1}\right) \\ \\ 0 \text{ if } x_n < x^*\left(a_{n-1}\right) \end{cases}, \tag{9}$$

where $x^*\left(a_{n-1}\right)$ is a decreasing function of a_{n-1}. Moreover, let σ be suffi-
ciently small according to

$$\sigma < \frac{\nu\left(2 - \beta\right)}{2\left(1 + \beta\right)}. \tag{10}$$

Then

$$x^*\left(0\right) = \hat{x} + \frac{\nu\left(1 - \beta p\right)}{2\left(1 + \beta\right)} \tag{11}$$

$$x^*\left(1\right) = \hat{x} - \frac{\nu\left(1 - \beta\left(1 - p\right)\right)}{2\left(1 + \beta\right)} \tag{12}$$

According to Proposition 1, monotone strategies, in type, are played at equilibrium. The cut-off points $x^*\left(a_{n-1}\right)$ specify the technology values at which user n is indifferent between technologies. These cutoffs depend on the predecessor's observed action and on the expected behavior of the immediate successor. Comparison between $\bar{x}$ and $\underline{x}$ in Lemma 1 and the cutoffs in (11-12) reveals that $x^*\left(0\right)$ and $x^*\left(1\right)$ belong to the region $[\underline{x}, \bar{x}]$. Proposition 1 yields $x^*\left(0\right) > x^*\left(1\right)$, so that when technology 1 is highly valuable relative to 0 –this occurs when a player observes a relatively high value for x_n– it is going to be adopted regardless of the predecessors' choices. On the other hand when technology 0 is more valuable, player n is more likely to purchase technology 1 only if he observes his predecessor choosing the same.

The inequality (10) does not play any role for the equilibrium unique-

ness result. Thanks to condition (10), there exists at least one value of the technology value process within the two cut-off points. This rules out the admittedly uninteresting situation in which the technology values are such that the game jumps from one equilibrium to the other, i.e. outside the region $[x^* (1), x^* (0)]$, every few periods.

Consider the time during which all types fall into one of the dominance regions, say $x_n > \bar{x}$. In this case network benefits are not strong enough to observe players that prefer technology 1 choosing the competing technology. When technology values fall into $[\bar{x}, \underline{x}]$ results from supermodular games allow to determine the unique equilibrium path. More specifically, when x_n is above $x^* (0)$ technology 1 is chosen regardless of the predecessors' actions (similarly, technology 0 is adopted for x_n below $x^* (1)$).

The interval $[x^* (0), x^* (1)]$ generates hysteresis, since player n's choice depends on the predecessors' actions (as well as the expectation of the successor's action) whenever x_n falls into the hysteresis band. In other words, when x_n lies between $x^* (1)$ and $x^* (0)$ individual n's choice is determined by his predecessors' actions, and equilibrium adoption is path dependent. Restriction (10) ensures that this happens for at least one player.

3.3   Comparative statics of equilibrium points

We can perform the following comparative statics over the hysteresis band: 1) it narrows with the discount rate β, and 2) it widens with the network benefit ν. High values for β mean that the importance of the future base is high relative to the installed base. Thus, individuals tend to disregard the

predecessor's action and the cut-off points get closer to each other, and closer to the pivotal point $\hat{x}$. On the other hand an increase in the network externality ν would increase the importance of the network component relative to the stand alone value in the individuals' expected utility. Future expected direct network values are discounted through β, implying that direct interaction with the predecessor becomes more important (in utility terms) with respect to interaction with the successor. As a result, player n attaches more importance to the technology adopted by the second generation thus widening the hysteresis band. These effects are summarized in figure 1. We fix the number of players to $N = 2,500$ and consider the following values for the technology value process: $p = 1/2$, $x_0 = 0$ and $\sigma = 0.05$. Panel 1a shows the equilibrium technology adoption path with $\beta = 0.5$ and $\nu = 2$. In Panel 1b

we increase the discount factor to 0.95, while panel 1c deals with $\nu = 4$.
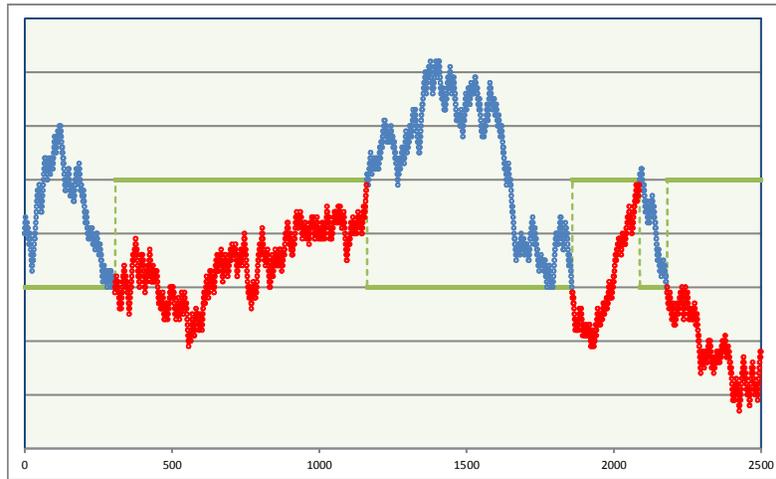
Panel 1a
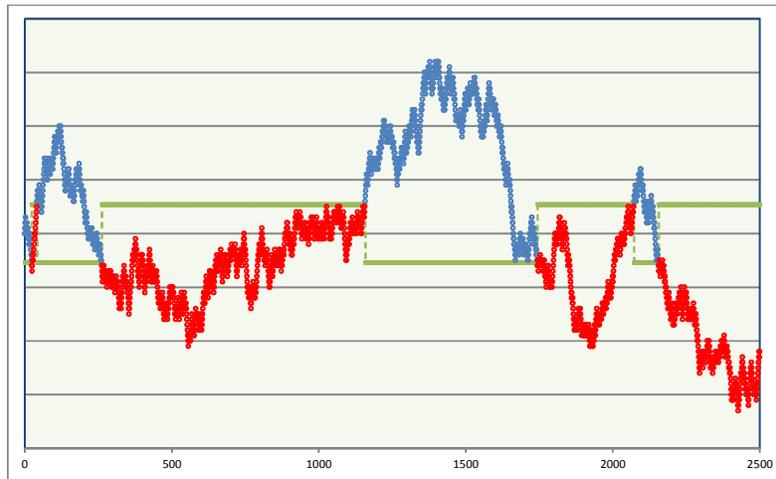
Panel 1b
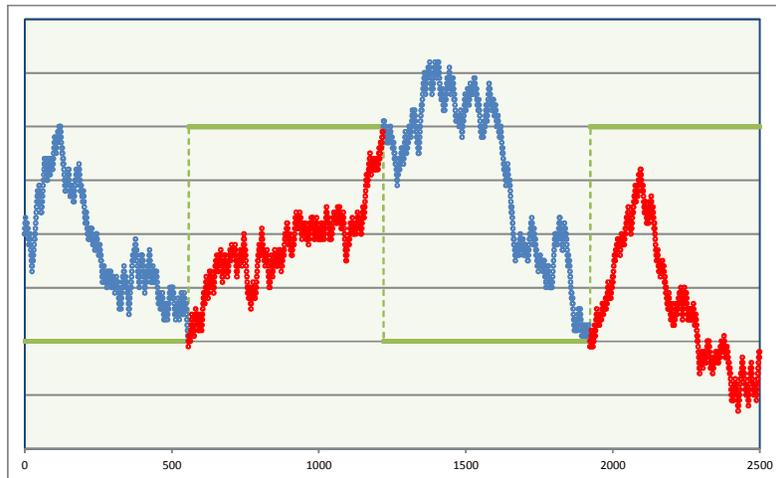
Panel 1c

Figure 1

4 Long run behavior and the expected time of adoption

We investigate lock-in effects in our OLG setup where individuals explicitly take into account the actions of future generations when choosing between two competing technologies. As mentioned in the Introduction, in Arthur (1989) one technology emerges as dominant as time goes by. In other words there exists a time after which *all* players opt for the same technology. In our setup, due to the stochastic nature of the individual types, the emergence of a technology as dominant is related to the likelihood of the stochastic process for x_n hitting the barriers $\bar{x}$, $\underline{x}$, $x^*(0)$ and $x^*(1)$. The long run characterization of our adoption game is thus captured by the limiting behavior of the technology process (5) like in Kandori, Mailath and Rob (1993). Technology 1 locks in if and only if x_n is always above $x^*(0)$ for large n (similarly technology 0 locks in if and only if x_n is below $x^*(1)$). More formally, let the adoption probabilities of the two technologies be defined as $\pi^1 = \lim_{n\to\infty} \Pr(x_n \geq x^*(0))$ and $\pi^0 = \lim_{n\to\infty} \Pr(x_n \leq x^*(1))$. Then technologies lock in if and only if either $\pi^1 = 1$ or $\pi^0 = 1$.

Proposition 3. No technology lock-in occurs in the long run.

The idea behind Proposition 3 is that technology values in (5) hit any barrier with positive probability, regardless of the uncertainty about future values σ. As a consequence, no technology can emerge as dominant in the long run, and lock-ins can occur only temporarily.

5   Discussion and extensions

We now briefly discuss the impact of alternative assumptions on our equilibrium outcomes.

First, equilibrium uniqueness is not driven by the linearity of the utility function in technology values as well as predecessor and successor's actions. The important feature of the model is that player n utility shows increasing differences in (a_n, x_n), (a_n, x_{n+1}), (a_n, a_{n-1}) and (a_n, a_{n+1}). A different specification of the payoffs would obviously imply different equilibrium cut-off points though. Similar, the binomial distribution of the random shocks in the technology value process can be dispensed -any random walk process would lead to equilibrium uniqueness.

Second, the important feature that must be imposed on the game is that it displays dominance regions, such that one can apply an iterative dominance argument and select a unique equilibrium. This means that payoffs should be specified in such a way that for *some* technology values the actions chosen by other players (via the installed and future base), play no role in determining player n's choice. The model in Arthur (**?**) does not belong to this class: it is not true for all n that one action is optimal no matter the technology value and the history. This happens because the stand alone value is bounded but the network value is unbounded and increasing in the actions of all the predecessors.

Finally, we consider the effect of converters enabling imperfect compatibility between technologies 1 and 0. Let $r \in (0, 1)$ denote the compatibility of technology 1 with 0 (s is defined similarly). The stand alone component

is not influenced by the existence of converters. Converters have an effect on the utility derived from networks, in that they allow agents to profit from the network even if no one else has chosen the same technology. Let's consider the following example: player n chooses technology 1 while both the previous and the following generations opt for technology 0. Compatibility results in a payoff of $\nu r (1 + \beta)$ contrasting with a null network benefit in the absence of converters. Similarly player n receives $\nu s (1 + \beta)$ if he adopts technology 0 and both agent $n - 1$ and $n + 1$ choose technology 1. Let $u_n^c = u^c (a_n, a_{n-1}, a_{n+1}, x_n, x_{n+1})$ denote player n overall utility with compatibility.

Using the payoffs defined in $(1, 2)$ for the incompatibility case gives:

$$
u_n^c = \begin{cases} u (1, a_{n-1}, a_{n+1}, x_n, x_{n+1}) + \nu r (1 - a_{n-1}) + \beta \nu r (1 - a_{n+1}) & \text{if } a_n = 1 \\ u (0, a_{n-1}, a_{n+1}, x_n, x_{n+1}) + \nu s a_{n-1} + \beta \nu s a_{n+1} & \text{if } a_n = 0 \end{cases}
$$

or equivalently:

$$
u_n^c = u_n + \nu r a_n (1 + \beta) + \nu (a_{n-1} + \beta a_{n+1}) (s - a_n (r + s)) \qquad (13)
$$

The analysis for compatible technologies can be found in the appendix. We summarize the main results as follows:

1) dominance regions widen: when technologies are compatible the gains of coordinating (i.e. the three generations choosing the same technology) are reduced, since player n profits from some network externalities even if he chooses a different technology relative to players $n-1$ and $n+1$ (see Lemma 2).

2) uniqueness of equilibrium and comparative statics: similar to Proposition 1, there exists a unique equilibrium in (type) monotone strategies that exhibits hystheresis. The hystheresis band narrows with β and widens with ν, similar to the case of incompatible technologies. In the presence of converters, however, an increase in β reduces the cut-off points less than under Proposition 1. This is because agents derive utility from the installed base even if they buy a technology that has not been chosen by the previous generation. Finally, an increase in either of the compatibility parameters r and/or s narrows the hystheresis band.

3) absence of lock-in: technologies cannot lock-in even in the presence of converters.

6 Conclusion

We have analyzed technology adoption choices of agents characterized by a forward looking behavior. The existing literature does not encompass users getting utility from the purchased technology over their whole life time. Due to this assumption, agents take into consideration the installed base only, but do not form expectations of the future base. The OLG model allows us to consider agents that: 1) receive benefits in all periods of their permanence in a network and 2) take them into account when choosing the technology in the first period. Agents must therefore form expectations about future behavior. This feature, together with the strategic complementarities arising from the network effects, yields multiple equilibria in technology adoption. Thanks to stochastic technology values, we are however able to prove uniqueness. This

is our main result: based on past observations, agents choose technologies through a unique switching strategy. A second result is that lock in cannot occur in our setup. The intuition is that, given the thresholds of the equilibrium strategy, it is always possible to find a technology value for which in any point in time agents switch from the most adopted technology to the least used one. Finally, we show that partially compatible technologies are characterized by lower path dependence.

The setup we consider lends itself to further extensions and modifications. For example, one can include agents living for more than two periods. In this case we would not expect the qualitative conclusions of our model to change. However, this extension would make the setup more realistic since at each point in time more than two generations are active in the market. Another promising direction is to introduce sponsored technologies produced by competing firms.

References

Adner, R. (2004) A demand-based perspective on technology life cycles, in *Business Strategy Over the Industry Lifecycle: Advances in Strategic Management* (J. Baum and A. McGahan, eds), 21, 25-43

Adner R. and Levinthal D. (2001) Demand heterogeneity and technology evolution: implication for product and process innovation, *Management Science*, 47, 611–628

Amir, R. (2005) Supermodularity and complementarity in economics: An elementary survey, *Southern Economics Journal*, 71, 636–660

Angeletos, George-Marios, Hellwig C. and Pavan. A (2007) Dynamic global games of regime change: learning, multiplicity, and the timing of attacks, *Econometrica*, 75, 711–756

Arthur, W.B. (1989), Competing technologies, increasing returns, and lock-in by historical events, *The Economic Journal*, 99, 116-131

Bass, F. (1974) The theory of stochastic preference and brand switching, *Journal of Marketing Research*, 9, 1-20

Bresnahan, T. and S. Greenstein (1999) Technological competition and the structure of the computer industry", *Journal of Industrial Economics*, 68, 1-40

Cabral, L. (2010) Dynamic price competition with network effects, mimeo, IESE Business School

Carlsson, H, E. van Damme (1993) Global games and equilibrium selection, *Econometrica*, 61, 989-1018

Choi, J.P. (1994), Irreversible choice of uncertain technologies with net-

work externalities, *RAND Journal of Economics*, 25, 382-401

Doganoglu, T. (2003) Dynamic price competition with consumption externalities, *Netnomics* 5, 656, 43–69

Doraszelski, U., J. Chen and J. Harrington (2009) Avoiding market dominance: product compatibility in markets with network effects, *RAND Journal of Economics*, 49, 455-485

Driskill, R. (2007) Monopoly and oligopoly supply of a good with dynamic network externalities, mimeo, Vanderbilt University

Economides, N. (1996) Network externalities, complementarities, and invitations to enter, *European Journal of Poilitical Economy*, 12, 211–233

Farrell, J. and P. Klemperer (1985) Coordination and lock-in: competition with switching costs and network effects, *Handbook of Industrial Organization* (M. Armstrong and R. Porter, eds.), 3, 1967-2072

Farrell, J. and G. Saloner (1985), Standardization, compatibility and innovation, *RAND Journal of Economics*, 16, 70-83

Fudenberg, D. and J. Tirole (2000) Pricing a network good to deter entry, *Journal of Industrial Economics,* 48, 373–390

Garcia, F. and J. Resende (2011), Dyamic games of network effects, *Dynamics, Games and Science II* (M. Peixoto, A. Pinto and D. Rang, eds), forthcoming

Heidhues, P. and N. Melissas (2006) Equilibria in a dynamic global game: the role of cohort effects, *Economic Theory*, 28, 531-557

Katz, M. and C. Shapiro (1986), Technology adoption in the presence of network externalities, *The Journal of Political Economy*, 94, 4, 822-841

Laussel, D. and J. Resende (2009) Does the absence of competition in the

market foster competition for the market? A dynamic approach to aftermarkets, mimeo, University of Porto

Liebowitz, S. and S.E. Margolis (1994), Network externality, an uncommon tragedy, *Journal of Economic Perspectives*, 8, 133-150

Liebowitz, S. and S.E. Margolis (1995), Path dependence, lock-in and history, *Journal of Law, Economics and Organization*, 11(1), 205-226

Markovich, S. (2008) Snowball: a dynamic oligopoly model with indirect network effects, *Journal of Economic Dynamics and Control*, 32, 909–938

Markovich, S. and J. Moenius (2009) Winning while losing: competition dynamics in the presence of indirect network effects, *International Journal of Industrial Organization*, 27, 346-357

Mitchell, M. and A. Skrzypacz (2006) Network externalities and long-run market shares, *Economic Theory*, 29, 621–648

Milgrom, P. and J. Roberts (1990). Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica*, 58, 1255-1277

Ochs, J. and I.-U. Park (2010), Overcoming the coordination problem, dynamic formation of networks, *Journal of Economic Theory*, 145, 689-720

Reny, P. J. (2011) On the existence of monotone pure strategy equilibria in bayesian games, *Econometrica*, forthcoming

Rohlfs, J. (1974) A theory of interdependent demand for a communication service, *Bell Journal of Economics*, 5, 16-37

Spitzer, F (2001) *Principles of random walk*, Springer-Verlag

Van Zandt, T. and X. Vives (2007), Monotone equilibria in bayesian games of strategic complementarities, *Journal of Economic Theory*, 134, 339-

360.

Appendix

Proof of Lemma 1. Let $\Delta_n = \Delta\left(a_{n-1}, a_{n+1}, x_n, x_{n+1}\right)$ be the difference in player n utility when he switches between technology 1 and 0:

$$\Delta_n = u\left(1, a_{n-1}, a_{n+1}, x_n, x_{n+1}\right) - u\left(0, a_{n-1}, a_{n+1}, x_n, x_{n+1}\right) \qquad (A.1)$$

which, using the expression for the player n utility in (4), becomes

$$\Delta_n = 2\left[x_n + \beta x_{n+1} + \nu\left(a_{n-1} + \beta a_{n+1}\right) - \frac{\nu\left(1 + \beta\right)}{2}\right] \qquad (A.2)$$

Let $\bar{\Delta}_n$ and $\underline{\Delta}_n$ denote respectively the value of Δ_n in (A.2) when both the users $n - 1$ and $n + 1$ choose technology 0 and 1, respectively, i.e. $\bar{\Delta}_n = \Delta\left(0, 0, x_n, x_{n+1}\right)$ and $\underline{\Delta}_n = \Delta\left(1, 1, x_n, x_{n+1}\right)$. From (A.2) one has:

$$\bar{\Delta}_n = 2\left[x_n + \beta x_{n+1} - \frac{\nu\left(1 + \beta\right)}{2}\right]$$

$$\underline{\Delta}_n = \bar{\Delta}_n + 2\nu\left(1 + \beta\right)$$

Finally, let $\bar{x}$ (resp. $\underline{x}$) be the technology value that makes player n indifferent between choosing one of the two technologies when both the predecessor and the successor coordinate on technology 0 (resp. 1), i.e. $E\left[\Delta\left(0, 0, \bar{x}, \bar{x} + \sigma\varepsilon_n\right)|x_n = \bar{x}\right] = 0$ (resp. $E\left[\Delta\left(1, 1, \underline{x}, \underline{x} + \sigma\varepsilon_n\right)|x_n = \bar{x}\right] = 0$). Using the above expressions for $\bar{\Delta}_n$ and $\underline{\Delta}_n$ and the fact that $E\left(x_{n+1}|x_n\right) = x_n + \sigma\left(2p - 1\right)$ under the Markov process (5) gives $\bar{x}$ and $\underline{x}$ in the main text.

Proof of Proposition 1. We prove this result in a series of steps. First we establish that the technology adoption game is monotone supermodular.

Invoking results in Van Zandt and Vives (2007), we then have that there exist a greatest and least BNE, and both are in monotone strategies. Second, we show that the greatest and least BNE coincide so that the equilibrium is unique.

Step 1: Consider the following properties of our technology adoption game:

i) From eq. (A.2) Δ_n is increasing in x_n, x_{n+1}, a_{n-1} and a_{n+1}

ii) Let $\Phi\left(\cdot\right)$ denote the cumultative distribution function for the technology value process in (5). Since ε_n is independent of x_{n-1} in (5), we have $\Phi\left(\varepsilon_n|x_{n-1} + \delta\right) = \Phi\left(\varepsilon_n|x_{n-1}\right) = \Phi\left(\varepsilon_n\right)$ for $\delta > 0$.

From property i) we have that player n utility $u\left(a_n, a_{n-1}, a_{n+1}, x_n, x_{n+1}\right)$ has increasing differences in $\left(a_n, a_{n-1}\right)$, $\left(a_n, a_{n+1}\right)$, $\left(a_n, x_n\right)$ and $\left(a_n, x_{n+1}\right)$, while from property ii) an increase in x_{n-1} increases the distribution of ε_n in the sense of first-order stochastic dominance. Therefore our technology adoption game is monotone supermodular (see Van Zandt and Vives (2007), p. 344), and there exist a greatest and least BNE, and both are in monotone strategies (Van Zandt and Vives (2007), Theorem 14).

Step 2: A monotone strategy for player n is $s\left(a_{n-1}, x_n\right)$ with the property that $s\left(a_{n-1}, x_n\right) = 1$ if and only if $x_n \geq x\left(a_{n-1}\right)$ where the cut-off $x\left(a_{n-1}\right)$ is a decreasing function of a_{n-1}. We now show that the cut-offs $x\left(0\right)$ and $x\left(1\right)$ are uniquely defined. Recall from Lemma 1 that $a_n = 1$ is dominant, i.e. $E\left(\Delta_n|x_n\right) > 0$, for $x_n \geq \bar{x}$ while $a_n = 0$ is dominant, i.e. $E\left(\Delta_n|x_n\right) < 0$, for $x_n < \underline{x}$. It remains to be shown that $E\left(\Delta_n|x_n\right)$ is strictly increasing in x_n between between $\underline{x}$ and $\bar{x}$. Let s_{n+1} be a monotone strategy for player

$n + 1$. By the definition of Δ_n in (A.1) we have that

$$
\begin{aligned}
E\left(\Delta_n | x_n\right) \;=\; & \Pr\left(x_{n+1} \geq x\left(0\right) | x_n\right) E\left[\Delta\left(a_{n-1}, 1, x_n, x_{n+1}\right) | x_n\right] \\
& + \Pr\left(x\left(0\right) > x_{n+1} \geq x\left(1\right) | x_n\right) \qquad\qquad\qquad\text{(A.3)} \\
& \times E\left[u\left(1, a_{n-1}, 1, x_n, x_{n+1}\right) - u\left(0, a_{n-1}, 0, x_n, x_{n+1}\right) | x_n\right] \\
& + \Pr\left(x_{n+1} < x\left(1\right) | x_n\right) E\left[\Delta\left(a_{n-1}, 0, x_n, x_{n+1}\right) | x_n\right]
\end{aligned}
$$

Since

$$
\Pr\left(x_{n+1} \geq x\left(0\right) | x_n\right) = \Pr_x\left(\varepsilon_{n+1} \geq \frac{x\left(0\right) - x_n}{\sigma}\right) = 1 - \Phi\left(\frac{x\left(0\right) - x_n}{\sigma}\right),
$$

$$
\Pr\left(x\left(0\right) > x_{n+1} \geq x\left(1\right) | x_n\right) = \Phi\left(\frac{x\left(0\right) - x_n}{\sigma}\right) - \Phi\left(\frac{x\left(1\right) - x_n}{\sigma}\right),
$$

and $\Pr\left(x_{n+1} < x\left(1\right) | x_n\right) = \Phi\left(\frac{x(1) - x_n}{\sigma}\right)$, the expectation in (A.3) rewrites

$$
\begin{aligned}
E\left(\Delta_n | x_n\right) \;=\; & E\left[\Delta\left(a_{n-1}, 1, x_n, x_{n+1}\right) | x_n\right] \\
& + \Phi\left(\frac{x\left(0\right) - x_n}{\sigma}\right) E\left[u\left(0, a_{n-1}, 1, x_n, x_{n+1}\right) - u\left(0, a_{n-1}, 1, x_n, x_{n+1}\right) | x_n\right] \\
& + \Phi\left(\frac{x\left(1\right) - x_n}{\sigma}\right) E\left[u\left(1, a_{n-1}, 0, x_n, x_{n+1}\right) - u\left(1, a_{n-1}, 1, x_n, x_{n+1}\right) | x_n\right]
\end{aligned}
$$

$$\text{(A.4)}$$

From (A.2) we have

$$
E\left[\Delta\left(a_{n-1}, 1, x_n, x_{n+1}\right) | x_n\right] = 2\left[x_n\left(1 + \beta\right) + \beta\sigma\left(2p - 1\right) + \nu\left(a_{n-1} + \beta\right) - \frac{\nu\left(1 + \beta\right)}{2}\right],
$$

while (4) gives

$$E\left[u\left(0, a_{n-1}, 1, x_n, x_{n+1}\right) - u\left(0, a_{n-1}, 1, x_n, x_{n+1}\right)|x_n\right] = -\nu\beta$$

$$= E\left[u\left(1, a_{n-1}, 0, x_n, x_{n+1}\right) - u\left(1, a_{n-1}, 1, x_n, x_{n+1}\right)|x_n\right],$$

so that (A.4) becomes

$$E\left(\Delta_n|x_n\right) = 2\left[x_n\left(1+\beta\right) + \beta\sigma\left(2p-1\right) + \nu\left(a_{n-1}+\beta\right) - \frac{\nu\left(1+\beta\right)}{2}\right]$$
$$-\nu\beta\left(\Phi\left(\frac{x\left(0\right)-x_n}{\sigma}\right) + \Phi\left(\frac{x\left(1\right)-x_n}{\sigma}\right)\right). \qquad (A.5)$$

Thus $E\left(\Delta_n|x_n\right)$ is strictly increasing in the current technology value x_n between $\underline{x}$ and $\bar{x}$, and the cut-offs are uniquely defined by $E\left(\Delta_n|x_n\right) = 0$. Suppose that σ is small enough so that $\sigma \le x\left(0\right) - x\left(1\right)$ (a condition that we verify later) so that $\Phi\left(\frac{x(0)-x(1)}{\sigma}\right) = 1$ and $\Phi\left(\frac{x(1)-x(0)}{\sigma}\right) = 0$. Then the cut-offs $x\left(0\right)$ and $x\left(1\right)$ solve

$$2\left[x\left(0\right)\left(1+\beta\right) + \beta\sigma\left(2p-1\right) - \frac{\nu\left(1-\beta\right)}{2}\right] - \nu\beta q = 0$$

$$2\left[x\left(1\right)\left(1+\beta\right) + \beta\sigma\left(2p-1\right) + \frac{\nu\left(1+\beta\right)}{2}\right] - \nu\beta\left(1+q\right) = 0$$

which give $x^*\left(0\right)$ and $x^*\left(1\right)$ in the main text. Finally, let $\bar{\sigma} = x^*\left(0\right) - x^*\left(1\right) = \frac{\nu(2-\beta)}{2(1+\beta)}$ so that $\sigma \le \bar{\sigma}$ is verified.

Proof of Proposition 2. This follows from the fact that every state in the technology value process is transient (if $p \ne q$) or (null) recurrent (if $p = 1/2$) (see Spitzer (2001))

Lemma 2. Let $\sigma < \nu - \frac{\nu(r+s)}{2}$. The space of technology values contains a region $[\bar{x}, +\infty)$ where technology 1 is dominant, and a region $(-\infty, \underline{x}]$ where technology 0 is dominant, i.e.

$$a_n(x_n) = \begin{cases} 1 & \text{if } x_n \geq \bar{x} = \hat{x} + \frac{\nu}{2} \\ 0 & \text{if } x_n < \underline{x} = \hat{x} - \frac{\nu}{2} \end{cases}$$

Proof. Let Δ_n^c be the difference in player n utility when he switches between technology 1 and 0 in the presence of converters:

$$\Delta_n^c = u^c(1, a_{n-1}, a_{n+1}, x_n, x_{n+1}) - u^c(0, a_{n-1}, a_{n+1}, x_n, x_{n+1})$$

which, using the expression for the player n utility in (13), becomes

$$\begin{aligned} \Delta_n^c &= 2\left[x_n + \beta x_{n+1} + \nu(a_{n-1} + \beta a_{n+1}) - \frac{\nu(1+\beta)}{2} \right] \\ &+ \nu r(1+\beta) - \nu(r+s)(a_{n-1} + \beta a_{n+1}) \end{aligned} \tag{A.6}$$

Similar to the proof of Lemma 1, we define as $\bar{\Delta}_n^c$ and $\underline{\Delta}_n^c$ the values of Δ_n^c when both the users $n-1$ and $n+1$ choose technology 0 and 1, so that

$$\bar{\Delta}_n^c = \bar{\Delta}_n + \nu r(1+\beta)$$
$$\underline{\Delta}_n^c = \bar{\Delta}_n - \nu s(1+\beta)$$

The dominance regions are defined by $\bar{x}^c$ and $\underline{x}^c$ that solve $E\left[\Delta^c(0, 0, \bar{x}, \bar{x} + \sigma\varepsilon_n) \,|\, x_n = \bar{x}^c\right] = 0$ and $E\left[\Delta^c(1, 1, \underline{x}, \underline{x} + \sigma\varepsilon_n) \,|\, x_n = \underline{x}^c\right] = 0$, which give $\bar{x}^c = \hat{x} + \frac{\nu(1-r)}{2}$ and $\underline{x}^c = \hat{x} - \frac{\nu(1-s)}{2}$.

Proposition 3. The game has a unique equilibrium, in which for all n

$$s\left(a_{n-1}, x_n\right) = \begin{cases} 1 \text{ if } x_n \geq x^{*,c}\left(a_{n-1}\right) \\ 0 \text{ if } x_n < x^{*,c}\left(a_{n-1}\right) \end{cases},$$

where $x^{*,c}\left(a_{n-1}\right)$ is a decreasing function of a_{n-1}. Moreover, let σ be sufficiently small according to

$$\sigma < \frac{\nu}{2\left(1+\beta\right)}\left(2 - \beta - p\left(r - \beta s\right) - q\left(s - \beta r\right)\right).$$

Then

$$\begin{aligned} x^{*,c}\left(0\right) &= \hat{x} + \frac{\nu\left(1 - r - p\beta\left(1 - s\right)\right)}{2\left(1+\beta\right)} \\ x^{*,c}\left(1\right) &= \hat{x} - \frac{\nu\left(1 - s - q\beta\left(1 - r\right)\right)}{2\left(1+\beta\right)} \end{aligned}$$

Proof. Inspection of (A.6) reveals that Δ_n^c is increasing in x_n, x_{n+1}, a_{n-1} and a_{n+1}. Since property first-order stochastic dominance is not affected by the presence of converters (see property ii in the proof of Proposition 1), we conclude that there exist a greatest and least BNE, and both are in monotone strategies (Van Zandt and Vives (2007), Theorem 14). Uniqueness comes from the dominance region and the fact that the function $E\left(\Delta_n^c | x_n\right)$ is strictly increasing in x_n between $\underline{x}^c$ and $\bar{x}^c$. To see the latter, we rewrite

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(A.5) as

$$
\begin{aligned}
E\left(\Delta_n | x_n\right) = {} & 2\left[x_n\left(1+\beta\right)+\beta\sigma\left(2p-1\right)+\nu\left(a_{n-1}+\beta\right)-\frac{\nu\left(1+\beta\right)}{2}\right] \\
& -\beta\nu\left(1-s\right)G\left(\frac{x\left(0\right)-x}{\sigma}\right)-\beta\nu\left(1-r\right)G\left(\frac{x\left(1\right)-x}{\sigma}\right)
\end{aligned}
$$

and the cut-off points $x^{*,c}\left(0\right)$ and $x^{*,c}\left(1\right)$ are obtained setting $E\left(\Delta_n | x_n\right)=0$ when player $n-1$ adopts technology 1 and 0.