A RANKING PROCEDURE FOR FUZZY DECISION-MAKING IN PRODUCT DESIGN

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Abstract: The decision-making process is a key issue for most manufacturing and services organizations, and forms an essential part of the product design process. Indeed, designers are always confronted with the dilemma of choosing between different alternatives with different parameters, often expressed in fairly vague terms. For this reason, many tools have been proposed offering more and better ways to improve company decision-making in this area. The ranking of fuzzy numbers is one of the most important of these tools, as fuzzy logic has a powerful capacity to manage the vague parameters usually associated with the expression of product requirements by the human personality. Improving the accuracy of such tools is a way to improve product design. In this vein, an extension of previous algorithms is proposed for the ranking procedure in fuzzy decision-making process. This extension is based on normal trapezoidal fuzzy numbers (and supports rectangular and triangular normal fuzzy numbers as well), instead of traditional triangular fuzzy numbers. The ranking procedure is based on the fuzzy preference relation. A product design example is provided.

Key words: product selection, fuzzy ranking, fuzzy decision-making, fuzzy preference relation, hamming distance

1- Introduction

The selection of a product for a specific customer is possible through a decision-making process in which the customer evaluates the advantages and disadvantages of each product relative to others. In the case where customers have specific requirements, the customers evaluate the products with the aim of identifying the product with the potential to provide the highest level of satisfaction. It may be that no product corresponds exactly to these requirements, but may correspond at some level of satisfaction. Fuzzy numbers have the capacity to represent such degrees of satisfaction. Of two different products, each providing a different degree of satisfaction, the customer selects the product that "dominates".

According to Tseng and Klein (1988, 1989), many ranking methods for fuzzy numbers have been developed. However, those methods did not consider many important factors, such as the shape, ranking order, and relative preference or dominance of fuzzy numbers, as well as the ease of computation of the ranking algorithm. It is therefore necessary to develop a new, accurate, effective, and efficient algorithm able to rank various shapes of fuzzy numbers.

Lee (2000) considered that the methods for ranking fuzzy numbers can be classified into two categories. The first is based on defuzzification and the second on the fuzzy preference relation. He maintains that a good ranking method should satisfy the following four criteria: (1) fuzzy preference presentation; (2) the rationality of preference ordering; (3) robustness; and (4) efficiency.

The algorithm for the ranking procedure in the fuzzy decision-making process presented in this work extends the algorithm proposed by Tseng and Klein (1989) which considers combinations of triangular fuzzy numbers. The proposed extension uses normal trapezoidal fuzzy numbers as a general model, among which rectangular and triangular fuzzy numbers are particular cases. Any pairwise combination of them is supported. A product design example proves its performance capacity in the product design decision-making process.

The paper is organized as follows. Section 2 presents a literature review. Section 3 describes the proposed ranking procedure. Section 4 gives an example applied to product design. Section 5 concludes the paper.

2- Literature review

2.1- Fuzzy logic in product design

Product design involves various phases, like evaluation, comparison, and decision-making. The application of fuzzy logic throughout all these phases makes it possible to include more accurate information related to customer desires. Several approaches, methods, and models have been proposed for the product design process which embrace different fuzzy models, such as the fuzzy goal programming model to determine the level of fulfilment of the design requirements (Chen and Weng, 2006), Green Fuzzy Design Analysis, which uses fuzzy logic to evaluate product design alternatives based on environmental considerations, and Fuzzy Multi-Attribute Decision-Making to select the most desirable design alternative (Kuo *et al*., 2006). (Shaowei, 2006) proposed a modified fuzzy model to coordinate and deal with the tangled fuzzy and non-fuzzy issues involved in product design.

Moreover, the Internet is now being used in this field, some examples of which include (Siddique and Ninan, 2005), who presented an Internet-based framework which uses a grammatical approach to represent and develop models of customized products, in order to integrate customer desires into the design of mass-customized products., The Internet has also been applied to develop a Web-based virtual design environment method which allows customers to participate in product design and help designers conveniently adjust the structure of their products. This method was published by (Shen *et al*., 2005).

During the last few years, a large number of application opportunities have appeared to take advantage of fuzzy logic in the product design process. It seems that product design is an area with great potential for the application of fuzzy logic (Barajas and Agard, 2008).

2.2- Fuzzy decision-making in product design

Product design is an engineering process involving iterative and complex decision-making. It usually starts with the identification of a need, proceeds through a sequence of activities to find an optimal solution to the problem, and ends with a detailed description of the product (Deciu *et al*., 2005).

Various approaches have been proposed for the fuzzy multi-criteria decision-making process based principally on the fuzzy preference relation (Fan *et al*., 2002; Büyüközkan and Feyzioglu, 2005; Işıklar and Büyüközkan, 2006; Zhang *et al*., 2007). Sun and Wu (2006) proposed an approach for the ranking process based on an easy and intuitive fuzzy simulation analysis method. Similarly, some fuzzy ranking methods for the decision-making process have been developed. Chen and Klein (1994) applied the α-cut and fuzzy subtraction operations to calculate the area under the new fuzzy number. Wang and Parkan (2005) presented three optimization models to assess the relative importance weights of attributes in a multiple-attribute decision-making problem. In all these approaches, the fuzzy ranking of fuzzy numbers plays an important role as a part of the decision-making process.

2.3- Ranking methods

The ranking of fuzzy numbers forms the basis of the decision-making process with fuzzy logic (Lee and You, 2003). Because of the importance and applicability of the ranking process, several methods have been developed, including the application of fuzzy ranking for multi-criteria decision-making (Baas and Kwakernaak, 1977; Chen and Klein, 1994). The fuzzy preference relation has been widely used for the fuzzy ranking procedure (Delgado *et al*., 1988; Lee, 2000; Modarres and Sadi-Nezhad, 2001). Other methods include the application of specific concepts like triangular membership functions (Chang, 1981), and maximization and minimization of sets (Chen, 1985). Lee and You (2003) presented a fuzzy ranking method for fuzzy numbers which considers various interesting functions and indices, such as the fuzzy satisfaction function, the fuzzy evaluation value, degree of defuzzification, degree of evaluation, and the relative index of defuzzification. A novel method incorporating fuzzy preferences and range reduction techniques was proposed by Ma and Li (2008). Yuan (1991) presented four criteria for evaluating fuzzy ranking methods, and he suggested an improved ranking method based on fuzzy preference representation, the rationality of fuzzy ordering, distinguishability, and robustness.

3- Ranking procedure in fuzzy decision-making

As mentioned in the previous section, the proposed algorithm for the ranking procedure in fuzzy decision-making is an extension of Tseng and Klein (1989), and consists of a more general model which considers different shapes of fuzzy numbers for ranking. Definitions of indifference and dominance are explained in section 3.1; section 3.2 explains the fuzzy preference relation; and section 3.3 presents the preference relation algorithm.

3.1- Indifference and dominance

Let A and B be two fuzzy numbers which are convex and normal fuzzy subsets.
If A and B are two fuzzy numbers, then there exist two notions, indifference and dominance, between the fuzzy numbers.

1- If there exists an area of overlap between fuzzy numbers A and B (intersection between A and B), then the overlap area is defined as indifference; that is A and B, are indifferent to one another in that area.
2- If there exist one or more non-overlap areas between fuzzy numbers A and B, then, for each non-overlap area, either A dominates B or B dominates A.

These notions for the general trapezoidal fuzzy numbers A and B can be seen in Figure 1. In case (a), B dominates A, and in case (c), A dominates B. Case (b) shows the notion of indifference represented by the intersection area between fuzzy numbers A and B.

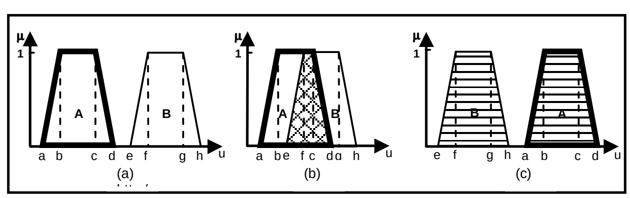

Figure 1: Dominance and indifference between A and B.

According to the above definitions of indifference and dominance, the non-overlap areas represent the domination areas for A or B. The domination between fuzzy numbers is given by the direction of A and B.

3.2- Fuzzy preference relation

If A and B are two fuzzy numbers, then the fuzzy preference relations R(A, B) and R(B, A) are defined as follows:

$$R(A, B) = \frac{\text{(areas where A dominates B) + (area where A and B are indifferent)}}{\text{(area of A)+(area of B)}}$$

$$R(B, A) = \frac{\text{(areas where B dominates A) + (area where A and B are indifferent)}}{\text{(area of A)+(area of B)}}$$

It is then obvious that R(A,B) + R(B,A) = 1,

where R(A, B) and R(B, A) are interpreted as the degree to which A is preferred to B and B is preferred to A respectively. The areas where A dominates B or B dominates A can be obtained using the Hamming distance.

Let S be an interval in the real line R. The Hamming distance between two fuzzy numbers A and B on S is then defined by

$$D(A,B \mid S) = \int_{u \in S} \left| \mu_A(u) - \mu_B(u) \right| du$$

where

$$S = \Re, D(A,B \mid \Re) = D(A,B)$$

The Hamming distance between fuzzy numbers A and B is actually the non-overlap area of the two numbers; that is, the sum of the areas where either A dominates B or B dominates A, or both. In this paper, the Hamming distance is used to obtain the preference relations between two fuzzy numbers. We illustrate this concept considering normal trapezoidal fuzzy numbers, as well as triangular and rectangular fuzzy numbers (see Figure 2).

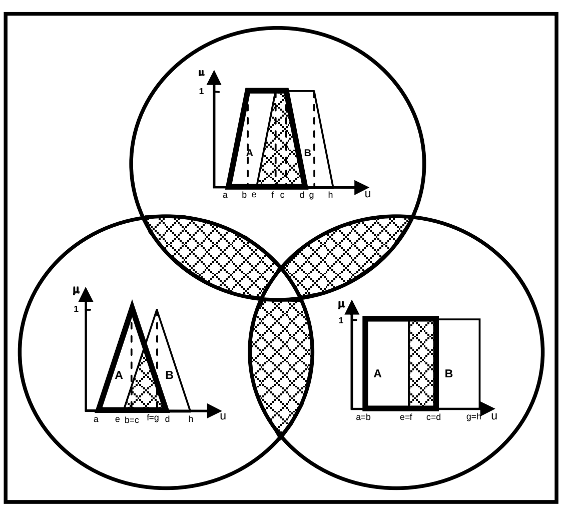

Figure 2: Fuzzy preference relations for trapezoidal, triangular, and rectangular fuzzy numbers.

Figure 2 also shows the possibility of comparing any pairwise combination of trapezoidal, triangular, and rectangular fuzzy numbers (e.g. triangular-triangular, triangular-trapezoidal, triangular-rectangular, and so on).

Let A and B be two normal trapezoidal fuzzy numbers where the support of A is the interval (a, d) and the support of B is the interval (e, h). The triangular fuzzy number is a particular case when b=c or f=g for A or B fuzzy numbers respectively. In the same way, Rectangular fuzzy numbers can be possible when a=b and c=d for fuzzy number A, and when e=f and g=h for fuzzy number B (see Figure 2).

Cases differ, depending on the fuzzy number (as illustrated in Figure 3). There may be different numbers of points of intersection, depending on the relative position of the fuzzy numbers. All these cases are depicted in Table 1. For each case, it is then necessary to consider the relevant areas when computing μ.

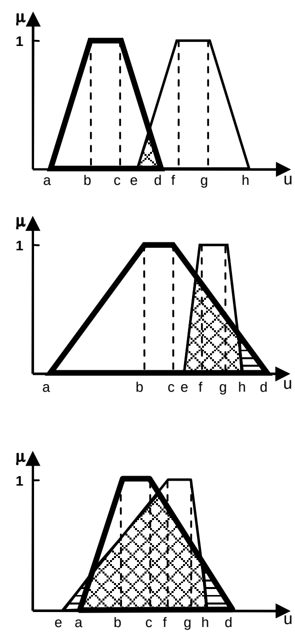

Figure 3: Some possible interaction points between fuzzy numbers.

3.3- Preference relation algorithm

Based on the definitions of dominance and indifference, the following algorithm can be used to determine a preference relation.

Step 1) Find the area where A and B are indifferent (intersection area)
Step 2) Find the areas where A dominates B
Step 3) Find the areas where B dominates A
Step 4) Find the areas of A and B
Step 5) Compute the fuzzy preference relations R(A,B) and R(B,A)

Let R(A,B) be the fuzzy preference relation and $\mu_R(A\ B)$ the membership function representation of R(A,B). The ordering of fuzzy numbers A and B is defined as follows.

Table 1: Number of points of intersection and relative position of the fuzzy numbers.

Points of intersection	Case statement based on normal trapezoidal fuzzy numbers
4	$a \geq e, b < f, c < g, d \geq h, c > f$
	$a \geq e, b < f, c > g, d \leq h$
	$a \leq e, b > f, c > g, d \leq h, b < g$
	$a \leq e, f \leq b, c \leq g, h \leq d$
3	$a \geq e, b < f, c < g, d \geq h, c < f$
	$a \geq e, b < f, c < g, d \leq h, c > f$
	$a \leq e, b < f, c < g, d \geq h, c > f$
	$a \geq e, b < f, c > g, d \geq h, b < g$
	$a \leq e, b < f, c > g, d \leq h, c > f$
	$a \geq e, b < f, c = g, d \geq h, b < g$
	$a \geq e, b < f, c = g, d \leq h, b < g$
	$e \leq a, f = b, g < c, d \leq h, b < g$
	$a \leq e, b = f, c > g, d \leq h, c > f$
	$a \leq e, b > f, c > g, d \leq h, b > g$
	$a \leq e, b > f, c > g, d \geq h, b < g$
	$a \leq e, f \leq b, c \leq g, d \leq h$
	$e \leq a, f \leq b, c \leq g, h \leq d$
2	$a \geq e, b < f, c < g, d \leq h, c < f$
	$a \leq e, b < f, c < g, d \geq h, c < f$
	$a \leq e, b \leq f, c \leq g, d \leq h, f \leq c$
	$a \leq e, b \leq f, g \leq c, h \leq d$
	$e \leq a, f = b, g = c, h \leq d$
	$a \geq e, b = f, c = g, d \leq h, b < g$
	$a \leq e, b > f, c > g, d \geq h, b > g$
	$a \geq e, b > f, c > g, d \leq h, b > g$
1	$a \leq e, b < f, c < g, d \geq e, c < f$
	$a \geq e, b > f, c > g, d \geq h, a \leq h, b > g$
0	$a < e, b < f, c < g, d \leq e$
	$a > e, b > f, c > g, a \geq h$

If the membership degree $\mu_R(A, B)$ is greater than 0.5, then the ordering is defined as A is preferred to B or A>B. If the membership degree $\mu_R(A, B)$ is equal to 0.5, then the ordering of A and B is defined as A is indifferent to B or A~B. If the membership degree $\mu_R(A, B)$ is less than 0.5, then the ordering is defined as B is preferred to A or B>A.

To extend the earlier definitions (Tseng and Klein, 1989), we apply here a pseudo-order preference model for the outranking of the fuzzy number procedure (Roy and Vincke, 1984; Wang, 1997; Gungor and Arikan, 2000; and Buyukozkan and Feyzioglu, 2004). Let the fuzzy preference relation between two ideas *a* and *b* for criterion *i* be obtained by pairwise comparison of $g_i(a)$ and $g_i(b)$, which will show the linguistic performance of ideas *a* and *b* respectively. $g_i(a)$ and $g_i(b)$ are represented by fuzzy numbers. Three types of preference relation are defined in terms of the fuzzy preference relations between two alternatives.

$$\forall a, b \in A, i \in C:$$
$$aP_ib \Leftrightarrow P(g_i(a), g_i(b)) - P(g_i(b), g_i(a)) > p_i,$$
$$aQ_ib \Leftrightarrow P(g_i(a), g_i(b)) - P(g_i(b), g_i(a)) \leq p_i,$$
$$aI_ib \Leftrightarrow \left| P(g_i(a), g_i(b)) - P(g_i(b), g_i(a)) \right| \leq q_i,$$

where P_i and Q_i depict strict and weak preference respectively, and *Ii* depicts indifference. The preference threshold p_i and indifference threshold q_i (defined by common sense, Roy and Vincke, 1984) are used to discriminate between the indifference, strict preference, and weak preference of two alternatives for criterion *i*.

4- Applications

Section 3 provides a ranking procedure in fuzzy decision-making which makes it possible to determine preference relations between two normal fuzzy numbers (trapezoidal, triangular, rectangular, and a mix of these). This could be useful in the decision-making process.

Consider the design of a product to satisfy a specific market share. A company's marketing department carried out an analysis to evaluate the customer's requirements for such a potential product. This analysis provides answers which describe the product's functionalities in rather vague descriptive terms, for example:

- The product should be a middle-range one.
- The price should be between x_1 and x_2.
- Most customers prefer characteristic C_1 to be small and characteristic C_2 to be high.
- And so on…

Dealing with such information may be difficult. It is sometimes possible to evaluate some criteria fairly precisely. An evaluation of the price, for example, is a current indicator of preference when designing the product. It could be more complicated when the characteristics are difficult to evaluate, however. C_1 could be the perception of product size and C_2 product reliability. Also, the terms "high" and "small" have to be defined.

It is usual to consider boundaries in such cases. C_1 may be considered acceptable, for example, if the product size is less than 50 mm in height, width, or diameter (see Figure 4).

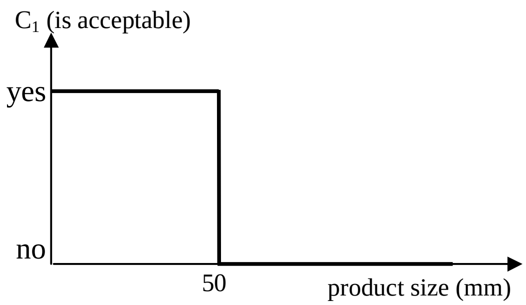

Figure 4: Discrete evaluation for customer requirement C_1.

But what happens if the product is 51 mm or 52 mm, or 55 mm in an specific size. Some customers may be willing to accept the product with more "fuzzy" boundaries. Maybe as it grows to 60 mm, fewer and fewer customers will consider the product to be "small". A fuzzy representation of the requirement is then best adapted (see Figure 5).

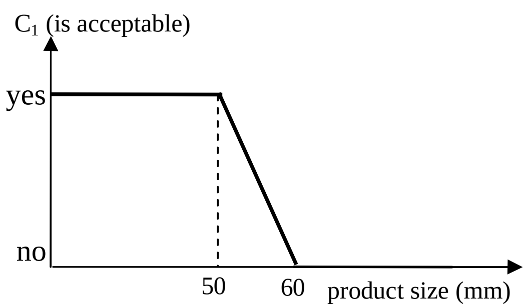

Figure 5: Fuzzy evaluation for customer requirement C_1.

The same applies for criterion C_2. Consider, for example, that criterion C_2 takes the following shape (Figure 6), evaluated on a [0, 10] scale (0 the product will never work…, 10 the product will always work!).

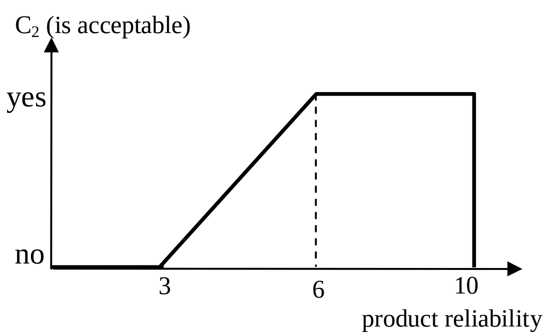

Figure 6: Fuzzy evaluation for customer requirement C_2.

Consider now that there are two alternatives when designing the product, and that the design team has to decide which one to select.

Alternative 1 is a product where C_1 is 53 mm and it has a reliability C_2 of 6.5, while alternative 2 is a product with C_1 = 47 mm and a reliability C_2 = 4 (see Table 2).

Table 2: Approximate characteristics of the product alternatives to compare.

	C_1 (size)	C_2 (reliability)
Alternative 1	53 mm	6.5
Alternative 2	47 mm	4

What is the best alternative for the design team to select, and why?

These questions may be difficult to answer without an appropriate method. Section 3 provides all the elements of such a method, which is a step-by-step approach. For each alternative, a ranking of each characteristic will be performed.

One possible fuzzy modelling of the characteristics for alternative 1 and alternative 2 is provided in Figure 7 (different shapes are possible).

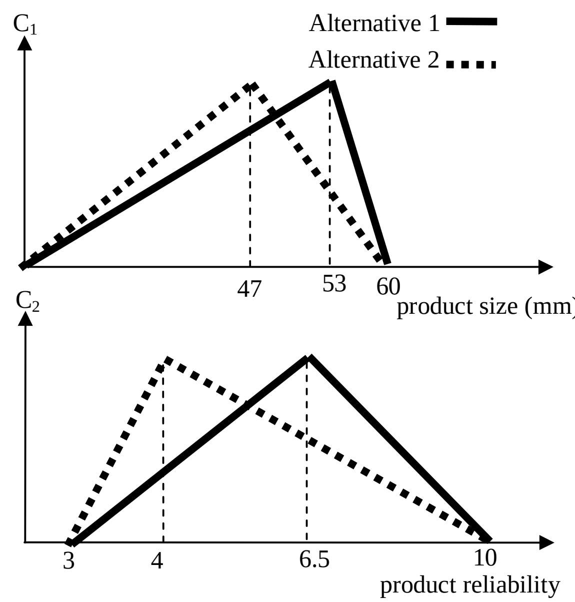

Figure 7: Fuzzy modelling of the product alternatives.

For alternative 1 and characteristic C_1, the method concerns the ranking of the following two fuzzy numbers (Figure 8).

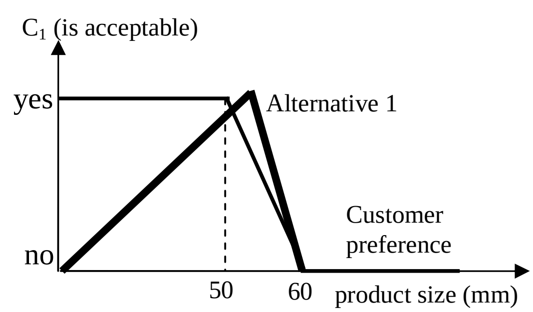

Figure 8: Fuzzy ranking of alternative 1 for characteristic C_1.

Denote A as the customer preference and B as Alternative 1. Then, Dom(A, B) is the area where fuzzy number A dominates fuzzy number B, Ind(A-B) is the area of indifference between A and B, Area(A) is the area of fuzzy number A, and Area(B) is the area of fuzzy number B.

To obtain the fuzzy preference relation (section 3.2), we need:

Dom(A, B) = 0

Ind(A-B) = 28.5714
Area(A) = 55
Area(B) = 30
Then:
R(A, B) = 0.3361

For alternative 2 and characteristic C_1, the same applies:
Dom(A, B) = 1.5
Ind(A-B) = 30
Area(A) = 55
Area(B) = 30
Then:
R(A, B) = 0.3706

For alternative 1 and characteristic C_2, we obtain:
Dom(A, B) = 1.7500
Ind(A-B) = 3.5
Area(A) = 5.5
Area(B) = 3.5
Then:
R(A, B) = 0.5833

For alternative 2 and characteristic C_2, we obtain:
Dom(A, B) =3.5556
Ind(A-B) =2.7222
Area(A) = 5.5
Area(B) = 3.5
Then:
R(A, B) = 0.6975

Table 3 summarizes those results.

Table 3: Comparative fuzzy preferences..

	C1 (size)	C2 (reliability)
Alternative 1	0.3361	0.5833
Alternative 2	0.3706	0.6975

If a characteristic of an alternative matches the customer requirement exactly, we will have:
Area(A) = Area(B) = Ind(A-B), and Dom(A, B) = 0
Then:
R(A, B) = 0.5.

For each characteristic, the best alternative is then the one closest to 0.5.

Table 4: Evaluation of the best alternative for each characteristic.

	C_1 (size)	C_2 (reliability)
Alternative 1	|0.5-0.336|= 0.1639	|0.5-0.5833|= 0.0833
Alternative 2	|0.5-0.3706|= 0,1294	|0.5-0.6975|= 0.1975

Table 4 shows that, for the first characteristic (C_1), alternative 2 is the best choice. For the second characteristic (C_2), alternative 1 is the best choice. At this point, it is not possible to determine the best alternative.

The design team has to prioritize characteristics C_1 and C_2. Consider the follows weight of importance (w_i) for each characteristic:

- w_1=0.4 for characteristic C_1
- w_2=0.6 for characteristic C_2

Then, the weighted average for each alternative is as follows:

$$R_j = \frac{\sum_{i=1}^{n} R_{ij}(A,B) * w_i}{\sum_{i=1}^{n} w_i}$$

For Alternative 1:
R_1= [(0.3361) (0.4) + (0.5833) (0.6)]/ (0.4+0.6) = 0.48442

For Alternative 2:
R_2= [(0.3706) (0.4) + (0.6975) (0.6)]/ (0.4+0.6) = 0.56674

This has to be compared to 0.5. We can now finally conclude that Alternative 1 is better than Alternative 2 for meeting the customer requirement.

5- Conclusions

In this paper, extended scope for the ranking procedure has been proposed, based on the fuzzy preference relation, in the fuzzy decision-making process. An example of how a product design decision is reached shows its applicability to the solution of real problems. With our procedure, it is possible to obtain the preference relation (strict, weak, or indifference) for any pairwise combination of normal fuzzy numbers. For the product design process, this contribution is highly important because many parameters from real applications are not strict, and modelling with fuzzy numbers (which are able to represent vague parameters as expressed by a human) will enrich the decision-making process. This powerful capacity is a critical aspect of product design which could significantly improve the design process. The example provided shows a possible application where different product alternatives are compared, where the parameters are evaluated with vague parameters (or parameters expressed in the form of preferences). Even with multiple variables and different preference relations for various parameters, the methodology is able to point to the best compromise. A future extension of our result here will address product configuration and the design of product families.

6- Acknowledgments

This research has been supported by funding from the Natural Sciences and Engineering Research Council of Canada (NSERC) and by the Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT).

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