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Decentralized observer-based fuzzy control for interconnected networked control systems with quantization and random packets dropout

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Abstract—**This paper addresses the problem of decentralized observer-based control for a class of discrete large-scale Takagi-Sugeno (TS) fuzzy systems in the presence of control quantization and random packets dropout, occurring in the controller-to-actuator, in the sense of stochastic stability. Thus, based on fuzzy Lyapunov candidate function, sufficient stability conditions formulated in Linear Matrix Inequalities (LMIs) terms are proposed. In order to solve the nonlinear problem, a cone complementarity formulation and variables changes are offered. Finally, a numerical example is considered ensuring the stability of the decentralized Networked control systems (DNCSs).**

Index Terms—**Decentralized observer-based networked control, Takagi-Sugeno (TS), quantization, random packets dropout**

I. Introduction

Interconnected systems (e.g., material processing systems, chemical processes) are composed into small-order sub-models to simplify the method development [12]. It has been seen that applying centralized controllers to this class of systems make the duty impractical and unprofitable [1]. These restrictions motivate the design of decentralized control schemes.

Decentralized control strategies have been studied for many years and there are many results concerning the development of decentralized schemes for interconnected systems including decentralized output feedback schemes [3]. Much existing work considers interconnected systems with nonlinearity. A class of nonlinear interconnected systems approximated by a (TS) fuzzy model is investigated in [14].

(TS) fuzzy models are considered as an efficient tool to approximate nonlinear Large-scale systems. We considered that these latter are systems in which communication in the feedback closed-loop system, from various components (sensors, controller, observer and actuators) is exchanged through a network with limited bandwidth and so called networked control systems (NCSs). It is emphasized that packet dropouts which have "random" characteristics, are one of the important to be considered in NCS analysis and synthesis [9], because they will probably degrade the performance of the NCS and also cause system instability. In fact, [6] presents a new approach on H_∞ filtering problem for uncertain network control system with distributed delay, quantization and packet dropout with Bernoulli distributions. On the other hand, the internal states of many industrial plants cannot be directly measured and only their outputs are available for control purposes, output feedback (TS) fuzzy controllers were considered [7]. To the authors best knowledge, decentralized observer-based controller design, has not been solved up to now for nonlinear large-scale discrete-time (TS) fuzzy systems, in the presence of control quantization and random packets dropout [13].

This work aims to develop a decentralized observer-based control conditions for nonlinear interconnected networked control systems. Effects of quantization phenomenon and random packets dropout are considered into control design problem. Based on fuzzy Lyapunov candidate function, sufficient stability conditions derived in LMIs terms for (TS) fuzzy systems are provided.

The paper is organized as follows. Section 2 is devoted to system description and preliminaries. Section 3, presents the main results, describing the control strategy and network with the synthesis conditions of decentralized control law that works through a communication network. Section 4, shows simulation results. Some conclusions and future works are presented at the end of the paper.

Notations: $sym(W)$ stands for $W + W^T$. Symbol (*) within a matrix represents the symmetric entries. $I \in R^n$ is the identity matrix. l_2 is the space of square integrable functions over $[0, \infty)$, and $||.||_2$ denotes the l_2-norm. $\mathcal{E}\{.\}$ denotes the expectation and $Prob\{.\}$ means the probability.

II. System Description and Preliminaries

Consider a class of large-scale system S composed of J interconnected subsystems $S_i, i = 1, 2, ..., J$. The resulting discrete-time (TS) fuzzy system model S_i shown in Fig. 1, with quantizer and data losses between the controller and the actuator, is inferred as the weighted average of the local models of the form

$$\begin{cases} x_i(t+1) = \sum_{l=1}^{r_i} h_i^l \Big\{ A_i^l x_i(t) + B_i^l U_{ci}(t) + \sum_{j=1}^{J} f_{ij}^l (x_j(t)) \Big\} \\ y_i(t) = C_i x_i(t) \end{cases} \tag{1}$$

We note $h_i^l(\theta_i(t)) = h_i^l$. For $i = 1, 2, ..., J$, $l = 1, 2..., r_i$, $x_i(t) \in R^n$ is the state vector; $y_i(t) \in R^p$ is the measured output; $\theta_{i1}(t), \theta_{i2}(t), ..., \theta_{ig}(t)$, are some measurable premise variables for subsystems S_i, $f_{ij}^l(x_j(t))$ represents the interconnection of fuzzy rules in subsystem S_i and subsystem S_j, and r_i represents the number of fuzzy rules in subsystem S_i. A_i^l, B_i^l and C_i are constant matrices with appropriate dimensions, where

$$h_i^l = \frac{v_i^l(\theta_i(t))}{\sum_{l=1}^{r_i} v_i^l(\theta_i(t))}, \quad v_i^l(\theta_i(t)) = \prod_{q=1}^{g} F_{iq}^l(\theta_{iq}(t)) \tag{2}$$

with $F_{iq}^l(\theta_{iq}(t))$ is the grade of membership of $\theta_{iq}(t)$ in the fuzzy set F_{iq}^l. $h_i^l(\theta_i(t))$ is membership function for each fuzzy rule, which represent normalized grade of membership, and satisfies

$$0 \le h_i^l \le 1, \text{ for } l = 1, 2, ..., r_i, \ \sum_{l=1}^{r_i} h_i^l = 1 \tag{3}$$

Assumptions

1) All pairs (A_i^l, B_i^l) $(i = 1, 2, ..., J$ and $l = 1, 2, ..., r_i)$ are stabilizable.
2) The interconnection $f_{ij}^l(x_j(t))$ is a smooth nonlinear function, which satisfies the following condition : $\|f_{ij}^l(x_j(t))\| \le \bar{f}_{ij}^l \|x_j(t)\|$, where $\bar{f}_{ii}^l = 0$, $\bar{f}_{ij}^l (i \neq j)$ is a positive constant.

Currently, the state variables are not always accessible to measurement. This problem can be overcome by introducing the fuzzy observer theory.
The overall fuzzy observer is represented as follows:

$$\begin{cases} \hat{x}_i(t+1) = \sum_{l=1}^{r_i} h_i^l [A_i^l \hat{x}_i(t) + B_i^l U_i(t) + L_i^l (y_i(t) - \hat{y}_i(t)] \\ \hat{y}_i(t) = C_i \hat{x}_i(t) \end{cases} \tag{4}$$

where $\hat{x}_i(t)$ is the state estimation, $\hat{y}_i(t)$ is the observer output, $U_i(t)$ is the control input vector and L_i^l are the constant observer gains to be determined.
The overall networked PDC fuzzy controller can be described as follows :

$$U_i(t) = \sum_{l=1}^{r_i} h_i^l K_i^l \hat{x}_i(t) \tag{5}$$

where K_i^l, are the constant feedback gains to be determined. The control signals will be quantized before they are transmitted to next nodes. The logarithmic quantizer is considered here. It is called logarithmic if the set of quantized levels is characterized by

$$u = \{u_i, u_i = \rho^i u_0, i = 0 \pm 1 \pm 2, ...\} \cup \{0\}, \quad u_0 > 0$$

where the parameter $0 < \rho < 1$ is called the quantization density, and the logarithmic quantizer $q(\nu)$ is

$$q(\nu) = \begin{cases} u_i \ \text{ if } \ \frac{1}{1+\delta}\rho^i u_0 < \nu \le \frac{1}{1-\delta}\rho^i u_0 \\ 0 \ \text{ if } \ \nu = 0 \\ -q(-\nu) \ \text{ if } \ \nu < 0 \end{cases} \tag{6}$$

where $\delta = \frac{1-\rho}{1+\rho}$.
By the sector bound method, $q(\nu)$ can be expressed as in [11] by

$$q(\nu) = (I + \Delta(t))\nu$$

where

$$\begin{aligned} \Delta(t) = diag\{\Delta_1(t), \Delta_2(t), ..., \Delta_n(t)\}, \\ \|\Delta_j(t)\| \le \delta, \quad j = 1, 2, ..., n \end{aligned} \tag{7}$$

In this paper, we model the data-missing phenomenon via a stochastic approach

$$U_{ci}(t) = (1 - \beta(t))q(U_i(t)) + \beta(t)U_{ci}(t-1)) \tag{8}$$

where the stochastic variable $\beta(t) \in \mathbb{R}$ is an independent Bernoulli distributed white sequence with the probability distribution as follows: [13]

$$\begin{cases} Prob\{\beta(t) = 1\} = \mathbb{E}\{\beta(t)\} = \bar{\beta}, \\ Prob\{\beta(t) = 0\} = 1 - \mathbb{E}\{\beta(t)\} = 1 - \bar{\beta}, \end{cases} \tag{9}$$

where the constant $\bar{\beta} \in [0, 1]$ reflects the occurrence probability of the event of data lost. More specifically, $\beta(t) = 1$ when the link fails. i.e., data are lost, and $\beta(t) = 0$ means successful transmission.

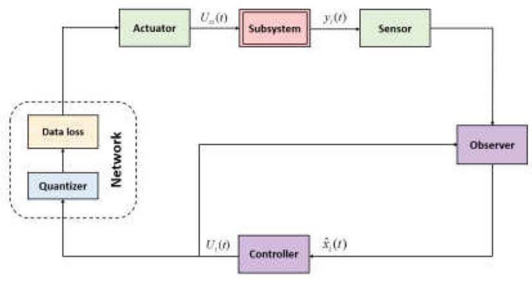

Fig. 1: Framework of networked control subsystem S_i

The objective is to determine feedback gains K_i^l and observer gains L_i^l such as the closed loop networked system with fuzzy observer (4) and fuzzy controller (5) is asymptotically stable. To do this, we define:

$$e_i(t) = x_i(t) - \hat{x}_i(t) \tag{10}$$

After introducing constraints of communication network namely quantization and packet dropouts and defining $\tilde{x}_i =$

$\begin{bmatrix} x_i^T & e_i^T & U_{ci}^T(t-1) \end{bmatrix}^T$, and from (1), (5) and (10) the augmented system can be expressed as the following form :

$$\begin{cases} \tilde{x}_i(t+1) = \tilde{\mathcal{A}}(t)\tilde{x}_i(t) + (\beta(t)-\bar{\beta})\tilde{\mathcal{M}}(t)\tilde{x}_i(t) + \Pi f(x(t)) \\ y_i(t) = \tilde{C}_1\tilde{x}_i(t) \end{cases} \tag{11}$$

It is seen that system (11) is a stochastic system with

$$\mathcal{E}\{\beta(t)-\bar{\beta}\} = 0, \quad \mathcal{E}\{(\beta(t)-\bar{\beta})^2\} = \bar{\beta}(1-\bar{\beta}) \tag{12}$$

$$\tilde{\mathcal{A}}(t) = \tilde{\mathcal{A}}_0 + \tilde{\mathcal{A}}_\Delta, \tag{13}$$

$$\tilde{\mathcal{A}}_0 = \sum_{l=1}^{r_i}\sum_{s=1}^{r_i} h_i^l h_i^s \begin{bmatrix} A_i^l + (1-\bar{\beta})B_i^l K_i^s & -(1-\bar{\beta})B_i^l K_i^s & \bar{\beta}B_i^l \\ -\bar{\beta}B_i^l K_i^s & A_i^l - L_i^l C_i + \bar{\beta}B_i^l K_i^s & \bar{\beta}B_i^l \\ (1-\bar{\beta})K_i^s & -(1-\bar{\beta})K_i^s & \bar{\beta} \end{bmatrix} \tag{14}$$

$$\tilde{\mathcal{A}}_\Delta = \sum_{l=1}^{r_i}\sum_{s=1}^{r_i} h_i^l h_i^s \begin{bmatrix} (1-\bar{\beta})B_i^l K_i^s\Delta(t) & -(1-\bar{\beta})B_i^l K_i^s\Delta(t) & 0 \\ (1-\bar{\beta})B_i^l K_i^s\Delta(t) & -(1-\bar{\beta})B_i^l K_i^s\Delta(t) & 0 \\ (1-\bar{\beta})K_i^s\Delta(t) & -(1-\bar{\beta})K_i^s\Delta(t) & 0 \end{bmatrix} \tag{15}$$

$$\begin{aligned} \tilde{\mathcal{M}}(t) &= \sum_{l=1}^{r_i}\sum_{s=1}^{r_i} h_i^l h_i^s \begin{bmatrix} -B_i^l\bar{K}_i^s & B_i^l\bar{K}_i^s & B_i^l \\ -B_i^l\bar{K}_i^s & B_i^l\bar{K}_i^s & B_i^l \\ -\bar{K}_i^s & \bar{K}_i^s & 1 \end{bmatrix}, \\ f(x(t)) &= \sum_{l=1}^{r_i} h_i^l \sum_{j=1}^{J} f_{ij}^l(x_j), \\ \tilde{C}_1 &= \begin{bmatrix} C_i & 0 & 0 \end{bmatrix}, \\ \Pi &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T, \\ h_i^l &= h_i^l(\theta_i(t)), \\ \bar{K}_i^l &= (I+\Delta(t))K_i^l \end{aligned} \tag{16}$$

$\tilde{\mathcal{A}}_0$ and $\tilde{\mathcal{A}}_\Delta$ represent, respectively, the nominal and the uncertain matrices.

In order to obtain the main results in this paper, the following lemmas are needed:

Lemma 2.1: [5] For each real vector ζ and ρ, we obtain that

$$2\zeta^T\rho \le \zeta^T Z\zeta + \rho^T Z^{-1}\rho \tag{17}$$

with $Z > 0$

Lemma 2.2: [15] The following inequality is verified for each real vector $\nu_i \in R^n$

$$[\sum_{i=1}^{m}\nu_i]^T[\sum_{i=1}^{m}\nu_i] \le m\sum_{i=1}^{m}\nu_i^T\nu_i. \tag{18}$$

III. MAIN RESULTS

Our main objective is to develop an observer-based fuzzy control scheme that uses a communication network to exchange actuator data transmission between the DNCS and its stability control system.

A. Stability analysis

The following quadratic Lyapunov functional candidate is used as well as :

$$\begin{aligned} \tilde{V}(\tilde{x}_i(t)) &= \sum_{i=1}^{J}\tilde{V}_i(\tilde{x}_i(t)),\ i = 1,2,...,J, \\ \tilde{V}_i(\tilde{x}_i(t)) &= \tilde{x}_i^T(t)P_i\tilde{x}_i(t) \end{aligned} \tag{19}$$

The following result is addressed for the robust closed-loop fuzzy system (11).

Lemma 3.1: For given scalars μ_1 and μ_2, closed-loop networked system (11) is asymptotically stable, if there exist positive matrices P_i and matrices $\tilde{G}_i$ with appropriate dimensions, such that the following condition holds

$$\Xi(t) = \begin{bmatrix} \Xi_{11}(t) & \Xi_{12}(t) \\ * & \Xi_{22} \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned} \Xi_{11}(t) &= -P_i + \mu_1 sym(\tilde{G}_i^T\tilde{\mathcal{A}}(t)) + \mu_1^2\tilde{G}_i^T\tilde{G}_i \\ &\quad + (2J\sum_{j=1}^{J}\hat{f}_{ji}^2\Pi\Pi^T), \\ \Xi_{12}(t) &= \mu_2\tilde{\mathcal{A}}^T(t)\tilde{G}_i - \mu_1\tilde{G}_i^T, \\ \Xi_{22} &= P_i - \mu_2 sym(\tilde{G}_i) + \mu_2^2\tilde{G}_i^T\tilde{G}_i \end{aligned}$$

Proof 3.1:

Using the quadratic Lyapunov function given in (19) and according to zero-value expression obtained from (11), we have :

$$\begin{aligned} \Delta\tilde{V}(\tilde{x}_i(t)) &= \sum_{i=1}^{J}\big(\tilde{V}_i(\tilde{x}_i(t+1)) - \tilde{V}_i(\tilde{x}_i(t))\big) \\ &\quad + \mathcal{E}\{2\sum_{i=1}^{J}\big[\mu_1\tilde{x}_i^T(t)\tilde{G}_i^T + \mu_2\tilde{x}_i^T(t+1)\tilde{G}_i^T\big] \\ &\quad \times\big[-\tilde{x}_i(t+1) + \tilde{\mathcal{A}}(t)\tilde{x}_i(t) + \Pi f(x(t)) \\ &\quad + (\beta(t)-\bar{\beta})\tilde{\mathcal{M}}(t)\tilde{x}_i(t)\big]\} \end{aligned} \tag{21}$$

Using Lemma 2.1 and 2.2, and considering the assumptions (1) and (2), and defining $\hat{f}_{ij} = \max_l \bar{f}_{ij}^l$, one can make the following developments [4]:

$$\Delta\tilde{V}(\tilde{x}_i(t)) \leq \sum_{i=1}^{J}\big(\tilde{x}_i^T(t+1)P_i\tilde{x}_i(t+1) - \tilde{x}_i^T(t)P_i\tilde{x}_i(t)\big)$$
$$+ \mathcal{E}\{2\sum_{i=1}^{J}\big[\mu_1\tilde{x}_i^T(t)\tilde{G}_i^T + \mu_2\tilde{x}_i^T(t+1)\tilde{G}_i^T\big]$$
$$\times\big[-\tilde{x}_i(t+1) + \tilde{\mathcal{A}}(t)\tilde{x}_i(t)$$
$$+ (\beta(t)-\bar{\beta})\tilde{\mathcal{M}}(t)\tilde{x}_i(t)\big]$$
$$+ 2J\tilde{x}_i^T(t)\sum_{j=1}^{J}\hat{f}_{ji}^2\Pi\Pi^T\tilde{x}_i(t)\} \tag{22}$$

From (22), we have

$$\Delta\tilde{V}(\tilde{x}_i(t)) \leq \chi_i^T(t)\Xi(t)\chi_i(t) \tag{23}$$

where $\chi_i^T(t) = \big[\tilde{x}_i^T(t) \quad \tilde{x}_i^T(t+1)\big]$. It is obvious from (20) that $\Delta\tilde{V}(\tilde{x}_i(t)) \leq 0$. Hence, the closed-loop system (11) is asymptotically stable.

To simplify the study, it is better to separate, in the following, the nominal part of the uncertain one in order to design controller and observer gains. The main result for the global asymptotic stability of the NCS, is summarized, according to Lemma 3.1 using (13-16) and Lemma 2.1:

$$\begin{aligned}\Xi(t) &= \hat{\Pi}(t) + sym(\Gamma_1^T\tilde{\Delta}_s\Gamma_{c1}) \\ &\leq \hat{\Pi}(t) + \Gamma_1^T N_1^{-1}\Gamma_1 + \Gamma_{c1}^T\bar{\Delta}_s^T v_1^{-1}\bar{\Delta}_s\Gamma_{c1} < 0\end{aligned} \tag{24}$$

where

$$\hat{\Pi}(t) = \begin{bmatrix}\hat{\Xi}_{11}(t) & \hat{\Xi}_{12}(t) \\ * & \Xi_{22}\end{bmatrix}$$

$$\hat{\Xi}_{11}(t) = -P_i + \mu_1 sym(\tilde{G}_i^T\tilde{\mathcal{A}}_0) + \mu_1^2\tilde{G}_i^T\tilde{G}_i + (2J\sum_{j=1}^{J}\hat{f}_{ji}^2)\Pi\Pi^T,$$

$$\hat{\Xi}_{12}(t) = \mu_2\tilde{\mathcal{A}}_0^T\tilde{G}_i - \mu_1\tilde{G}_i^T,$$

$$\Gamma_1 = \begin{bmatrix}\mu_1 sym(\Gamma_{11}^T\tilde{G}_i) & \mu_2\Gamma_{11}^T\tilde{G}_i \\ 0 & 0\end{bmatrix},$$

$$\Gamma_{11} = \sum_{l=1}^{r_i}\sum_{s=1}^{r_i}h_i^l h_i^s\begin{bmatrix}(1-\bar{\beta})B_i^l K_i^l & -(1-\bar{\beta})B_i^l K_i^l & 0 \\ (1-\bar{\beta})B_i^l K_i^l & -(1-\bar{\beta})B_i^l K_i^l & 0 \\ (1-\bar{\beta})K_i^l & -(1-\bar{\beta})K_i^l & 0\end{bmatrix},$$

$$\Gamma_{c1} = \sum_{l=1}^{r_i}h_i^l\begin{bmatrix}I & 0 \\ 0 & 0\end{bmatrix},$$

$$\tilde{\Delta}_s = diag(\Delta_s,\Delta_s) \leq \bar{\Delta}_s$$

B. Controller and observer design

The goal now is to determine the gain matrices K_i^l and L_i^l such that the closed-loop system (11) is asymptotically stable.

Theorem 3.1: For given scalars μ_1, μ_2, and δ_s, the closed-loop networked system (11) is asymptotically stable, if there exist positive matrices $\tilde{P}_i$, $\mathfrak{G}_i$, $\hat{G}_i$, g_i, Y_i^l and F_i^l with appropriate dimensions, and positive matrices N_1 and v_1 such that the following conditions hold for $1 \leq i,j \leq J$ and $1 \leq l < s \leq r_i$

$$\tilde{\Lambda}_{ij}^{ll} < 0 \tag{25}$$
$$\tilde{\Lambda}_{ij}^{ls} + \tilde{\Lambda}_{ij}^{sl} < 0 \tag{26}$$
$$v_1 * N_1 = I \tag{27}$$

where

$$\tilde{\Lambda}_{ij}^{ls} = \begin{bmatrix}\tilde{\Xi}_{ij} & \tilde{\Gamma}_1^T & \tilde{\Gamma}_{c1}^T \\ * & -N_1 & 0 \\ * & * & -v_1\end{bmatrix} < 0 \tag{28}$$

and

$$\tilde{\Xi}_{ij} = \begin{bmatrix}\tilde{\Xi}_{11i} & \tilde{\Xi}_{12i} & \mathfrak{G}_i & \mu_1\mathcal{G}_i^T & 0 \\ * & \tilde{\Xi}_{22i} & 0 & 0 & \mu_2\mathcal{G}_i^T \\ * & * & \tilde{\Xi}_{33ij} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I\end{bmatrix} \tag{29}$$

$$\tilde{\Xi}_{11i} = -\tilde{P}_i + \mu_1 sym(\mathbf{A}_i),$$
$$\tilde{\Xi}_{12i} = \mu_2\mathbf{A}_i - \mu_1\mathcal{G}_i,$$
$$\tilde{\Xi}_{22i} = \tilde{P}_i - \mu_2 sym(\mathbf{G}_i),$$
$$\tilde{\Xi}_{33ij} = -(2J\sum_{j=1}^{J}\hat{f}_{ji}^2\Pi\Pi^T)^{-1},$$

$$\mathbf{A}_i = \begin{bmatrix}A_i^l g + B_i^l Y_i^s & A_i^l & \bar{\beta}B_i^l \\ 0 & \hat{G}_i^T A_i^l - F_i^l C_i & \bar{\beta}\hat{G}_i^T B_i^l \\ Y_i^s & 0 & \bar{\beta}\end{bmatrix},$$

$$\mathbf{G}_i = \begin{bmatrix}g_i & I & 0 \\ 0 & \hat{G}_i^T & 0 \\ Y_i^s & 0 & I\end{bmatrix},$$

$$\tilde{\Gamma}_1 = \begin{bmatrix}\mu_1 sym(\tilde{Y}_1) & \mu_2\tilde{Y}_1^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{Y}_1 = (1-\bar{\beta})\begin{bmatrix}B_i^l Y_i^s & 0 & 0 \\ B_i^l Y_i^s & 0 & 0 \\ Y_i^s & 0 & 0\end{bmatrix},$$

$$\tilde{\Gamma}_{c1} = \bar{\Delta}_s\begin{bmatrix}1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\bar{\Delta}_s = diag(\bar{\delta}_s,\bar{\delta}_s,\bar{\delta}_s,\bar{\delta}_s,\bar{\delta}_s),$$
$$\mathfrak{G}_i = \mathcal{G}_i^T\mathbb{G}_i^T.$$

Where $Y_i^l = K_i^l g_i$ and $F_i^l = \hat{G}_i^T L_i^l$.

Proof 3.2:

Under the conditions of Theorem 3.1, we have $\tilde{\Xi}_{22i} < 0$. Thus $sym(\mathbf{G}_i) > 0$ and $\mathcal{G}_i$ is nonsingular. Letting $\mathcal{G}_i^{-1} = \begin{bmatrix}I & 0 & 0 \\ 0 & G_i & 0 \\ 0 & 0 & I\end{bmatrix}$ with $\hat{G}_i = G_i^{-1}$. Then, after using the complement of Schur and checking a congruence transformation to (28) by $diag(\mathcal{G}_i^{-1},\mathcal{G}_i^{-1},I,I)$ and defining $\bar{P}_i = \mathcal{G}_i^{-T}\tilde{P}_i\mathcal{G}_i^{-1}$, we find that

$$\bar{\Lambda}(t) = \sum_{l=1}^{r_i}\sum_{s=1}^{r_i}h_i^l h_i^s\bar{\Lambda}_{ij} = \begin{bmatrix}\bar{\Pi}(t) & \Gamma_1^T & \Gamma_{c1}^T \\ * & -N_1 & 0 \\ * & * & -v_1\end{bmatrix} < 0 \tag{30}$$

where

$$\bar{\Pi}(t) = \begin{bmatrix} \bar{\Xi}_{11}(t) & \bar{\Xi}_{12}(t) \\ * & \bar{\Xi}_{22} \end{bmatrix}$$

$$\bar{\Xi}_{11}(t) = -\bar{P}_i + \mu_1 sym(\bar{\mathbb{A}}) + \mu_1^2 I + (2J\mathbb{G}_i^T \sum_{j=1}^{J} \hat{f}_{ji}^2)\Pi\Pi^T\mathbb{G}_i,$$
$$\bar{\Xi}_{12}(t) = \mu_2\bar{\mathbb{A}}^T - \mu_1\mathbb{G}_i,$$
$$\bar{\Xi}_{22} = \bar{P}_i - \mu_2 sym(\mathbb{G}_i) + \mu_2^2 I,$$
$$\bar{\mathbb{A}} = \tilde{A}_0\mathbb{G}_i$$

Defining $\mathbb{G}_i = \tilde{G}_i^{-1} = \begin{bmatrix} g_i & G_i & 0 \\ 0 & G_i & 0 \\ Y_i^s & 0 & I \end{bmatrix}$. We check a congruence transformation to (30) by $diag(\tilde{G}_i, \tilde{G}_i, I, I)$, considering (3), we can conclude that condition (24) holds.

Remark 3.1:
It should be noted that the last equation (27) is non convex. To solve such a problem a convergent algorithm using the cone complementarity formulation proposed in [2], [8], [10], replacing (27) with

$$\begin{bmatrix} v_1 & I \\ I & N_1 \end{bmatrix} \geq 0 \quad (31)$$

solving the nonconvex LMI (27) is converted into solving the following nonlinear minimization problem involving LMI conditions.

Minimize $Trace(v_1 * N_1)$ subject to (25) and (31).

IV. Simulation Results

Example 4.1: Consider a large-scale system S composed of three fuzzy subsystems $S_i,\ i = 1, 2, 3$, given as follows :

- Subsystem S_1:

$$x_1(t+1) = \sum_{l=1}^{2} [A_1^l x_1(t) + B_1^l u_1(t) + B_{w1}^l w(t) + \sum_{j=1}^{3} f_{1j}(x_j(t))]$$
$$y_1(t) = C_1 x_1(t)$$

where

$$A_1^1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B_1^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$
$$B_1^2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad B_{w1}^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{w1}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad f_{11} = 0,$$
$$f_{12} = \begin{bmatrix} 0.08 \\ 0.05 \end{bmatrix}^T x_2, \quad f_{13} = \begin{bmatrix} 0.09 \\ 0.06 \end{bmatrix}^T x_3, \quad x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}$$

Consider the following membership functions: $h_1^1(t) = sin^2(x_{11}(t))$, $h_1^2(t) = cos^2(x_{11}(t))$

- Subsystem S_2:

$$x_2(t+1) = \sum_{l=1}^{2} [A_2^l x_2(t) + B_2^l u_2(t) + B_{w2}^l w(t) + \sum_{j=1}^{3} f_{2j}(x_j(t))]$$
$$y_2(t) = C_1 x_2(t)$$

where

$$A_2^1 = \begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}, \quad A_2^2 = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}, \quad B_2^1 = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix},$$
$$B_2^2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, \quad B_{w2}^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{w2}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$f_{21} = \begin{bmatrix} 0.02 \\ 0.012 \end{bmatrix}^T x_1, \quad f_{23} = \begin{bmatrix} 0.06 \\ 0.036 \end{bmatrix}^T x_3, \quad f_{22} = 0,$$
$$x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

Consider the following membership functions: $h_2^1(t) = cos^2(x_{21}(t))$, $h_2^2(t) = sin^2(x_{21}(t))$

- Subsystem S_3:

$$x_3(t+1) = \sum_{l=1}^{2} [A_3^l x_3(t) + B_3^l u_3(t) + B_{w3}^l w(t) + \sum_{j=1}^{3} f_{3j}(x_j(t))]$$
$$y_3(t) = C_1 x_3(t)$$

where

$$A_3^1 = \begin{bmatrix} -3 & 1 \\ 5 & -3 \end{bmatrix}, \quad A_3^2 = \begin{bmatrix} -2 & 1 \\ 3 & -0.3 \end{bmatrix}, \quad B_3^1 = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix},$$
$$B_3^2 = \begin{bmatrix} 0.2 \\ 1.2 \end{bmatrix}, \quad B_{w3}^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{w3}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$f_{31} = \begin{bmatrix} 0.48 \\ 0.64 \end{bmatrix}^T x_1, \quad f_{32} = \begin{bmatrix} 0.024 \\ 0.032 \end{bmatrix}^T x_2, \quad f_{33} = 0,$$
$$x_3(t) = \begin{bmatrix} x_{31}(t) \\ x_{32}(t) \end{bmatrix}$$

Consider the following membership functions: $h_3^1(t) = cos^2(x_{31}(t))$, $h_3^2(t) = sin^2(x_{31}(t))$

For a given quantization density $\rho = 0.1$ and $\bar{\beta} = 0.8$, by applying Theorem 3.1, the solutions of LMIs can be obtained as follows :

$$K_1^1 = \begin{bmatrix} -3.4998 & -4.1447 \end{bmatrix}, \quad K_1^2 = \begin{bmatrix} -2.6949 & -3.4758 \end{bmatrix},$$
$$K_2^1 = \begin{bmatrix} -7.8856 & 4.8444 \end{bmatrix}, \quad K_2^2 = \begin{bmatrix} -6.5905 & 3.8839 \end{bmatrix},$$
$$K_3^1 = \begin{bmatrix} 3.0671 & -3.1237 \end{bmatrix}, \quad K_3^2 = \begin{bmatrix} 2.4193 & -2.8052 \end{bmatrix}$$
$$L_1^1 = \begin{bmatrix} -12.2811 & -0.0989 \\ 0.9499 & -12.1620 \end{bmatrix}, \quad L_1^2 = \begin{bmatrix} -13.3189 & -0.1232 \\ 0.9599 & -12.1539 \end{bmatrix},$$
$$L_2^1 = \begin{bmatrix} -7.7536 & -10.1564 \\ 15.9187 & -7.9330 \end{bmatrix}, \quad L_2^2 = \begin{bmatrix} -7.2759 & -9.8463 \\ 14.5677 & -7.4151 \end{bmatrix},$$
$$L_3^1 = \begin{bmatrix} -7.9072 & -6.6345 \\ 13.5026 & -8.6494 \end{bmatrix}, \quad L_3^2 = \begin{bmatrix} -7.4932 & -6.1365 \\ 10.9400 & -6.5337 \end{bmatrix}.$$

For simulation, initial conditions are $x_1(0) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$, and $x_3(0) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$.

The state variables evolution of NCSs systems and control inputs are shown in Figures 2, 3, 4 and 5 from which, we can note that all the observed states converge toward the original state variables. This shows the good estimation of the states and effectiveness of the approach after the first 4s in simulation, due to the different initial conditions between the estimated and real states. Thus, we can know that the proposed decentralized observer-based fuzzy control make the nonlinear interconnected system in network communication satisfy an asymptotic stability.

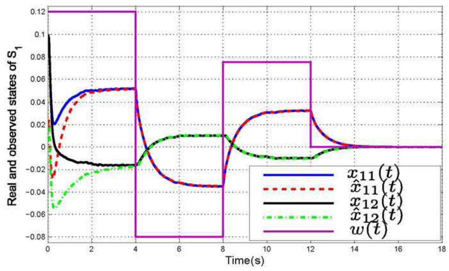

Fig. 2: State responses and state estimate in S_1

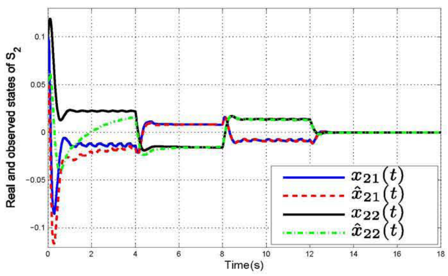

Fig. 3: State responses and state estimate in S_2 .

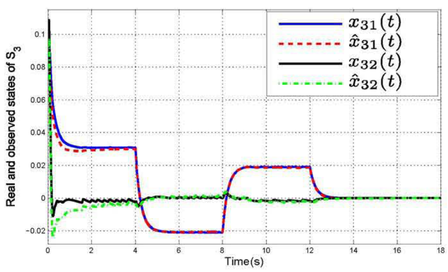

Fig. 4: State responses and state estimate in S_3.

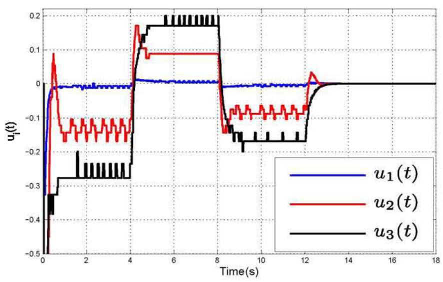

Fig. 5: Control signal trajectories $u_i(t)$.

V. Conclusion

In this paper, the problem of observer-based control of nonlinear systems with random measurement, where the measurement channel is assumed to subject to quantization and random packet dropout has been studied. A simulation example is exploited to illustrate the advantages of the proposed design procedure. Future works include: (1) our approach will be tested in the case of the application to automotive vehicle, (2) parametric uncertainties will be integrated into the controlled system to achieve better performance.

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