

# Answer Set Programming Modulo ‘Space-Time’

Carl Schultz<sup>1</sup>, Mehul Bhatt<sup>2,3</sup>, Jakob Suchan<sup>3</sup>, and Przemysław Wałęga<sup>4</sup>

Spatial Reasoning. [www.spatial-reasoning.com](http://www.spatial-reasoning.com)

<sup>1</sup> Aarhus University, Denmark, <sup>2</sup> Örebro University, Sweden

<sup>3</sup> University of Bremen, Germany, <sup>4</sup> University of Warsaw, Poland

**Abstract.** We present ASP Modulo ‘Space-Time’, a declarative representational and computational framework to perform commonsense reasoning about regions with both spatial and temporal components. Supported are capabilities for mixed qualitative-quantitative reasoning, consistency checking, and inferring compositions of space-time relations; these capabilities combine and synergise for applications in a range of AI application areas where the processing and interpretation of spatio-temporal data is crucial. The framework and resulting system is the only general KR-based method for declaratively reasoning about the dynamics of ‘space-time’ regions as first-class objects. We present an empirical evaluation (with scalability and robustness results), and include diverse application examples involving interpretation and control tasks.

## 1 INTRODUCTION

Answer Set Programming (ASP) has emerged as a robust declarative problem solving methodology with tremendous application potential [8, 16, 17, 33]. Most recently, there has been heightened interest to extend ASP in order to handle specialised domains and application-specific knowledge representation and reasoning (KR) capabilities. For instance, ASP Modulo Theories (ASPMT) go beyond the propositional setting of standard answer set programs by the integration of ASP with Satisfiability Modulo Theories (SMT) thereby facilitating reasoning about continuous domains [3, 16, 20]; using this approach, integrating knowledge sources of *heterogeneous semantics* (e.g., infinite domains) becomes possible. Similarly, CLINGCON [14] combines ASP with specialised constraint solvers supporting non-linear finite integers. Other most recent extensions include the ASPMT founded *non-monotonic spatial reasoning* extensions in ASPMT(QS) [34]; ASP modulo *acyclicity* [6]; *probabilistic* extensions to ASP [36]. Indeed, being rooted in KR, in particular non-monotonic reasoning, ASP can theoretically characterise —and promises to serve in practice as— a modern foundational language for several domain-specific AI formalisms, and offer a uniform computational platform for solving many of the classical AI problems involving planning, explanation, diagnosis, design, decision-making, control [8, 24, 33]. In this line of research, this paper presents ASP Modulo ‘Space-Time’, a specialised formalism and computational backbone enabling generalised commonsense reasoning about ‘*space-time objects*’ and their spatio-temporal dynamics directly within the answer set programming paradigm.

**Reasoning about ‘Space-Time’ (Motion)** Imagine a moving object within 3D space. Here, the complete trajectory of motion of the moving object within a space-time localisation framework constitutes a 4D space-time history consisting of both spatial and

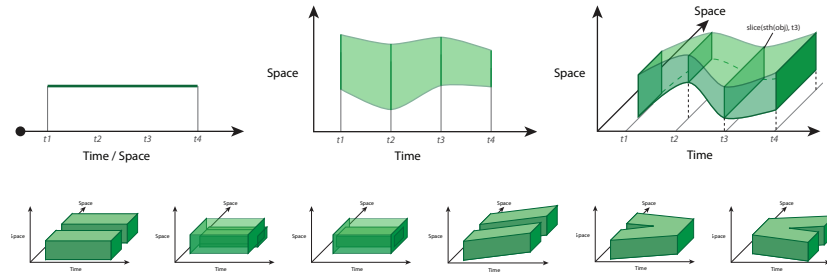


Fig. 1: Space-Time Histories in 1D and 2D; Spatio-temporal patterns and events, i.e. discrete, overlapping, inside, parallel movement, merge, and split.

temporal components – i.e., it is a region in *space-time* (Fig. 1). Regions in *space*, *time*, and *space-time* have been an object of study across a range of disciplines such as ontology, cognitive linguistics, conceptual modeling, KR (particularly qualitative spatial reasoning), and spatial cognition and computation. Spatial knowledge representation and reasoning can be classified into two groups: topological and positional calculi [1, 22]. With topological calculi such as the Region Connection Calculus (RCC) [27], the primitive entities are spatially extended regions of space, and could be arbitrarily (but uniformly) dimensioned space-time histories. For the case of ‘space-time’ representations, the main focus in the state of the art has been on axiom systems (and the study of properties resulting therefrom) aimed at pure *qualitative reasoning*. In particular, axiomatic characterisations of mereotopologically founded theories with spatio-temporal regions as primitive entities are very well-studied [18, 23]. Furthermore, the dominant method and focus within the field of spatial representation and reasoning — be it for topological or positional calculi — has been primarily on relational-algebraically founded semantics [22] in the absence of (or by discarding available) quantitative information. Pure qualitative spatial reasoning is very valuable, but it is often counterintuitive to not utilise or discard quantitative data if it is available (numerical information is typically available in domains involving sensing, interaction, interpretation, and control).

**Answer Set Modulo ‘Space-Time’** Within the state of the art, it is not possible for AI applications (e.g., involving reasoning about moving objects in a vision system, control in robotic manipulation) to directly exploit commonsense representation and reasoning with ‘space-time’ objects and their mutual spatial-temporal relationships as first-class entities within a robust KR framework such as ASP. The main contributions of the paper are: (1). Developing a systematic formal account and associated computational characterisation of a ‘space-time’ theory as a general language founded in answer set programming; the focus is on declarative modelling, commonsense inference and question-answering with space-time objects and their mutual relationships as first-class objects; (2). Support of mixed qualitative-quantitative reasoning and dynamic *quantification* (i.e., grounding of real world parameters); this is very powerful, e.g., when only partial information is available, (sensor) data is noisy, or when *quantification* — is not needed or can be delayed; (3). Demonstrating, by running examples and an empirical evaluation, the applicability of the resulting general reasoning system to support reasoning about space-time histories in diverse application scenarios focussing on interpretation and control. The proposed model is implemented using CLINGO [13, 15];

to the best of our knowledge, no systematic realisation of a general declarative method supporting native space-time histories and relationships thereof currently exists (be it mixed qualitative-quantitative reasoning, or even purely qualitative reasoning).<sup>1</sup>

## 2 ASP MODULO ‘SPACE-TIME’

### 2.1 Space-Time Histories

The spatio-temporal domain ( $ST$ ) that we focus on in our formal framework consists of the following ontology:

**Spatial Domains.** Spatial domain entities include *points* and *simple polygons*: a *2D point* is a pair of reals  $x, y$ ; a *simple polygon*  $P$  is defined by a list of  $n$  vertices (points)  $p_0, \dots, p_{n-1}$  such that the boundary is non-self-intersecting, i.e., no two edges of the polygon intersect. We denote the number of vertices in  $P$  with  $|P|$ . A polygon is *ground* if all vertices are assigned real values. A translation vector  $t$  is a pair of reals  $t_x, t_y$ . Given point  $p = (x, y)$  and translation vector  $t$  then  $p + t = (x + t_x, y + t_y)$ . A *translation* is a ternary relation between two polygons  $P, Q$  and a translation vector  $t$  such that:  $|P| = |Q| = n$  and  $p_i = q_i + t$  where  $p_i$  is the  $i^{\text{th}}$  vertex in  $P$  and  $q_i$  is the  $i^{\text{th}}$  vertex in  $Q$ , for  $0 \leq i < n$ . A translation vector  $t$  is *ground* if  $t_x, t_y$  are assigned real values, otherwise it is *unground*.<sup>2</sup>

**Temporal domain  $\mathcal{T}$ .** The temporal dimension is constituted by an infinite set of time points – each time point is a real number. The time-line is given by a linear ordering  $<$  of time-points.

**$ST$  Histories.** Consider a moving two-dimensional spatial object  $s$ , e.g. represented by a polygon at each time point. If we treat time as an additional dimension, then we can represent  $s$  as a three-dimensional object in space-time. Intuitively, at each time point, the corresponding space-time region of  $s$  has a 2D spatial representation (a spatial *slice*). The space-time object is formed by taking all such slices over time.

An  *$ST$  object*  $o \in O$  is a variable associated with an  $ST$  domain  $D$  (e.g. the domain of 2D polygons over time). An *instance* of an object  $i \in D$  is an element from the domain. Given  $O = \{o_1, \dots, o_n\}$ , and domains  $D_1, \dots, D_n$  such that  $o_i$  is associated with domain  $D_i$ , then a *configuration* of objects  $\psi$  is a one-to-one mapping between object variables and instances from the domain,  $\psi(o_i) \in D_i$ . For example, a variable  $o_1$  is associated with the domain  $D_1$  of moving 2D points over time. An  *$ST$  point* moving in a straight line starting at spatial coordinates  $(0, 0)$  at time 0 and arriving at 2D spatial coordinates  $(10, 0)$  at time 1 is an instance of  $D_1$ . A configuration is defined that maps  $o_1$  to a 3D line with end points  $(0, 0, 0), (10, 0, 1)$  i.e.  $\psi(o_1) = [(0, 0, 0), (10, 0, 1)]$ .

**$ST$  Relations.** Let  $D_1, \dots, D_n$  be spatio-temporal domains. A spatio-temporal relation  $r$  of arity  $n$  ( $0 < n$ ) is defined as  $r \subseteq D_1 \times \dots \times D_n$ . That is, each spatio-temporal relation is an equivalence class of instances of  $ST$  objects. Given a set of objects  $O$ , a relation  $r$  of arity  $n$  can be asserted as a constraint that must hold between objects

<sup>1</sup> Implementation and examples may be consulted here: <http://think-spatial.org/ASP-ST.zip>.

<sup>2</sup> For brevity we focus on 2D spatial entities; our approach also readily extends to 3D spatial entities, and in general  $nD$  points, polytopes, and translation vectors.

Relation	Definition
<b>Topology</b>	
disconnects (DC)	$\forall t \text{ dc}(s_1(t), s_2(t))$
discrete from (DR)	$\forall t \text{ dr}(s_1(t), s_2(t))$
part of (P)	$\forall t \text{ p}(s_1(t), s_2(t))$
non-tangential proper part (NTPP)	$\forall t \text{ ntpp}(s_1(t), s_2(t))$
equal (EQ)	$\forall t \text{ eq}(s_1(t), s_2(t))$
contacts (C)	$\exists t \text{ c}(s_1(t), s_2(t))$
overlaps (O)	$\exists t \text{ o}(s_1(t), s_2(t))$
partially overlaps (PO)	$\exists t \text{ po}(s_1(t), s_2(t))$
externally connects (EC)	$\text{dr}(s_1, s_2) \wedge \exists t \text{ ec}(s_1(t), s_2(t))$
proper part (PP)	$\text{p}(s_1, s_2) \wedge \exists t \text{ pp}(s_1(t), s_2(t))$
tangential proper part (TPP)	$\text{p}(s_1, s_2) \wedge \exists t \text{ tpp}(s_1(t), s_2(t))$
split	$\text{p}(s_1(t_0), s_2(t_0)) \wedge \text{dc}(s_1(t_N), s_2(t_N))$
merge	$\text{dc}(s_1(t_0), s_2(t_0)) \wedge \text{p}(s_1(t_N), s_2(t_N))$
<b>Size</b>	
fixed size	$\forall t \forall t' (\text{area}(s(t)) = \text{area}(s(t')))$
grows	$\neg \text{fixed\_size}(s) \wedge$ $\forall t \forall t' (\text{area}(s(t)) \leq \text{area}(s(t')))$
shrinks	$\text{reverse}(\text{grows}(s_1, s_2))$
<b>Movement</b>	
moves	$\exists t \exists t' \text{ p}(t) \neq \text{p}(t')$
move parallel	$\text{moves}(s_1) \wedge$ $\forall t \forall t' (p_2(t) - p_1(t)) = (p_2(t') - p_1(t'))$
towards	$\text{moves}(s_1) \wedge \neg \text{moves\_parallel}(s_1, s_2) \wedge$ $\forall t \forall t' \Delta(p_1(t), p_2(t)) \geq \Delta(p_1(t'), p_2(t'))$
away	$\text{reverse}(\text{towards}(s_1, s_2))$
follows	$\forall t' \exists t \text{ duration}(t, t') \leq \alpha$ $\wedge \Delta(p_1(t), p_2(t)) > \Delta(p_1(t'), p_2(t))$ $\wedge \Delta(p_1(t), p_2(t)) < \Delta(p_1(t), p_2(t'))$

Table 1: Relations between  $\mathcal{ST}$  regions  $s_1, s_2$  over time interval  $I = [t_0, t_N]$ ;  $t, t'$  range over  $I$  with  $t \leq t'$ ;  $\text{reverse}(R)$  denotes the definition of relation  $R$  with reversed temporal ordering,  $t' \leq t$ ;  $p_i(t_j)$  is the centre point of  $s_i$  at  $t_j$ ;  $\Delta$  is the Euclidean distance between two points;  $\alpha$  is a user-specified temporal threshold.

$o_1, \dots, o_n \in O$ , denoted  $r(o_1, \dots, o_n)$ . The constraint  $r(o_1, \dots, o_n)$  is satisfied by configuration  $\psi$  if  $(\psi(o_1), \dots, \psi(o_n)) \in r$ . For example, if  $pp$  is a topological relation *proper part*, and  $O = \{o_1, o_2\}$  is a set of moving polygon objects, then  $pp(o_1, o_2)$  is the constraint that moving polygon  $o_1$  is a proper part of  $o_2$ .

Table 1 presents definitions for  $\mathcal{ST}$  relations that hold between  $s_1$  and  $s_2$ , where  $t, t'$  range over a (dense) time interval with start and end time points  $t_0$  and  $t_N$  in which  $s_1$  and  $s_2$  occur and  $t \leq t'$ . We define mereotopological relations using the Region Connection Calculus (RCC) [27]: all spatio-temporal RCC relations between  $\mathcal{ST}$  regions are defined based on the RCC relations of their slices (for simplicity we use the same names for spatial and spatio-temporal RCC relations).  $\mathcal{ST}$  regions *split* (conversely, *merge*) if their spatial slices are initially parts and end up disconnected.  $\mathcal{ST}$  region  $s$  *grows* or *shrinks* if the area monotonically increases or decreases, respectively.  $\mathcal{ST}$  region  $s$  *moves* if the centre point changes, and region  $s_1$  moves *away* from, *towards*  $s_2$  if the centre point distance ( $\Delta$ ) increases, decreases, and *parallel* if the vector between centre points does not change. An  $\mathcal{ST}$  region  $s_1$  *follows*  $\mathcal{ST}$  region  $s_2$  if, at each time step,  $s_1$  moves towards a previous location of  $s_2$ , and  $s_2$  moves away from a previous location of  $s_1$ ; we introduce a user-specified maximum duration threshold  $\alpha$  between these two time points to prevent unwanted scenarios being defined as *follows* events such as  $s_1$  taking one step towards  $s_2$  and then stopping while  $s_2$  continues to move away from  $s_1$ .

## 2.2 Space-Time Semantics as Polynomial Constraints

One approach for formalising the semantics of spatial reasoning is by encoding qualitative spatial relations as systems of polynomial equations and inequalities [4, 34]. The task of determining whether a set of spatial relations is consistent is then equivalent to determining whether the set of polynomial constraints are satisfiable. Given a system of polynomial constraints over real variables  $X$ , the constraints are satisfiable if there exists some real value for each variable in  $X$  such that all the polynomial constraints are simultaneously satisfied.<sup>3</sup> For example, let point  $p$  be defined by real coordinates  $x_p, y_p$ , and let circle  $c$  be defined by the centre point  $x_c, y_c$  and real radius  $r_c$ . A point  $p$  is incident to the interior of a circle  $c$  if the distance between  $p$  and the centre of  $c$  is less than the radius of  $c$ :  $(x_p - x_c)^2 + (y_p - y_c)^2 < r_c^2$ . If there exists an assignment of real values to the variables (e.g.,  $x_p = 3.5, x_c = 10.5$ , etc.) that satisfies all polynomial constraints, then the qualitative spatial relations are consistent. Continuing with the example, if we now add the relation that point  $p$  is also incident to the boundary of  $c$ :  $(x_p - x_c)^2 + (y_p - y_c)^2 = r_c^2$  and we reformulate the system of constraints we get:  $(d_{pc} < r_c) \wedge (d_{pc} = r_c)$ . Distance  $d_{pc}$  cannot be both less than and equal to the radius  $r_c$ , and thus the system of polynomial constraints is inconsistent, and no configuration of points and circles (within Euclidean space) exists that can satisfy this set of qualitative spatial relations.

## 2.3 Spatio-Temporal Consistency

Consider the topological *disconnected* relation. There is no polygon that is *disconnected* from itself, i.e. the relation is *irreflexive*. Algebraic properties of *ST* relations are expressed as the following ASP rules and constraints.<sup>4</sup>

$$\begin{aligned}
 r \text{ is reflexive} &: r(A, A) \leftarrow \text{entity}(A) \\
 r \text{ is irreflexive} &: \neg r(A, A) \leftarrow \text{entity}(A) \\
 r \text{ is symmetric} &: r(B, A) \leftarrow r(A, B) \\
 r \text{ is asymmetric} &: \neg r(B, A) \leftarrow r(A, B) \\
 r_2 \text{ is converse of } r_1 &: r_2(B, A) \leftarrow r_1(A, B) \\
 r_1 \text{ is implies of } r_2 &: r_2(A, B) \leftarrow r_1(A, B) \\
 r_1, r_2 \text{ are mutually inconsistent} &: \perp \leftarrow r_1(A, B), r_2(A, B) \\
 r_1, r_2, r_3 \text{ are transitively inconsistent} &: \perp \leftarrow r_1(A, B), r_2(B, C), r_3(A, C)
 \end{aligned} \tag{1}$$

<sup>3</sup> The worst case complexity of solving a system of non-linear polynomial constraints over  $n$  real variables is  $O(2^{2^n})$  [2] owing to the Cylindrical Algebraic Decomposition algorithm [9], which is implemented in the solver z3 [10]. Although not relevant to this paper, it is worth pointing out that we use a (sound and complete) polynomial constraint solver that determines whether a system of non-linear polynomial constraints is satisfiable, based on an integration of Satisfiability Modulo Theories solver z3 [10] and numerical optimisation [30] with the library NLOpt [19] using BOBYQA [25]. The employed polynomial encodings are highly optimised (e.g., by symmetry-based pruning heuristics [29]) for the specific spatio-temporal context.

<sup>4</sup> Standard stable model semantics is applicable [12], [17], and [11]. An ASP program  $P$  consists of a finite set of universally quantified *rules* of the form  $h \leftarrow b_1, \dots, b_n, \text{not } c_1, \dots, \text{not } c_m$  such that  $h$  is an atom, and the expression  $b_1, \dots, b_n, \text{not } c_1, \dots, \text{not } c_m$  is a conjunction of atoms. ASP *facts* are rules of the form  $h \leftarrow \top$ , and ASP *constraints* are rules of the form  $\perp \leftarrow b_1, \dots, b_n, \text{not } c_1, \dots, \text{not } c_m$ .

We have automatically derived these properties using our polynomial constraint solver *a priori* and generated the corresponding ASP rules. A violation of these properties corresponds to *3-path inconsistency* [22], i.e. there does not exist any combination of polygons that can violate these properties. In particular, a total of 1586 space-time constraints result.<sup>5</sup>

**Ground Polygons.** We can determine whether *ST* relation  $r$  holds between two ground polygons  $P, Q$  by directly checking whether the corresponding polynomial constraints are satisfied, i.e. polynomial constraint variables are replaced by the real values assigned to the ground polygon vertices. This is accomplished during the *grounding* phase of ASP. E.g. two ground polygons are *disconnected* if the distance between them is greater than zero.

**Unground Translation.** Given ground polygons  $P_0, P_1$ , *unground* polygon  $P'_0$ , and unground translation  $t = (t_x, t_y)$ , let  $P'_0$  be a  $t$  translation of  $P_0$  such that  $r$  holds between  $P'_0, P_1$ . The (exact) set of real value pairs that can be assigned to  $(t_x, t_y)$  such that  $P'_0, P_1$  satisfy  $r$  is precisely determined using the Minkowski sum method [35]; we refer to this set as the *solution set* of  $t$  for  $r$ . Given  $n$  ground polygons  $P_1, \dots, P_n$ , and  $n$  relations  $r_1, \dots, r_n$  such that relation  $r_i$  is asserted to hold between polygon  $P_0, P_i$ , for  $1 \leq i \leq n$ , let  $M_i$  be the solution set of  $t$  for  $r_i$ . The conjunction of relations  $r_1, \dots, r_n$  is consistent if the *intersection* of solution sets  $M_1, \dots, M_n$  is non-empty. Computing and intersecting solution sets is accomplished during the *grounding* phase of ASP.

***ST* Relation Consistency.** In the following tasks the input is a set of objects  $O$  and a set of qualitative spatio-temporal relations  $R$  between those objects: (1) *Consistency*. Determine whether there exists a configuration  $\psi$  of  $O$  that satisfies all relation constraints in  $R$ . Such a configuration is called a *consistent configuration*; (2). *Generating configurations*. Return a consistent configuration  $\psi$  of  $O$ .

### 3 REASONING WITH ASP MODULO SPACE-TIME

We have implemented our *ST* reasoning module in Clingo (v5.1.0) [13, 15]. Table 2 presents our system’s predicate interface. Our system provides special predicates for (1) declaring spatial objects, and (2) relating objects spatio-temporally. Each *ST* object is represented with *st\_object/3* relating the identifier of the *ST* entity, time point of this slice, and identifier of the associated geometric representation.

```
st_object(EntityId, at(Time), id(PolygonId)).
```

Polygons are represented using the *polygon/2* predicate that relates an identifier of the geometric representation with a list of  $x, y$  vertex coordinate pairs, e.g.:

```
polygon(id(pgBx2_0), (268, 0, 303, 0, 303, 5)).
```

**Deriving *ST* relations.** the predicate *spacetime/3* is used to specify the entities between which *ST* relations should be derived:

<sup>5</sup> These may be consulted in the files “spatial\_invariance.lp” and “movement\_invariance.lp” in the submitted source code.

Predicate	Description
<b>ST Entities</b>	
<code>polygon(Pg, (X1, Y2, ..., Xn, Yn))</code>	Polygon Pg has $n$ ground vertices $(x_1, y_1), \dots, (x_n, y_n)$ .
<code>translation(Pg1, Pg2)</code>	Polygon Pg2 is an unground translation of Pg1.
<code>st_object(E)</code>	E is a spatio-temporal entity.
<code>st_object(E, at(Time), id(Pg))</code>	2D polygon Pg is a spatial <i>slice</i> of spatio-temporal entity E at time point Time.
<b>ST Relations</b>	
<code>spacetime(STAspect, E, time(T1, T2))</code>	Derive unary ST relations for STAspect (topology, size, or movement) for entity E from time T1 to T2.
<code>spacetime(STAspect, E1, E2, time(T1, T2))</code>	Derive binary ST relations for STAspect (topology, size, or movement) between entities E1, E2 from time T1 to T2.
<code>topology(Rel, E1, E2, time(T1, T2))</code>	Topological relation Rel is asserted to hold between ST entities E1, E2 from time T1 to T2.
<code>size(Rel, E1, E2, time(T1, T2))</code>	Size relation Rel is asserted to hold between ST entities E1, E2 from time T1 to T2.
<code>movement(Rel, E, time(T1, T2))</code>	Unary movement relation Rel is asserted to hold for ST entity E from time T1 to T2.
<code>movement(Rel, E1, E2, time(T1, T2))</code>	Binary movement relation Rel is asserted to hold between ST entities E1, E2 from time T1 to T2.
<code>spatial(witness, E, EWitness)</code>	Ground entity EWitness is a consistent witness for unground entity E.

Table 2: ST entities and relation predicates.

```
% derive properites of entity e1 during time 25 to 75
spacetime(movement,e1, time(25,75)).
% derive relations between entities e5,e6 time 1 to 10
spacetime(movement,e5,e6, time(1,10)).
% derive relations between all entities for time 10 to 20
spacetime(movement,Entity1,Entity2,time(1,10)):-
    st_object(Entity1,_,_), st_object(Entity2,_,_).
```

**Purely qualitative reasoning.** if no geometric information for slices is given then our system satisfies 3-consistency, e.g. the following program includes transitively inconsistent spatio-temporal relations:

```
st_object(s1). st_object(s2). st_object(s3).
topology(ntpp,s1,s2). topology(pp,s2,s3). topology(dc,s1,s3).
UNSATISFIABLE
```

**Mixed qualitative-numerical reasoning.** a new ST object can be specified that consists of *translated* slices of a given ST object. Our system determines whether translations exist that satisfy all given spatio-temporal constraints. Our system produces the solution set and a spatial witness that minimises the translation distance.

```
translation(st1, translated_st1).
topology(pp, translated_st1, st2).
spatial(witness, translated_st1, witness_st1).
```

### 3.1 Application Examples: Interpretation and Control

► **1. INSECT BEHAVIOUR.** In this section we describe how spatio-temporal relations are derived from a large dataset of fly movement video data used to study the social interactions of flies.<sup>6</sup> The dataset consists of 20 flies in a bowl, captured in 200 image

<sup>6</sup> Data provided by K. Branson from Janelia Research Campus: <https://www.janelia.org/lab/branson-lab>; accessible from the *ilastik* website: <http://ilastik.org/download.html>



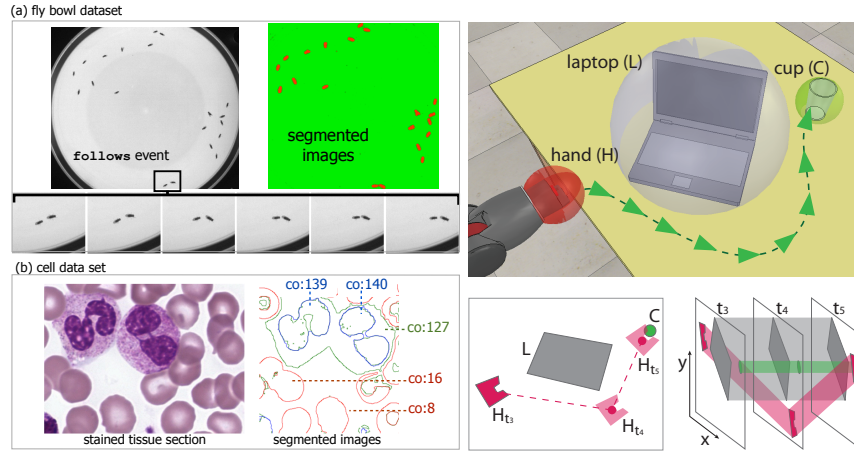


Fig. 2: Application: Insect Behaviour, Cell Biology, and Cognitive Robotics

frames (130 MB). Figure 2(a) illustrates example images of the dataset and segmentation. We performed initial image segmentation and animal tracking using the *ilastik* interactive toolkit [31]. We then parse the output into our ASP predicates: *st\_object/3* and *polygon/2*.

**Example 1.1.** Derive *ST* movement relations between all pairs of flies for the first time step:

```
%% define movement thresholds
stationary_threshold(2). away_threshold(2).
%% derive relations between entities E1,E2
spacetime(movement, E1,E2, time(0,1)) :-
    st_object(E1, _, _), st_object(E2, _, _), E1 < E2.
#show movement/3. #show movement/4.
```

The result is:

```
movement(stationary, fly11, unit_time(0,1))
movement(moving, fly4, unit_time(0,1))
movement(towards, fly15, fly37, unit_time(0,1))
movement(follows, fly10, fly24, unit_time(0,1))
...
SATISFIABLE
Models: 1
Time : 1.533s (Solving: 0.0s 1st Model: 0.0s Unsat: 0.0s)
```

The extract of the results shows that, during the first time step: *fly11* is stationary; *fly4* is moving; *fly15* is moving towards *fly37*; *fly10* is following *fly24*.

**Example 1.2.** Derive all spacetime movement relations between flies *fly25*, *fly24* for the entire video:

```
spacetime(movement, fly25, fly24, time(0,199)).
```

The result is:



```

movement(follows, fly25, fly24, time(29, 35))
movement(away, fly24, fly25, time(28, 35))
movement(towards, fly24, fly25, time(38, 42))
...
SATISFIABLE
Time: 1.718s (Solving: 0.0s 1st Model: 0.0s Unsat: 0.0s)

```

The extract of the results shows that: during time period [29, 35] *fly25* is following *fly24*; during time period [28, 35] *fly24* is moving away from *fly25*; during time period [38, 42] *fly24* is moving towards *fly25*.

**Example 1.3.** Find flies that are *near* each other at time 25 and exhibit *follows* behaviour for at least 3 time units during period from time 25 to 35:

```

near_threshold(120). %% define nearness threshold
%% derive distance and movement relations
spacetime(distance, E1, E2, time(25)) :- st_object(E1, _, _), st_object(E2, _, _).
spacetime(movement, E1, E2, time(25, 35)) :- spacetime(near, E1, E2, _).
%% show follows events lasting at least 3 time units
#show movement(follows, E1, E2, time(T1, T2)) :
      movement(follows, E1, E2, time(T1, T2)), T2 - T1 > 3.

```

The result is:

```

movement(follows, objects(fly38, fly4), time(25, 31))
movement(follows, objects(fly25, fly24), time(29, 35))
...
Time: 1.730s (Solving: 0.0s 1st Model: 0.0s Unsat: 0.0s)

```

The extract of the results shows that: *fly38* follows *fly4* during time [25, 31]; *fly25* follows *fly24* during time [29, 35].

► **2. CELL FUNCTION.** In this section we demonstrate how to solve spatial reasoning problems by translating polygons. Figure 2(b) presents a stained tissue section of red and white blood cells from a patient with chronic myelogenous leukemia. We analyse the relationships between the physical structures of cell components, in particular whether certain cell components could move and fit inside other cell components. We segment the image, which assigns a class type to each segment, and apply standard contour detection algorithms to convert the raster image into polygons. We then parse the output as ASP facts including *st\_object/3* and *polygon/2*.

**Example 2.1.** Firstly we determine whether a cell with the same shape as “*co:8*” might also fit inside the cytoplasm region by creating a new polygon “*tr:8*” that is a *translation* of polygon “*co:8*”. We translate “*tr:8*” so that it is a *proper part* (pp) of “*co:127*”.

```

...
translation(id("co:8"), id("tr:8")).
topology(pp, id("tr:8"), id("co:127")).
spatial(witness, id("tr:8")).

```

The result is:

```

solution_set(id("tr:8"), (((352, 151, ...),), ()))
witness(id("tr:8"), translate(-93, -186),
        polygon(191, 297, ...))
Time: 3.124s (Solving: 0.0s 1st Model: 0.0s Unsat: 0.0s)

```

The result shows that indeed a cell with a polygon contour “*co:8*” could be a proper part of the cytoplasm region with polygon contour “*co:8*”, and we are given a ground polygon as a witness that is a translation  $t = (-93, -186)$  of polygon “*co:8*” (by default, the witness given is the minimum translation required to satisfy the relation).

**Example 2.2.** We now demonstrate going beyond purely qualitative reasoning by taking polygon *shape* into account. We check whether “*tr:8*” can be disconnected from *both* “*co:139*” and “*co:140*” simultaneously (which is impossible due to the particular polygons in the dataset).

```
topology(pp, id("tr:8"), id("co:127")).
topology(dc, id("tr:8"), id("co:139")).
topology(dc, id("tr:8"), id("co:140")).
```

The result is:

```
UNSATISFIABLE
Time: 3.788s (Solving: 0.0s 1st Model: 0.0s Unsat: 0.0s)
```

The result shows that no translation of polygon “*co:8*” exists that satisfies all given topological constraints, due to the shapes of the polygons, i.e. this is an example of mixed qualitative-numerical reasoning.

► **3. MOTION PLANNING.** We show how *ST* regions can be used for motion planning, e.g. in robotic manipulation tasks using abduction.

**Example 3.** An agent (a robot with a manipulator) is at a desk in front of a laptop. A cup of coffee is positioned behind the laptop and the agent wants to get the cup of coffee without the risk of spilling the coffee on the laptop. The agent should not hit the computer while performing the task.

This task requires abducting intermediate states that are consistent with the domain constraints. We model the laptop, hand, and cup from a top-down perspective as *ST* regions with polygonal slices, and give the initial shapes.

```
%% domain objects
desk_object(laptop). desk_object(hand). desk_object(cup).
```

The initial configuration is given for time 0:

```
%% polygonal shapes of objects
polygon(shape(laptop), (0,0, ...)).
polygon(shape(hand), (-105, 3, ...)).
polygon(shape(cup), (205, 54, ...)).

%% initial position of objects at time 0
st_object(Object, at(0), id(shape(Object))) :- desk_object(Object).
```

We model the scenario from time 0 to 2.

```
%% modelling two time steps
time(1..2).

%% it is possible that objects move at each time step
{moves(Object, at(T))} :- desk_object(Object), time(T).

%% objects that move are represented by an (unground) translation of their polygon
translation(shape(Object), translated_shape(Object, at(T))) :- moves(Object, at(T)).
%% at the end of the time step we need a witness of moving objects
spatial(witness, translated_shape(Object, at(T)),
        shape(Object, at(T))) :- moves(Object, at(T)).

%% slice of moving object at time T is a translated polygon
st_object(Object, at(T), id(translated_shape(Object, at(T)))) :-
    moves(Object, at(T)).
%% slice of stationary object at time T is polygon from last time step
st_object(Object, at(T), id(shape(Object, at(LastT)))) :-
    desk_object(Object), time(T), LastT = T - 1, not moves(Object, at(T)).
```

The goal is for the hand to make contact with the cup:

```
topology(c, hand, cup, time(1,2)).
```

We model default domain assumptions, e.g., the cup does not move by default. We express this by assigning costs to interpretations where objects move.

```
cost(0, Object) :- Object = (cup; laptop), time(T), -moves(Object, at(T)).
cost(1, Object) :- Object = (cup; laptop), time(T), moves(Object, at(T)).
#minimize{ C, X : cost(C, X) }.
```

The spatio-temporal constraints for planning the motion trajectory are that the hand and cup must remain disconnected from the laptop.

```
topology(dc, laptop, hand, time(0,2)). topology(dc, laptop, cup, time(0,2)).
```

Our system finds a consistent and optimal answer set where neither the laptop nor cup move in the period before the robot hand has made contact with the cup. Given the spatio-temporal constraints in this optimal answer set, our system then produces a consistent motion trajectory witness of the solution set (Fig. 2).

### 3.2 Empirical Evaluation

In the previous section we demonstrated applicability and runtime results of our system on real world data. We now empirically evaluate our system on synthetic data to more precisely assess runtime scalability and robustness against missing data in the following tests  $T1 - T4$ .<sup>7</sup>

► **T1 (scalability / qualification).** Measuring runtime of deriving spacetime relations between  $n$   $ST$  objects over  $m$  time steps (Table 3). Each  $ST$  object is assigned a randomly generated polygon slice (with between 5 and 10 vertices) for each time step. Each object has a direction vector, speed, fixed angular speed, and fixed acceleration (fixed values randomly selected from  $[-0.1, 0.1]$ ). At each time step the object position is updated according to the direction and speed, and the direction and speed are updated according to the angular speed and acceleration. It is useful to identify semantically relevant object pairs based on other spatio-temporal relations, e.g. with social flies (Fig. 2) the *follow* event is only meaningful when the flies are *near*. We therefore measure (a) average time to compute relations between one pair of  $ST$  objects for all time steps, (b) average time to compute relations between all  $ST$  objects for one time step. Results show that our approach is practical within  $n = 40$   $ST$  objects and  $m = 40$  timesteps.

► **T2 (robustness).** Measuring accuracy of derived spacetime relations when slices are randomly deleted from  $ST$  objects (Table 4). Tests are created as in T1 with 10 objects over 20 time steps. In each such test  $t$  there are  $m \times n$  polygon slices. We copy  $t$  to create test  $t'$ , randomly select  $k$  slices and delete them from  $t'$ . We then compare  $ST$  relations derived from  $t$  and  $t'$  and record the number of matching relations as a measure of accuracy. Our results indicate that linearly interpolating between slices is satisfactorily robust against missing data. This also implies that using ASP to sample large datasets to reduce the search space when identifying meaningful spatio-temporal relations is a viable approach.

<sup>7</sup> Experiments were run on a MacBook Pro, OSX 10.8 2.6 GHz, Intel Core i7, 16GB RAM. Runtime results are ASP grounding time plus solving time, as reported by clingo.

Table 3: T1: Average runtime (seconds) for deriving spacetime relations.

$n$ $ST$ objects (time steps $m = 40$ )	10	20	30	40
One pair, all timesteps	0.47s	1.11s	2.08s	3.3s
All pairs, one timestep	0.46s	8.12s	7.72s	18.29s
$m$ time steps ( $ST$ objects $n = 40$ )	10	20	30	40
One pair, all timesteps	0.33s	0.92s	1.88s	3.24s
All pairs, one timestep	15.47s	15.89s	16.87s	18.14s

Table 4: T2: Accuracy of derived relations from interpolation when  $k$  slices are deleted ( $k \in \{10, 20, 30, 40\}$ ) from 200 slices.

Deleted slices:	5%	10%	15%	20%
Correct relations	97.32%	95.90%	94.41%	93.02%

► **T3 (scalability / translation).** Measuring runtime of determining (in)consistency of translating a polygon to satisfy given spacetime constraints (mixed-numerical reasoning problem) (Table 5). For each test,  $n$   $ST$  objects are created as in T1, and a new  $ST$  object  $g$  is declared and assigned  $m = 10$  randomly generated polygon slices that can be translated. We measure time taken to find the first 10,000 models (and solution sets of all consistent translations) where one mereotopological relation is asserted between  $g$  and each other object (i.e. each model has  $n$  relations). The large number of models is due to existential  $ST$  relations, e.g. two  $ST$  objects have *contact* if *at least one slice* has contact, thus leading to many alternative models. The results show that our approach is practical up to  $n = 20$  objects.

► **T4 (scalability / inconsistency).** Measuring runtime for determining (in)consistency of  $n$  qualitatively constrained  $ST$  objects with no numerical information (purely qualitative reasoning) (Table 6). Each object  $i \in \{1, 2, \dots, n\}$  is declared with no polygonal slices. Object pairs are randomly selected and assigned 4 randomly chosen alternative  $ST$  relations using the algorithm described in [28] (mean degree of constraint network  $d = 5$ ). Each test with  $n$  objects is run 10 times, we report mean runtime and number of models (i.e. consistent constraint networks). Our results show that our approach is practical up to  $n = 30$  objects before combinatorial explosion occurs.

Table 5: T3: translating  $m = 10$  polygon slices of  $ST$  object  $g$  to satisfy qualitative constraints (find first 10,000 models).

$n$ $ST$ objects:	5	10	15	20
Runtime (sec)	1.30s	6.15s	17.55s	39.38s

Table 6: T4: Runtime (seconds) for determining inconsistencies in purely qualitative constraints.

$n$ ST objects:	10	20	30	40
Models (mean)	20%	10%	10%	20%
Runtime (sec)	0.8562	3.7995	10.7358	205.941

## 4 DISCUSSION AND RELATED WORK

ASP Modulo extensions for handling specialised domains and abstraction mechanisms provides a powerful means for the utilising ASP as a foundational knowledge representation and reasoning (KR) method for a wide-range of application contexts. This approach is clearly demonstrated in work such as ASPMT [3, 16, 20], CLINGCON [14], ASPMT(QS) [34]. Most closely related to our research is the ASPMT founded *non-monotonic spatial reasoning* system ASPMT(QS) [34]. Whereas ASPMT(QS) provides a valuable blueprint for the integration and formulation of geometric and spatial reasoning within answer set programming modulo theories, the developed system is a first-step and lacks support for a rich spatio-temporal ontology or an elaborate characterisation of complex ‘space-time’ objects as native (the focus there has been on enabling non-monotonicity with a basic spatial and temporal ontology). In addition to the ontological extensions for a much richer ‘space-time’ component, our system pipeline –based on CLINGO [13] — has the following additional advantages over the standard ASPMT / ASPMT(QS) pipeline: (1). we generate *all* spatially consistent models compared to only one model in the standard ASPMT pipeline; (2). we compute optimal answer sets, e.g. add support preferences, which allows us to rank models, specify weak constraints; (3). unlike ASPMT(QS) we support quantification of space-time regions.

Within the relation algebraic driven (qualitative) spatial reasoning community, researchers have investigated translating qualitative spatial calculi into ASP programs e.g. [7, 21]. The primary difference with our line of research is we emphasise both purely qualitative and mixed qualitative-quantitative constraints and efficiently deriving *ST* relations from large datasets, and that spatio-temporal entities and relations have natively encoded semantics within the KR framework being employed, namely answer set programming. More broadly, this research is driven by a departure from the use of relational-algebra, and instead focussing on *declarative spatial reasoning* directly within KR frameworks such as constraint logic programming, answer set programming, and inductive logic programming [5, 32, 34].

## 5 SUMMARY AND OUTLOOK

A novel method and corresponding system for declaratively modelling and reasoning about the dynamics of space-time histories —*regions with spatial and temporal components*— as first-class objects within answer set programming is developed. The framework is implemented as an extension of the CLINGO ASP solver [13], whereas the crux of the method relies on leveraging upon the semantics of (mereotopological) spatio-temporal relations using specialised and highly optimised systems of polynomials. We

have presented an empirical evaluation, and demonstrated several reasoning features in the context of select applications domains requiring interpretation and control tasks. The outlook of this work is geared towards enhancing the application of the developed specialised ASP Modulo Space-Time component specifically for non-monotonic spatio-temporal reasoning about large datasets in the domain of visual stimulus interpretation, as well as constraint-based motion control in the domain of home-based and industrial robotics. The reasoning system is also slated for deployment as an open-source robotics domain specific library as part of the ROS [\[26\]](#) robotics framework.

## Bibliography

- [1] Aiello, M., Pratt-Hartmann, I., van Benthem, J.: Handbook of spatial logics. Springer (2007)
- [2] Arnon, D.S., Collins, G.E., McCallum, S.: Cylindrical Algebraic Decomposition I: The basic algorithm. *SIAM Journal on Computing* **13**(4), 865–877 (1984)
- [3] Bartholomew, M., Lee, J.: System `aspmt2smt`: Computing ASPMT Theories by SMT Solvers. In: *Logics in Artificial Intelligence*, pp. 529–542. Springer (2014)
- [4] Bhatt, M., Lee, J.H., Schultz, C.: CLP(QS): A Declarative Spatial Reasoning Framework. In: *COSIT 2011 - Spatial Information Theory*. pp. 210–230. Springer-Verlag, Berlin, Heidelberg (2011)
- [5] Bhatt, M., Lee, J.H., Schultz, C.P.L.: CLP(QS): A declarative spatial reasoning framework. In: *Spatial Information Theory - 10th International Conference, COSIT 2011, Belfast, ME, USA, September 12-16, 2011. Proceedings. Lecture Notes in Computer Science*, vol. 6899, pp. 210–230. Springer (2011)
- [6] Bomanson, J., Gebser, M., Janhunen, T., Kaufmann, B., Schaub, T.: Answer set programming modulo acyclicity\*. *Fundam. Inform.* **147**(1), 63–91 (2016)
- [7] Brenton, C., Faber, W., Batsakis, S.: Answer Set Programming for Qualitative Spatio-Temporal Reasoning: Methods and Experiments. In: *Technical Communications of ICLP*. vol. 52, pp. 4:1–4:15 (2016)
- [8] Brewka, G., Eiter, T., Truszczyński, M.: Answer set programming at a glance. *Commun. ACM* **54**(12), 92–103 (Dec 2011)
- [9] Collins, G.E., Hong, H.: Partial cylindrical algebraic decomposition for quantifier elimination. *Journal of Symbolic Computation* **12**(3), 299 – 328 (1991)
- [10] De Moura, L., Bjørner, N.: Z3: An efficient smt solver. In: *Tools and Algorithms for the Construction and Analysis of Systems*, pp. 337–340. Springer (2008)
- [11] Ferraris, P.: Answer sets for propositional theories. In: *Logic Programming and Nonmonotonic Reasoning*, pp. 119–131. Springer (2005)
- [12] Ferraris, P., Lee, J., Lifschitz, V.: Stable models and circumscription. *Artificial Intelligence* **175**(1), 236–263 (2011)
- [13] Gebser, M., Kaminski, R., Kaufmann, B., Schaub, T.: *Clingo* = ASP + control: Preliminary report. In: Leuschel, M., Schrijvers, T. (eds.) *Technical Communications of ICLP*. vol. 14(4-5) (2014), *theory and Practice of Logic Programming, Online Supplement*
- [14] Gebser, M., Ostrowski, M., Schaub, T.: Constraint answer set solving. In: Hill, P., Warren, D. (eds.) *Proceedings of the Twenty-fifth International Conference on Logic Programming (ICLP’09)*. *Lecture Notes in Computer Science*, vol. 5649, pp. 235–249. Springer-Verlag (2009)
- [15] Gebser, M., Kaufmann, B., Kaminski, R., Ostrowski, M., Schaub, T., Schneider, M.: Potassco: The Potsdam answer set solving collection. *AI Communications* **24**(2), 107–124 (2011)
- [16] Gelfond, M.: Answer sets. *Handbook of knowledge representation* **1**, 285 (2008)
- [17] Gelfond, M., Lifschitz, V.: The stable model semantics for logic programming. In: *ICLP/SLP*. vol. 88, pp. 1070–1080 (1988)
- [18] Hazarika, S.M.: Qualitative spatial change: space-time histories and continuity. Ph.D. thesis, The University of Leeds (2005)
- [19] Johnson, S.G.: The NLopt nonlinear-optimization package. <http://ab-initio.mit.edu/nlopt>



- [20] Lee, J., Meng, Y.: Answer set programming modulo theories and reasoning about continuous changes. In: IJCAI 2013, Proceedings of the 23rd International Joint Conference on Artificial Intelligence, Beijing, China, August 3-9, 2013 (2013)
- [21] Li, J.J.: Qualitative spatial and temporal reasoning with answer set programming. In: Tools with Artificial Intelligence (ICTAI), 2012 IEEE 24th International Conference on. vol. 1, pp. 603–609. IEEE (2012)
- [22] Ligozat, G.: Qualitative Spatial and Temporal Reasoning. Wiley-ISTE (2011)
- [23] Muller, P.: A qualitative theory of motion based on spatio-temporal primitives. KR **98**, 131–141 (1998)
- [24] Neubauer, K., Wanko, P., Schaub, T., Haubelt, C.: Exact multi-objective design space exploration using ASPmT. In: 2018 Design, Automation & Test in Europe Conference & Exhibition, DATE 2018, Dresden, Germany, March 19-23, 2018. pp. 257–260. IEEE (2018)
- [25] Powell, M.J.: The BOBYQA algorithm for bound constrained optimization without derivatives. Cambridge NA Report TR NA2009/06, University of Cambridge, Cambridge (2009)
- [26] Quigley, M., Conley, K., Gerkey, B.P., Faust, J., Foote, T., Leibs, J., Wheeler, R., Ng, A.Y.: Ros: an open-source robot operating system. In: ICRA Workshop on Open Source Software (2009)
- [27] Randell, D.A., Cui, Z., Cohn, A.G.: A spatial logic based on regions and connection. KR **92**, 165–176 (1992)
- [28] Renz, J., Nebel, B.: Efficient methods for qualitative spatial reasoning. J. Artif. Intell. Res.(JAIR) **15**, 289–318 (2001)
- [29] Schultz, C., Bhatt, M.: Spatial symmetry driven pruning strategies for efficient declarative spatial reasoning. In: Spatial Information Theory - 12th International Conference, COSIT 2015, Santa Fe, NM, USA, October 12-16, 2015, Proceedings. Lecture Notes in Computer Science, vol. 9368, pp. 331–353. Springer (2015)
- [30] Schultz, C., Bhatt, M.: A numerical optimisation based characterisation of spatial reasoning. In: International Symposium on Rules and Rule Markup Languages for the Semantic Web. pp. 199–207. Springer (2016)
- [31] Sommer, C., Straehle, C., Koethe, U., Hamprecht, F.A.: Ilastik: Interactive learning and segmentation toolkit. In: Biomedical Imaging: From Nano to Macro, 2011 IEEE International Symposium on. pp. 230–233. IEEE (2011)
- [32] Suchan, J., Bhatt, M., Schultz, C.P.L.: Deeply semantic inductive spatio-temporal learning. In: Cussens, J., Russo, A. (eds.) Proceedings of the 26th International Conference on Inductive Logic Programming (Short papers), London, UK, 2016. CEUR Workshop Proceedings, vol. 1865, pp. 73–80. CEUR-WS.org (2016)
- [33] Suchan, J., Bhatt, M., Walega, P.A., Schultz, C.: Visual Explanation by High-Level Abduction: On Answer-Set Programming Driven Reasoning About Moving Objects. In: McIlraith, S.A., Weinberger, K.Q. (eds.) Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, New Orleans, Louisiana, USA, February 2-7, 2018. AAAI Press (2018)
- [34] Wałęga, P., Bhatt, M., Schultz, C.: ASPMT(QS): Non-Monotonic Spatial Reasoning with Answer Set Programming Modulo Theories. In: LPNMR: Logic Programming and Non-monotonic Reasoning - 13th International Conference. Lexington, KY, USA (2015)
- [35] Wallgrün, J.O.: Topological adjustment of polygonal data. In: Advances in Spatial Data Handling, pp. 193–208. Springer (2013)
- [36] Wang, Y., Lee, J.: Handling uncertainty in answer set programming. In: Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, Texas, USA. pp. 4218–4219. AAAI Press (2015)