

Towards Stabilization of Distributed Systems under Denial-of-Service

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Abstract—In this paper, we consider networked distributed systems in the presence of Denial-of-Service (DoS) attacks, namely attacks that prevent transmissions over the communication network. First, we consider a simple and typical scenario where communication sequence is purely Round-robin and we explicitly calculate a bound of attack frequency and duration, under which the interconnected large-scale system is asymptotically stable. Second, trading-off system resilience and communication load, we design a hybrid transmission strategy consisting of Zeno-free distributed event-triggered control and Round-robin. We show that with lower communication loads, the hybrid communication strategy enables the systems to have the same resilience as in pure Round-robin.

I. INTRODUCTION

Cyber-physical systems (CPSs) are increasingly appealing for industry nowadays thanks to the development of computation and communication infrastructures. The application of CPSs ranges from local control systems to large-scale systems, examples being house temperature control systems and regional grid control systems. Owing to the advances in economic and possibly reliability reasons, systems tend to be large-scale, interconnected and spatially distributed, among which communications are operated via wireless network [1]. This triggers the attention towards networked control of large-scale interconnected systems, which are possibly safety-critical and potentially exposed to malicious attacks [?].

The concept of cyber-physical security mostly concerns security against intelligent attacks. One usually classifies these attacks as either deceptive attacks or Denial-of-Service (DoS). Deceptive attacks affect the trustworthiness of transmitted data [3], [4]. Instead, DoS compromises the timeliness of information exchange, *e.g.* in the presence of DoS, communications are not possible [5], [6].

This paper investigates DoS attacks. We consider a large-scale system composed of interconnected subsystems, which are possibly spatially distributed. The information exchange between distributed systems and controllers takes place over a shared communication channel, which implies that all the communication attempts can be denied in the presence of DoS.

The literature on distributed/decentralized networked control [7]–[13] and centralized system under DoS attacks [6], [14]–[26] is large and diversified. In [12], based on a small-gain approach, the authors propose a parsimonious event-triggered design, which is able to prevent Zeno behavior and

stabilize nonlinear distributed systems asymptotically. In [8], [10], event-triggered approaches are discussed within large-scale interconnected systems. By introducing a constant in the triggering condition, the authors prove that the system converges to a region around equilibrium without the occurrence of Zeno behavior. In [14], the authors consider a scenario where malicious attacks and genuine packet losses coexist, where the effect of malicious attacks and random packet losses are merged and characterized by an overall packet drop ratio. In [25], the authors formulate a two-player zero-sum stochastic game framework to consider a remote secure estimation problem, where the signals are transmitted over a multi-channel network under DoS attacks. A problem similar to zero-sum games between controllers and strategic jammers is considered in [16]. In [18], the authors investigate DoS from the attacker’s viewpoint where the objective is to consume limited energy and maximize the effect induced by DoS attacks. The paper [26] considers a stabilization problem where transmissions are event-based and the network is corrupted by periodic DoS attacks. In [20], [21], a framework is introduced where DoS attacks are characterized by *frequency* and *duration*. The contribution is an explicit characterization of DoS frequency and duration under which stability can be preserved through state-feedback control. Extensions have been considered dealing with dynamic controllers [22], [23] and nonlinear system [24].

In this paper, we consider networked distributed systems under DoS attacks, which has not been investigated so far under the class of DoS attacks introduced in [20], [21]. Previously in [20]–[24], the authors analyze the behavior of systems in a centralized-system manner, where the major characteristic is that all the states are assumed to be collected and sent in one transmission attempt. In this paper, we analyze the problem from the distributed system point of view, where the interconnected subsystems share one communication channel and transmission attempts of the subsystems take place asynchronously. The contribution of this paper is twofold. First, we consider a simple but typical scenario where the communication sequence is purely Round-robin and we explicitly compute a bound on attack frequency and duration, under which the large-scale system is asymptotically stable. Second, trading-off system resilience and communication load, we design a hybrid transmission strategy. Specifically, in the absence of DoS attacks, we design a distributed event-triggered control using small gain argument, which guarantees practical stability of the closed-loop system while preventing the occurrence of Zeno behavior. During DoS-active periods, communication switches to Round-robin, aiming at quick communication restore. This

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hybrid communication strategy surprisingly ends up with the same bound as pure Round-robin transmission but promotes the possibility to save communication resources.

The paper is organized as follows. In Section II, we introduce the framework of interest along with the considered family of DoS attacks. In Section III, we present the main result. We first use small gain approach to study large-scale system under Round-robin. Subsequently, we introduce the main result of this paper: the characterization of frequency and duration of DoS attacks, under which the large-scale system is asymptotically stable. Section IV briefly introduces a hybrid transmission design, which achieves the same result as in Section III with lower communication load. Section V discusses numerical simulations and Section VI ends the paper with conclusions and possible future research directions.

A. Notation

We denote by \mathbb{R} the set of reals. Given $\alpha \in \mathbb{R}$, we let $\mathbb{R}_{>\alpha}$ ($\mathbb{R}_{\geq\alpha}$) denote the set of reals greater than (greater than or equal to) α . We let \mathbb{N}_0 denote the set of nonnegative integers, $\mathbb{N}_0 := \{0, 1, \dots\}$. The prime denotes transpose. Given a vector $v \in \mathbb{R}^n$, $\|v\|$ is its Euclidean norm. Given a matrix M , $\|M\|$ is its spectral norm. Given two sets A and B , we denote by $B \setminus A$ the relative complement of A in B , *i.e.*, the set of all elements belonging to B , but not to A .

II. FRAMEWORK

A. Networked distributed system

Consider a large-scale system consisting of N interacting subsystems, whose dynamics satisfy

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j \in N_i} H_{ij} x_j(t) \quad (1)$$

where A_i , B_i and H_{ij} are matrices with appropriate dimensions and $t \in \mathbb{R}_{\geq 0}$. $x_i(t)$ and $u_i(t)$ are state and control input of subsystem i , respectively. Here we assume that all the subsystems are full state output. N_i denotes for the set of neighbors of subsystem i . Subsystem i physically interacts through $\sum_{j \in N_i} H_{ij} x_j(t)$ with its neighbor subsystem(s) $j \in N_i$. Here we consider bidirectional edges, *i.e.* $j \in N_i$ when $i \in N_j$.

The distributed systems are controlled via a shared networked channel, through which distributed plants broadcast the measurements and controllers send control inputs. The computation of control inputs is based on the transmitted measurements. The received measurements are in sample-and-hold fashion such as $x_i(t_k^i)$ where t_k^i represents the sequence of transmission instants of subsystem i . We assume that there exists a feedback matrix K_i such that $\Phi_i = A_i + B_i K_i$ is Hurwitz. Therefore, the control input applied to subsystem i is given by

$$u_i(t) = K_i x_i(t_k^i) + \sum_{j \in N_i} L_{ij} x_j(t_k^j) \quad (2)$$

where L_{ij} is the coupling gain in the controller. Here we assume that the channel is noiseless and there is no quantization. Moreover, we assume that the network transmission delay and the computation time of control inputs are zero.

B. DoS attacks—frequency and duration

We refer to Denial-of-Service as the phenomenon for which transmission attempts may fail. In this paper, we do not distinguish between transmission failures due to channel unavailability and transmission failures because of DoS-induced packet corruption. Since the network is shared, DoS simultaneously affects the communication attempts of all the subsystems.

Clearly, the problem in question does not have a solution if the DoS amount is allowed to be arbitrary. Following [21], we consider a general DoS model that constrains the attacker action in time by only posing limitations on the frequency of DoS attacks and their duration. Let $\{h_n\}_{n \in \mathbb{N}_0}$, $h_0 \geq 0$, denote the sequence of DoS *off/on* transitions, *i.e.*, the time instants at which DoS exhibits a transition from zero (transmissions are possible) to one (transmissions are not possible). Hence,

$$H_n := \{h_n\} \cup [h_n, h_n + \tau_n[\quad (3)$$

represents the n -th DoS time-interval, of a length $\tau_n \in \mathbb{R}_{\geq 0}$, over which the network is in DoS status. If $\tau_n = 0$, then H_n takes the form of a single pulse at h_n . If $\tau_n \neq 0$, $[h_n, h_n + \tau_n[$ represents an interval from the instant h_n (include h_n) to $(h_n + \tau_n)^-$ (arbitrarily close to but exclude $h_n + \tau_n$). Similarly, $[\tau, t[$ represents an interval from τ to t^- . Given $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, let $n(\tau, t)$ denote the number of DoS *off/on* transitions over $[\tau, t[$, and let

$$\Xi(\tau, t) := \bigcup_{n \in \mathbb{N}_0} H_n \cap [\tau, t] \quad (4)$$

denote the subset of $[\tau, t]$ where the network is in DoS status. The subset of time where DoS is absent is denoted by

$$\Theta(\tau, t) := [\tau, t] \setminus \Xi(\tau, t) \quad (5)$$

We make the following assumptions.

Assumption 1: (DoS frequency). There exist constants $\eta \in \mathbb{R}_{\geq 0}$ and $\tau_D \in \mathbb{R}_{> 0}$ such that

$$n(\tau, t) \leq \eta + \frac{t - \tau}{\tau_D} \quad (6)$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

Assumption 2: (DoS duration). There exist constants $\kappa \in \mathbb{R}_{\geq 0}$ and $T \in \mathbb{R}_{> 1}$ such that

$$|\Xi(\tau, t)| \leq \kappa + \frac{t - \tau}{T} \quad (7)$$

for all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

Remark 1: Assumptions 1 and 2 do only constrain a given DoS signal in terms of its *average* frequency and duration. Actually, τ_D can be defined as the average dwell-time between consecutive DoS off/on transitions, while η is the chattering bound. Assumption 2 expresses a similar requirement with respect to the duration of DoS. It expresses

the property that, on the average, the total duration over which communication is interrupted does not exceed a certain *fraction* of time, as specified by $1/T$. Like η , the constant κ plays the role of a regularization term. It is needed because during a DoS interval, one has $|\Xi(h_n, h_n + \tau_n)| = \tau_n > \tau_n/T$. Thus κ serves to make (7) consistent. Conditions $\tau_D > 0$ and $T > 1$ imply that DoS cannot occur at an infinitely fast rate or be always active. ■

III. MAIN RESULT

In this section, our objective is to find stability conditions for the networked distributed systems under DoS attacks. We first study the stabilization problem of large-scale systems under a digital communication channel in the absence of DoS.

A. A small-gain approach for large-scale systems under networked communication

For each subsystem i , we denote by $e_i(t)$ the error between the value of the state transmitted to its neighbors and the current state, *i.e.*,

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad i = 1, 2, \dots, N \quad (8)$$

Then combine (1), (2) and (8), the dynamics of subsystem i can be written as

$$\begin{aligned} \dot{x}_i(t) &= \Phi_i x_i(t) + B_i K_i e_i(t) + \sum_{j \in N_i} (B_i L_{ij} + H_{ij}) x_j(t) \\ &\quad + B_i \sum_{j \in N_i} L_{ij} e_j(t) \end{aligned} \quad (9)$$

from which one sees that the dynamics of subsystem i depend on the interconnected neighbors $x_j(t)$ as well as $e_i(t)$, $e_j(t)$ and the coupling parameters. Intuitively, if the couplings are weak and e remains small, then stability can be achieved. Here, the notion ‘‘smallness’’ of e can be characterized by the x -dependent bound $\|e_i(t)\| \leq \sigma_i \|x_i(t)\|$, in which σ_i is a suitable design parameter. Notice that this is not the network update rule.

We implement a periodic sampling protocol, *e.g.* Round-robin, as our update law. In this respect, we make the following hypothesis.

Assumption 3: (Inter-sampling of Round-robin). In the absence of DoS attacks, there exists an inter-sampling interval Δ such that

$$\|e_i(t)\| \leq \sigma_i \|x_i(t)\| \quad (10)$$

holds, where σ_i is a suitable design parameter. ■

For centralized settings, values of Δ satisfying a bound like (10) can be explicitly determined. On the other hand, in [11], [27], the authors compute and apply a lower bound of time elapsed between two events to prevent Zeno behavior, where the distributed/decentralized systems are asymptotically stable. The problem of obtaining Δ is left for future research.

As mentioned in the foregoing argument, σ_i should be designed carefully. Otherwise, even if there exists a Δ under

which (10) holds, in the event of an inappropriate σ_i , stability can be lost as well.

Given any symmetric positive definite matrix Q_i , let P_i be the unique solution of the Lyapunov equation $\Phi_i^T P_i + P_i \Phi_i + Q_i = 0$. For each i , consider the Lyapunov function $V_i = x_i^T P_i x_i$, which satisfies

$$\lambda_{\min}(P_i) \|x_i(t)\|^2 \leq V_i(x_i(t)) \leq \lambda_{\max}(P_i) \|x_i(t)\|^2 \quad (11)$$

where $\lambda_{\min}(P_i)$ and $\lambda_{\max}(P_i)$ represent the smallest and largest eigenvalue of P_i , respectively. The following lemma presents the design of σ_i guaranteeing stability.

Lemma 1: Consider a distributed system as in (1) along with a control input as in (2). Suppose that the spectral radius $r(A^{-1}B) < 1$. The distributed system is asymptotically stable if σ_i satisfies

$$\sigma_i < \sqrt{\frac{l_i}{j_i}} \quad (12)$$

where l_i is the i -th entry of row vector $L := \mu^T(A - B) = [l_1, l_2, \dots, l_N]$ and j_i is the j -th entry of row vector $J := \mu^T \Gamma = [j_1, j_2, \dots, j_N]$. $\mu \in \mathbb{R}_+^N$ is an arbitrary column vector satisfying $\mu^T(-A + B) < 0$. The matrices A , B and Γ are given by

$$A = \begin{bmatrix} \alpha_1 & & & \\ & \ddots & & \\ & & \alpha_N & \end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix} 0 & \beta_{12} & \cdots & \beta_{1N} \\ \beta_{21} & 0 & \beta_{23} & \beta_{2N} \\ \vdots & \vdots & 0 & \vdots \\ \beta_{N1} & \beta_{N2} & \cdots & 0 \end{bmatrix} \quad (14)$$

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NN} \end{bmatrix} \quad (15)$$

with

$$\alpha_i = \lambda_{\min}(Q_i) - \delta - \sum_{j \in N_i} 2\delta \quad (16)$$

$$\beta_{ij} = \frac{\|P_i\|^2 \|B_i L_{ij} + H_{ij}\|^2}{\delta} \quad (17)$$

$$\gamma_{ii} = \frac{\|P_i\|^2 \|B_i K_i\|^2}{\delta} \quad (18)$$

$$\gamma_{ij} = \frac{\|P_i\|^2 \|B_i L_{ij}\|^2}{\delta} \quad (19)$$

where δ is a positive real such that $\alpha_i > 0$ and $\lambda_{\min}(Q_i)$ is the smallest eigenvalue of Q_i for $i = 1, 2, \dots, N$.

Proof. Recalling that $V_i = x_i^T P_i x_i$, the derivative of V_i

along the solution to (9) satisfies

$$\begin{aligned} \dot{V}_i(x_i(t)) &\leq -\lambda_{\min}(Q_i)\|x_i(t)\|^2 \\ &\quad + \|2P_i B_i K_i\| \|x_i(t)\| \|e_i(t)\| \\ &\quad + \sum_{j \in N_i} \|2P_i(B_i L_{ij} + H_{ij})\| \|x_i(t)\| \|x_j(t)\| \\ &\quad + \sum_{j \in N_i} \|2P_i B_i L_{ij}\| \|x_i(t)\| \|e_j(t)\| \end{aligned} \quad (20)$$

Observe that for any positive real δ , the Young's inequalities yield

$$\begin{aligned} &\|2P_i B_i K_i\| \|x_i(t)\| \|e_i(t)\| \\ &\leq \delta \|x_i(t)\|^2 + \frac{\|P_i\|^2 \|B_i K_i\|^2}{\delta} \|e_i(t)\|^2 \end{aligned} \quad (21)$$

$$\begin{aligned} &\|2P_i(B_i L_{ij} + H_{ij})\| \|x_i(t)\| \|x_j(t)\| \\ &\leq \delta \|x_i(t)\|^2 + \frac{\|P_i\|^2 \|B_i L_{ij} + H_{ij}\|^2}{\delta} \|x_j(t)\|^2 \end{aligned} \quad (22)$$

$$\begin{aligned} &\|2P_i B_i L_{ij}\| \|x_i(t)\| \|e_j(t)\| \\ &\leq \delta \|x_i(t)\|^2 + \frac{\|P_i\|^2 \|B_i L_{ij}\|^2}{\delta} \|e_j(t)\|^2 \end{aligned} \quad (23)$$

Hence, the derivative of V_i along the solution to (9) satisfies

$$\begin{aligned} \dot{V}_i(x(t)) &\leq -\alpha_i \|x_i(t)\|^2 + \sum_{j \in N_i} \beta_{ij} \|x_j(t)\|^2 \\ &\quad + \gamma_{ii} \|e_i(t)\|^2 + \sum_{j \in N_i} \gamma_{ij} \|e_j(t)\|^2 \end{aligned} \quad (24)$$

where α_i , β_{ij} , γ_{ii} and γ_{ij} are as in Lemma 1. Notice that one can always find a δ such that $\alpha_i > 0$ for $i = 1, 2, \dots, N$.

By defining vectors

$$\begin{aligned} V_{vec}(x_i(t)) &:= [V_1(x_1(t)), V_2(x_2(t)), \dots, V_N(x_N(t))]^T \\ \|x(t)\|_{vec} &:= [\|x_1(t)\|^2, \|x_2(t)\|^2, \dots, \|x_N(t)\|^2]^T \\ \|e(t)\|_{vec} &:= [\|e_1(t)\|^2, \|e_2(t)\|^2, \dots, \|e_N(t)\|^2]^T \end{aligned}$$

the inequality (24) can be compactly written as

$$\dot{V}_{vec}(x_i(t)) \leq (-A + B)\|x(t)\|_{vec} + \Gamma\|e(t)\|_{vec} \quad (25)$$

with A , B and Γ being as in Lemma 1.

If the spectral radius satisfies $r(A^{-1}B) < 1$, there exists a positive vector $\mu \in \mathbb{R}_+^n$ such that $\mu^T(-A + B) < 0$. We refer readers to [28] for more details. We select the Lyapunov function $V(x(t)) := \mu^T V_{vec}(x_i(t))$. Then the derivative of V yields

$$\begin{aligned} \dot{V}(x(t)) &= \mu^T \dot{V}_{vec}(x_i(t)) \\ &\leq \mu^T(-A + B)\|x(t)\|_{vec} + \mu^T \Gamma \|e(t)\|_{vec} \end{aligned} \quad (26)$$

By noticing that $\mu^T(-A + B) < 0$, we have

$$\dot{V}(x(t)) \leq -L\|x(t)\|_{vec} + J\|e(t)\|_{vec} \quad (27)$$

where $L := \mu^T(A - B)$ and $J := \mu^T \Gamma$ are row vectors. Denote l_i and j_i as the entries of L and J , respectively.

Then, (27) yields

$$\begin{aligned} \dot{V}(x(t)) &\leq -\sum_{i \in N} l_i \|x_i(t)\|^2 + \sum_{i \in N} j_i \|e_i(t)\|^2 \\ &= -\sum_{i \in N} (l_i \|x_i(t)\|^2 - j_i \|e_i(t)\|^2) \end{aligned} \quad (28)$$

which implies asymptotic stability with $\sigma_i < \sqrt{\frac{l_i}{j_i}}$. ■

Remark 2: Lemma 1 can only deal with the case where $j_i > 0$. The case $j_i = 0$ is only possible whenever every entry in the column i of Γ is zero. In fact, $j_i = 0$ implies that the error $\|e_i(t)\|$ never contributes to the system dynamics via (28), which in turn implies that $\|e_i(t)\|$ does not affect stability at all. Therefore, in the case $j_i = 0$, no constraint on $\|e_i(t)\|$ is imposed. ■

B. Stabilization of distributed systems under DoS

In the previous analysis, we have introduced the design of a suitable σ_i and hence error bound, under which the system is asymptotically stable in the absence of DoS. By hypothesis, we also assumed the existence of a Round-robin transmission that satisfies such error bound. In the presence of DoS, (10) is possibly violated even though the sampling strategy is still Round-robin. Under such circumstances, stability can be lost. Hence, we are interested in the stabilization problem when the Round-robin network is under DoS attacks.

Theorem 1: Consider a distributed system as in (1) along with a control input as in (2). The plant-controller information exchange takes place over a shared network, in which the communication protocol is Round-robin with sampling interval Δ as in Assumption 3. The large-scale system is asymptotically stable for any DoS sequence satisfying Assumption 1 and 2 with arbitrary η and κ , and with τ_D and T if

$$\frac{1}{T} + \frac{\Delta_*}{\tau_D} < \frac{\omega_1}{\omega_1 + \omega_2} \quad (29)$$

in which $\Delta_* = N\Delta$, $\omega_1 := \min\{\frac{l_i - \sigma_i^2 j_i}{\lambda_{\max}(P_i) \mu_i}\}$ and $\omega_2 := \frac{4 \max\{j_i\}}{\min\{\mu_i \lambda_{\min}(P_i)\}} \cdot l_i$, j_i , μ_i and σ_i are as in Lemma 1.

Proof. The proof is divided into three steps:

Step 1. Lyapunov function in DoS-free periods. In DoS-free periods, by hypothesis of Assumption 3, (10) holds true with σ_i as in Lemma 1 and (28) is negative. Therefore, the derivative of the Lyapunov function satisfies

$$\begin{aligned} \dot{V}(x(t)) &\leq -\sum_{i \in N} (l_i - j_i \sigma_i^2) \|x_i(t)\|^2 \\ &\leq -\sum_{i \in N} \frac{l_i - \sigma_i^2 j_i}{\lambda_{\max}(P_i) \mu_i} \mu_i V_i \\ &= -\omega_1 V \end{aligned} \quad (30)$$

where $\omega_1 := \min\{\frac{l_i - \sigma_i^2 j_i}{\lambda_{\max}(P_i) \mu_i}\}$. Thus for $t \in [h_n + \tau_n, h_{n+1}[$ (DoS-free time), the Lyapunov function yields

$$V(x(t)) \leq e^{-\omega_1(t - h_n - \tau_n)} V(x(h_n + \tau_n)) \quad (31)$$

Step 2. Lyapunov function in DoS-active periods. Here we let z_n^i denote the last successful sampling instant before

the occurrence of DoS. Recalling the definition of $e_i(t)$, we obtain that

$$e_i(t) = x_i(z_m^i) - x_i(t) = x_i(h_n) - x_i(t) \quad (32)$$

and

$$\|e_i(t)\|^2 \leq \|x_i(h_n)\|^2 + 2\|x_i(t)\|\|x_i(h_n)\| + \|x_i(t)\|^2 \quad (33)$$

for $t \in H_n$. By summing up $\|e_i(t)\|^2$ for $i \in N$, we obtain

$$\begin{aligned} \sum_{i \in N} \|e_i(t)\|^2 &\leq \sum_{i \in N} \|x_i(h_n)\|^2 + \sum_{i \in N} \|x_i(t)\|^2 \\ &\quad + \sum_{i \in N} (\|x_i(h_n)\|^2 + \|x_i(t)\|^2) \\ &= 2 \sum_{i \in N} \|x_i(h_n)\|^2 + 2 \sum_{i \in N} \|x_i(t)\|^2 \end{aligned} \quad (34)$$

If $\sum_{i \in N} \|x_i(h_n)\|^2 \leq \sum_{i \in N} \|x_i(t)\|^2$, we have that $\sum_{i \in N} \|e_i(t)\|^2 \leq 4 \sum_{i \in N} \|x_i(t)\|^2$. Otherwise, we have $\sum_{i \in N} \|e_i(t)\|^2 \leq 4 \sum_{i \in N} \|x_i(h_n)\|^2$

Recalling (28), it is simple to see that

$$\dot{V}(x(t)) \leq \sum_{i \in N} j_i \|e_i(t)\|^2 \quad (35)$$

Thus, for all $t \in H_n$ (DoS-active time) in the case that $\sum_{i \in N} \|x_i(h_n)\|^2 \leq \sum_{i \in N} \|x_i(t)\|^2$, the derivative of the Lyapunov function yields

$$\begin{aligned} \dot{V}(x(t)) &\leq \max\{j_i\} \sum_{i \in N} \|e_i(t)\|^2 \\ &\leq 4 \max\{j_i\} \sum_{i \in N} \|x_i(t)\|^2 \\ &\leq \frac{4 \max\{j_i\}}{\min\{\mu_i \lambda_{\min}(P_i)\}} \sum_{i \in N} \mu_i V(x_i(t)) \\ &= \omega_2 V(x(t)) \end{aligned} \quad (36)$$

with $\omega_2 := \frac{4 \max\{j_i\}}{\min\{\mu_i \lambda_{\min}(P_i)\}}$. On the other hand, for all $t \in H_n$ such that $\sum_{i \in N} \|x_i(h_n)\|^2 > \sum_{i \in N} \|x_i(t)\|^2$, one has

$$\dot{V}(x(t)) \leq \omega_2 V(x(h_n)) \quad (37)$$

Thus, (36) and (37) imply the Lyapunov function during H_n satisfies

$$V(x(t)) \leq e^{\omega_2(t-h_n)} V(x(h_n)) \quad (38)$$

Step 3. Switching between stable and unstable modes. Consider a DoS attack with period τ_n , at the end of which the overall system has to wait an additional period with length $N\Delta$ to have a full round of communications. Hence, the period where at least one subsystem transmission is not successful can be upper bounded by $\tau_n + N\Delta$. For all $\tau, t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$, the total length where communication is not possible over $[\tau, t]$, say $|\bar{\Xi}(\tau, t)|$, can be upper bounded by

$$\begin{aligned} |\bar{\Xi}(\tau, t)| &\leq |\Xi(\tau, t)| + (1 + n(\tau, t))\Delta_* \\ &\leq \kappa_* + \frac{t - \tau}{T_*} \end{aligned} \quad (39)$$

where $\Delta_* = N\Delta$, $\kappa := \kappa + (1 + \eta)\Delta_*$ and $T_* := \frac{\tau_D T}{\tau_D + T\Delta_*}$. Considering the additional waiting time due to Round-robin, the Lyapunov function in (31) yields $V(x(t)) \leq e^{-\omega_1(t-h_n-\tau_n-N\Delta)} V(h_n + \tau_n + N\Delta)$ for $t \in [h_n + \tau_n + N\Delta, h_{n+1}[$ and $V(x(t)) \leq e^{\omega_2(t-h_n)} V(h_n)$ for $t \in [h_n, h_n + \tau_n + N\Delta[$.

Thus, the overall behavior of the closed-loop system can be regarded as a switching system with two modes. Applying simple iterations to the Lyapunov functions in and out of DoS status, one has

$$\begin{aligned} V(x(t)) &\leq e^{-\omega_1|\bar{\Theta}(0,t)|} e^{\omega_2|\bar{\Xi}(0,t)|} V(x(0)) \\ &\leq e^{\kappa_*(\omega_1+\omega_2)t} e^{-\beta_*t} V(x(0)) \end{aligned} \quad (40)$$

where $\beta_* := \omega_1 - (\omega_1 + \omega_2)(\frac{\Delta_*}{\tau_D} + \frac{1}{T})$. By constraining $\beta_* < 0$, one obtains the desired result in (29). Hence, stability is implied at once. ■

Remark 3: The resilience of the distributed systems depends on the largeness of ω_1 and the smallness of ω_2 . To achieve this, one can try to find K_i and L_{ij} such that $\|B_i K_i\|$ and $\|B_i L_{ij}\|$ are small. On the other hand, the sampling interval of Round-robin also affects stability in the sense that it determines how fast the overall system can restore the communication. One can always apply smaller Round-robin inter-sampling time to reduce the left-hand side of (29) at the expense of higher communication load.

IV. APPROXIMATION OF RESILIENCE WITH REDUCED COMMUNICATION: HYBRID TRANSMISSION STRATEGY

In the foregoing argument (*cf. Remark 3*), we have shown that system resilience depends on the sampling rate of Round-robin. The faster the sampling rate of Round-robin, the quicker the overall system restores the communication. On the other hand, in DoS-free periods, we are interested in the possibility of reducing communication load while maintaining the comparable robustness as in Section III. To realize this, we propose a hybrid transmission strategy: in the absence of DoS, the communications of the distributed systems are event-based; if DoS occurs, the communications switch to Round-robin until the moment where every subsystem has one successful update.

The advantage of event-triggered control is saving communication resources. However, the effectiveness of prolonging transmission intervals, in turn, appears to be a disadvantage in the presence of DoS. The main shortcoming concerns that event-triggered control could potentially prolong DoS status. For example, consider that the sampling strategy is purely event-based. After a DoS attack, there is a short period where communications are possible, during which the error bounds as in (10) are not violated so that systems do not update. If DoS appears soon, this is equivalent to the scenario that systems face a longer DoS attack. This indicates that a better strategy is to save communications in the absence of DoS and restore communications as soon as possible when DoS is over, which leads indeed to a hybrid communication strategy.

A. Zeno-free event-triggered control of distributed systems in the absence of DoS

Abusing the notation, in this section we denote $\{t_k^i\}$ as the triggering time sequence of subsystem i under event-triggered control scheme. For a given initial condition $x_i(0)$, if t_k^i converges to a finite t^{i*} , we say that the event-triggered control induces Zeno behavior [11], [12]. Hence, Zeno-freeness implies an event-triggered control scheme preventing the occurrence of Zeno behavior. The following lemma addresses the Zeno-free event-triggered control.

Lemma 2: Consider a distributed system as in (1) along with a control input as in (2). Suppose that the spectral radius $r(A^{-1}B) < 1$. In the absence of DoS, the distributed system is practically stable and Zeno-free if the event-triggered law satisfies

$$\|e_i(t)\| \leq \max\{\sigma_i \|x_i(t)\|, c_i\} \quad (41)$$

in which c_i is a positive finite real and

$$\sigma_i < \min\left\{\sqrt{\frac{l_i}{j_i}}, 1\right\} \quad (42)$$

where l_i and j_i are the same as in Lemma 1.

Proof. By Lemma 1 if spectral radius $r(A^{-1}B) < 1$, (28) holds true. Then one can observe that the event-triggered control law (41) would lead (28) to

$$\begin{aligned} \dot{V}(x(t)) &\leq -\sum_{i \in N} (l_i \|x_i(t)\|^2 - j_i \max\{\sigma_i^2 \|x_i(t)\|^2, c_i^2\}) \\ &\leq \max\left\{-\sum_{i \in N} (l_i - j_i \sigma_i^2) \|x_i(t)\|^2, \right. \\ &\quad \left. -\sum_{i \in N} l_i \|x_i(t)\|^2 + \sum_{i \in N} j_i c_i^2\right\} \\ &\leq -\sum_{i \in N} (l_i - j_i \sigma_i^2) \|x_i(t)\|^2 + \sum_{i \in N} j_i c_i^2 \end{aligned} \quad (43)$$

which implies practical stability with $\sigma_i < \min\left\{\sqrt{\frac{l_i}{j_i}}, 1\right\}$ and finite c_i .

Then we introduce the analysis about Zeno-freeness of this distributed event-triggered control law. Since $\dot{e}_i(t) = -\dot{x}_i(t)$, then the dynamics of e_i satisfy

$$\begin{aligned} \dot{e}_i(t) &= A_i e_i(t) - \Phi_i x_i(t_k^i) - \sum_{j \in N_i} (B_i L_{ij} + H_{ij}) x_j(t_k^j) \\ &\quad + \sum_{j \in N_i} H_{ij} e_j(t) \end{aligned} \quad (44)$$

From the triggering law (41), one can obtain $\|x_i(t_k^i) - x_i(t)\| \leq \max\{\sigma_i \|x_i(t)\|, c_i\}$ and further calculations yield $\|x_i(t)\| - \|x_i(t_k^i)\| \leq \sigma_i \|x_i(t)\| + c_i$. Thus, it is simple to verify that $\|e_i(t)\| \leq \bar{\sigma}_i \|x_i(t_k^i)\| + \bar{\sigma}_i c_i$, where $\bar{\sigma}_i := \frac{\sigma_i}{1-\sigma_i}$.

For each i , at the instant t_{k+1}^i , $\|e_i(t)\|$ satisfies

$$\begin{aligned} \|e_i(t_{k+1}^i)\| &\leq f_i \|\Phi_i\| \|x_i(t_k^i)\| \\ &\quad + f_i \sum_{j \in N_i} \|B_i L_{ij} + H_{ij}\| m \\ &\quad + f_i \sum_{j \in N_i} \|H_{ij}\| \bar{\sigma}_j (m + c_j) \end{aligned} \quad (45)$$

where $f_i := \int_{t_k^i}^{t_{k+1}^i} e^{A(t_{k+1}^i - \tau)} d\tau$, $m = \max\{\|x_j(t_p^j)\|\}$ for $t_k^i \leq t_p^j < t_{k+1}^i$ and $j \in N_i$. Meanwhile, the triggering law in (41) implies that $\|e_i(t_{k+1}^i)\| \geq c_i$. Then, one immediately sees that

$$\begin{aligned} t_{k+1}^i - t_k^i &\geq z_i, & \text{if } \mu_{A_i} \leq 0, \\ t_{k+1}^i - t_k^i &\geq \frac{1}{\mu_{A_i}} \log(z_i \mu_{A_i} + 1), & \text{if } \mu_{A_i} > 0, \end{aligned} \quad (46)$$

in which

$$z_i := \frac{c_i}{\|\Phi_i\| \|x_i(t_k^i)\| + m \sum_{j \in N_i} \zeta_{ij} + \sum_{j \in N_i} \|H_{ij}\| \bar{\sigma}_j c_j}$$

where $\zeta_{ij} := \|B_i L_{ij} + H_{ij}\| + \|H_{ij}\| \bar{\sigma}_j$ and μ_{A_i} is the logarithmic norm of A_i . Notice that the system is practically stable, so that $\|x_i(t_k^i)\|$ and m are bounded. This implies that $z_i > 0$ and hence $t_{k+1}^i - t_k^i > 0$. ■

B. Stabilization of distributed systems with hybrid transmission strategy under DoS

As a counterpart of Assumption 3, here we assume that there exists a Round-robin sampling interval Δ satisfying (41). Now we are ready to present the following result.

Theorem 2: Consider a distributed system as in (1) along with a control input as in (2). The plant-controller information exchange takes place over a shared network implementing the event-triggered control law (41) in the absence of DoS. Suppose that there exists a Round-robin sampling interval Δ such that (41) holds. The network is subject to DoS attacks regulated by Assumption 1 and 2, during which the communication switches to Round-robin until every subsystem updates successfully. Then the distributed system is practically stable if (29) holds true.

Proof. Similar to the proof of Theorem 1, considering the additional waiting time $N\Delta$ due to Round-robin for the restoring of communications, in DoS-free periods the Lyapunov function satisfies

$$\begin{aligned} V(x(t)) &\leq e^{-\omega_1(t-h_n-\tau_n-N\Delta)} V(x(h_n+\tau_n+N\Delta)) \\ &\quad + \frac{c}{\omega_1} \end{aligned} \quad (47)$$

for $t \in [h_n+\tau_n+N\Delta, h_{n+1}[$, where ω_1 is as in Theorem 1 and $c := \sum_{i=1}^N j_i c_i^2$. On the other hand, (38) still holds for $t \in [h_n, h_n+\tau_n+N\Delta[$.

Applying the very similar calculation as in Step 3 in the proof of Theorem 1, we obtain

$$\begin{aligned} V(x(t)) &\leq e^{-\omega_1|\bar{\Theta}(0,t)|} e^{\omega_2|\bar{\Xi}(0,t)|} V(x(0)) \\ &\quad + \sum_{n=0}^q e^{-\omega_1|\bar{\Theta}(h_n,t)|} e^{\omega_2|\bar{\Xi}(h_n,t)|} \frac{c}{\omega_1} + \frac{c}{\omega_1} \\ &\leq e^{\kappa_*(\omega_1+\omega_2)} e^{-\beta_* t} V(x(0)) \\ &\quad + e^{\kappa_*(\omega_1+\omega_2)} \sum_{n=0}^q e^{-\beta_*(t-h_n)} \frac{c}{\omega_1} + \frac{c}{\omega_1} \end{aligned} \quad (48)$$

where $n \in \mathbb{N}_0$, $q := \sup\{q \in \mathbb{N}_0 | h_q \leq t\}$ and β_* is as in the proof of Theorem 1. Notice that $t - h_n \geq \tau_D n(h_n, t) - \tau_D n$

by exploiting Assumption 1. Then, the Lyapunov function yields

$$\begin{aligned} V(x(t)) &\leq e^{\kappa_*(\omega_1+\omega_2)} e^{-\beta_* t} V(x(0)) \\ &\quad + e^{\kappa_*(\omega_1+\omega_2)+\beta_* \tau_D \eta} \sum_{n=0}^q e^{-\beta_* \tau_D n} \frac{c}{\omega_1} \\ &\quad + \frac{c}{\omega_1} \end{aligned} \quad (49)$$

Recalling the definition of Assumption 1, one has that $n(h_n, t) - n(h_{n+1}, t) \geq 1$ for $t \geq h_{n+1}$. This implies that

$$\sum_{n=0}^q e^{-\beta_* \tau_D n} \leq \frac{1}{1 - e^{-\beta_* \tau_D}} \quad (50)$$

Finally, (49) can be written as

$$\begin{aligned} V(x(t)) &\leq e^{\kappa_*(\omega_1+\omega_2)} e^{-\beta_* t} V(x(0)) \\ &\quad + \frac{e^{\kappa_*(\omega_1+\omega_2)+\beta_* \tau_D \eta}}{1 - e^{-\beta_* \tau_D}} \frac{c}{\omega_1} + \frac{c}{\omega_1} \end{aligned} \quad (51)$$

If (29) holds, it is simple to verify that $\beta_* < 0$, which implies practical stability. ■

V. SIMULATION

A. Example 1

The numerical example is taken from [29]. The systems are open-loop unstable such as

$$\begin{aligned} \dot{x}_1(t) &= x_1(t) + u_1(t) + x_2(t) \\ \dot{x}_2(t) &= x_2(t) + u_2(t) \end{aligned}$$

under distributed control inputs such that

$$\begin{aligned} u_1(t) &= -4.5x_1(t_k^1) - 1.4x_2(t_k^2) \\ u_2(t) &= -6x_2(t_k^2) - x_1(t_k^1) \end{aligned}$$

Solutions of the Lyapunov equation $\Phi_i^T P_i + P_i \Phi_i + Q_i = 0$ with $Q_i = 1$ ($i = 1, 2$) yields $P_1 = 0.1429$ and $P_2 = 0.1$. The matrices are $A = [0.7 \ 0; 0 \ 0.9]$, $B = [0 \ 0.0327; 0.1 \ 0]$ and $\Gamma = [4.1327 \ 0.4; 0.1 \ 3.6]$ according to Lemma 1. From these parameters, we obtain that the spectral radius $r(A^{-1}B) = 0.072$, $\sigma_1 < 0.3765$ and $\sigma_2 < 0.4657$. We let $\sigma_1 = \sigma_2 = 0.2$. Based on Assumption 3, we choose Round-robin sampling interval $\Delta = 0.01$ s.

With those parameters, we obtain the bound $\frac{\omega_1}{\omega_1+\omega_2} \approx 0.0175$ with $\omega_1 \approx 3.0149$ and $\omega_2 \approx 169.3061$. This implies that a maximum duty cycle of 1.75% of a sustained DoS would not destabilize our systems in the example. Actually, this bound is conservative. The systems in inspection can endure more DoS without losing stability. As shown in Figure 1, lines represent states and gray stripes represent the presence of DoS. Over a simulation horizon of 20s, the DoS corresponds to parameters of $\tau_D \approx 1.8182$ and $T \approx 2.5$, and $\sim 40\%$ of transmission failures. According to (29), we obtain $\frac{\Delta_*}{\tau_D} + \frac{1}{T} = 0.411$, which violates the theoretical bound, but the system is still stable.

Meanwhile, the hybrid transmission strategy is able to reduce communications effectively. As shown in Figure 1. the transmissions with the hybrid transmission strategy is only 10% of the transmissions with the pure Round-robin strategy.

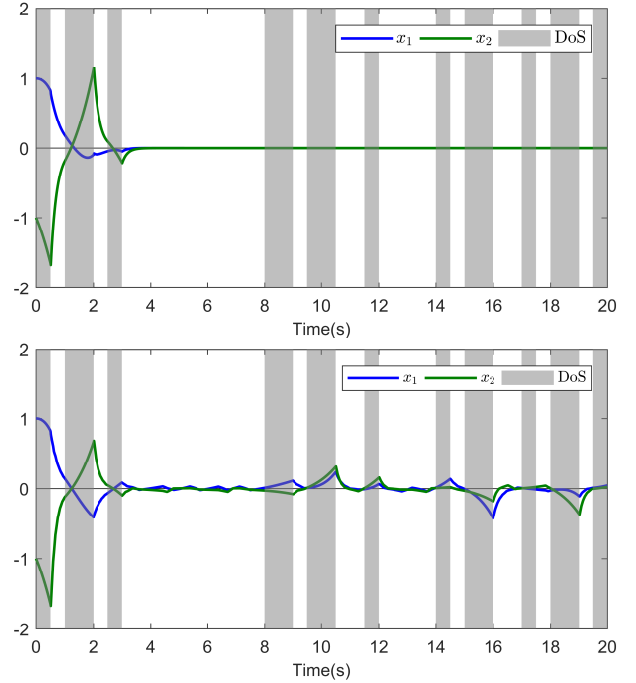


Fig. 1. Example 1: Top picture—States under pure Round-robin communication where there are 1200 transmissions in total; Bottom picture—States under hybrid communication strategy where there are 112 transmissions.

B. Example 2

In this example, we consider a physical system in [30]. The system is composed of N inverted pendulums interconnected as a line by springs, whose states are $x_i = [\bar{x}_i, \dot{\bar{x}}_i]^T$ for $i = 1, 2, \dots, N$. Here, we consider a simple case where $N = 3$. The parameters of the pendulums are

$$\begin{aligned} A_1 = A_3 &= \begin{bmatrix} 0 & 1 \\ -3.75 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -2.5 & 0 \end{bmatrix} \\ B_1 = B_2 = B_3 &= \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} \end{aligned}$$

$$H_{12} = H_{21} = H_{23} = H_{32} = \begin{bmatrix} 0 & 0 \\ 1.25 & 0 \end{bmatrix}$$

The parameter of designed controllers are given by

$$\begin{aligned} K_1 = K_3 &= [-23 \ -12], \quad K_2 = [-18 \ -12] \\ L_{12} = L_{32} &= [-5 \ 0.25], \quad L_{21} = L_{23} = [-4.75 \ -0.25] \end{aligned}$$

With the solutions of Lyapunov function $\Phi_i^T P_i + P_i \Phi_i + Q_i = 0$ where $Q_i = I$ and $i = 1, 2, 3$, we obtain

$$\begin{aligned} A &= \begin{bmatrix} 0.67 & 0 & 0 \\ 0 & 0.45 & 0 \\ 0 & 0 & 0.67 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0.0608 & 0 \\ 0.1217 & 0 & 0.1217 \\ 0 & 0.0608 & 0 \end{bmatrix} \\ \Gamma &= \begin{bmatrix} 47.7983 & 24.4007 & 0 \\ 22.0276 & 33.2386 & 22.0276 \\ 0 & 24.4007 & 47.7983 \end{bmatrix} \end{aligned}$$

With A , B and Γ we obtain that $r(A^{-1}B) = 0.2216$, $\sigma_1 < 0.0646$, $\sigma_2 < 0.0844$ and $\sigma_3 < 0.0646$. We select $\sigma_1 = \sigma_2 = \sigma_3 = 0.01$. The Round-robin sampling interval is chosen as $\Delta = 0.001s$ according to Assumption 3. Follow the same procedures as in Example 1, we obtain $\frac{\omega_1}{\omega_1 + \omega_2} \approx 0.00012$, which is considerably conservative. In fact, if the systems are under the same DoS attacks as in Example 1, they are still stable, which can be seen from Figure 2. The conservativeness is due to the unstable dynamics of the inverted pendulums, the feedback gain K_i and the coupling parameter L_{ij} in the controllers. It is worth investigating how to design suitable K_i and L_{ij} to mitigate this effect (cf. Remark 3).

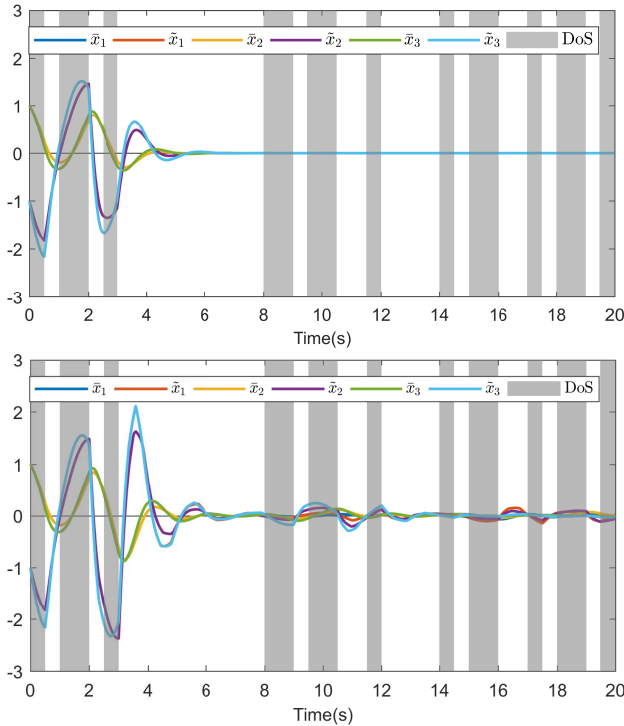


Fig. 2. Example 2: Top picture—States under Pure Round-robin communication during which there are 11997 transmissions; Bottom picture—States under hybrid communication strategy where there are 254 transmissions .

VI. CONCLUSIONS

In this work, we investigated the problem of stabilizing distributed systems under Denial-of-Service, characterizing DoS frequency and duration under which stability can be preserved. In order to save communication resources, we also consider a hybrid communication strategy. It turns out that the hybrid transmission strategy can reduce communication load effectively and prevent Zeno behavior while preserving the same robustness as pure Round-robin protocol.

An interesting research direction is the stabilization problem of networked distributed systems, where only a fraction of subsystems, possibly time-varying are under DoS. It is also interesting to investigate the problem where DoS attacks imposing on systems are asynchronous with different frequencies and durations. Finally, in the hybrid transmission

strategy, the effect of event-triggered control with communication collision can be an interesting direction from a practical viewpoint.

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