# Refactoring Neural Networks for Verification

David Shriver, Dong Xu, Sebastian Elbaum, Matthew B. Dwyer

Department of Computer Science, University of Virginia

{dlshriver, dx3yy, selbaum, matthewbdwyer}@virginia.edu

Abstract—Deep neural networks (DNN) are growing in capability and applicability. Their effectiveness has led to their use in safety critical and autonomous systems, yet there is a dearth of cost-effective methods available for reasoning about the behavior of a DNN.

In this paper, we seek to expand the applicability and scalability of existing DNN verification techniques through DNN refactoring. A DNN refactoring defines (a) the transformation of the DNN's architecture, i.e., the number and size of its layers, and (b) the distillation of the learned relationships between the input features and function outputs of the original to train the transformed network. Unlike with traditional code refactoring, DNN refactoring does not guarantee functional equivalence of the two networks, but rather it aims to preserve the accuracy of the original network while producing a simpler network that is amenable to more efficient property verification.

We present an automated framework for DNN refactoring, and demonstrate its potential effectiveness through three case studies on networks used in autonomous systems.

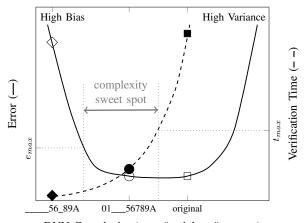
Index Terms—neural networks, verification, architecture, knowledge distillation

#### I. INTRODUCTION

Deep learning has emerged over the past decade as a means of synthesizing implementations of complex classification [1], clustering [2], and control functions [3], [4]. These implementations, in the form of Deep Neural Networks (DNN), offer the potential for new or enhanced system capability. For example, functions that compute control outputs from bit-mapped images have been synthesized that can outperform a trained human on a range of complex "game playing" tasks [3], [4] and those techniques have been adapted to replace heuristic and human control in autonomous systems [5], [6].

As these techniques mature they are being deployed in safety critical systems. However, there is a lack of cost-effective techniques for reasoning about the behavior of DNNs which, in turn, limits the development of safety arguments for such systems and is a cause for concern, e.g., automotive industry standards are under scrutiny for not addressing DNNs [7]. To address this need, researchers have been actively exploring algorithms for verifying that the behavior of a trained DNN meets explicitly stated correctness properties. *ReLuplex* [8] and *Planet* [9] were the first such tools, but more than 18 DNN verification algorithms [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25] have been developed in the past two years.

Despite this progress, the state-of-the-art in DNN verification is far from adequate for addressing the scale and complexity of DNNs used in autonomous systems. As we demonstrate in §IV, networks such as DAVE-2 [26] and DroNet [27],



DNN Complexity (e.g., #weights, #neurons)

Fig. 1: Conceptual diagram depicting (a) the bias-variance tradeoff in deep-learning and (b) the growth in verification time with model complexity.

which are used in autonomous driving and autonomous UAV navigation, are too complex for cost-effective verification.

Balancing Error and Verifiability: Fig. 1 illustrates, with the solid curve, a conceptual plot of the classic bias-variance tradeoff in training a machine learning model [28]. While there are many possible measures of complexity of a machine learning model, for training a DNN one can understand complexity in terms of the number of parameters, or weights, that are learned. This plot captures the intuition that: (a) low complexity networks have insufficient capacity to fit the training data (high-bias), and (b) high complexity networks risk overfitting the training data (high-variance). The plot also reflects that, in practice, DNNs tend not to overfit until their complexity becomes very large relative to the training data, e.g., [29], [30], [31]. This has the effect of extending the region of low error far to the right in terms of Fig. 1.

For developers who are focused exclusively on low error (high accuracy) this means that there is little disincentive to adding neurons, and thereby additional parameters, to the DNN in the hopes of achieving lower error. However, for developers who seek both low error and the ability to verify properties, increasing DNN complexity, e.g., by adding neurons, can substantially increase verification time. Fig. 1 illustrates, with the dashed curve, a conceptual plot of the growth in verification time with increasing DNN complexity – Fig. 5 in §IV reports data substantiating the growth in verification time with the number of neurons.

Consider a developer who needs a DNN with error less than

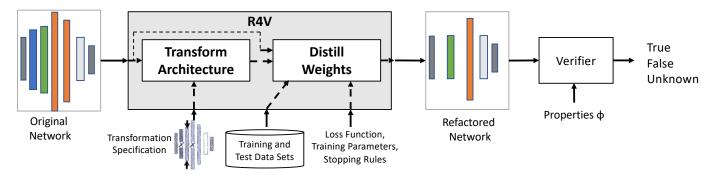


Fig. 2: The R4V Approach

 $e_{max}$  and who is willing to wait  $t_{max}$  for verification results. Conceptually, these constraints define a lower bound on DNN complexity, i.e., the complexity necessary to achieve lower error than  $e_{max}$ , and an upper bound on DNN complexity, i.e., the complexity beyond which verification time exceeds  $t_{max}$ . These bounds define a *complexity sweet spot* within which a DNN is both accurate and verifiable. In this paper, we propose automated support that developers can use to navigate error-verifiability tradeoffs resulting from DNN complexity to produce a network within this sweet spot.

**Refactoring for Verification**: To achieve this we adapt the concept of *refactoring* to deep neural networks. Code refactoring [32] typically seeks to (a) restructure a software system to facilitate subsequent development activities, while (b) preserving the behavior of the original software system. Fowler's original focus was on improving maintainability and extensibility, but researchers have adapted the concept in myriad ways. For example, to enhance the testability [33], [34] and verification [35] of software.

DNN refactoring for verification (R4V) applies and adapts the basic principles of code refactoring. As depicted in Fig. 2, given an original trained DNN, refactoring proceeds in two phases: architectural transformation and weight distillation.

The architecture of a new DNN is defined as a transformation of the original network's architecture based on a developer provided *transformation specification*. As described in §III, a specification designates layers to drop, layers to be downscaled to eliminate neurons, and layers to be transformed to meet verifier requirements. DNN transformation proceeds by propagating inter-layer constraints to produce a well-defined, but untrained, DNN that is less complex than the original and thereby facilitates verification.

This transformed DNN architecture is trained using knowledge distillation which seeks to train the transformed DNN to match the accuracy of the original, trained DNN [36]. Like traditional training, distillation requires the specification of training and test data sets and training parameters, such as the specific loss function and stopping rules.

Unlike code refactoring, where exact preservation of observable behavior is often a goal, the stochastic nature of DNN training makes this difficult. However, for many applications the behavior of a DNN is captured by its test error. Since the distillation process drives training of the transformed DNN to

minimize its difference with the original, it preserves behavior by trying to match the error of the networks.

**R4V in Action**: Fig. 1 illustrates how DNN refactoring can help the developers of the DAVE-2 network verify properties and achieve desired error using a binary-search on the complexity of refactored DNNs. We illustrate this by referencing data on refactored versions of DAVE-2 reported in Fig. 5.

The original DAVE-2 network computes steering angles from images using a 13 layer DNN comprised of 82669 neurons. Unfortunately, on average it takes 20,000 seconds to run the state-of-the-art *Neurify* [19] verifier to check a single property and even then the verifier returns an *unknown* result – it is unable to prove or disprove property conformance. Fig 1 depicts the error of the network and its verification time with an unfilled square and a filled square, respectively. The high-cost of verifying this network means it is too complex and we should refactor it to reduce its complexity.

An extreme aggresive refactoring might drop 6 of the layers resulting in a very small DNN with only 161 neurons, denoted \_\_\_\_\_56\_89A^1. This refactored DNN is easy to verify, taking only 30 seconds on average, which we depict as a filled diamond. Unfortunately, this network lacks sufficient complexity to match the training data. Its error, which we denote with a diamond, has an MSE of 0.062 relative to the original network – this network has not preserved the original networks behavior. Clearly refactoring has swung too far towards low DNN complexity.

Directing refactoring back towards greater complexity splits the difference between the first two networks by dropping only fewer layers. The resulting 8 layer DNN,  $01\__56789$ A, has 74045 neurons. On average, properties of this network can be verified in 5000 seconds and it achieves a relative Mean Squared Error (MSE) of 0.008. If  $t_{max}$  and  $e_{max}$  are 3 hours and a relative MSE of 0.01 then, as depicted by the filled and unfilled circles, this refactored network would lie within the complexity sweet spot. The ability to cost-effectively verify DNN  $01\__56789$ A, allows developers to determine whether it meets critical properties and, if so, to deploy since it has acceptably low error.

The contributions of this work lie in (1) identifying the

<sup>&</sup>lt;sup>1</sup>This DNN naming scheme uses "\_" to denote a dropped layer – the original network would be denoted 0123456789A. Since DNN refactorings always preserve the input and output layers we do not include them in the name.

notion of a complexity sweet spot that allows developers to balance DNN error and verifiability; (2) developing automated DNN refactoring techniques that can be used to side-step the limitations of existing DNN verifiers and adjust DNN complexity to meet desired error and verification constraints; and (3) demonstrating the potential of this approach on a series of case studies using two substantial DNNs deployed in autonomous systems and four state-of-the-art DNN verifiers.

#### II. BACKGROUND

In this section, we give background on DNNs, DNN verification, and DNN distillation as a prelude to describing R4V.

#### A. Deep Neural Networks

A deep neural network  $\mathcal{N}$  encodes an approximation of a target function  $f: \mathbb{R}^n \to \mathbb{R}^m$ . DNNs are comprised of a sequence of layers,  $l_0, l_1, \ldots, l_k$ , where  $l_0$  has an input domain of  $\mathbb{R}^n$ ,  $l_k$  has an output domain of  $\mathbb{R}^m$ , and the output domain of  $l_i$  is the same as the input domain of  $l_{i+1}$ . Each layer defines an independent function and the overall network's function approximation is their composition  $l_k \circ l_{k-1} \ldots \circ l_0$ . As discussed below, layers may vary in their internal structure, but all are comprised of a set of *neurons* that accumulate a weighted sum of other neurons and then apply an *activation function* (e.g., ReLU, Sigmoid) to the neuron output to introduce non-linearity.

The literature on machine learning has developed a broad range of *layer types* and explored the benefits of different layer combinations in realizing accurate approximations of different target functions, e.g., [37]. For instance, layer types have been defined that specialize in processing input image data, e.g., convolutional and max pooling, in providing flexible expressive power for function approximation, e.g., fully-connected, and in producing output classifications, e.g., softmax. A layer is defined by instantiating a layer type with parameter definitions. For instance, a convolutional layer type might define the size of its input, the number of output channels, its kernel size, and its stride.

Layers can also have a more complex structure, with branches of computation that are recombined to produce the layer output. For example, residual blocks, e.g., [38], have 2 computation paths. One path applies a set of computations, such as convolution and batch normalization, while the other path is an identity function, mapping the input to the final recombination operation. The two paths are combined by adding their outputs. In this work we represent these complex blocks as block(C,S), where C is the recombination operation (e.g. Add) and S is the set of computation paths. Residual blocks can be represented using this notation as  $block(Add, \{I, f\})$ , where I is the identity operation, and f is the computation to apply to the computation path.

The definition of a set of layers and their interconnections defines the *architecture* of the network. An architecture is *trained* using a population of labeled training data, D, consisting of training instances,  $(\mathbf{x}, y)$ . Training instances are evaluated,  $\mathcal{N}(\mathbf{x})$ , and the *loss* relative to the expected

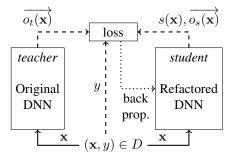


Fig. 3: DNN Distillation

output, y, is used to drive a gradient-descent process, called backpropagation, which updates the network weights based on their degree of contribution to the loss. The term training parameters refers collectively to the set of choices made in instantiating this training process, including: how the  $(\mathbf{x}, y)$  are sampled from D, the loss function (e.g., mean squared error, cross entropy), and when to stop training.

The *accuracy* of the network estimates the degree to which  $\mathcal{N}(\mathbf{x}) \approx f(\mathbf{x})$  by computing the percentage of labels that are matched by  $\mathcal{N}$  on a set of labeled instances that are disjoint from D. It can be measured to determine the progress of training, e.g., validation accuracy, or to determine generalization of the network after training is completed, e.g., test accuracy. The *error* of a network is 1 minus its accuracy.

#### B. DNN Verification

Given a network  $\mathcal{N}$  and a correctness property,  $\phi$ , that defines a set of constraints over the inputs  $\phi_x$ , and a set of constraints over the outputs  $\phi_y$ , verification of  $\mathcal{N}$  seeks to prove:  $\forall x \in \mathbb{R}^n \ \phi_x(x) \Rightarrow \phi_y(\mathcal{N}(x))$ .

A widely studied class of properties express the *local* robustness of a DNN. Such properties state that inputs within an  $\epsilon$ -ball centered at  $\mathbf{x}$  should be mapped to the same value,  $\forall \delta \in [0, \epsilon]^n \ \mathcal{N}(\mathbf{x}) = \mathcal{N}(\mathbf{x} \pm \delta)$ , where n is the dimension of  $\mathbf{x}$ . For example, a network that classifies road signs should still classify an image of a stop sign correctly, even after a small amount of noise is applied to the image.

A recent survey [39] describes a variety of algorithmic approaches for verifiying such properties. Reachability methods that calculate approximations of the sets of values that can be computed as output of the network given an input constraint include: MaxSens [10], Ai2 [11], DeepGo [12], and ERAN [13], [40], [41]. Methods that formulate property violations as a threshold for an objective function and use optimization algorithms to determine failure to meet that threshold include: MIPVerify [14], ILP [15], Duality [16], ConvDual [17], and Certify [18]. Search methods that explore regions of the input space where they then formulate reachability or optimization sub-problems include: Neurify [19], ReluVal [20], Fast-lin& Fast-lip [21], DLV [22], CROWN [42], CNN-CERT [23], Sherlock [24], Bab [25], ReLuplex [8], and Planet [9]. In §IV we report on the use of four of these DNN verifiers in combination with R4V.

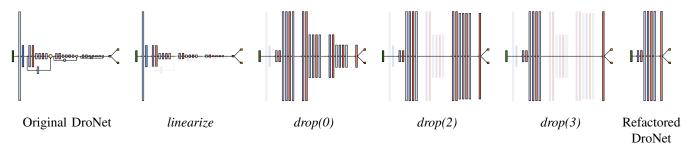


Fig. 4: Refactoring the DroNet DNN.

#### C. Network Distillation

Hinton et al. [36] introduced *knowledge distillation* as a general approach to addressing the differing requirements in training and deploying DNNs (e.g., less computation, lower energy consumption). Distillation has proven useful in addressing varied deployment requirements [43], [44], [45]. In R4V, we employ distillation to address the requirement that DNNs be verified prior to deployment in safety critical systems.

As depicted in Fig. 3, distillation trains a *student* DNN to match the accuracy of an trained *teacher* DNN. Training inputs,  $\mathbf{x}$ , are fed to each network. In traditional training, loss would be computed between the label y – a *hard* training target – and the output inferred by the network being trained,  $s(\mathbf{x})$ . Distillation can also include differences in *soft* training targets in computing loss. Soft targets are vectors of values computed in the output layers of the networks,  $o_t(\mathbf{x})$  and  $o_s(\mathbf{x})$ ; for classification networks these would be the normalized logits in the softmax layer. The computed loss drives backpropagation to update the student's weights, but the addition of soft targets can better help the student match the accuracy of the teacher.

# III. APPROACH

The goal of our approach, R4V, is to increase the applicability and scalability of verification through refactoring – transferring the knowledge of a complex DNN into a smaller simpler network that is more amenable to verification. In this section we discuss the components of R4V, as outlined in Fig. 2, describe its implementation, and present recommendations on how to systematize its application.

# A. Transforming DNN Architecture

Different strategies can be applied to reduce the complexity of a DNN. We define a simple set of transformations over DNN layers. These transformations permit dropping layers, scaling layers, and changing certain types of layers that are not supported by verification tools. We also define a utility operation to apply transformations to layers that satisfy a given predicate.

**Drop layers**: The drop layer operation (Eq. 1) takes in a set of layer indices L, and removes those layers from the DNN.

$$drop(L): \forall l \in L \ l' = \bot$$
 (1)

After dropping a layer, the output shape of the preceding layer is used to update successive layers, to ensure that the new architecture is valid. The drop operation can be applied to layers that perform some computation on their input. We prohibit its

application to the output layer to ensure that the transformed architecture produces outputs with the same dimensions as the original architecture. We also prohibit dropping reshaping layers, such as *Transpose* and *Flatten*, since they are required to ensure that the input to a layer are the correct shape.

**Scale layers**: The scale layer operation (Eq. 2) takes in a set of layer indices L, and a scale factor f, and scales the number of neurons in each specified layer by the factor f.

$$scale(L,f): \forall l \in L \ (l \neq \bot) \rightarrow (\#l' = \lfloor f \cdot \#l \rfloor)$$
 (2) where  $\#$  denotes the size of a layer. After scaling a layer, its new output shape is used to update the parameterization of successive layers, to ensure that the new architecture is valid. Our current framework supports the scale operation on the input layer, fully-connected layers, and convolutional layers. Fully-connected layers are scaled by changing the number of neurons in the layer. Convolutional layers are scaled by changing the number of kernels. The input layer is scaled by its height and width dimensions.

**Linearize layers**: In general, residual blocks have one path that performs a computation on the input, and a second path that performs the identity operation, or resizes the input to match the output shape of the computation path. The linearize operation removes the non-computation path from the residual block so that the layer involves a single path. It (Eq. 3) takes a set of indices L of residual layers, and removes the identity connection from each.

$$linearize(L): \forall l \in L \ l = block(Add, \{I, f\}) \rightarrow l' = f$$
 (3)

**Forall layers**: We also define a utility operation *forall* (Eq. 4), that applies a partially parameterized operation  $\lambda$  over all layers of a DNN  $\mathcal{N}$ , that satisfy predicate  $\varphi$ .

$$forall(\mathcal{N}, \varphi, \lambda) = \lambda(\{l \in \mathcal{N} \mid \varphi(l)\}) \tag{4}$$

Fig. 4 illustrates the individual transformations resulting from this composition of operators:

forall(forall(DroNet, isResidual, linearize),

$$isLayer(\{0,2,3\}), drop)$$

where *isResidual* and *isLayer* are predicates that determine whether a layer is residual and whether a layer's identifier is in a specified set, respectively. The colors in the figure depict different layer types and their height corresponds to the number of neurons. The transformation is non-trivial, because whenever a layer's shape is modified the adjacent layers have to be updated.

We have applied and investigated these operations individually and in combination over several networks, including the ones that we report on in §IV. We have considered extending the transformation framework with additional capabilities, for example, to *add* a layer. To date, however, we have focused on capabilities that reduce the complexity of refactored networks – adding layers works against that goal – and that are necessary to support existing DNN verifiers.

# B. Distilling Weights in the Transformed DNN

We employ knowledge distillation to refactor DNNs that retain accuracy relative to the original network. Since distillation is a flexible, highly parameterized, DNN cross-training framework, our approach identifies three degrees of freedom in adapting distillation for verification.

**Datasets.** These nominally include training, validation, and test sets. Usually, the sets used on the teacher will be used on the student as well. However, R4V supports the utilization of different sets that may be needed to, for example, accelerate distillation or performing additional checks on external representativeness.

**Training parameters.** As with other DNN training approaches, there is a large hyperparameter space for tuning performance. Our approach has been to adopt the training parameters of the original network as a starting point. For refactoring classification networks, we follow prior work beginning with Hinton et al. [36], and incorporate soft targets, but hard targets can be used to provide additional information for computing loss. For refactoring regression networks, we distill strictly using the outputs of the teacher as hard targets. **Stopping rules.** In addition to establishing a timeout for distillation, R4V can terminate distillation when the relative student-teacher error is below a given threshold. For example, in a regression setting, the MSE between the teacher and student outputs may be used to measure relative accuracy.

Once parameterized, R4V trains the student network automatically. The resulting network can be assessed for accuracy and subjected to property verification.

# C. Implementation

Our implementation of R4V consists of two main components: a transformation component, and a distillation component. Both components are parameterized using a single configuration file, specified using the TOML language [46], such as the one shown in Listing 1.

The transformation component is parameterized with a teacher architecture and a set of transformation strategies. The teacher model is specified using the Open Neural Network Exchange (ONNX) format [47], which can be produced from many of the most popular machine learning frameworks. The configuration file in Listing 1 will transform the teacher architecture (specified in lines 22-25) by dropping layers 2 and 3 (lines 26-27), linearizing all residual blocks in the network (lines 28-30), and scaling the first layer by a factor 0.5 (lines 31-33). After transforming the architecture, the R4V tool will automatically begin distillation.

The distillation component was implemented in Python, with PyTorch as the underlying machine learning framework [48] used for training. The default distillation settings

Listing 1: An example R4V configuration file.

```
[distillation]
maxmemorv="32G
threshold=0.01
type="regression"
[distillation.parameters]
epochs=100
optimizer="adam"
loss="MSE"
[distillation.data]
format="dronet"
batchsize=32
[distillation.data.transform]
grayscale=true
[distillation.data.train]
shuffle=true
student.path="artifacts/dronet.200/training"
teacher.path="artifacts/dronet.200/training"
[distillation.data.validation]
shuffle=false
student.path="artifacts/dronet.200/validation"
teacher.path="artifacts/dronet.200/validation"
[distillation.teacher]
model="networks/dronet/model.onnx"
input_shape=[1, 200, 200, 1]
input_format="NHWC"
[[distillation.strategies.drop_layer]]
layer id=[2, 3]
[[distillation.strategies.forall]]
layer_type="ResidualConnection"
strategy="linearize"
[[distillation.strategies.scale_layer]]
layer_id=[0]
```

are based on the knowledge distillation method of Hinton et al. [36], and can be configured through the configuration file provided by the user. R4V allows users to choose the training algorithm and loss function (lines 7 and 8), as well as set hyper parameters such as the learning rate, and the batch size (line 11). We also extend distillation with a relative-performance-based stopping criteria (such as the error threshold specified in line 3), as well as a memory limit (line 2) and timeout. These permit developers to enforce a distillation budget to allow for fast partial distillation training to rapidly explore the space of refactorings; something we plan to explore in more depth in future work. Our distillation procedure also allows for different datasets to be specified for the student and the teacher (lines 14-21). This allows the student to be trained on smaller input sizes or on inputs that have been pre-processed differently than the teacher. For example, in the second case study below, we transform the DroNet network to operate on smaller input sizes, so we update lines 16 and 20 to point to a pre-processed dataset with the smaller image sizes. The distilled student model is saved in the ONNX format.

We also developed several tools for translating the ONNX models to the input formats required by the verifiers used in this work. §IV provides details on these translators.

We will make our implementation and study artifacts available before the work is published.

# D. Recommended Practices

Our experience to date with R4V has led to several broad, and likely evolving, recommended practices to explore the space of DNN refactorings possible with the approach.

First, the configuration language provides a simple and unified way to specify architectural transformations, and R4V removes potential errors associated with the transformation itself by automating this process. Still, when specifying architectural transformations, a developer may be challenged to identify the complexity sweet spot given the possible number of applicable transformations. We recommend to initially use a binary search to explore the complexity spectrum, starting with the original network and applying coarse *drop* transformations to generate networks with half the complexity and determine whether the accuracy and verifiability rendered are acceptable. Once the sweet spot is identified, further transformations can be applied to refine this initial approximation. For example, scale transformations can provide finer gradations in DNN complexity than using *drop* alone. We explore this recommendation further in §V, and certainly a key part of future work is to provide further support for this search process including making the binary search process multidimensional to account not just for the number of layers to drop but also for the layer type and its location in the DNN.

Second, the configuration language provides a means to specify how the distillation process should work, including training and validation data sets, loss functions, and training parameters. With regard to datasets for R4V, we advocate a strategy of exploiting, whenever available, datasets used for the original network. The intuition here is that the knowledge learned by the teacher, has been informed by repeated application of training data from those sets and distilling will effect the transfer of that well-informed knowledge to the student.

Third, the sweet spot is likely to vary across verifiers. As described in §II, many verifiers are emerging with different underlying approaches that may present performance and applicability tradeoffs, and it is unlikely that a single best verifier across the space of DNNs and properties will emerge. To address such richness, we recommend an R4V portfolio approach, where an ensemble of strategies can be employed to generate a portfolio of refactored networks, each of which can then be subjected to verification by a portfolio of verifiers. Selection of the best refactored DNN, e.g., the most accurate verifiable network, can be made from the portfolio results. Furthermore, such refactoring and verification portfolios would be comprised of independent tasks which can be easily parallelized – a practice employed in conducting the case studies discussed in §IV.

#### IV. STUDY

This section sets up the three case studies we conduct to explore the potential of R4V to support DNN verification.

The first case study focuses on Verifier Applicability. Verifiers are constantly playing catch-up to the latest architectural features introduced for DNNs. As a result, it is common to find that a favorite verifier does not support the features of a network of interest. For example, some verifiers, such as CROWN and ReluVal, are restricted to fully connected layers. Other verifiers, such as MIPVerify and DeepGo, add support for convolutional and maxpooling, but cannot handle more

TABLE I: Case Studies

Case Study	Artifacts	Verifier
Verifier Applicability	DAVE-2 DroNet	ReLuplex ERAN
Verification Efficiency	DAVE-2 DroNet	Neurify Planet
Error-Verifiability Trade-offs	DAVE-2	Neurify

complex layers such as residual blocks. In this case study, we showcase how R4V can be used to overcome this kind of limitation by refactoring the network to one that satisfies the verifiers feature constraints.

The second case study focuses on Verifier Efficiency. When faced with modern complex networks, even when verifiers can support all the features in a network, they often struggle to provide results in actionable time. In such cases R4V can be used to refactor the network into one that can be verified significantly more efficiently.

The third case study focuses on Error-Verifiability Tradeoffs. While the first two studies investigate verification applicability and efficiency enabled by R4V under a small number of refactorings, this study provides a much broader exploration of the refactoring space. It does so by performing a binary search for refactored versions of the original DNNs that lie in the *complexity sweet spot* where error and verification time meet acceptable thresholds, and then it follows up with a more extensive characterization providing concrete evidence of the tradeoffs illustrated earlier in the paper.

The studies are performed on two large DNN artifacts, four state-of-the-art verifiers, and multiple refactored DNNs in order to provide a broader characterization of the approach. Table I summarizes the case studies and how they pair networks and verifiers.

#### A. Artifacts

The study applies R4V to two large DNN artifacts used for autonomous vehicles, DAVE-2 and DroNet, both with more than 10 layers of different types, and an average of 250,000 neurons respectively. Prior DNN verification efforts have typically focused on relatively simple DNNs (e.g., ACAS has 8 layers and hundreds of neurons [8], MNIST has four layers and thousands of neurons [9]) or manually simplified versions of larger DNNs (e.g., a version of DAVE-2 with half of the layers [49]) to assess their performance. Our goal is to explore the performance of R4V under modern and challenging DNNs that push the state-of-the-art.

**DAVE-2** is trained to drive an autonomous ground vehicle. The DAVE-2 architecture [26] is made up of over 80000 neurons spread over 13 layers, including five convolutional layers and four fully connected layers, and was designed to provide steering control for an autonomous car. Because the original trained DAVE-2 model is not publicly available, in this study we use the model trained by other researchers [50] with the Udacity self-driving car dataset [51]. The dataset is comprised of 101,396 training images (of which we use 90% for training, and 10% for validation) and 15837 test

images. Images are taken from three cameras (center, left, right) looking out the windshield of a car. For each image, there is also a corresponding steering angle, indicating the position of the steering wheel at the time the image was taken. The DAVE-2 model that we use was trained by Pei et al. [50] on color images that were scaled down to a height and width of 100 by 100. We use the same input pre-processing when refactoring the network. The resulting network has a MSE of 0.008.

Following the process of *Neurify* [49], we randomly generated 10 safety properties for the DAVE-2 networks. The properties specify that images with a Chebyshev distance of at most 2 from a given test image must produce an output steering angle within a target range. Different from *Neurify* [49], in order to make the properties independent of an individual network, we use the test data label as the target steering angle. Another difference is that we restricted the output to a tighter range (from  $\pm 30$  to  $\pm 10$  degrees) to make it more realistic for the problem domain.

**DroNet** is trained to provide control signals for an autonomous drone [27]. DroNet is based on the ResNet architecture, and is made up of a convolutional and maxpooling layer, followed by 3 residual blocks, and a fully connected layer, totalling over 475,000 neurons. It consumes a 200x200 grayscale image and outputs a steering angle, as well as a probability that the drone is about to collide with an obstacle. The model is trained on a modified version of the Udacity self-driving car dataset. The dataset only uses the images from the center camera, and their corresponding steering angles from the Udacity dataset. It augments these examples with a set of images taken from a camera mounted to a bicycle as it is ridden on the streets of a city. These images each have a label indicating a corresponding probability of collision. The resulting network has a MSE of 0.021 on steering angle and an accuracy of 0.983 on collision avoidance.

We randomly generated 20 safety properties for the DroNet networks, 10 for the steering angle regression task, and 10 for the collision probability classification task. Similarly to the properties for DAVE-2, the DroNet steering properties specify that images within a Chebyshev distance of 2 of a given test image must produce a steering angle within 10 degrees of the steering angle of the original. The collision probability properties specify that images within a Chebyshev distance of 2 of a given test image must produce a collision classification that is the same as the target classification.

#### B. Verifiers

Given our studies' objectives, we selected the following verifiers ReLuplex, ERAN, Neurify, and Planet. ReLuplex and Planet are the most studied DNN verifiers, while ERAN and Neurify represent the state-of-the-art. All these verifiers also have freely available open source implementations. The ReLuplex verifier is based on an adaptation of the simplex method that provides support for DNN architectures with fully-connected layers with ReLU activation functions. The

ERAN verifier uses abstract interpretation to overapproximate the reachable output region. In this study we use ERAN with the DeepPoly abstract domain, which can handle convolutional, fully-connected, and maxpooling layers with ReLU, tanh, or sigmoid activation functions [40]. The Neurify verifier combines symbolic interval analysis with symbolic linear relaxation, and supports convolutional and fully-connected layers with ReLU activations. The Planet verifier blends techniques from SMT and search, and supports layers that can be specified as a linear combination of neurons, as well as ReLU activations and maxpooling layers.

For each verifier, we wrote a translation tool to convert networks in the ONNX format (the output format of R4V) to the specific input format required by each verifier.

Both ReLuplex and Planet take in a single file that combines a description of the network and the property specification. In the 'nnet' file format of ReLuplex, the property is encoded as a set of layers on the end of the network. The types of properties supported by this format are ones in which the input constraints are intervals, and the output constraint is a linear inequality over the output neurons. In the 'rlv' format used by Planet, layers of a network are specified as linear combinations of neurons, and the property is specified as a list of linear assertions, with each input required to have a lower and upper bound. We wrote translation tools to combine an ONNX network and property into the corresponding file type. For ReLuplex, we support translating networks comprised only of fully connected layers. For Planet, we support translating any networks comprised of fully connected and convolutional layers with ReLU activations, as well as maxpooling, batch normalization, and residual blocks.

Rather than combining the network and property into a single file, Neurify and ERAN take in both a network specification and several parameters to specify the property to be checked, such as the input and output bounds. The extended 'nnet' format used by Neurify accepts networks composed of convolutional and fully connected layers that must have ReLU activations. The ERAN verifier accepts networks specified as a subset of the operations in the TensorFlow machine learning framework. We built translation tools to convert the ONNX models output by R4V to the network specification format accepted by each verifier. For Neurify, we support translating networks comprised of convolutional and fully connected layers. For ERAN, we support fully connected and convolutional layers with ReLU or sigmoid activations, as well as batch normalization layers, by converting them to fully connected layers.

For some of the verifiers we had to make small modifications to get the tool to work on the artifacts used in this study. For *ReLuplex*, we use the version of the tool modified by [25] to support generic properties. This version of the tool works by checking whether the output neuron is less than 0. Properties can be specified by encoding them as a set of layers at the end of the network being verified. Because the *Neurify* tool is hard coded to check a constant set of properties, we modified the version of the tool built to operate on DAVE-2 for use in

our study. We made the tool more general by allowing a user specified input and output interval to be specified at runtime. Finally, we modified *ERAN* to support regression networks. The original tool was designed to verify robustness properties for classification networks. We made modifications to the tool to return the lower and upper bounds that it had computed for the outputs. We then checked whether these bounds were within the bounds specified by the property.

#### C. Methodology

For each case study, we use R4V to refactor the original DNN. The transformations in each scenario are meant to demonstrate how R4V can be applied to increase verifiability. Distillation parameters are set to correspond to the training parameters reported for the original DAVE-2 and DroNet, including the number of epochs we used (50 for DAVE-2 and 100 for DroNet), the optimization method (Adadelta for DAVE-2 and Adam for DroNet), and the batch size (256 for DAVE-2 and 32 for DroNet). After training, we select the model from the epoch with the best mean squared error (MSE) performance (relative to the teacher), evaluated on the validation set. For DAVE-2, the performance is measured as the MSE with respect to the target steering angle. Because DroNet has 2 outputs, we report the performance as 2 values: the MSE with respect to the target steering angle, and the accuracy with respect to the probability of collision.

For each scenario in each case study, we use the corresponding verifier to check the safety properties described above on the corresponding refactored network. We measure the verification time and report the verification results (true, false, unknown, "out of resources" (OOR)) as well as the mean and standard deviation of the verification time.

Distillation took around 11 hours on average to complete all the epochs. Note that we made no effort to optimize this time, such as using early stopping or transfer learning. As an example of the potential for optimization, the average best epoch for the accuracy-verifiability trade-off study below was 37 (out of 50 total). This suggests that simply using early stopping would have reduced distillation time by 20%. More aggressive training optimizations, such as those used in neural architecture search [52], apply a strict training budget, e.g., 5% of the epochs, when making preliminary assessments about the suitability of an architecture and then only using larger budgets for very promising architectures. This would work well for the approach described in Case Study III and would reduce the time for most distillation runs to a matter of minutes.

Distillation was run on GPU computing nodes with 64GB of memory, and using NVIDIA 1080Ti GPUs for network training. Verification tasks were run on compute nodes with 2.3GHz Xeon processors and 64GB of memory running CentOS Linux 7. Verification tasks had a time out of 24 hours.

# V. RESULTS

In this section, we present the results of each case study.

TABLE II: Results for Case Study I.

	Property Count ≃			Mean Verif. Time (sec.)			
Artifact/Verifier	T	F	U	00	T	F	U
DAVE-2/ReLuplex R4V(DAVE-2)/ReLuplex	DAV 10	E-2 0	Not (	Supp 0	orted by	ReLu	plex -
DroNet/ERAN R4V(DroNet)/ERAN	Dr 8	oNet	Not 0	Supp 0	orted b	y <i>ERA</i> 525	N -

# A. Verification Applicability

Artifacts and Verifiers. This case study pairs the DAVE-2 and DroNet artifacts with the *ReLuplex* and *ERAN* (using the DeepPoly abstract domain) verifiers. The pairing is designed to showcase the potential value of R4V to overcome a verifier's limited support for certain DNN features. The original DAVE-2 network has several convolutional layers, which are not supported by *ReLuplex*. The original DroNet network has complex residual blocks. *ERAN*, however, does not support residual blocks when using the DeepPoly domain (indeed this increasingly common layer type is only supported by a limited set of verifers, e.g., CNN-Cert).

Refactoring. Since ReLuplex only supports fully-connected layers, DAVE-2 must be refactored to remove all its convolutional layers. The refactoring uses the transformation  $drop(\{0,1,2,3,4\})$  to remove all unsupported layers, leaving only the final 5 fully-connected layers. To enable ERAN (DeepPoly) to run on DroNet, we first drop the convolutional and maxpooling layers at the beginning of DroNet, as well as the last 2 residual blocks through  $drop(\{0,2,3\})$ . We then linearize the remaining residual connection  $linearize(\{1\})$ . The distillation for both artifacts uses the parameters described in the previous section, resulting in a refactored DAVE-2 with an MSE of 0.062 relative to the original (other refactorings of DAVE-2 that render much smaller errors are presented in the last case study), and a refactored DroNet with a relative MSE on the steering angle of 0.020 and a relative accuracy on the collision avoidance of 0.979.

Findings. The results from this case study are shown in Table II. The first column of this table is the paired artifact and verifier, the next four columns are the number of properties that were checked that resulted in either proving or falsifying the property, returning unknown, or running out of resources. The next three columns show the mean verification time across all properties for each result type.

While *ReLuplex* can not be applied to the original DAVE-2 artifact, and *ERAN* (DeepPoly) could not be applied to the original DroNet artifact, both verifiers could be applied to the refactored networks. In the case of DAVE-2 and *ReLuplex*, all 10 properties where shown to hold in less than a minute per property. In the case of DroNet and *ERAN* (DeepPoly), the refactoring enabled the verification of all 20 properties, eight deemed true and 12 deemed false, taking on the order of 10 minutes to complete the verification of each property.

The most significant finding is that R4V enabled the application of previously inapplicable verifiers.

TABLE III: Results for Case Study II

	Property Count			Mean Verif. Time (sec.)			
Artifact/Verifier	Т	F	U	OOR	Т	F	U
DAVE-2/Neurify R4V(DAVE-2)/Neurify	0 10	0	10 0	0	2449	-	20318*
DroNet/Planet R4V(DroNet)/Planet	0	0 2	0	20 17	1155	45423	

## B. Verification Efficiency

Artifacts and Verifiers. This case study pairs DAVE-2 and DroNet with the *Neurify* and *Planet* verifiers. These verifiers can run these networks, but on the original architectures *Neurify* takes on average over 5.6 hours to check a single property while *Planet* cannot verify any DroNet property within 24 hours.

Refactoring. To improve the efficiency of *Neurify* on the DAVE-2 network, we apply R4V to refactor DAVE-2 by applying the transformation  $drop(\{2,3,4,7,9\})$ , which matches the one manually generated by [49]. To increase the efficiency of *Planet* on DroNet, we apply R4V to remove the first convolutional and maxpooling layers, as well as the last two residual blocks  $(drop(\{0,2,3\}))$ . We also scale the size of the input layer to a 10 by 10 grayscale image  $(scale(\{input\}, 0.05))$ . This last step is necessary to enable planet to verify any of the properties within the time limit.

Findings. The results from this case study are summarized in Table III. The first column of this table is the artifact and verifier pair, the next four columns are the number of properties that were checked that resulted in either proving or falsifying the property, or returning unknown or running out of resources. The next three columns show the mean verification time for each kind of returned result.

While *Neurify* could mostly handle the original DAVE-2 network, its results were flaky for some properties, alternating segmentation errors with verification completions. Hence, the reported numbers on the original DAVE-2 with *Neurify* correspond to the best of up to five tries. But even when *Neurify* completed its execution after an average of 339 minutes per property, the complexity of the original network introduced significant imprecision into the analysis, causing all 10 property checks to return an *unknown* result. In contrast, *Neurify* was able to return true for all properties of the refactored network, taking about 41 minutes on average to verify a property - an 8 times speedup.

Similarly, although *Planet* can accept the original DroNet architecture, it could not complete any of the property checks within one day. After applying R4V to refactor the network, *Planet* was able to finish checking three out of the 10 properties, taking just under 20 minutes to verify a property and a little over 16.5 hours to falsify a property.

The most significant finding is that R4V can speed up the verification time and enable verifiers to provide more useful results.

#### C. Accuracy and Verifiability Trade-offs

Artifacts and Verifiers. In this case study, we explore the error-verifiability trade-offs using the DAVE-2 architecture and the *Neurify* verifier. We generate multiple refactored versions of DAVE-2 to explore the trade-offs.

Refactoring. We start our exploration of the refactoring space using the coarser granularity transformation drop, and performing a binary search to determine the number of layers to drop based on the refactored network error and verification time. We arbitrary set the acceptable error in terms of the MSE relative to the original network to be under 0.01. In terms of verifiability, we require for all the properties to be verified as true or falsified in under three hours.

Findings. The binary search process we followed is summarized in Table IV. The first column describes the search action taken in terms of the refactoring operation. The second column shows the name of the network. In DAVE-2 the first five layers are convolutional layers, layers five and six are reshaping layers, which cannot be modified by R4V, and the last four layers are fully-connected. The name of each network includes all of the layers that are kept after transformation, and replaces the names of dropped layers with an underscore. The third column shows one measure of the network complexity in terms of the number of neurons. The fourth column shows the relative mean square error when compared with the original network. The fifth column provides a summary of the verification outcome (more details are provided next). The last column contains the assessment to guide the search process.

The process shows how a systematic binary search, starting from the original network and repeatedly bisecting the space towards the direction that seems to offer the best tradeoffs, renders a refactored network 0123456\_9A that performs according the target criteria. This network can verify all 10 properties in less than three hours per property and do so while retaining the behavior of the original network with a MSE under 0.01. The process also hints at how even when considering a single transformation, many options arise for exploration and refinement, including some options that we have illustrated and others that we have yet to explore such as the combination of *drop* with *scale* transformations.

To complement the data in Table IV and to provide a more extensive characterization of the space of tradeoffs involved we use R4V to refactor the original network into another 11 networks. The results are depicted in Fig. 5. Each row in this figure is a refactored network, and each column is a property to verify. The networks names and their number of neurons appears on the first and second column. The networks are sorted by the number of neurons, so networks towards the bottom of the plot are simpler. The size of the circle represents the time to verify a given network and property, and the color and pattern of the circle indicate the verification result. The relative MSE of each network appears on the right of the plot.

Starting from the top, we notice that the original DAVE-2 network takes over 10,000 seconds to verify every single property, and for the second property as much as 39306

TABLE IV: Case Study III: Searching for Viable Architectures with R4V.

Search Action	Network	#Neurons	Rel.MSE	Verification	Assessment	
Start with existing network	Original	82669	0.0	10/10 Unknowns Over 3 hours per property	Verification returns too many unknowns and takes too long X	
Drop all but one computation layer	56A	11	0.062	Verified 10/10 in seconds	Quickly verifiable but error is too large X	
Drop 4 layers (convolutional)	056789A	56621	0.008	Verified 10/10 in less than 3 hours	Verifiable but error is borderline ✓	
Drop 4 layers (fully-connected)	0123456	81345	0.008	Verifier cannot process	Incompatible with verifier X	
Drop 2 layers (convolutional)	01256789A	77933	0.005	Verified 6/10, 3 Unknowns, 1 TO	Partially verifiable	
Drop 2 layers (fully-connected)	01234569A	81405	0.003	Under 3 hours per property Verified 10/10 Just under 3 hours per property	and acceptable error ✗ Verifiable and acceptable error ✓	

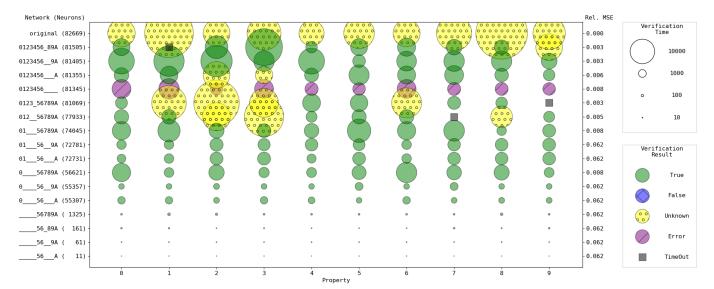


Fig. 5: Results for Case Study III

seconds, and in all cases it returns unknown. When R4V is applied to refactor DAVE-2 by removing the fully-connected layer labeled as '7' resulting in 0123456\_89A (second row in the graph), we observe a 2.1x speedup in verification time and also more useful returned results (eight properties were shown to be true, one unknown, and one timed-out). The third row shows network 0123456\_9A, which has another fully-connected layer removed, resulting in a speedup of 3.6x in verification time and all properties checked as true. The following two rows of networks keep dropping fully-connected layers, and when all fully-connected layers are dropped we notice that Neurify returns an error as it cannot deal with resulting architecture of 0123456 which is correct but has no fully connected layers, which was unexpected by Neurify. The following rows start the removal of convolutional layers, resulting in another noticeable reduction in network complexity. Network 01 56789A (highlighted in §I) renders a speedup of 6.3x in verification time while retaining a MSE of 0.008 relative to the original network. That is also the case 56789A, with 9.4x speedup and still an acceptable error rate. The rows below have too large of an error to be deemed viable.

In total, 3 of the 16 refactored DAVE-2 networks in Fig 5 meet the error-verifiability criteria, suggesting that the complexity sweet spot for DAVE-2 contains a variety of acceptable

DNNs.

The most significant finding of this study is that, without prior domain knowledge, R4V can enable the effective exploration of the error and verifiability tradeoffs and converge towards a network that meets both criteria.

# VI. CONCLUSION

In this work, we proposed automated support for refactoring DNNs to facilitate verification while retaining the behavior of the original network. R4V couples specification-driven architecture transformation support with knowledge distillation to produce refactored DNNs. We demonstrate, through a series of case studies, that R4V can help developers overcome the limitations of individual DNN verification tools, reduce the time to produce verification results, and, most importantly, to navigate the accuracy-verifiability tradeoff to arrive at a network that meets stated accuracy requirements and verification budgets. These findings suggest that R4V has the potential for improving the development of critical systems involving DNNs, since refactored DNNs can be subjected to property verification and have acceptable accuracy for deployment.

In future work, we plan to expand the range of refactorings, e.g., adding *add* and *replace* operators, extend existing operations, e.g., linearization of dense networks, and supporting additional optimizations, e.g., transferring weights from the

original network, using inexpensive training parameters during the search for a good refactoring. In addition, we plan to explore different strategies for automating the search for refactored DNNs that lie within the complexity sweet spot, which has the potential to make R4V broadly applicable by practitioners.

#### ACKNOWLEDGMENT

This material is based in part upon work supported by the National Science Foundation under Grant Number 1617916, 1900676, and 1901769, and the U. S. Army Research Laboratory and the U. S. Army Research Office under contract/grant number W911NF-19-1-0054-P00001.

#### REFERENCES

- A. Krizhevsky, I. Sutskever, and G. E. Hinton, "Imagenet classification with deep convolutional neural networks," in Advances in Neural Information Processing Systems 25, F. Pereira, C. J. C. Burges, L. Bottou, and K. Q. Weinberger, Eds. Curran Associates, Inc., 2012, pp. 1097–1105. [Online]. Available: http://papers.nips.cc/paper/ 4824-imagenet-classification-with-deep-convolutional-neural-networks. pdf
- [2] R. Duda, P. Hart, and D. Stork, Pattern classification, ser. Pattern Classification and Scene Analysis: Pattern Classification. Wiley, 2001. [Online]. Available: https://books.google.com/books?id=YoxQAAAAMAAJ
- [3] V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. Riedmiller, "Playing atari with deep reinforcement learning," in NIPS Deep Learning Workshop, 2013.
- [4] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. A. Riedmiller, A. Fidjeland, G. Ostrovski, S. Petersen, C. Beattie, A. Sadik, I. Antonoglou, H. King, D. Kumaran, D. Wierstra, S. Legg, and D. Hassabis, "Human-level control through deep reinforcement learning," *Nature*, vol. 518, no. 7540, pp. 529–533, 2015. [Online]. Available: https://doi.org/10.1038/nature14236
- [5] N. Imanberdiyev, C. Fu, E. Kayacan, and I. Chen, "Autonomous navigation of uav by using real-time model-based reinforcement learning," in 2016 14th International Conference on Control, Automation, Robotics and Vision (ICARCV), Nov 2016, pp. 1–6.
- [6] H. Bou-Ammar, H. Voos, and W. Ertel, "Controller design for quadrotor uavs using reinforcement learning," in 2010 IEEE International Conference on Control Applications, Sept 2010, pp. 2130–2135.
- [7] R. Salay, R. Queiroz, and K. Czarnecki, "An analysis of iso 26262: Machine learning and safety in automotive software," in SAE Technical Paper. SAE International, 04 2018. [Online]. Available: https://doi.org/10.4271/2018-01-1075
- [8] G. Katz, C. W. Barrett, D. L. Dill, K. Julian, and M. J. Kochenderfer, "Reluplex: An efficient SMT solver for verifying deep neural networks," in Computer Aided Verification 29th International Conference, CAV 2017, Heidelberg, Germany, July 24-28, 2017, Proceedings, Part 1, 2017, pp. 97–117. [Online]. Available: https://doi.org/10.1007/978-3-319-63387-9\_5
- [9] R. Ehlers, "Formal verification of piece-wise linear feed-forward neural networks," in *Automated Technology for Verification and Analysis* - 15th International Symposium, ATVA 2017, Pune, India, October 3-6, 2017, Proceedings, 2017, pp. 269–286. [Online]. Available: https://doi.org/10.1007/978-3-319-68167-2\_19
- [10] W. Xiang, H. Tran, and T. T. Johnson, "Output reachable set estimation and verification for multilayer neural networks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 11, pp. 5777–5783, Nov 2018.
- [11] T. Gehr, M. Mirman, D. Drachsler-Cohen, P. Tsankov, S. Chaudhuri, and M. Vechev, "Ai2: Safety and robustness certification of neural networks with abstract interpretation," in 2018 IEEE Symposium on Security and Privacy (SP), May 2018, pp. 3–18.
- [12] W. Ruan, X. Huang, and M. Kwiatkowska, "Reachability analysis of deep neural networks with provable guarantees," in *IJCAI*. ijcai.org, 2018, pp. 2651–2659.

- [13] G. Singh, T. Gehr, M. Mirman, M. Püschel, and M. Vechev, "Fast and effective robustness certification," in *Advances in Neural Information Processing Systems 31*, S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, Eds. Curran Associates, Inc., 2018, pp. 10802–10813. [Online]. Available: http://papers.nips.cc/paper/8278-fast-and-effective-robustness-certification.pdf
- [14] V. Tjeng, K. Y. Xiao, and R. Tedrake, "Evaluating robustness of neural networks with mixed integer programming," in *International Conference on Learning Representations*, 2019. [Online]. Available: https://openreview.net/forum?id=HyGIdiRqtm
- [15] O. Bastani, Y. Ioannou, L. Lampropoulos, D. Vytiniotis, A. V. Nori, and A. Criminisi, "Measuring neural net robustness with constraints," in *Proceedings of the 30th International Conference on Neural Information Processing Systems*, ser. NIPS'16. USA: Curran Associates Inc., 2016, pp. 2621–2629. [Online]. Available: http://dl.acm.org/citation.cfm?id=3157382.3157391
- [16] K. Dvijotham, R. Stanforth, S. Gowal, T. Mann, and P. Kohli, "A dual approach to scalable verification of deep networks," in *Proceedings* of the Thirty-Fourth Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-18). Corvallis, Oregon: AUAI Press, 2018, pp. 162–171.
- [17] E. Wong and J. Z. Kolter, "Provable defenses against adversarial examples via the convex outer adversarial polytope," in *ICML*, ser. Proceedings of Machine Learning Research, vol. 80. PMLR, 2018, pp. 5283–5292.
- [18] A. Raghunathan, J. Steinhardt, and P. Liang, "Certified defenses against adversarial examples," in *ICLR*. OpenReview.net, 2018.
- [19] S. Wang, K. Pei, J. Whitehouse, J. Yang, and S. Jana, "Efficient formal safety analysis of neural networks," in *NeurIPS*, 2018, pp. 6369–6379.
- [20] ——, "Formal security analysis of neural networks using symbolic intervals," in *USENIX Security Symposium*. USENIX Association, 2018, pp. 1599–1614.
- [21] T. Weng, H. Zhang, H. Chen, Z. Song, C. Hsieh, L. Daniel, D. S. Boning, and I. S. Dhillon, "Towards fast computation of certified robustness for relu networks," in *ICML*, ser. Proceedings of Machine Learning Research, vol. 80. PMLR, 2018, pp. 5273–5282.
- [22] X. Huang, M. Kwiatkowska, S. Wang, and M. Wu, "Safety verification of deep neural networks," in CAV (1), ser. Lecture Notes in Computer Science, vol. 10426. Springer, 2017, pp. 3–29.
- [23] A. Boopathy, T.-W. Weng, P.-Y. Chen, S. Liu, and L. Daniel, "Cnn-cert: An efficient framework for certifying robustness of convolutional neural networks," in AAAI, Jan 2019.
- [24] S. Dutta, S. Jha, S. Sankaranarayanan, and A. Tiwari, "Output range analysis for deep feedforward neural networks," in NFM, ser. Lecture Notes in Computer Science, vol. 10811. Springer, 2018, pp. 121–138.
- [25] R. R. Bunel, I. Turkaslan, P. H. S. Torr, P. Kohli, and P. K. Mudigonda, "A unified view of piecewise linear neural network verification," in *NeurIPS*, 2018, pp. 4795–4804.
- [26] M. Bojarski, D. D. Testa, D. Dworakowski, B. Firner, B. Flepp, P. Goyal, L. D. Jackel, M. Monfort, U. Muller, J. Zhang, X. Zhang, J. Zhao, and K. Zieba, "End to end learning for self-driving cars," *CoRR*, vol. abs/1604.07316, 2016. [Online]. Available: http://arxiv.org/abs/1604.07316
- [27] A. Loquercio, A. I. Maqueda, C. R. D. Blanco, and D. Scaramuzza, "Dronet: Learning to fly by driving," *IEEE Robotics and Automation Letters*, 2018.
- [28] S. Geman, E. Bienenstock, and R. Doursat, "Neural networks and the bias/variance dilemma," *Neural Computation*, vol. 4, no. 1, pp. 1–58, 1992
- [29] W. Zhou, V. Veitch, M. Austern, R. P. Adams, and P. Orbanz, "Non-vacuous generalization bounds at the imagenet scale: A pac-bayesian compression approach," in *Proceedings of the International Conference on Learning Representations (ICLR)*, 2019, to appear.
- [30] B. Neyshabur, Z. Li, S. Bhojanapalli, Y. LeCun, and N. Srebro, "Towards understanding the role of over-parametrization in generalization of neural networks," in *Proceedings of the International Conference on Learning Representations (ICLR)*, 2019, to appear.
- [31] Y. Cheng, D. Wang, P. Zhou, and T. Zhang, "Model compression and acceleration for deep neural networks: The principles, progress, and challenges," *IEEE Signal Processing Magazine*, vol. 35, no. 1, pp. 126– 136, 2018.
- [32] M. Fowler, Refactoring: improving the design of existing code. Addison-Wesley Professional, 2018.

- [33] M. Ó. Cinnéide, D. Boyle, and I. H. Moghadam, "Automated refactoring for testability," in 2011 IEEE Fourth International Conference on Software Testing, Verification and Validation Workshops. IEEE, 2011, pp. 437–443.
- [34] M. Harman, "Refactoring as testability transformation," in 2011 IEEE Fourth International Conference on Software Testing, Verification and Validation Workshops. IEEE, 2011, pp. 414–421.
- [35] X. Yin, J. Knight, and W. Weimer, "Exploiting refactoring in formal verification," in 2009 IEEE/IFIP International Conference on Dependable Systems & Networks. IEEE, 2009, pp. 53–62.
- [36] G. Hinton, O. Vinyals, and J. Dean, "Distilling the knowledge in a neural network," in NIPS Deep Learning and Representation Learning Workshop, 2015. [Online]. Available: http://arxiv.org/abs/1503.02531
- [37] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*. MIT Press, 2016, http://www.deeplearningbook.org.
- [38] K. He, X. Zhang, S. Ren, and J. Sun, "Deep residual learning for image recognition," in 2016 IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2016, Las Vegas, NV, USA, June 27-30, 2016, 2016, pp. 770–778.
- [39] C. Liu, T. Arnon, C. Lazarus, C. Barrett, and M. J. Kochenderfer, "Algorithms for verifying deep neural networks," *CoRR*, vol. abs/1903.06758, 2019.
- [40] G. Singh, T. Gehr, M. Püschel, and M. T. Vechev, "An abstract domain for certifying neural networks," *PACMPL*, vol. 3, no. POPL, pp. 41:1– 41:30, 2019
- [41] G. Singh, T. Gehr, M. Püschel, and M. Vechev, "Boosting robustness certification of neural networks," in *Proceedings of the International Conference on Learning Representations (ICLR)*, 2019.
- [42] H. Zhang, T.-W. Weng, P.-Y. Chen, C.-J. Hsieh, and L. Daniel, "Efficient neural network robustness certification with general activation functions," in *Advances in Neural Information Processing Systems* (NuerIPS), dec 2018.
- [43] A. Romero, N. Ballas, S. E. Kahou, A. Chassang, C. Gatta, and Y. Bengio, "Fitnets: Hints for thin deep nets," in *Proceedings of the International Conference on Learning Representations (ICLR)*, 2015.
- [44] G. Chen, W. Choi, X. Yu, T. Han, and M. Chandraker, "Learning efficient object detection models with knowledge distillation," in *Advances in Neural Information Processing Systems 30*, I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, Eds. Curran Associates, Inc., 2017, pp. 742–751.
- [45] A. G. Howard, M. Zhu, B. Chen, D. Kalenichenko, W. Wang, T. Weyand, M. Andreetto, and H. Adam, "Mobilenets: Efficient convolutional neural networks for mobile vision applications," *CoRR*, vol. abs/1704.04861, 2017. [Online]. Available: http://arxiv.org/abs/1704.04861
- [46] T. Preston-Werner, "TOML," https://github.com/toml-lang/toml, 2013.
- [47] ONNX, "Open Neural Network Exchange," https://github.com/onnx/ onnx, 2017.
- [48] A. Paszke, S. Gross, S. Chintala, G. Chanan, E. Yang, Z. DeVito, Z. Lin, A. Desmaison, L. Antiga, and A. Lerer, "Automatic differentiation in pytorch," in NIPS-W, 2017.
- [49] S. Wang, K. Pei, J. Whitehouse, J. Yang, and S. Jana, "Efficient formal safety analysis of neural networks," in *Advances in Neural Information Processing Systems 31*, S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, Eds. Curran Associates, Inc., 2018, pp. 6367–6377. [Online]. Available: http://papers.nips.cc/paper/7873-efficient-formal-safety-analysis-of-neural-networks.pdf
- [50] K. Pei, Y. Cao, J. Yang, and S. Jana, "Deepxplore: Automated whitebox testing of deep learning systems," in *Proceedings of* the 26th Symposium on Operating Systems Principles, Shanghai, China, October 28-31, 2017, 2017, pp. 1–18. [Online]. Available: https://doi.org/10.1145/3132747.3132785
- [51] Udacity, "Self driving car," https://github.com/udacity/self-driving-car, 2016.
- [52] H. Pham, M. Y. Guan, B. Zoph, Q. V. Le, and J. Dean, "Efficient neural architecture search via parameter sharing," in *Proceedings of* the 35th International Conference on Machine Learning, ICML 2018, Stockholmsmässan, Stockholm, Sweden, July 10-15, 2018, 2018, pp. 4092–4101.