

A Local Density-Based Approach for Local Outlier Detection

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Abstract

This paper presents a simple but effective density-based outlier detection approach with the local kernel density estimation (KDE). A *Relative Density-based Outlier Score* (RDOS) is introduced to measure the local outlierness of objects, in which the density distribution at the location of an object is estimated with a local KDE method based on extended nearest neighbors of the object. Instead of using only k nearest neighbors, we further consider reverse nearest neighbors and shared nearest neighbors of an object for density distribution estimation. Some theoretical properties of the proposed RDOS including its expected value and false alarm probability are derived. A comprehensive experimental study on both synthetic and real-life data sets demonstrates that our approach is more effective than state-of-the-art outlier detection methods.

1. Introduction

Advances in data acquisition have created massive collections of data, capturing valuable information to science, government, business, and society. However, despite of the availability of large amount of data, some events are rare or exceptional, which are usually called “outliers” or “anomalies”. Compared with many other knowledge discovery problems, outlier detection is sometimes more valuable in many applications, such as network intrusion detection, fraudulent transactions, and medical diagnostics. For example, in network intrusion detection, the number of intrusions or attacks (“bad” connections) is much less than the “good” and normal connections. Similarly, the abnormal behaviors are usually rare in many other cases. Although these

outliers are only a small portion of the whole data set, it is much more costly to misunderstand them compared with other events.

In recent decades, many outlier detection approaches have been proposed. Usually an outlier detection method can be categorized into the following four types of method [1][2]: distribution-based, distance-based, clustering-based, and density-based. In distribution-based methods, an object is considered as the outlier if it deviates from a standard distribution (e.g., normal, Poisson, etc.) too much [3]. The problem of the distribution-based method is that the underlying distribution is usually unknown and does not follow a standard distribution for many practical applications.

The distance-based methods detect outliers by computing distances among all objects. An object is considered as the outlier when it has d_0 distance away from p_0 percentage of objects in the data set [4]. In [5], the distance among objects is calculated in feature subspace through projections for high dimensional data sets. The problem of these methods is that the local outliers are usually misdetected for the data set with multiple components or clusters. To detect the local outliers, a top- n k -th nearest neighbor distance is proposed in [6], in which the distance from an object to its k -th nearest neighbor indicates outlierness of the object. The cluster-based methods detect the outlier in the process of finding clusters. The object does not belong any cluster is considered as the outlier [7][8][9].

In density-based methods, an outlier is detected when its local density differs from its neighborhood. Different density estimation methods can be applied to measure the density. In Local Outlier Factor (LOF) [10], an outlierness score, indicating how an object differs from its locally reachable neighborhood, is measured. Previous studies [11][12] have shown that it is more reliable to consider the objects with the highest LOF scores as outliers, instead of comparing the LOF score with a threshold. Several variations of the LOF are also proposed [12][13]. In [12], a Local Distance-based Outlier Factor (LDOF) using the relative distance from an object to its neighbors is proposed for outlier detection in scattered datasets. In [13], a INFLUenced Outlierness (INFLO) score is measured by considering both neighbors and reverse neighbors of an object when estimating its relative density distribution [13]. To address the issue that the LOF method and its variants do not consider the underlying pattern of data, Tang et. al. proposed a connectivity-based outlier factor (COF) scheme in [14]. While the LOF-based and COF-based outlier detection methods use the relative distance to estimate the density, several other density-based methods are proposed

based on kernel density estimation [15][16][17]. For example, Local Density Factor (LDF) [15] extends the LOF by using kernel density estimation. In [17], similar to the LOCI, a relative density score termed KDEOS is calculated using kernel density estimation and applies the z -score transformation for score normalization.

In this paper, we propose an outlier detection method based on the local kernel density estimation for robust local outlier detection. Instead of using the whole data set, the density of an object is estimated with the objects in its neighborhood. Three kinds of neighbors: k nearest neighbors, reverse nearest neighbors, and shared nearest neighbors, are considered in our local kernel density estimation. A simple but efficient relative density calculation, termed Relative Density-based Outlier Score (RDOS), is introduced to measure the outlierness. Theoretical properties of the RDOS, including the expected value and the false alarm probability are derived, which suggests parameter settings in practical applications. We further employ the top- n scheme to rank the objects with their outlierness, i.e., the objects with the highest RDOS values are considered as the outliers. Simulation results on both synthetic data sets and real-life data sets illustrate superior performance of our proposed method.

The paper is organized as follows: In Section 2, we introduce the definition of the RDOS and present the detailed descriptions of our proposed outlier detection approach. In Section 3, we derive theoretical properties of the RDOS and discuss the parameter settings. In Section 4, we present experimental results and analysis, which show superior performance compared with previous approaches. Finally, conclusions are given in Section 5.

2. Proposed Outlier Detection

2.1. Local Kernel Density Estimation

We use the KDE method to estimate the density at the location of an object based on the given data set. Given a set of objects $\mathcal{X} = \{X_1, X_2, \dots, X_m\}$, where $X_i \in \mathbb{R}^d$ for $i = 1, 2, \dots, m$, the KDE method estimates the distribution as follows:

$$p(X) = \frac{1}{m} \sum_{i=1}^m \frac{1}{h^d} K\left(\frac{X - X_i}{h}\right) \quad (1)$$

where $K\left(\frac{X-X_i}{h}\right)$ is the defined kernel function with the kernel width of h , which satisfies the following conditions:

$$\int K(u)du = 1, \int uK(u)du = 0, \text{ and } \int u^2K(u)du > 0 \quad (2)$$

A commonly used multivariate Gaussian kernel function is given by

$$K\left(\frac{X-X_i}{h}\right)_{\text{Gaussian}} = \frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{\|X-X_i\|^2}{2h}\right) \quad (3)$$

where $\|X-X_i\|$ denotes the Euclidean distance between X and X_i . The distribution estimate in Eq. (1) offers many nice properties, such as its non-parametric property, continuity and differentiability [18]. Also it is an asymptotic unbiased estimator of the density.

To estimate the density at the location of the object X_p , we only consider its neighbors of X_p as kernels, instead of using all objects in the data set. The reason for this is twofold: firstly, many complex real-life data sets usually have multiple clusters or components, which are the intrinsic patterns of the data. The density estimation using the full data set may lose the local difference in density and fail to detect the local outliers; secondly, the outlier detection will calculate the score for each object, and using the full data set would lead to a high computational cost, which has the complexity of $O(N^2)$ where N is the total number of objects in the data set.

To better estimate the density distribution in the neighbourhood of an object, we propose to use k nearest neighbors, reverse nearest neighbors and shared nearest neighbors as kernels in KDE. Let $NN_r(X_p)$ be the r -th nearest neighbors of the object X_p , we denote the set of k nearest neighbors of X_p as $\mathcal{S}_{KNN}(X_p)$:

$$\mathcal{S}_{KNN}(X_p) = \{NN_1(X_p), NN_2(X_p), \dots, NN_k(X_p)\} \quad (4)$$

The *reverse nearest neighbors* of the object X_p are those objects who consider X_p as one of their k nearest neighbors [19], i.e., X is one reverse nearest neighbor of X_p if $NN_r(X) = X_p$ for any $r \leq k$. The *shared nearest neighbors* of the object X_p are those objects who share one or more nearest neighbors with X_p , in other words, X is one shared nearest neighbor of X_p if $NN_r(X) = NN_s(X_p)$ for any $r, s \leq k$. We show these three types of nearest neighbors in Fig. 1.

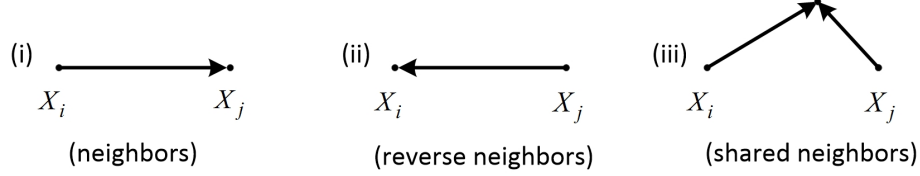


Figure 1: Three types of nearest neighbors considered. Arrows from X_i and X_j to $NN_r(X_i)$ and $NN_s(X_j)$, respectively.

We denote $\mathcal{S}_{RNN}(X_p)$ and $\mathcal{S}_{SNN}(X_p)$ by the sets of reverse nearest neighbors and shared nearest neighbors of X_p , respectively. For an object, there would be always k nearest neighbors in $\mathcal{S}_{KNN}(X_p)$, while the sets of $\mathcal{RNN}(X_p)$ and $\mathcal{SNN}(X_p)$ can be empty or have one or more elements. Given the three data sets $\mathcal{S}_{KNN}(X_p)$, $\mathcal{S}_{RNN}(X_p)$ and $\mathcal{S}_{SNN}(X_p)$ for the object X_p , we form an extended local neighborhood by combining them together, denoted by $\mathcal{S}(X_p) = \mathcal{S}_{KNN}(X_p) \cup \mathcal{S}_{RNN}(X_p) \cup \mathcal{S}_{SNN}(X_p)$. Thus, the estimated density at the location of X_p is written as

$$p(X_p) = \frac{1}{|\mathcal{S}(X_p)| + 1} \sum_{X \in \mathcal{S}(X_p) \cup \{X_p\}} \frac{1}{h^d} K\left(\frac{X - X_p}{h}\right) \quad (5)$$

where $|\mathcal{S}|$ denotes the number of elements in the set of \mathcal{S} .

2.2. Relative Density-based Outlier Factor

After estimating the density at the locations of all objects, we propose a novel relative density-based outlier factor (RDOS) to measure the degree to which the density of the object X_p deviates from its neighborhood, which is defined as follows:

$$RDOS_k(X_p) = \frac{\sum_{X_i \in \mathcal{S}(X_p)} p(X_i)}{|\mathcal{S}(X_p)| p(X_p)} \quad (6)$$

The RDOS is the ratio of the average neighborhood density to the density of interested object X_p . If $RDOS_k(X_p)$ is much larger than 1, then the object X_p would be outside of a dense cluster, indicating that X_p would be an outlier. If $RDOS_k(X_p)$ is equal or smaller than 1, then the object X_p would be surrounded by the same dense neighbors or by a sparse cloud, indicating that X_p would not be an outlier. In practice, we would like to rank the RDOS values and detect top- n outliers. We summarize our algorithm

in Algorithm 1, which takes the KNN graph as input. The KNN graph is a directed graph in which each object is a vertex and is connected to its k nearest neighbors with an outbound direction. In the KNN graph, an object will have k outbound edges to the elements in \mathcal{S}_{KNN} , and have none, one or more inbound edges. The KNN graph construction using the brute-force method has the computational complexity of $O(N^2)$ for N objects, and it can be reduced to $O(N \log N)$ using the $k - d$ trees [20]. Using the KNN graph KNN-G, it is easy to obtain the k nearest neighbors \mathcal{S}_{KNN} , reverse nearest neighbors \mathcal{S}_{RNN} and shared nearest neighbors \mathcal{S}_{SNN} with an approximate computational cost of $O(N)$. For each object, we form a set of local nearest neighbors \mathcal{S} with the combination of \mathcal{S}_{KNN} , \mathcal{S}_{RNN} and \mathcal{S}_{SNN} , and calculate the density at the location of the object based on the set of \mathcal{S} . Then, we calculate the RDOS value of each object based on the densities of local neighbors in \mathcal{S} . The top- n outliers are obtained by sorting the RDOS values in a descending way. If one wants to determine whether an object X_p is outlier, we can compare the value of $RDOS_k(X_p)$ with a threshold τ , i.e., we determine an object is outlier if $RDOS_k(X_p)$ satisfies

$$RDOS_k(X_p) > \tau \quad (7)$$

where the threshold τ is usually a constant value that is pre-determined by users.

3. Theoretical Properties

In this section, we analyze several nice properties of the proposed outlierlieness metric. In Theorem 1, we give the expected value of RDOS when the object and its neighbors are sampled from the same distribution, which indicates the lower bound of RDOS for outlier detection.

Theorem 1. *Let the object X_p be sampled from a continuous density distribution. For $N \rightarrow \infty$, the RDOS equals 1 with probability 1, i.e., $RDOS_k(X_p) = 1$, when the kernel function K is nonnegative and integrable.*

Proof. For a fixed k , $N \rightarrow \infty$ indicates that the objects in $\mathcal{S}(X_p)$ locate in the local neighborhood of X_p with the radius $r \rightarrow 0$. Considering data sampled from a continuous density distribution $f(x)$, the expectation of the density estimation at X_p exists and is consistent to the true one [21]:

$$\mathbb{E}[p(X_p)] = f(X_p) \int K(u) du = f(X_p) \quad (8)$$

Algorithm 1: RDOS for top- n outlier detection based on the KNN graph

INPUT: k, \mathcal{X}, d, h , the KNN graph KNN-G.

OUTPUT: Top- n objects in \mathcal{X} .

ALGORITHM:

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foreach object  $X_p \in \mathcal{X}$  do
1 |  $\mathcal{S}_{KNN}(X_p) = \text{getOutboundObjects}(\text{KNN-G}, X_p)$ : get  $k$  nearest
   | neighbors of  $X_p$ ;
2 |  $\mathcal{S}_{RNN}(X_p) = \text{getInboundObjects}(\text{KNN-G}, X_p)$ : get reverse nearest
   | neighbors of  $X_p$ ;
3 |  $\mathcal{S}_{SNN}(X_p) = \emptyset$ : initialize shared nearest neighbors of  $X_p$ ;
4 | foreach object  $X \in \mathcal{S}_{KNN}(X_p)$  do
5 | |  $\mathcal{S}_{RNN}(X) = \text{getInboundObjects}(\text{KNN-G}, X)$ ;
6 | |  $\mathcal{S}_{SNN}(X_p) = \mathcal{S}_{SNN}(X_p) \cup \mathcal{S}_{RNN}(X)$ : get objects who share  $X$ 
   | | as nearest neighbors with  $X_p$ ;
   | end
7 |  $\mathcal{S}(X_p) = \mathcal{S}_{KNN}(X_p) \cup \mathcal{S}_{RNN}(X_p) \cup \mathcal{S}_{SNN}(X_p)$ ;
8 |  $p(X_p) = \text{getKernelDensity}(\mathcal{S}(X_p), X_p, h)$ : estimate the local
   | kernel density at the location of  $X_p$ ;
   end
foreach object  $X_p \in \mathcal{X}$  do
9 | Calculate  $RDOS_k(X_p)$  for  $X_p$  according to Eq. (6);
   end
10 Sort RDOS in a descending way and output the top- $n$  objects.

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and its asymptotic variance is given by [21]

$$\text{Var} [p(X_p)] = 0 \quad (9)$$

Meanwhile, the average density at the neighborhood of X_p with the radius of $r \rightarrow 0$ can be given by

$$\mathbb{E}[\bar{p}(X_p)] = \mathbb{E} \left[\frac{\sum_{X_i \in \mathcal{S}(X_p)} p(X_i)}{|\mathcal{S}(X_p)|} \right] = \mathbb{E} [p(X_p)] = f(X_p) \quad (10)$$

Taking the ratio, we get

$$\mathbb{E}[\bar{p}(X_p)] / \mathbb{E} [p(X_p)] = 1 \quad (11)$$

□

This theorem shows that when $RDOS_k(X_p) \approx 1$, we could say that the object X_p is not an outlier. Since RDOS is always positive, when $0 < RDOS_k(X_p) < 1$, we could say the object X_p can be ignored in outlier detection. Only these objects whose RDOS values are larger than 1 are possible to be outliers.

Following the work in [12], we next examine the upper-bound false detection probability to give a sense of threshold selection in practice.

Theorem 2. *Let $\mathcal{S}(X_p)$ be the set of local neighbors of X_p in RDOS, which are assumed to be uniformly distributed in ball B_r centered at X_p with the radius of r . Using the Gaussian kernel, the probability of false detecting X_p as an outlier is given by*

$$P[RDOS_k(X_p) > \gamma] \leq \exp\left(-\frac{2(\gamma-1)^2(|\mathcal{S}|+1)^2(2\pi)^d h^{2d}}{|\mathcal{S}|(2|\mathcal{S}|+\gamma+1)^2 V^2}\right) \quad (12)$$

where h is the kernel width and V is the volume of ball B_r .

Proof. For simplicity of notation, we use \mathcal{S} for $\mathcal{S}(X_p)$ and consider $X_p = 0$. Then, the density estimation at X_p given the local neighbors $X_1, X_2, \dots, X_{|\mathcal{S}|}$ is written as

$$p(X_p) = \frac{1}{|\mathcal{S}|+1} \sum_{X_i \in \mathcal{S} \cup X_p} \frac{1}{(2\pi)^{d/2} h^d} \exp\left(-\frac{\|X_i\|^2}{2h}\right) \quad (13)$$

and the average density estimation in the neighborhood of X_p is written as

$$\begin{aligned} \bar{p}(X_p) &= \frac{1}{|\mathcal{S}|} \sum_{X_i \in \mathcal{S}} p(X_i) \\ &= \frac{1}{|\mathcal{S}|(|\mathcal{S}|+1)} \sum_{X_i \in \mathcal{S}} \sum_{X_j \in \mathcal{S} \cup X_p} \frac{1}{(2\pi)^{d/2} h^d} \exp\left(-\frac{\|X_i - X_j\|^2}{2h}\right) \end{aligned} \quad (14)$$

For $X_i, i = 1, 2, \dots, |\mathcal{S}|$, uniformly distributed in ball B_r , we can compute the expectation of both $p(X_p)$ and $\bar{p}(X_p)$ from Theorem 1, which is given by:

$$\mathbb{E}[\bar{p}(X_p)] = \mathbb{E}[p(X_p)] = \frac{1}{V} = \frac{\pi^{n/2} r^n}{\Gamma(n/2 + 1)} \quad (15)$$

where V is the volume of n -sphere B_r and $n = d - 1$. The rest of proof follows the McDiarmid's Inequality which gives the upper bound of the probability that a function of i.i.d. variables $f(X_1, X_2, \dots, X_{|\mathcal{S}|})$ deviates from its expectation. Let $f : \mathbb{R}^d \rightarrow \mathbb{R}$, $\forall i, \forall x_1, \dots, x_{|\mathcal{S}|}, x'_i \in \mathcal{S}$,

$$|f(x_1, \dots, x_i, \dots, x_{|\mathcal{S}|}) - f(x_1, \dots, x'_i, \dots, x_{|\mathcal{S}|})| \leq c_i \quad (16)$$

Then, for all $\epsilon > 0$,

$$\mathbb{P}[f - \mathbb{E}(f) \geq \epsilon] \leq \exp\left(\frac{-2\epsilon^2}{\sum_{i=1}^{|\mathcal{S}|} c_i^2}\right) \quad (17)$$

For $f_1 = p(X_p)$, we have

$$\begin{aligned} & |f_1(x_1, \dots, x_i, \dots, x_{|\mathcal{S}|}) - f_1(x_1, \dots, x'_i, \dots, x_{|\mathcal{S}|})| \\ &= \frac{K(X_i/h) - K(X'_i/h)}{h^d(|\mathcal{S}| + 1)} \leq \frac{1 - \exp(-r^2/2h)}{(2\pi)^{d/2}h^d(|\mathcal{S}| + 1)} = c_1 \end{aligned} \quad (18)$$

For $f_2 = \bar{p}(X_p)$, we have

$$\begin{aligned} & |f_2(x_1, \dots, x_i, \dots, x_{|\mathcal{S}|}) - f_2(x_1, \dots, x'_i, \dots, x_{|\mathcal{S}|})| \\ &= \frac{K(\frac{X_i}{h}) - K(\frac{X'_i}{h}) + 2 \sum_{j=1, j \neq i}^{|\mathcal{S}|} \left[K(\frac{X_i - X_j}{h}) - K(\frac{X'_i - X_j}{h}) \right]}{h^d(|\mathcal{S}| + 1)} \\ &\leq \frac{1 - \exp(-r^2/2h) + 2|\mathcal{S}|(1 - \exp(-2r^2/h))}{(2\pi)^{d/2}h^d(|\mathcal{S}| + 1)} = c_2 \end{aligned} \quad (19)$$

We define a new function $f = f_2 - \gamma f_1$, which is bounded by

$$|f| \leq |f_2| + \gamma |f_1| \leq c_2 + \gamma c_1 \leq \frac{2|\mathcal{S}| + \gamma + 1}{(2\pi)^{d/2}h^d(|\mathcal{S}| + 1)} = c \quad (20)$$

Then, the probability of false alarm is written as

$$\begin{aligned} \mathbb{P}[RDOS_k(X_p) > \gamma] &= \mathbb{P}[\bar{p}(X_p) - \gamma p(X_p)] \\ &= \mathbb{P}[f - \mathbb{E}(f) > t] \end{aligned} \quad (21)$$

where $t = (\gamma - 1)/V$. From Theorem 1, we are only interested in the case of $RDOS_k(X_p) > 1$, i.e., $\gamma > 1$, and $t > 0$. Using the McDiarmid's Inequality,

we have

$$\begin{aligned} \mathbb{P}[RDOS_k(X_p) > \gamma] &\leq \exp\left(-\frac{2t^2}{\sum_{i=1}^{|\mathcal{S}|} c^2}\right) = \exp\left(-\frac{2t^2}{|\mathcal{S}|c^2}\right) \\ &\leq \exp\left(-\frac{2(\gamma-1)^2(|\mathcal{S}|+1)^2(2\pi)^d h^{2d}}{|\mathcal{S}|(2|\mathcal{S}|+\gamma+1)^2 V^2}\right) \end{aligned} \quad (22)$$

□

4. Experimental Results and Analysis

4.1. Synthetic Data Sets

We first test the proposed RDOS in two synthetic data sets for outlier detection. Our first synthetic data set includes two Gaussian clusters centered at $(0.5, 0.8)$ and $(2, 0.5)$, respectively, each of which has 100 data samples. There are three outliers around these two clusters, as indicated in Fig. 2. To calculate the RDOS, we use $k = 21$ nearest neighbors and $h = 0.01$ in kernel functions. In Fig. 3, we show the RDOS value of all data samples, where the color and the radius of circles denote the value of RDOS. It can be shown that the RDOS of these three outliers is significantly larger than that of non-outliers.

The second synthetic data set used in our simulation consists of data samples uniformly distributed around a cosine curve, which can be written as

$$x_2 = \cos(x_1) + w \quad (23)$$

where $w \sim \mathcal{N}(0, \sigma^2)$. In our simulation, we use $\sigma^2 = 0.1$, and generate four outliers in this data set, as shown in Fig. 4. The RDOS value of all data samples is shown in Fig. 5, where both the color and the radius of circles indicate the value of RDOS. It is still shown that the RDOS-based method can effectively detect the outliers.

4.2. Real-Life Data Sets

We also conduct outlier detection experiments on four real-life data sets to demonstrate the effectiveness of our proposed RDOS approach. All of these four data sets are originally from the UCI repository [22], including BREAST CANCER, PEN-LOCAL, PEN-GLOBAL, and SATELLITE, but are modified for

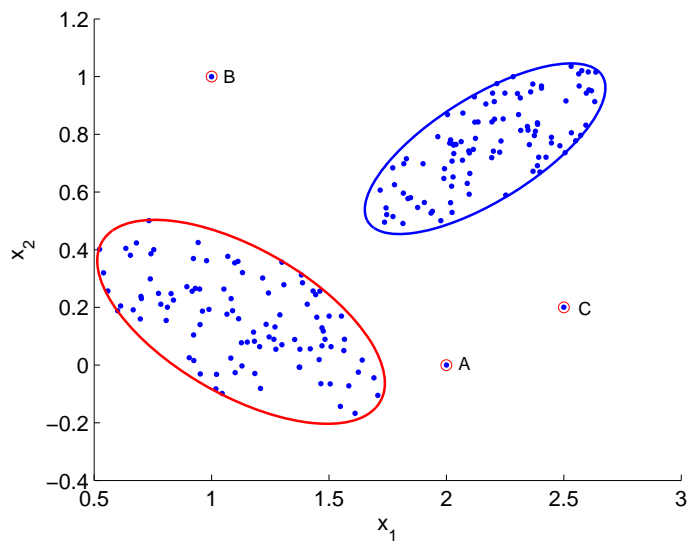


Figure 2: The distribution of normal data and outliers, where the objects: A , B , and C are outliers.

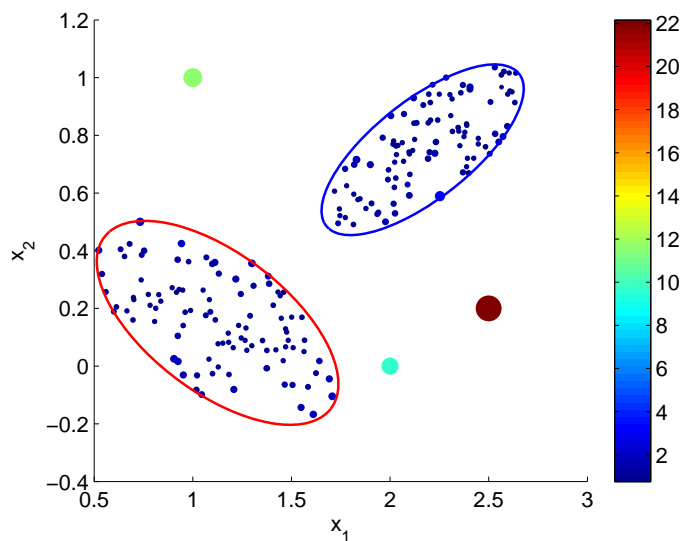


Figure 3: The outliersness scores of all data samples, where the value of RDOS is illustrated by the color and the radius of the circle.

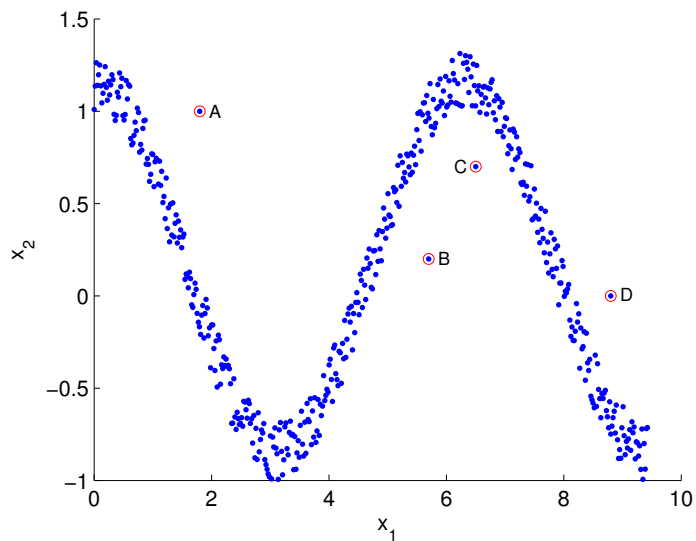


Figure 4: The distribution of normal data and outliers, where A , B , C and D are considered as outliers.

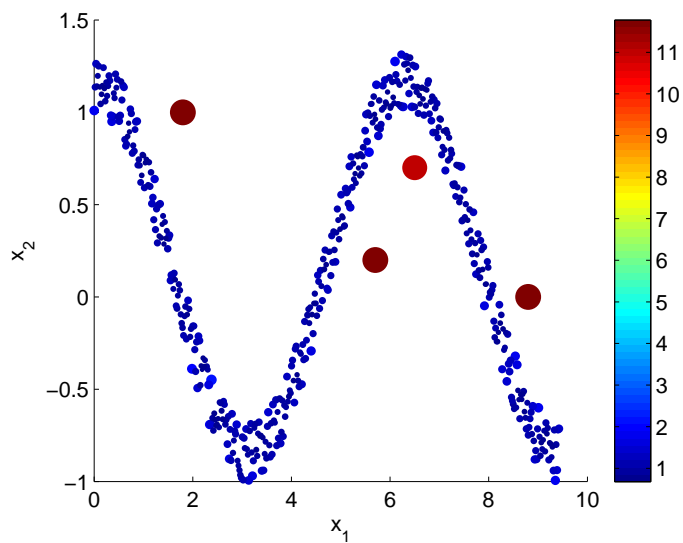


Figure 5: The outliersness scores of all data samples, where the value of RDOS is illustrated by the color and the radius of the circle.

Table 1: The characteristics of four data sets

Dataset	# of features	# of outliers	# of data
BREAST CANCER	30	10	357
PEN-LOCAL	16	10	6714
PEN-GLOBAL	16	90	719
SATELLITE	36	75	5025

local and global outlier detection [23]. We summarize the characteristics of these four data sets in Table 1. Prior to calculating the RDOS, we first normalize the data ranging from 0 to 1. In Fig. 6, we show the first two principle components of these four data sets, where the outliers are denoted by the solid circle.

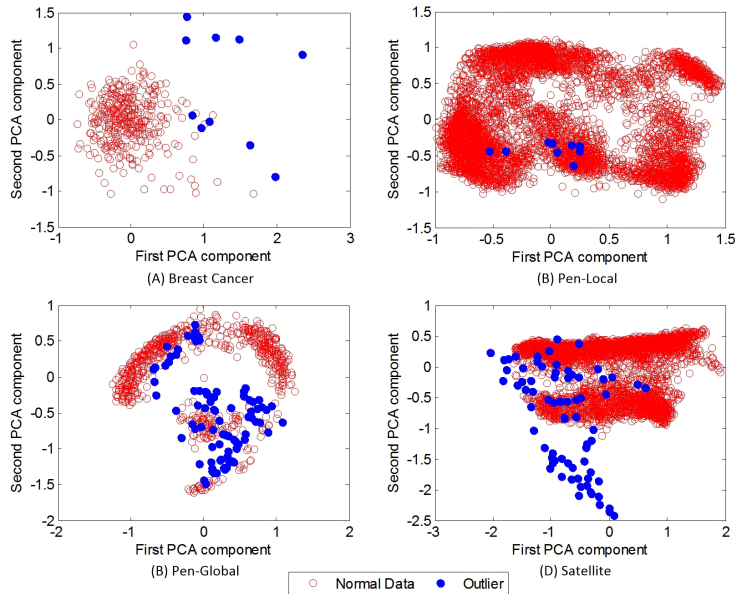


Figure 6: The normal data and outliers in four real-life data sets: (A) BREAST CANCER, (B) PEN-LOCAL, (C) PEN-GLOBAL, and (D) SATELLITE. Only the first two principle components are shown.

For each data sample, we calculate its RDOS and compare it with a

threshold to determine whether it is an outlier. Since all these data sets are highly imbalanced, the use of overall accuracy is not appropriate. In our experiments, we use the metric of AUC (area under the ROC curve) for performance comparison. The ROC curve examines the performance of a binary classifier with different thresholds, leading to different pairs of false alarm rate and true positive rate. We compare our RDOS approach with another four widely used outlier detection approaches: Outlier Detection using Indegree Number (ODIN) [24], LOF [10], INFLO [13], and Mutual Nearest Neighbors (MNN) [9]. Since all of these examined methods are nearest neighbors-based methods, we evaluate the outlier detection performance with different k values. Fig. 7 shows the performance of AUC for the data set of BREAST CANCER. It can be shown that our proposed RDOS approach, in general, performs better than other four approaches, and has a similar performance to the approaches of LOF and INFLO when k is larger than 7. When $k = 5$, the performance improvement of the proposed RDOS approach is largest, as illustrated in Fig. 8.

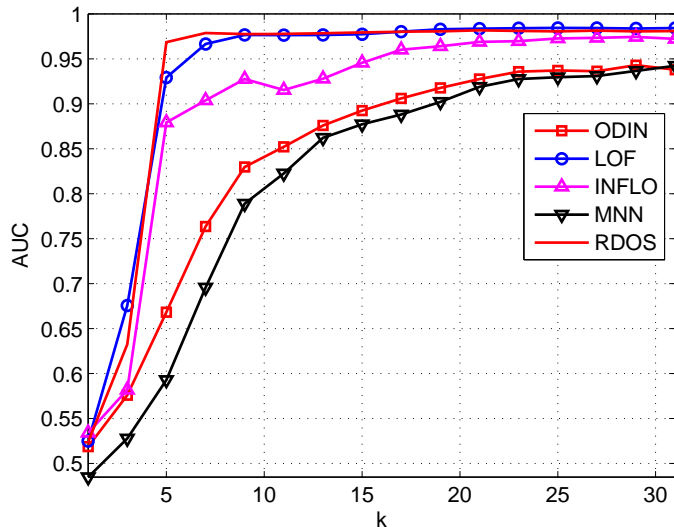


Figure 7: The performance of AUC with different k values for the data set of BREAST CANCER

In Fig. 9, we show the performance of AUC for the data set of PEN-LOCAL. It also shows that our RDOS approach generally outperforms other four approaches when k is less than 7. Specifically, in Fig. 8, we show the

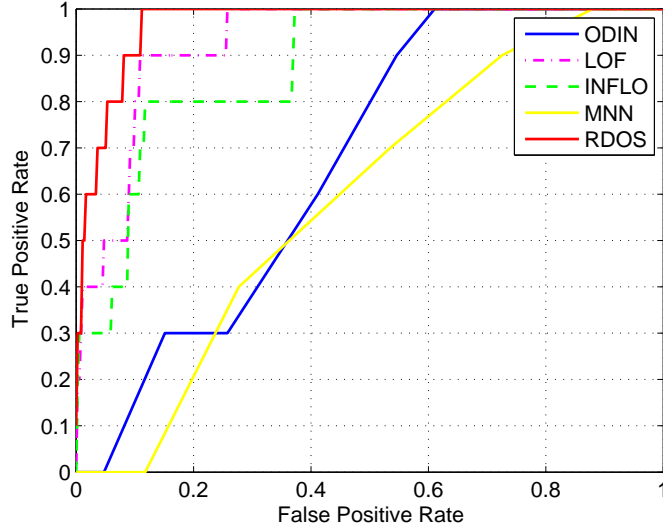


Figure 8: The ROC for the data set of BREAST CANCER, where $k = 5$

ROC curve for $k = 5$. Compared to the LOF and INFLO approaches, the performance difference is close to zero for a large k value.

In Fig. 11, we show the performance of AUC for the data set of PEN-GLOBAL. It shows a large performance improvement of our RDOS approach when the number of nearest neighbors k increases. In Fig. 12, the ROC curves of all the five approaches are compared, when $k = 15$. From Fig. 7, 9 and 11, it can be shown that $\text{RDOS} > \text{LOF} > \text{INFLO} > \text{ODIN} > \text{MNN}$, where the symbol “ $>$ ” means “performs better than”, for the data sets of BREAST CANCER, PEN-LOCAL, and PEN-GLOBAL.

Fig. 13 shows the performance of AUC for the data set of SATELLITE. When the number of nearest neighbors k is less than 11, three approaches of RDOS, LOF and INFLO have a similar AUC. When the number of nearest neighbors k is larger than 11 and less than 23, the approach of INFLO performs the best and our RDOS approach is the second. When the number of nearest neighbors k is larger than 25, our RDOS approach has the best performance. Specifically, we show the ROC curve of all the five approaches in Fig. 14. In general, we observe the following phenomena in our experiments: Firstly, the performance of all the five approaches is usually poor for a small k , and the improvement of our RDOS approach is not significant. When a small number of nearest neighbors are considered, the relative density in a

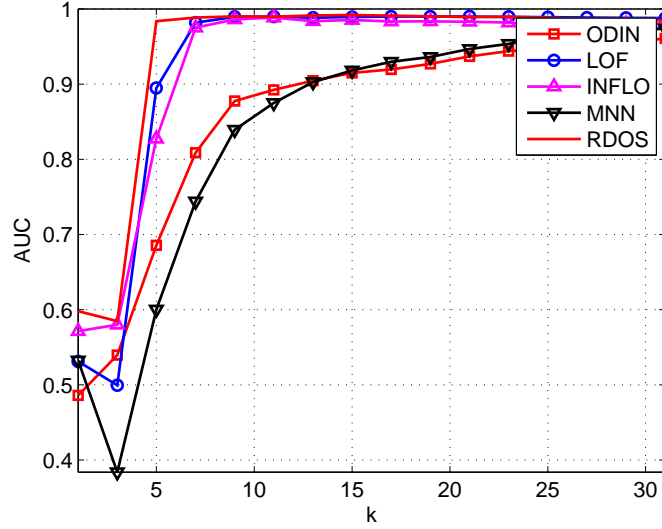


Figure 9: The performance of AUC with different k values for the data set of PEN-LOCAL

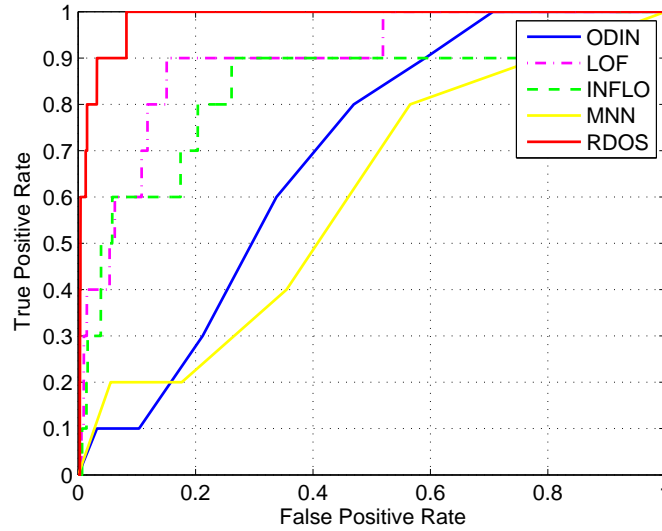


Figure 10: The ROC for the data set of PEN-LOCAL, where $k = 5$

neighborhood might not be well represented. Secondly, the proposed RDOS

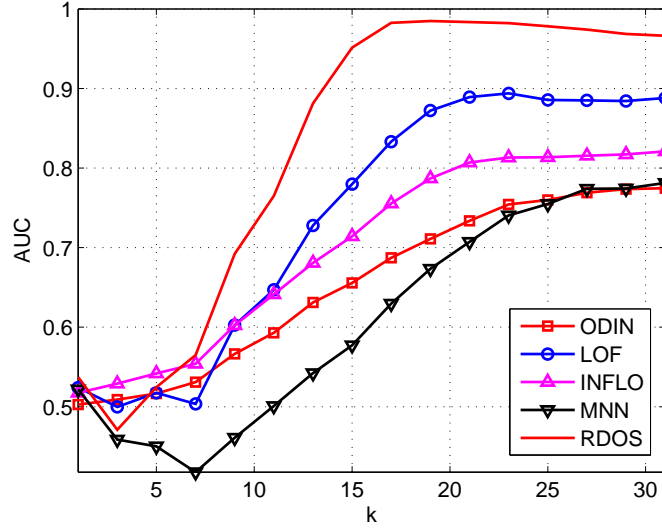


Figure 11: The performance of AUC with different k values for the data set of PEN-GLOBAL

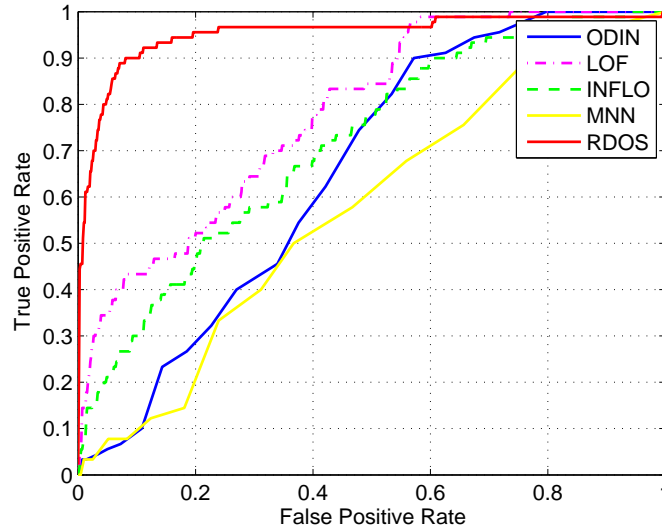


Figure 12: The ROC for the data set of PEN-GLOBAL, where $k = 15$

approach performs the best for specific k values. Thirdly, we observe that

the MNN approach has the worst performance, compared with other four approaches, for these four data sets.

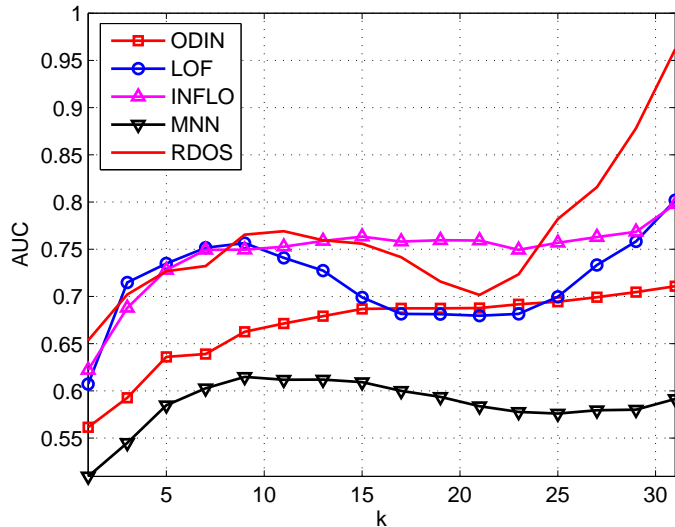


Figure 13: The performance of AUC with different k values for the data set of SATELLITE

5. Conclusions and Future Work

This paper presented a novel local outlier detection method based on local kernel density estimation. Instead of only considering the k nearest neighbors of a data sample, we considered three kinds of neighbors: k nearest neighbors, reverse nearest neighbors, and shared nearest neighbors, for local kernel density estimation. A simple but efficient relative density calculation, termed Relative Density-based Outlier Score (RDOS), was introduced to measure the outlierness. We further derived theoretical properties of the proposed RDOS measure, including the expected value and the false alarm probability. The theoretical results suggest parameter settings for practical applications. Simulation results on both synthetic data sets and real-life data sets illustrate superior performance of our proposed method. One drawback of kernel-based density estimation is its kernel width selection. Along this research direction, new density estimation methods such as exponentially embedded families [25, 26, 27] and PDF projection theorem [28, 29] will be

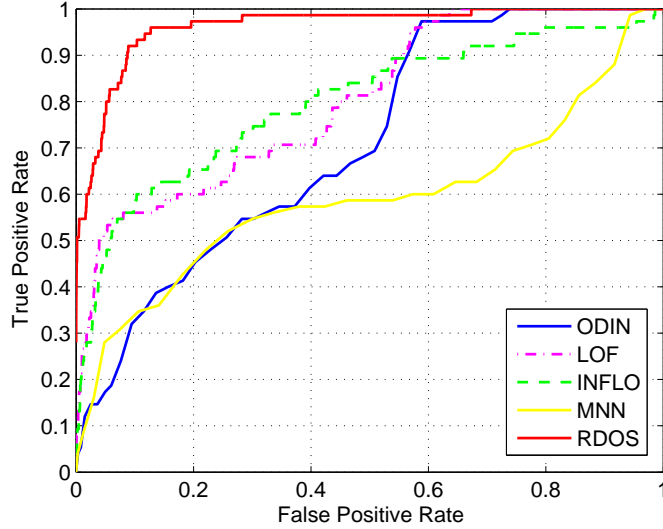


Figure 14: The ROC for the data set of SATELLITE, where $k = 31$

investigated in our future work.

Reference:

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