

Regularity Normalization: Constraining Implicit Space with Minimum Description Length

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Abstract

Inspired by the adaptation phenomenon of biological neuronal firing rate, we propose regularity normalization: a reparametrization of the activation in a neural network that take into account the statistical regularity in the implicit space. The implicit space is constrained by the normalizing factor, the minimum description length of the optimal universal code. We introduce an incremental version of computing this universal code as normalized maximum likelihood and demonstrated its flexibility to include data prior such as top-down attention and other oracle information and its compatibility to be incorporated into batch normalization and layer normalization. The preliminary empirical results are inconclusive regarding its advantages in tackling the limited and imbalanced data from a non-stationary distribution benchmarked on computer vision task. This biologically plausible normalization, however, has the potential to deal with other complicated real-world scenarios - further research are proposed to discover these scenarios and explore the behaviors among different variants.

1 Introduction

The Minimum Description Length (MDL) principle asserts that the best model given some data is the one that minimizing the combined cost of describing the model and describing the misfit between the model and data [Rissanen, 1978] with a goal to maximize regularity extraction for optimal data compression, prediction and communication [Grünwald, 2007]. Most unsupervised learning algorithms can be understood using the MDL principle [Rissanen, 1989], treating the neural network as a system communicating the input to a receiver.

If we consider the neural network training as the optimization process of a communication system, each input at each layers of the system can be described as a point in a low-dimensional continuous constraint space [Zemel and Hinton, 1999]. If we consider the neural networks as population codes, the constraint space can be subdivided into the input-vector space, the hidden-vector space, and the *implicit space*, which represents the underlying dimensions of variability in

the other two spaces, i.e., a reduced representation of the constraint space. For instance, if we are given a image of an object, the rotated or scaled version of the same image still refers to the same objects, then each image instance of the same object can be represented by a code assigned position on a 2D implicit space with one dimension as orientation and the other as size of the shape [Zemel and Hinton, 1999]. The relevant information about the implicit space can be constrained to ensure a minimized description length of the neural networks.

This type of constraint can also be found in biological systems. In biological brains of primates, high-level brain areas are known to send top-down feedback connections to lower-level areas to encourage the selection of the most relevant information in the current input given the current task [Ding *et al.*, 2017], a process similar to the communication system above. This type of modulation is performed by collecting statistical regularity in a hierarchical encoding process between these brain areas. One feature of the neural coding during the hierarchical processing is the adaptation: in vision neuroscience, vertical orientation reduce their firing rates to that orientaiton after the adaptation [Blakemore and Campbell, 1969], while the cell responses to other orientations may increase [Dragoi *et al.*, 2000]. These behaviors contradict to the Bayesian assumption that the more probable the input, the larger firing rate should be, but instead, well match the information theoretical point-of-view that the most relevant information (saliency), which depends on the statistical regularity, have higher “information”, just as the firing of the neurons. The more regular the input features are, the lower it should yield the activation. We introduce the minimum description length (MDL), such that the activation of neurons can be analogous to the code length of the model (a specific neuron or neuronal population) - a shorter code length would be assigned to a more regular input (such as after adaptation), and a longer code length to a more rare input or event.

In this paper, we adopt the similar definition of implicit space as in [Zemel and Hinton, 1999], but extend it beyond unsupervised learning, into a generical neural network optimization problem in both supervised and unsupervised setting. In addition, we consider the formulation and computation of description length differently, given the neuroscience inspiration described above. Instead of considering neural networks as population codes, we formulate each layer of neural networks during training a state of module selection.

In our setup, the description length is computed not in the scale of the entire neural networks, but by the unit of each layer of the network. In addition, the optimization objective is not to minimize the description length, but instead, to take into account the minimum description length as part of the normalization procedure to reparameterize the activation of each neurons in each layer. The computation of the description length (or model cost as in [Zemel and Hinton, 1999]) aims to minimize it, while we directly compute the minimum description length in each layer not to minimize anything, but to reassign the weights based on statistical regularities. Finally, we compute the description length by an optimal universal code obtained by the batch input distribution in an on-line incremental fashion.

We begin our presentation in section 2, with a short overview of related works in normalization methods and MDL in neural networks. Section 3 formulated the problem setting in neural networks where we consider the training as a layer-specific model selection process under MDL principle. We then introduce the proposed regularity normalization (RN) method, its formulation, implementation, and the online incremental tricks for batch computation. We also present several variants of the regularity normalization by incorporating batch and layer normalizations, termed regularity batch normalization (RBN) and regularity layer normalization (RLN), as well as including the data prior as a top-down attention mechanism during the training process, termed saliency normalization (SN). In section 5, we present the preliminary results on the imbalanced MNIST dataset and demonstrated that our approach might be advantageous over batch normalization and weight normalization, but not as robust as layer normalization. In the last section, we conclude our methods and point out several future work directions as the next step of this research.

2 Related work

Normalization.

Batch normalization (BN) performs global normalization along the batch dimension such that for each neuron in a layer, the activation over all the mini-batch training cases follows standard normal distribution, reducing the internal covariate shift [Ioffe and Szegedy, 2015]. Similarly, layer normalization (LN) performs global normalization over all the neurons in a layer, and have shown effective stabilizing effect in the hidden state dynamics in recurrent networks [Ba *et al.*, 2016]. Weight normalization (WN) applied normalization over the incoming weights, offering computational advantages for reinforcement learning and generative modeling [Salimans and Kingma, 2016]. Like BN and LN, we apply the normalization on the activation of the neurons, but as an element-wise reparameterization (over both the layer and batch dimension). In section 4.2, we also proposed the variant methods based on our approach with batch-wise and layer-wise reparameterization, the regularity batch normalization (RBN) and regularity layer normalization (RLN).

Description length in neural networks.

[Hinton and Van Camp, 1993] first introduce the description length to quantify neural network simplicity and develop an

optimization method to minimize the amount of information required to communicate the weights of the neural network. [Zemel and Hinton, 1999] considered the neural networks as population codes and used MDL to develop highly redundant population code. They showed that by assuming the hidden units reside in low-dimensional implicit spaces, optimization process can be applied to minimize the model cost under MDL principle. Our proposed method adopt a similar definition of implicit space, but consider the implicit space as data-dependent encoding statistical regularities. Unlike [Zemel and Hinton, 1999] and [Hinton and Van Camp, 1993], we consider the description length as a indicator of the data input and assume that the implicit space is constrained when we normalize the activation of each neurons given its statistical regularity. Unlike the implicit approach to compute model cost, we directly compute the minimum description length with optimal universal code obtained in an incremental style.

3 Problem Setting

3.1 Minimum Description Length

Given a model class Θ consisting of a finite number of models parameterized by the parameter set θ . Given a data sample x , each model in the model class describes a probability $P(x|\theta)$ with the code length computed as $-\log P(x|\theta)$. The minimum code length given any arbitrary θ would be given by $L(x|\hat{\theta}(x)) = -\log P(x|\hat{\theta}(x))$ with model $\hat{\theta}(x)$ which compresses data x most efficiently and offers the maximum likelihood $P(x|\hat{\theta}(x))$ [Grünwald, 2007].

However, the compressibility of the model will be unattainable for multiple inputs, as the probability distributions are different. The solution relies on a universal code, $\bar{P}(x)$ defined for a model class Θ such that for any data sample x , the shortest code for x is always $L(x|\hat{\theta}(x))$ [Rissanen, 2001].

3.2 Normalized Maximum Likelihood

Among the optimal universal code, the normalized maximum likelihood (NML) probability minimizes the worst-case regret and avoids assigning an arbitrary distribution to Θ . The minimax optimal solution is given by [Myung *et al.*, 2006]:

$$P_{NML}(x) = \frac{P(x|\hat{\theta}(x))}{\sum_{x'} P(x'|\hat{\theta}(x'))} \quad (1)$$

where the summation is over the entire data sample space. Figure 1 describes the optimization problem of finding optimal model $P(x_i|\hat{\theta}_i)$ given data sample x_i among model class Θ . The models in the class, $P(x|\theta)$, are parameterized by the parameter set θ . x_i are data sample from data X . With this distribution, the regret is the same for all data sample x given by [Grünwald, 2007]:

$$\begin{aligned} COMP(\Theta) &\equiv \text{regret}_{NML} \\ &\equiv -\log P_{NML}(x) + \log P(x|\hat{\theta}(x)) \\ &= \log \sum_{x'} P(x'|\hat{\theta}(x')) \end{aligned} \quad (2)$$

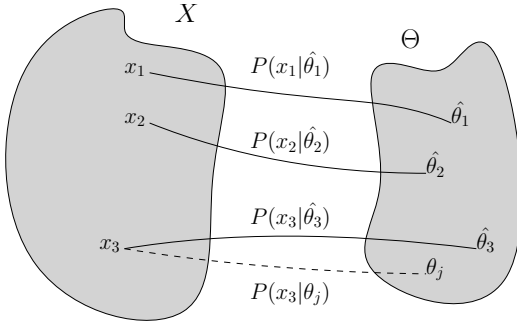


Figure 1: **Normalized maximal likelihood.** Shown here in the schematic illustration, data sample x_i are drawn from the entire data distribution X and model $\hat{\theta}_i$ is the optimal model that describes data x_i with the shortest code length. θ_j is an arbitrary model that is not $\hat{\theta}_3$, so $P(x_3|\theta_j)$ is not considered when computing optimal universal code according to NML formulation.

which defines the model class complexity as it indicates how many different data samples can be well explained by the model class Θ .

3.3 Neural networks as model selection

In the neural network setting where optimization process are performed in batches (as incremental data sample x_j with j denoting the batch j), the model selection process is formulated as a partially observable problem (as in Figure 2). Herein to illustrate our approach, we consider a feedforward neural network as an example, without loss of generalizability to other architecture (such as convolutional layers or recurrent modules). x_j^i refers to the activation at layer i at time point j (batch j). θ_j^i is the parameters that describes x_j^i (i.e. weights for layer $i - 1$) optimized after $j - 1$ steps (seen batch 0 through $j - 1$). Because one cannot exhaust the search among all possible θ , we assume that the optimized parameter $\hat{\theta}_j^i$ at time step j (seen batch 0 through $j - 1$) is the optimal model $P(x_j^i|\hat{\theta}_j^i)$ for data sample x_j^i . Therefore, we generalize the optimal universal code with NML formulation as:

$$P_{NML}(x_{[0, \dots, i]}) = \frac{P(x_i|\hat{\theta}_{[0, \dots, i-1]}(x_i))}{\sum_{x_j^{0, \dots, i}} P(x_j|\hat{\theta}_{[0, \dots, j-1]}(x_j))} \quad (3)$$

where $\hat{\theta}_{[0, \dots, i-1]}(x_i)$ (or denoted $\hat{\theta}_i(x_i)$) as in Figure 2) refers to the model parameter already optimized for $i - 1$ steps and have seen sequential data sample x_0 through x_{i-1} . This distribution is updated every time a new data sample is given, and can therefore be computed incrementally, as in batch-based training.

4 Regularity Normalization

4.1 Formulation

Regularity normalization is outlined in Algorithm 1, where the input would be the activation of each neurons in certain layer and batch. Parameters $COMP$ and θ are updated after each batch, through the incrementation in the normalization

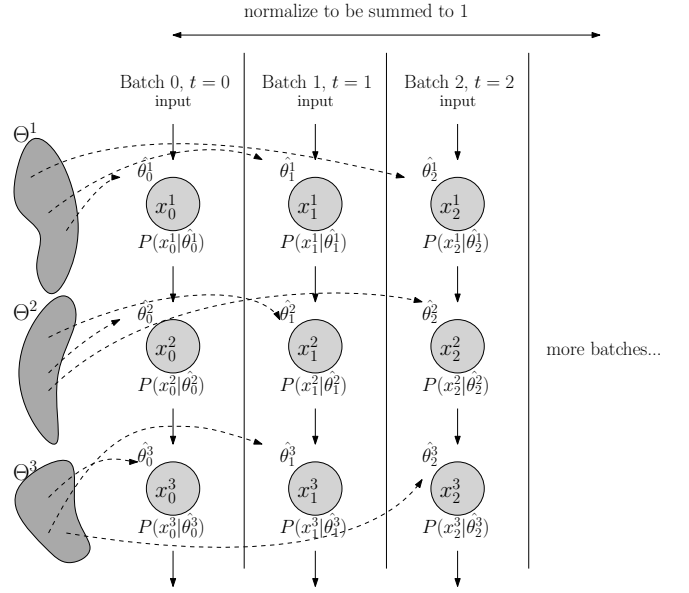


Figure 2: **Model selection in neural network.** If we consider each time step of the optimization (drawn here to be batch-dependent) as the process of choose the optimal model from model class Θ^i for i th layer of the neural networks, the optimized parameter $\hat{\theta}_j^i$ with subscript j as time step $t = j$ and superscript i as layer i can be assumed to be the optimal model among all models in the model class Θ^i . The normalized maximum likelihood can be computed by choosing $P(x_j^i|\hat{\theta}_j^i)$, the ‘‘optimal’’ model with shortest code length given data x_j^i , as the summing component in the normalization.

and optimization in the training respectively. The incrementation step involves computing the log sum of two values, which can be easily stabilized with the log-sum-exp trick. The normalization factor is then computed as the shortest code length L given the NML formulation.

4.2 Variant: Saliency Normalization

NML distribution can be modified to also include a data prior function, $s(x)$, given by [Zhang, 2012]:

$$P_{NML}(x) = \frac{s(x)P(x|\hat{\theta}(x))}{\sum_{x'} s(x')P(x'|\hat{\theta}(x'))} \quad (4)$$

where the data prior function $s(x)$ can be anything, ranging from the emphasis of certain inputs, to the cost of certain data, or even top-down attention. For instance, we can introduce the prior knowledge of the fraction of labels (say,

Algorithm 1 Regularity Normalization (RN)

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_1, \dots, x_m\}$;

Parameter: $COMP_t, \hat{\theta}_t$

Output: $y_i = RN(x_i)$

$COMP_{t+1} = \text{increment}(COMP_t, P(x_i|\hat{\theta}_t(x_i)))$

$L_{x_i} = COMP_{t+1} - \log P(x_i|\hat{\theta}_t(x_i))$

$y_i = L_{x_i} * x_i$

Table 1: Imbalanced MNIST 784-1000-1000-10 task

	Epoch 1	Epoch 2	Epoch 3	Epoch 4	Epoch 5
baseline	0.8730	0.9101	0.0308	0.9560	0.9566
BN	0.8731	0.9062	0.9266	0.9399	0.9454
LN	0.9089	0.9441	0.9404	0.9666	0.9771
WN	0.8304	0.9206	0.9398	0.9577	0.9600
RN	0.9209	0.9385	0.8818	0.9070	0.9394

in an imbalanced data problem where the oracle informs the model of the distribution of each label in the training phase); or in a scenario where we wish the model to focus specifically on certain feature of the input, say certain texture or color (just like a convolution filter); or in the case where the definition of the regularity drifts (such as the user preferences over years): in all these possible applications, the normalization procedure can be more strategic given these additional information. Therefore, we formulate this additional functionality into our regularity normalization, to be saliency normalization (SN), where the P_{NML} is computed with the addition of a pre-specified data prior function $s(x)$.

4.3 Variant: Beyond Elementwise Normalization

In our current setup, the normalization is computed elementwise, considering the implicit space of the model parameters to be one-dimensional (i.e. all activations across the batch and layer are considered to be represented by the same implicit space). Instead, the definition of the implicit can be more than one-dimensional to increase the expressibility of the method, and can also be user-defined. For instance, we can also perform the normalization over the dimension of the batch, such that each neuron in the layer should have an implicit space to compute the universal code. We term this variant regularity batch normalization (RBN). Similarly, we can perform regularity normalization over the layer dimension, as the regularity layer normalization (RLN). These two variants have the potential to inherit the innate advantages of batch normalization and layer normalization.

5 Preliminary Results

As a proof of concept, we evaluated our approach on MNIST dataset [LeCun, 1998] and computed the total number of classification errors as a performance metric. As we specifically wish to understand the behavior where the data inputs are non-stationary and highly imbalanced, we created an imbalanced MNIST benchmark to test four methods: batch normalization (BN), layer normalization (LN), weight normalization (WN), and regularity normalization (RN). Given the nature of regularity normalization, it should better adapt to the regularity of the data distribution than other methods, tackling the imbalanced data issue by upweighting the activation of the rare samples and downweighting those of the dominant samples.

To simulate changes in the context (input) distribution, in each epoch we randomly choose three classes out of the ten, and set their sampling probability to be 0.05 (only 5% of those three classes are used in the training). In this way, the training data may trick the models into preferring to classifying into the dominant classes. For simplicity, we consider

the classical 784-1000-1000-10 feedforward neural network with ReLU activation functions for all four normalization methods, as well as the baseline neural network without normalization. Five epochs of trainings are being recorded (all model face the same randomized imbalanced epochs). As shown in Table 1, RN performed the best in the first epoch (the most imbalanced data), and over time, it was surpassed by other models (as the imbalanced issue is alleviated over epochs). This also suggest that the element-wise universal codes might not be optimal, and Regularity Layer Normalization (RLN) and Regularity Batch Normalization (RBN) should be included into evaluations in the next step. Further analysis and experiments need to be included to fully understand these behaviour.

6 Conclusions

Inspired by the neural code adaptation of biological brains, we propose a biologically plausible normalization method taking into account the regularity (or saliency) of the activation distribution in the implicit space, and normalize it to upweight activation for rarely seen scenario and downweight activation for commonly seen ones. We introduce the concept from MDL principle and proposed to consider neural network training process as a model selection problem. We compute the optimal universal code length by normalized maximum likelihood in an incremental fashion, and showed this implementation can be easily incorporated with established methods like batch normalization and layer normalization. In addition, we proposed saliency normalization, which can introduce top-down attention and data prior to facilitate representation learning. Fundamentally, we implemented with an incremental update of normalized maximum likelihood, constraining the implicit space to have a low model complexity and short universal code length. Preliminary results offers a proof of concept to the proposed method, but given the limited experiments at the current state, we cannot conclude its advantage in the imbalanced or limited data scenario as hypothesized. Next steps of this research include experiments with variants of the regularity normalization (SN, RLN, RBN etc.), as well as the inclusion of top-down attention given by data prior (such as feature extracted from signal processing, or task-dependent information). We are also currently exploring this method to convolutional and recurrent neural networks, and applying to popular state-of-the-art neural network architectures in multiple modality of datasets.

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