Robust Cooperative Relay Beamforming

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Abstract—In this paper, the robust distributed relay beamforming problem is solved using the worst case approach, where the problem solution has been involved because of the effect of uncertainty of channel knowledge on the quality of service (QoS) constraints. It is shown that the original robust design, which is a non-convex semi-infinite problem (SIP), can be relaxed and reformed to a semi-definite problem (SDP). Monte-Carlo simulations are presented to verify the performance improvement of our proposed robust problem over existing robust and non-robust problems in terms of transmit power and symbol error probability.

Index Terms—Distributed relay beamforming, Semidefinite programming, Robust optimization, Channel uncertainty

I. Introduction

Relay networks is one of the main novel feasible techniques which can increase the capacity of wireless networks by multi-hopping [1–3] or parallel relaying [4] in regenerative setting or nonregenerative settings [5]. Recently, distributed relay beamforming has been found to be appealing because of the simplicity of non-regenerative relays hardware and also achieving the favorite diversity order offered by multiple route dirverity or . In such systems, relay nodes organize a single virtual MIMO node and transmit the linearly beamformed version of their received signal in distributed fashion without communicating to each other. The distributed beamforming systems for single user [6] and multi-users [7, 8], minimize the total relays transmit power with signal to interference and noise ratio (SINR) constraints at the destinations. In [6–8], the instantaneous channel state information (CSI) have to be perfectly available at the relays to maintain the instantaneous SINR above a threshold. In all of the proposed beamforming systems in [6–8], it is assumed that the perfect CSI is available at the relay nodes; However, this is an idealistic assumption, since the CSI is often subject to uncertainties because of the channel estimation or quantization error. If the statistical information of channels uncertainty (i.e. the probability density function) are available, a probabilistic or statistical approach can be used which the SINR constraints of the problem are often formulated based on outage probability and it is recently investigated in [9, 10]. In the other case which the unknown perturbation is subject to unknown probability distribution with bounded variation, worst-case robust approach is used commonly. The worst case robust beamforming has been well presented in [11] for basic multiple antenna system with only simple power constraints. The robust formulation in [12] and [13] has redesigned respectively the distributed beamforming problems of [7] and [8] by worst-case approach, which

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demonstrates visible performance improvement with respect to the non-robust systems when the channels are perturbed. The contributions of our work are as follows

- All channels are subject to uncertainty, while in [12, 13] the sources-to-relays channels are not perturbed.

- The approach in [12, 13] used a conservative approximation for SINR constraints, Min over Max (MoM), to avoid semi-infinite programming (SIP) appeared due to the uncertainty region in the QoS constraints of the robust problem. But because of more accurate formulation, our work doesn't utilize conservative MoM approximation and it outperforms [12, 13]. Instead of MoM, we propose a new equivalent Semi-Definite representable (SDr) problem to the original SIP.

II. Robust Design Formulation

Assume d single antenna sources and destinations are communicating without direct link through R single antenna amplify-and-forward (AF) relays. The one-shot received signals at the relays in the scalar and vector forms are written as

$$x_r = \sum_{p=1}^{d} f_{rp} s_p + v_r, \quad \mathbf{x} = \sum_{p=1}^{d} \mathbf{f}_p s_p + \mathbf{v}. \qquad (1)$$

where s_p is p^{th} user's transmit signal with the transmit power $P_p = E|s_p|^2$, x_r and v_r are r^{th} relay received signal and noise, f_{rp} is the complex channel coefficient from p^{th} source to r^{th} relay. For ease of vector based formulation, we define

$$\mathbf{f}_p \triangleq [f_{1,p}..., f_{R,p}]^T, \mathbf{x} \triangleq [x_1..., x_R]^T, \mathbf{v} \triangleq [v_1..., v_R]^T. \quad (2)$$

To perform relay beamforming, a complex weight coefficient, denoted as w_r^*, is used at the rth relay to amplify its received signal. By denoting $\mathbf{w} \triangleq [w_1, w_2, ..., w_R]^T$, $\mathbf{W} \triangleq diag(\mathbf{w})$, the output signal vector of the relays is $\mathbf{t} = \mathbf{W}^H \mathbf{x}$ and the received signal at the kth destination is given by

$$y_k = \mathbf{g}_k^T \mathbf{t} + n_k = \mathbf{g}_k^T \mathbf{W}^H \sum_{p=1}^{d} \mathbf{f}_p s_p + \mathbf{g}_k^T \mathbf{W}^H \mathbf{v} + n_k$$

$$= \mathbf{g}_k^T \mathbf{W}^H \mathbf{f}_k s_k + \mathbf{g}_k^T \mathbf{W}^H \sum_{p=1,p\neq k}^{d} \mathbf{f}_p s_p + \left(\mathbf{g}_k^T \mathbf{W}^H \mathbf{v} + n_k \right). \quad (3)$$

where g_{rk} is the channel coefficient from r^{th} relay to k^{th} destination, n_k is the noise at the kth destination and $\mathbf{g}_k \triangleq [g_{1k}..., g_{Rk}]^T$. The three last terms in (3) are respectively desired received signal, interference and noise. By denoting P_T as the total transmit relays power, γ_{th} as the specified SINR threshold, Γ_k as the SINR at the kth destination and P_s^k, P_i^k and P_n^k respectively as the desired signal, interference

and noise powers at the kth destination, they can be computed as

$$P_T = \underset{(s_1,..,s_d)}{E}\left\{\mathbf{t}^H\mathbf{t}\right\}=Tr\left\{\mathbf{W}^H\underset{(s_1,..,s_d)}{E}\left\{\mathbf{x}^H\mathbf{x}\right\}\mathbf{W}\right\}$$

$$=Tr\left\{\mathbf{W}^H\mathbf{R}_x\mathbf{W}\right\}=\sum_{r=1}^{R}|w_r|^2[\mathbf{R}_x]_{r,r}=\mathbf{w}^H\mathbf{D}\mathbf{w} \quad (4)$$

$$P_s^k = \underset{s_k}{E}\left|\mathbf{g}_k^T\mathbf{W}^H\mathbf{f}_k s_k\right|^2 = P_k\mathbf{w}^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w} = \mathbf{w}^H\mathbf{R}_h^k\mathbf{w}$$

$$P_i^k = \underset{(s_1,..,s_d)}{E}\left|\mathbf{g}_k^T\mathbf{W}^H\sum_{p=1,p\neq k}^{d}\mathbf{f}_p s_p\right|^2 =\left|P_k\mathbf{w}^H(\mathbf{g}_k\odot\mathbf{f}_p)\right|^2 = \mathbf{w}^H\mathbf{Q}_k\mathbf{w}$$

$$P_n^k = \underset{\mathbf{v},n_k}{E}\left|\mathbf{g}_k^T\mathbf{W}^H\mathbf{v} + n_k\right|^2 = \mathbf{w}^H\mathbf{D}_k\mathbf{w} + \sigma_n^2$$

$$\Gamma_k =\frac{P_s^k}{P_i^k+P_n^k} = \frac{\mathbf{w}^H\mathbf{R}_h^k\mathbf{w}}{\mathbf{w}^H\left(\mathbf{Q}_k+\mathbf{D}_k\right)\mathbf{w}+\sigma_n^2} \quad (5)$$

where in the above formulation we have used the following auxiliary matrix and vector parameters

$$\mathbf{R}_x \triangleq \sum_{p=1}^{d}P_p\mathbf{R}_f^p+\sigma_v^2\mathbf{I}, \ \mathbf{R}_f^p\triangleq\mathbf{f}_p\mathbf{f}_p^H, \ \mathbf{R}_g^k\triangleq\mathbf{g}_k\mathbf{g}_k^H \quad (6)$$

$$\mathbf{R}_h^k \triangleq P_k\mathbf{h}_k\mathbf{h}_k^H, \quad \mathbf{h}_k\triangleq\mathbf{g}_k\odot\mathbf{f}_k \quad (7)$$

$$\mathbf{Q}_k \triangleq \sum_{p\in\{1,...,d\}\setminus\{k\}}P_p\mathbf{h}_k^p(\mathbf{h}_k^p)^H , \quad \mathbf{h}_k^p\triangleq\mathbf{g}_k\odot\mathbf{f}_p \quad (8)$$

$$\mathbf{D} \triangleq \text{diag}\left([\mathbf{R}_x]_{1,1},[\mathbf{R}_x]_{2,2},\cdots,[\mathbf{R}_x]_{R,R}\right) \quad (9)$$

$$\mathbf{D}_k \triangleq \sigma_v^2\text{diag}\left([\mathbf{R}_g^k]_{1,1},[\mathbf{R}_g^k]_{2,2},\cdots,[\mathbf{R}_g^k]_{R,R}\right). \quad (10)$$

The robust beamforming aims to minimize the total relay transmit power P_T subject to holding the SINR of each user Γ_k, above a predefined γ_k, while the state information of $\mathbf{f}_k$ and $\mathbf{g}_k$ are incomplete. Now the robust problem can be written as

$$\underset{w}{\text{minimize}}\left\{\max \hat{P}_T\right\} = \left\{\max \mathbf{w}^H\widehat{\mathbf{D}}\mathbf{w}\right\} \quad (11a)$$

$$\text{s.t. } \Gamma_k = \frac{\mathbf{w}^H\widehat{\mathbf{R}}_h^k\mathbf{w}}{\mathbf{w}^H\left(\widehat{\mathbf{Q}}_k+\widehat{\mathbf{D}}_k\right)\mathbf{w}+\sigma_n^2} \geq \gamma_k \, \forall\, k=1...,\text{d} \quad (11b)$$

$$\forall\widehat{\mathbf{Q}}_k\in\mathbf{S}^{\widehat{\mathbf{Q}}_k}, \forall\widehat{\mathbf{D}}_k\in\mathbf{S}^{\widehat{\mathbf{D}}_k}, \widehat{\mathbf{D}}\in\mathbf{S}^{\widehat{\mathbf{D}}}, \forall\widehat{\mathbf{R}}_k\in\mathbf{S}^{\widehat{\mathbf{R}}_k}. \quad (11c)$$

where $\widehat{\mathbf{D}} = \mathbf{D} + \mathbf{\Delta_D}$, $\widehat{\mathbf{R}}_h^k = \mathbf{R}_h^k + \mathbf{\Delta}_k^{R^h}$, $\widehat{\mathbf{Q}}_k = \mathbf{Q}_k + \mathbf{\Delta}_k^{\mathbf{Q}}$, $\widehat{\mathbf{D}}_k = \mathbf{D}_k + \mathbf{\Delta}_k^{\mathbf{D}}$ are the perturbed versions of the matrices defined in (6)-(10). The block perturbation of the matrices is an approach to simplify the algebraic complexity appears due to propagation of perturbation from $\mathbf{f}_k$ and $\mathbf{g}_k$ to the matrices defined in (6)-(10). This approach is proposed by [12, 13]. In their works $\mathbf{f}_k$ is left unperturbed.

The matrices $\mathbf{\Delta_D}$, $\mathbf{\Delta}_k^{R^h}$, $\mathbf{\Delta}_k^{\mathbf{Q}}$ and $\mathbf{\Delta}_k^{\mathbf{D}}$ are the random uncertainty Hermitian matrices which are added to $\mathbf{D}$, $\mathbf{R}_{k}^h$, $\mathbf{Q}_k$ and $\mathbf{D}_k$, respectively. The sets of all possible values of $\widehat{\mathbf{D}}$, $\widehat{\mathbf{R}}_h^k$, $\widehat{\mathbf{Q}}_k$, $\widehat{\mathbf{D}}_k$ are denoted by $\mathbf{S}^{\widehat{\mathbf{D}}}$, $\mathbf{S}^{\widehat{\mathbf{R}}_h^k}$, $\mathbf{S}^{\widehat{\mathbf{Q}}_k}$ and $\mathbf{S}^{\widehat{\mathbf{D}}_k}$ respectively, which cover all cases of the perturbed channel coefficients. It is assumed that the $\mathbf{\Delta_D}$ and $\mathbf{\Delta}_k^{\mathbf{D}}$ are diagonal random matrices, because they are perturbing $\mathbf{D}$ and $\mathbf{D}_k$ matrices which are also diagonal. Furthermore, according to the worst-case approach,

we assume that the channel coefficient uncertainties are norm bounded by some known constants as

$$\|\mathbf{\Delta_D}\|\leq\varepsilon_{\mathbf{D}} \quad (12a)$$

$$\left\|\mathbf{\Delta}_k^{\mathbf{Q}}\right\| \leq \varepsilon_k^{\mathbf{Q}}, \left\|\mathbf{\Delta}_k^{\mathbf{D}}\right\| \leq \varepsilon_k^{\mathbf{D}}, \left\|\mathbf{\Delta}_k^{\mathbf{R}^h}\right\| \leq \varepsilon_k^{\mathbf{R}^h}. \quad (12b)$$

Since the perturbation matrices are not independent in general, the problem formulated by replacing (11c) by (12), results in suboptimal solution for (11). In order to guarantee positive power quantities and the convexity of our problem, the estimated channel matrices should be positive semi-definite as

$$\mathbf{D} + \mathbf{\Delta_D}\succeq 0 \quad (13a)$$

$$\mathbf{R}_h^k + \mathbf{\Delta}_k^{\mathbf{R}^h}\succeq 0, \mathbf{Q}_k + \mathbf{\Delta}_k^{\mathbf{Q}}\succeq 0, \mathbf{D}_k + \mathbf{\Delta}_k^{\mathbf{D}}\succeq 0. \quad (13b)$$

The first intractability of the robust optimization problem in (11b) is the infinite number of the constraints which the problem should be solved subject to them. In fact, the problem is a SIP [14]. Since SIP have some solutions [14], but computationally complex, it is preferred to avoid these problems by converting them to another standard form. To simplify the SIP robust problem (11b), first we need to maximize the objective function over the uncertainty matrix $\mathbf{\Delta_D}$. Then, the objective function for the robust problem is reformulated as

$$\underset{\mathbf{\Delta_D}}{\max}\left\{\mathbf{w}^H\left(\mathbf{D}+\mathbf{\Delta_D}\right)\mathbf{w}\right\} \quad (14)$$

Second, since all constraints of (11b) must be satisfied for all values of perturbed matrices, we can equivalently say that minimum value of the left side of the inequality should always be greater than the requested SINR. Considering all of the constraints, the robust optimization problem in (11b) can be expressed as

$$\underset{\mathbf{w}}{\text{Minimize}}\ \underset{\mathbf{\Delta_D}}{\max}\mathbf{w}^H\left(\mathbf{D}+\mathbf{\Delta_D}\right)\mathbf{w} \quad (15a)$$

$$\text{s.t.}\ \underset{\mathbf{\Delta}_k^{\mathbf{D}},\mathbf{\Delta}_k^{\mathbf{R}^h},\mathbf{\Delta}_k^{\mathbf{Q}}}{\min}\left\{\mathbf{w}^H\left(\mathbf{T}_k+\mathbf{\Delta}_k^{\mathbf{T}}\right)\mathbf{w}_k\right\} \geq \sigma_n^2\gamma_k, \forall k \quad (15b)$$

$$\text{Inequalities of} \quad (12),(13)\,\forall k=1,\cdots,d. \quad (15c)$$

where $\mathbf{T}_k = \mathbf{R}_k^h - \gamma_k\left(\mathbf{Q}_k+\mathbf{D}_k\right)$ and $\mathbf{\Delta}_k^{\mathbf{T}} = \mathbf{\Delta}_k^{\mathbf{R}^h} - \gamma_{th}\left(\mathbf{\Delta}_k^{\mathbf{Q}}+\mathbf{\Delta}_k^{\mathbf{D}}\right)$.

In contrast to our constraint formulation (15b), the approach of [12, 13] has differently qualified $\Gamma_k \geq \gamma_k$ by approximating $\frac{\min P_s^k}{\max\left(P_i^k+P_n^k\right)} \geq \gamma_k$ which is named MoM in our work.

In order to use the maximum term in the objective function of (15), along with its active constraint $\mathbf{D} + \mathbf{\Delta_D} \succeq 0$, we use the Rayleigh-Ritz theorem. Actually, when $\mathbf{\Delta_D}$ is in the same direction of $\mathbf{w}\mathbf{w}^H$, the objective function is maximized. The maximization over $\mathbf{\Delta_D}$ subject to its related constraints in (15) is

$$\underset{\mathbf{\Delta_D}}{\max.}\ \mathbf{w}^H\left(\mathbf{D}+\mathbf{\Delta_D}\right)\mathbf{w}$$
$$\text{s.t.}\ \mathbf{D}+\mathbf{\Delta_D}\succeq 0,\ \|\mathbf{\Delta_D}\|\leq\varepsilon_{\mathbf{D}} \quad (16)$$

where the worst case value for $\mathbf{\Delta_D}$ is

$$\mathbf{\Delta_D} = \frac{\mathbf{w}\mathbf{w}^H}{\|\mathbf{w}\|^2}. \quad (17)$$

Using (17), the robust form of the objective function of (15) will be

$$\mathbf{w}^H \left(\mathbf{D} + \varepsilon_\mathbf{D}\mathbf{I}\right)\mathbf{w} \tag{18}$$

Since the objective function and QoS constraints of (15) do not have any common constrains, the QoS and positive semi-definite constraints of (15) form an optimization problem over all of the perturbation matrices, which is independent from the objective function in (16). Therefore, we focus on the optimization of the QoS constraint in (15) for $k = 1, ..., d$. The constraint in (15) can be written as follows

$$\min_{\mathbf{\Delta}_k^\mathbf{D}, \mathbf{\Delta}_k^{\mathbf{R}^h}, \mathbf{\Delta}_k^\mathbf{Q}} \quad \mathbf{w}^H \left(\mathbf{T}_k + \mathbf{\Delta}_k^\mathbf{T}\right)\mathbf{w}_k - \sigma_n^2\gamma$$
$$\text{s.t.} \quad (12\text{b}), (13\text{b}) \,\forall\ k = 1, ..., d, \tag{19}$$

Note that the positive semi-definite (PSD) constraint of (19) also satisfies the corresponding constraints on its related instantaneous covariance matrices, $\mathbf{R}_h^k$, $\mathbf{Q}_k$ and $\mathbf{D}_k$. To solve the problem, we look for a relaxation scheme to convert the problem into a convex form and investigate the gap between the relaxed and non-relaxed problems. Applying Lagrange duality technique, the solution of (19) is equal to the solution of the following problem

$$\inf_{\mathbf{\Delta}_k^\mathbf{D}, \mathbf{\Delta}_k^{\mathbf{R}^h}, \mathbf{\Delta}_k^\mathbf{Q}} L \left(\lambda_k^\mathbf{Q}, \lambda_k^\mathbf{D}, \lambda_k^{\mathbf{R}^h}, \mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}, \mathbf{\Delta}_k^\mathbf{Q}, \mathbf{\Delta}_k^\mathbf{D}, \mathbf{\Delta}_k^{\mathbf{R}^h}\right)$$
$$\text{s.t.} \quad \mathbf{Z}_k^\mathbf{Q} \succeq 0, \mathbf{Z}_k^\mathbf{D} \succeq 0, \mathbf{Z}_k^{\mathbf{R}^h} \succeq 0, \lambda_k^\mathbf{Q} \geq 0, \lambda_k^\mathbf{D} \geq 0, \lambda_k^{\mathbf{R}^h} \geq 0$$
$$\text{and} \quad \mathbf{Z}_k^\mathbf{D} \text{ is diagonal matrix} \tag{20}$$

where the requirement of being diagonal on the dual variable $\mathbf{Z}_k^\mathbf{D}$ comes from the fact that $\mathbf{\Delta}_k^\mathbf{D}$ needs to be diagonal. Since the problem in (20) is convex, we can use the dual Lagrange function and Karush-Kuhn-Tucker (KKT) conditions to find the global optimum value of the problem [15]. Applying the KKT conditions, the Lagrange function will be

$$L = \mathbf{w}^H \Big(\mathbf{R}_k^h - \gamma_k\left(\mathbf{Q}_k + \mathbf{D}_k\right) + \mathbf{\Delta}_k^{\mathbf{R}^h} - \gamma_{th}\left(\mathbf{\Delta}_k^\mathbf{Q} + \mathbf{\Delta}_k^\mathbf{D}\right)\Big)\mathbf{w}$$
$$+ \lambda_k^\mathbf{Q}\left(\left\|\mathbf{\Delta}_k^\mathbf{Q}\right\|^2 - \left(\varepsilon_k^\mathbf{Q}\right)^2\right) + \lambda_k^\mathbf{D}\left(\left\|\mathbf{\Delta}_k^\mathbf{D}\right\|^2 - \left(\varepsilon_k^\mathbf{D}\right)^2\right)$$
$$+ \lambda_k^{\mathbf{R}^h}\left(\left\|\mathbf{\Delta}_k^{\mathbf{R}^h}\right\|^2 - \left(\varepsilon_k^{\mathbf{R}^h}\right)^2\right) - Tr\left(\mathbf{Z}_k^\mathbf{Q}\left(\mathbf{Q}_k + \mathbf{\Delta}_k^\mathbf{Q}\right)\right)$$
$$+ Tr\left(\mathbf{Z}_k^\mathbf{D}\left(\mathbf{D}_k + \mathbf{\Delta}_k^\mathbf{D}\right) + Z_k^{\mathbf{R}^h}\left(\mathbf{R}_h^k + \mathbf{\Delta}_k^{\mathbf{R}^h}\right)\right) - \sigma_n^2\gamma_k \tag{21}$$

where $\left\{\lambda_k^\mathbf{Q}\right\}_{k=1}^d$, $\left\{\lambda_k^\mathbf{D}\right\}_{k=1}^d$ and $\left\{\lambda_k^{\mathbf{R}^h}\right\}_{k=1}^d$ are the non-negative dual variables. For the sake of simplicity, we denote the above function by L_k. Using [16] and setting zero the derivatives of L_k with respect to $\mathbf{\Delta}_k^\mathbf{Q}$, $\mathbf{\Delta}_k^\mathbf{D}$ and $\mathbf{\Delta}_k^{\mathbf{R}^h}$, we have

$$\mathbf{\Delta}_k^\mathbf{Q} = \frac{\mathbf{Z}_k^\mathbf{Q} + \gamma_{th}\mathbf{X}}{2\lambda_k^\mathbf{Q}}, \ \mathbf{\Delta}_k^\mathbf{D} = \frac{\mathbf{Z}_k^\mathbf{D} + \gamma_{th}\mathbf{X}}{2\lambda_k^\mathbf{D}} \odot \mathbf{I}, \ \mathbf{\Delta}_k^{\mathbf{R}^h} = \frac{\mathbf{Z}_k^{\mathbf{R}^h} - \mathbf{X}}{2\lambda_k^{\mathbf{R}^h}}$$

Inserting the derived values for $\mathbf{\Delta}_k^\mathbf{Q}$, $\mathbf{\Delta}_k^\mathbf{D}$, $\mathbf{\Delta}_k^{\mathbf{R}^h}$ in (21) and maximizing the relation with respect to $\lambda_k^\mathbf{Q}$, $\lambda_k^\mathbf{D}$ and $\lambda_k^{\mathbf{R}^h}$, leads to the following form for the Lagrange dual function after some algebraic manipulations and using $\mathbf{X} = \mathbf{w}\mathbf{w}^H$

$$\underset{\mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}}{\text{Maximize}} M\left(\mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}, \mathbf{X}\right) \tag{22a}$$
$$\text{s.t.} \ \mathbf{Z}_k^\mathbf{Q} \succeq 0, \ \mathbf{Z}_k^\mathbf{D} \succeq 0, \ \mathbf{Z}_k^{\mathbf{R}^h} \succeq 0 \ \forall k = 1, ..., d \tag{22b}$$

where

$$M(.) = Tr\left(\mathbf{X}\mathbf{T}_k\right) - \frac{1}{2}\left\|\left(\mathbf{Z}_k^\mathbf{D} + \gamma_{th}\mathbf{X}\right) \circ \mathbf{I}\right\|\varepsilon_k^\mathbf{D} +$$
$$Tr\left(-\mathbf{Z}_k^{\mathbf{R}^h}\mathbf{R}_h^k - \mathbf{Z}_k^\mathbf{Q}\mathbf{Q}_k - \mathbf{Z}_k^\mathbf{D}\mathbf{D}_k\right) - \frac{\varepsilon_k^{\mathbf{R}^h}}{2}\left\|\mathbf{Z}_k^{\mathbf{R}^h} - \mathbf{X}\right\|$$
$$- \frac{\varepsilon_k^\mathbf{Q}}{2}\left\|\mathbf{Z}_k^\mathbf{Q} + \gamma_{th}\mathbf{X}\right\| - \sigma_n^2\gamma_k$$

By substituting (22) and (18) in (15), we can write our main robust optimization problem as

$$\min_{\mathbf{X}} \quad Tr\left(\mathbf{X}\left(\mathbf{D} + \varepsilon_\mathbf{D}\mathbf{I}\right)\right)$$
$$\text{s.t.} \quad \max_{\mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}} M\left(\mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}\right) \geq 0$$
$$\text{Constraints (22b), and } \text{Rank}\left(\mathbf{X}\right) = 1 \tag{23}$$

Consider the fact that maximum of the expression $Tr\left(-\mathbf{Z}_k^{\mathbf{R}^h}\mathbf{R}_h^k - \mathbf{Z}_k^\mathbf{Q}\mathbf{Q}_k - \mathbf{Z}_k^\mathbf{D}\mathbf{D}_k\right)$ is equal to zero (since $\mathbf{Z}_k^\mathbf{Q} \succeq 0$, $\mathbf{Z}_k^\mathbf{D} \succeq 0$, $\mathbf{Z}_k^{\mathbf{R}^h} \succeq 0$, $\forall\ k = 1, ..., d$.) and setting zero all the norm bounds in (23), the problem in (23) is transformed to the non-robust problem stated in [7]. If all the constraints in (23) be above zero, then the maximum constraint will also satisfy the inequality, so (23) can be simplified as follows

$$\min_{\mathbf{X}} \quad Tr\left(\mathbf{X}\left(\mathbf{D} + \varepsilon_\mathbf{D}\mathbf{I}\right)\right)$$
$$\text{s.t.} \quad M\left(\mathbf{Z}_k^\mathbf{Q}, \mathbf{Z}_k^\mathbf{D}, \mathbf{Z}_k^{\mathbf{R}^h}\right) \geq 0 \quad \forall k = 1, \cdots, d,$$
$$\text{Constraints (22b), and } \text{Rank}\left(\mathbf{X}\right) = 1 \tag{24}$$

Note that the objective function in (24) is linear and all the constraints except the last one are conic convex. We drop this non-convex constraint to relax the problem into a convex optimization problem. The well known semi-definite problem (SDP) solvers such as SeDuMi or CVX can be used for solving the above problem by semi-definite programming in polynomial time using interior point methods. Since the solution of (24) is not always rank one, randomization techniques [7] can be used to obtain an approximate solution of the original problem from the solution of the relaxed problem. However, our simulation results show that the rank of $\mathbf{X}$ is always one when $d < 3$. This has been also reported in [17] analytically for $d < 3$. If the optimum solution is a rank-one matrix, the principal value of the matrix is used to determine $\mathbf{w}$, otherwise the best rank one approximation is obtained using the procedure described in [18]. Note that the minimum value of the relaxed form (without rank constraint) of the problem in (24) is a lower bound for the minimum value of the original problem in (24). As mentioned after (12), the optimal solution of (24) is still suboptimal solution for (11) as it happens for [12, 13] too, but if uncertainties of the perturbation matrices are occurred due to the quantization noise of related matrices, they can be assumed independent and both problems have the same optimal solutions.

III. SIMULATION AND NUMERICAL RESULTS

In this section, we use Monte-Carlo simulations to compare our accurate robust with MoM robust [13] and non-robust power allocation methods. In all simulations, we assume 15 relays ($R = 15$) and 2 users ($d = 2$). The noise power
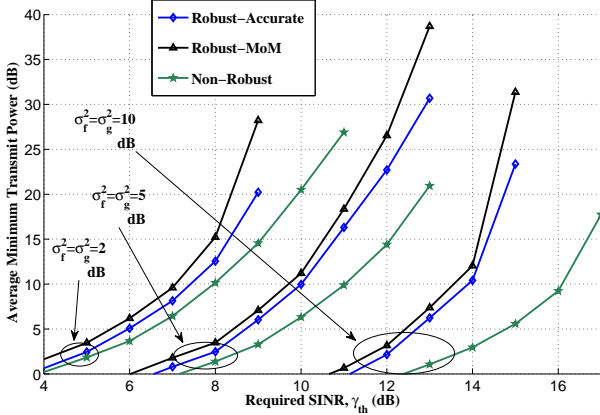
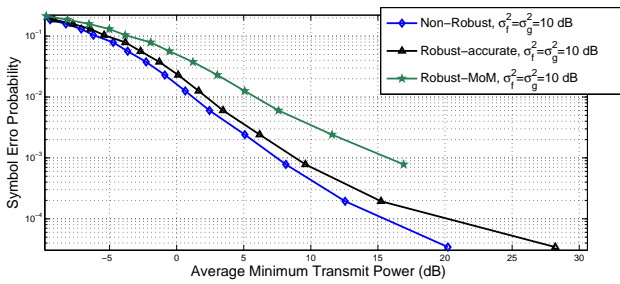Fig. 1. Average minimum transmit power versus the required SINR for $\varepsilon = 1\%\|.\|_F$ perturbation

Fig. 2. Comparison of worst-case symbol error probability between non-robust and robust systems by sweeping γ_{th}.

and source transmit power are assumed to be $0dB$ Watt and the channels are realized randomly with Rayleigh fading distribution. For the robust case, we suppose that the maximum perturbation norm of $\mathbf{D}$, $\mathbf{D}_k$, $\mathbf{R}_k^h$ and $\mathbf{Q}_k$ matrices are 1% of their Frobenius norms which we denote $\varepsilon = 0.01\|.\|_F$ by meaning that $\varepsilon_{\mathbf{X}} = 0.01\|\mathbf{X}\|_F$ for perturbation bound of any perturbed matrix $\mathbf{X}$.

In the first simulation setup, the average transmitted powers for the non-robust and both of the robust methods (MoM and Accurate) are depicted in Fig. 1 with respect to the required received SINR for different channel variances. Figure 1 shows the extra amount of the power that the robust methods consume as the expense of resistance against channel perturbations, while holding the QoS constraint in its desired range. It can be seen that our proposed accurate robust method outperforms MoM robust method for the difference channel variances.

In the last scenario, the efficiency of our proposed accurate robust method is compared with MoM robust and non-robust methods in terms of symbol error probability (SEP) and average transmitted power. Figure 2 shows the SEP versus average transmit power of the relays for the same variance $\sigma_f^2 = \sigma_g^2 = 10dB$ of the elements of the channels vector $\mathbf{f}, \mathbf{g}$ over 3000 iterations and 1% perturbation while at least 20% of the realization are feasible. Please note that the non-robust method is evaluated in non-perturbed settings to draw a benchmark for computing the power loss of other two robust methods. It is observed that for a specific amount of SEP, the accurate robust design outperforms the MoM and non-robust designs in terms of power consumption.

IV. CONCLUSION

In this paper, the robust optimal power allocation algorithm for multi-user cooperative networks was solved in a more accurate approach compared to the previous works. The proposed approach assumed uncertaintiy on all channels in the QoS aware beamforming problem to perform a robust design. The simulation results have shown the superiority of the proposed robust method compared to the previous robust and non-robust methods.

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