

Enforcing Almost-Sure Reachability in POMDPs

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Abstract

Partially-Observable Markov Decision Processes (POMDPs) are a well-known formal model for planning scenarios where agents operate under limited information about their environment. In safety-critical domains, the agent must adhere to a policy satisfying certain behavioral constraints. We study the problem of computing policies that almost-surely reach some goal state while a set of bad states is never visited. In particular, we present an iterative symbolic approach that computes a so-called winning region, that is, a set of system configurations such that all policies that stay within this set are guaranteed to satisfy the constraints. The empirical evaluation demonstrates the scalability and efficacy of our approach. In addition, we show the applicability to the safe exploration of POMDPs by restricting the agent behavior to these winning regions.

1 Introduction

Partially observable Markov decision processes (POMDPs) constitute the standard model for agents acting under partial information in uncertain environments (Kaelbling, Littman, and Cassandra 1998; Thrun, Burgard, and Fox 2005). A common, but undecidable, problem is to find a policy for the agent that maximizes a reward objective (Madani, Hanks, and Condon 1999). In safety-critical domains, however, one seeks a *safe* policy that exhibits strict behavioral guarantees, for instance in the form of temporal logic constraints (Pnueli 1977). To that end, we employ almost-sure reach-avoid specifications, where the probability to reach a set of *avoid* states is zero, and the probability to *reach* a set of goal states is one. Our **Problem 1** is to compute a policy that adheres to such specifications. The ability to solve this problem *for each state of the POMDP* is essential for satisfying general temporal logic constraints. Furthermore, we aim to ensure the *safe exploration of a POMDP*, with safe reinforcement learning (García and Fernández 2015) as direct application. **Problem 2** is then to compute a large set of safe policies for the agent to choose from at any state of the POMDP. Such sets of policies are referred to as *permissive policies*. These EXPTIME-complete problems require an exponential (in the number of states) amount of memory (Chatterjee, Doyen, and Henzinger 2010).

Approach. In a nutshell, our approach iteratively computes so-called *winning regions* (also: controllable or attracter re-

gions) in a backward fashion to solve the two aforementioned problems. A winning region is a part of the potentially infinite belief space of the POMDP from which there exists a (permissive) policy to satisfy the specification. For almost-sure specifications, such regions are sufficiently captured within the finite belief-support MDP. The states of this MDP are formed by the set of supports for all possible belief states of the POMDP. Starting from the belief support states that shall be reached almost-surely, further states are added to the winning region if there exists a known policy that reaches these states without visiting those that are to avoid. We iteratively employ an encoding based on *Boolean satisfiability solving* (SAT) (Biere et al. 2009). This symbolic encoding avoids an expensive explicit construction of the belief support states. The computed winning region directly translates to a sufficient set of constraints on the set of safe policies, i.e., each policy satisfying these constraints satisfies, by construction, the specification. The key idea is to successively add short-cuts that correspond to the already known policies satisfying the specification. These changes to the structure of the POMDP are performed implicitly on the SAT encoding.

Shielding. An agent that stays within a winning region is guaranteed to adhere to the almost-sure specification. It can be enforced to do so by *shielding* or *masking* any action that potentially leads the agent out of the winning region (Nam and Alur 2010; Pecka and Svoboda 2014; Akametalu et al. 2014). We stress that the shape of the winning region is independent of the transition probabilities or rewards in the POMDP. This independence means that the only prior knowledge we need to assume is the topology, that is, the graph of the POMDP. A pre-computation of the winning region thus yields a shield and allows us to restrict an agent to safely explore environments, which is the essential requirement for safe reinforcement learning (García and Fernández 2015; Ful-ton and Platzer 2018) of POMDPs. The shield can be used with any RL agent (Alshiekh et al. 2018).

Comparison with the state-of-the-art. Similar to our approach, (Chatterjee, Chmelik, and Davies 2016) solves almost-sure specifications using SAT. Intuitively, the aim is to find a so-called *simple policy* that is Markovian (aka memoryless). Such a policy may not exist, yet, the method can

be applied to a POMDP that has an extended state space to account for finite memory (Meuleau et al. 1999; Junges et al. 2018). There are three shortcomings that we overcome in this paper. First, one needs to pre-define the memory a policy has at its disposal, as well as a fixed lookahead on the exploration of the POMDP. Instead, we implicitly work on a fragment of the belief-support MDP. Second, the approach is only feasible if these bounds are small. Our approach scales beyond these memory bounds. Third, the approach finds a single simple policy starting from a pre-defined initial state. For safe exploration, this means that we may exclude many policies and never explore important parts of the system, harming the final performance of the agent. Shielding MDPs is not new (Bloem et al. 2015; Alshiekh et al. 2018; Jansen et al. 2020). However, those methods do neither take partial observability into account, nor can they guarantee reaching desirable states. Nam and Alur (2010) cover partial observability and reachability, but do not account for uncertainty.

Experiments. To showcase the feasibility of our method, we adopted a number of typical POMDP environments. We demonstrate that our method scales better than the state of the art. We evaluate the shield by letting an agent explore the POMDP environment according to the permissive policy, thereby enforcing the satisfaction of the almost-sure specification. We visualize the resulting behavior of the agent in those environments with a set of videos.

Contribution. Our paper makes three key contributions: (1) We present a new approach to computing policies satisfying almost-sure properties that can handle large environments requiring memory-based decision-making; the method is amenable to POMDPs whose belief-support states count billions; (2) We not only compute a single policy that satisfies the specification, but we determine a permissive policy, that is, a set of policies, and (3) using our method, we directly construct a shield for almost-sure specifications on POMDPs which enforces at runtime that *no unsafe states are visited* and that, under mild assumptions, *the agent almost-surely reaches the set of desirable states*.

Further related work. (Chatterjee et al. 2016) computes winning regions for minimizing a reward objective via an explicit state representation, or consider almost-sure reachability using an explicit state space (Chatterjee et al. 2015; Svorenová et al. 2015). The problem of determining any winning policy is related to strong cyclic planning, using, e.g., decision diagrams (Bertoli, Cimatti, and Pistore 2006).

Quantitative variants of reach-avoid specifications have gained attention in, e.g., (Norman, Parker, and Zou 2017; Horák, Bosanský, and Chatterjee 2018; Carr et al. 2019), often for simple policies (Poupart and Boutilier 2003; Amato, Bernstein, and Zilberstein 2010; Junges et al. 2018). (Wang, Chaudhuri, and Kavvaki 2018) uses an iterative Satisfiability Modulo Theories (SMT) (Barrett and Tinelli 2018) approach for quantitative finite-horizon specifications, which requires computing beliefs. Various general POMDP approaches exist, e.g., (Hauskrecht 2000; Shani, Pineau, and Kaplow 2013;

Walraven and Spaan 2017; Silver and Veness 2010; Jaakkola, Singh, and Jordan 1994; Wierstra et al. 2007; Hausknecht and Stone 2015). The underlying approaches depend on discounted reward maximization and are able to satisfy almost-sure specifications with high reliability. However, enforcing probabilities that are close to 0 or 1 requires a discount factor close to 1, drastically reducing the scalability of the aforementioned approaches (Horák, Bosanský, and Chatterjee 2018). Moreover, probabilities in the underlying POMDP need to be precisely given, which is not always realistic (Burns and Brock 2007).

Another line of work (for example (Turchetta, Berkenkamp, and Krause 2019)) uses an idea similar to winning regions with uncertain specifications, but in a fully observable setting. Finally, complementary to shielding, there are approaches that guide reinforcement learning (with full observability) via temporal logic constraints (Hasanbeig, Abate, and Kroening 2020; Hahn et al. 2019).

2 Winning Beliefs, Winning Regions, Shields

The support of a discrete probability distribution μ over X is denoted $\text{supp}(\mu) = \{x \in X \mid \mu(x) > 0\}$, with $\text{Distr}(X)$ the set of all distributions. A *Markov decision process* (MDP) is a tuple $\mathcal{M} = \langle S, \text{Act}, \mu_{\text{init}}, \mathbf{P} \rangle$ with a set S of states, an initial distribution $\mu_{\text{init}} \in \text{Distr}(S)$, a finite set Act of actions, and a transition function $\mathbf{P}: S \times \text{Act} \rightarrow \text{Distr}(S)$ for all $s \in S$ and $\alpha \in \text{Act}$. Let $\text{post}_s(\alpha) = \text{supp}(\mathbf{P}(s, \alpha))$ denote the states that may be the successors of the state $s \in S$ for action $\alpha \in \text{Act}$ under the distribution $\mathbf{P}(s, \alpha)$. If $\text{post}_s(\alpha) = \{s\}$ for all actions α , s is called *absorbing*.

A *partially observable MDP* (POMDP) is a tuple $\mathcal{P} = \langle \mathcal{M}, \Omega, \text{obs} \rangle$ where $\mathcal{M} = \langle S, \text{Act}, \mu_{\text{init}}, \mathbf{P} \rangle$ is the underlying MDP with finite S , Ω is a finite set of observations, and $\text{obs}: S \rightarrow \Omega$ is an observation function. More general observation functions $\text{obs}: S \rightarrow \text{Distr}(\Omega)$ are possible via a (polynomial) reduction (Chatterjee et al. 2016). A path through an MDP is a sequence π , of states and actions. The observation function obs applied to a path yields a *trace*: a sequence $\text{obs}(\pi)$ of observations and actions.

A policy $\sigma: (S \times \text{Act})^* \times S \rightarrow \text{Distr}(\text{Act})$ maps a path π to a distribution over actions. A policy is *observation-based*, if for each two paths π, π' it holds that $\text{obs}(\pi) = \text{obs}(\pi') \Rightarrow \sigma(\pi) = \sigma(\pi')$. For POMDPs, the notion of a *belief* describes the probability of being in certain state based on an observation sequence. Formally, a belief b is a distribution $b \in \text{Distr}(S)$ over the states. A state s with positive belief $b(s)$ is in the *belief support*, $s \in \text{supp}(b)$.

2.1 Problem statement

The policy synthesis problem usually consists in finding a policy that satisfies a certain specification for a POMDP. We consider *reach-avoid* specifications, a subclass of indefinite horizon properties (Puterman 1994). For a POMDP \mathcal{P} with states S , such a specification is $\varphi = \langle \text{REACH}, \text{AVOID} \rangle \subseteq S \times S$. We assume that states in *AVOID* and in *REACH* are (made) absorbing. Let $\text{Pr}_b^\sigma(S')$ denote the probability to reach a set $S' \subseteq S$ of states from belief b under the policy σ . More precisely, $\text{Pr}_b^\sigma(S')$ denotes the probability of all paths

that reach S' from b when all nondeterminism is resolved by the policy σ .

Definition 1 (Winning). *A policy σ is winning for φ from belief b , iff $Pr_b^\sigma(\text{AVOID}) = 0$ and $Pr_b^\sigma(\text{REACH}) = 1$, i.e., if it reaches AVOID with probability zero and REACH with probability one (almost-surely) from all states within the current belief. Belief b is winning for φ , if there exists a winning policy from b .*

Problem 1: Given a POMDP, a belief b , and a specification φ , decide whether b is winning.

The problem is EXPTIME-complete (Chatterjee, Doyen, and Henzinger 2010). We emphasize that this problem setting may be applied to general specifications in linear temporal logic formulas, by extending the POMDP with an associated automaton. This standard construction is for instance described in (Bouton, Tumova, and Kochenderfer 2020).

We now consider sets of winning beliefs.

Definition 2 (Winning region). *Let σ be a policy. A set $W_\varphi^\sigma \subseteq \text{Distr}(S)$ of beliefs is a winning region for φ and σ , if σ is winning from each $b \in W_\varphi^\sigma$.*

We make three key observations. First, for qualitative reach-avoid specifications the belief probabilities are irrelevant—*only the belief support is important*. Second, additionally, if a policy is winning for a belief with support B , *this policy is also winning for a belief whose support is contained in B* . Third, winning policies for individual beliefs may be composed to another winning policy that is winning for all of these beliefs, using the individual action choices.

Lemma 1. (1): *If belief b with support $\text{supp}(b) = B$ is winning, then belief b' with support $\text{supp}(b') = B'$ and $B' \subseteq B$ is winning.* (2): *If the policies σ and σ' are winning for the beliefs b and b' , respectively, then there exists a policy σ'' that is winning for both b and b' .*

These observations allow us to formulate the following problem without depending on a concrete policy. They are fundamental for the key idea presented later.

Problem 2: Given a POMDP \mathcal{P} and a specification φ , find a (large) winning region W_φ .

2.2 From winning regions to shields

We aim to define a *shield* that imposes restrictions on policies to satisfy the specification. Technically, we adapt so-called *permissive* policies (Dräger et al. 2015; Junges et al. 2016) for a belief-support MDP. For a POMDP $\mathcal{P} = \langle \mathcal{M}, \Omega, \text{obs} \rangle$ with $\mathcal{M} = \langle S, \text{Act}, \mu_{\text{init}}, \mathbf{P} \rangle$, the finite state space of the *belief-support MDP* is given by $S^b = \{B \subseteq S \mid \forall s, s' \in B: \text{obs}(s) = \text{obs}(s')\}$, that is, each state is the support of a belief state. Action α in B leads (with an irrelevant positive probability) to a state B' , if there is an $s \in B$ and $s' \in B'$ such that $s' \in \text{post}_s(\alpha)$, that is, transitions between states within B and B' are mimicked.

A winning region can be interpreted as a fragment of a belief-support MDP. To force an agent to stay within a winning region W_φ for specification φ , we define a φ -*shield* $\nu: S^b \rightarrow 2^{\text{Act}}$ such that for any B that is winning for φ we

have $\nu(B) \subseteq \{\alpha \in \text{Act} \mid \text{post}_B(\alpha) \subseteq W_\varphi\}$, that is, an action is part of the shield $\nu(B)$ if it exclusively leads to belief support states that are inside the winning region.

A shield restricts the set of actions an arbitrary policy may take. We call such restricted policies *admissible*. Specifically, let B_τ be the belief support after observing a sequence of observations τ . Then policy σ is ν -admissible if $\text{supp}(\sigma(\tau)) \subseteq \nu(B_\tau)$ for every observation-sequence τ . Consequently, a policy is *not* admissible if for some observation sequence τ , the policy selects an action $\alpha \in \text{Act}$ which is not allowed by the shield.

Some admissible policies may choose to stay in the winning region without progressing towards the *REACH* states. Such a policy adheres to the avoid-part of the specification, but violates the reachability part. To enforce *progress*, we adapt a notion from formal methods called *fairness*. A policy is fair if it takes every action infinitely often at any belief support state that appears infinitely often along a trace (Baier and Katoen 2008). For example, a policy that randomizes (arbitrarily) over all actions is fair. If φ is a specification where *REACH* = \emptyset , we can drop the fairness assumption.

Theorem 1. *For a φ -shield, any fair φ -admissible policy satisfies φ .*

3 Iterative SAT-Based Computation of Winning Regions

In the remainder of the paper, we consider the computation of winning regions. In particular, we devise an approach for iteratively computing an increasing sequence of winning regions. The approach delivers a compact symbolic encoding of winning regions: For a belief (or belief-support) state from a given winning region, we can efficiently decide whether the outcome of an action emanating from the state stays within the winning region. This operation is essential for the functioning of a shield.

For modeling flexibility, we allow actions to be unavailable in a state (e.g., opening doors is only available when at a door), and it turned out to be crucial to handle this explicitly in the following algorithms. Technically, the transition function is a partial function, and the enabled actions are a set $\text{EnAct}(s) = \{\alpha \in \text{Act} \mid \text{post}_s(\alpha) \neq \emptyset\}$. To ease the presentation, we assume that states s, s' with the same observation share a set of enabled actions $\text{EnAct}(s) = \text{EnAct}(s')$.

We start our discussion by briefly recapping how to compute memoryless winning policies in Sect. 3.1, before we present a novel solution in Sect. 3.2.

3.1 One-shot approach to find small policies from a single belief (support) state

Let us first consider Problem 1, i.e., how to find a winning policy for a fixed belief support B . The number of policies is exponential in the actions and the exponentially many belief support states. Searching among doubly exponentially many possibilities is intractable in general. However, Chatterjee et al. (Chatterjee, Chmelik, and Davies 2016) observe that often much simpler winning policies exist and provides a *one-shot approach* to find them. Concretely, a memoryless observation-based policy $\sigma: \Omega \rightarrow \text{Distr}(\text{Act})$ is com-

puted that is winning for (initial) belief support B and an almost-sure reachability specification φ . This problem is NP-complete, and it is thus natural to encode the problem as a satisfiability query in propositional logic. We mildly extend the original encoding of winning policies (Chatterjee, Chmelik, and Davies 2016). It is both necessary and sufficient that the policy ensures *progress* with positive probability, which is encoded by means of a *ranking* of states, where reaching a lower ranked state means progress.

We introduce three sets of Boolean variables: $A_{z,\alpha}$, C_s and $P_{s,j}$. If a policy takes action $\alpha \in \text{Act}$ with positive probability upon observation $z \in \Omega$, then and only then, $A_{z,\alpha}$ is true. If under this policy a state $s \in S$ is reached from initial belief support B with positive probability, then and only then, C_s is true. We define a maximal rank k to assure progress. For each state s and rank $0 \leq j \leq k$, variable $P_{s,j}$ indicates rank j for s , that is, a path from s leads to $s' \in \text{REACH}$ within j steps. A winning policy is then obtained by finding a satisfiable solution (via a SAT solver) to the conjunction $\Psi_P^\varphi(B, k)$ of the constraints (1)–(5), where $S_\gamma = S \setminus \text{AVOID} \setminus \text{REACH}$.

$$\bigwedge_{z \in \Omega} \bigvee_{\alpha \in \text{EnAct}(z)} A_{z,\alpha} \wedge \bigwedge_{s \in B} C_s \quad (1)$$

$$\bigwedge_{\substack{s \in S \\ \alpha \in \text{EnAct}(s)}} C_s \wedge A_{\text{obs}(s),\alpha} \rightarrow \bigwedge_{s' \in \text{post}_s(\alpha)} C_{s'} \quad (2)$$

$$\bigwedge_{s \in \text{AVOID}} \neg C_s \wedge \bigwedge_{s \in S_\gamma} C_s \rightarrow P_{s,k} \quad (3)$$

The conjunction in (1) ensures that in every observation, at least one action is taken, that the states in B are in the set of reached states. The conjunction (2) ensures that this set is transitively closed under reachability (for the policy described by $A_{z,\alpha}$). Finally, conjunctions (3) ensure that no states in AVOID are reached, and that any state that is reached almost-surely reaches a state in REACH .

$$\bigwedge_{s \notin \text{REACH}} \neg P_{s,0} \quad (4)$$

$$\bigwedge_{\substack{s \in S_\gamma \\ 1 \leq j \leq k}} P_{s,j} \leftrightarrow \left(\bigvee_{\alpha \in \text{EnAct}(s)} (A_{\text{obs}(s),\alpha} \wedge \left(\bigvee_{s' \in \text{post}_s(\alpha)} P_{s',j-1} \right)) \right) \quad (5)$$

Conjunctions (4) and (5) describe a ranking function. Only states in REACH have rank zero, and a state with positive probability to reach a state with rank $j-1$ has rank at most j .

By (Chatterjee, Chmelik, and Davies 2016, Thm. 2), it holds that $\Psi_P^\varphi(b, k)$ is satisfiable, if there is a memoryless observation-based policy such that φ is satisfied. If $k = |S|$, then the reverse direction also holds. If $k < |S|$, we may miss states with a higher rank. Large k values are practically intractable (Chatterjee, Chmelik, and Davies 2016), as the encoding grows significantly with k . Pandey and Rintanen (Pandey and Rintanen 2018) propose extending SAT-solvers with a dedicated handling of ranking constraints.

In order to apply this to small-memory policies, one can unfold $\log(m)$ bits of memory of such a policy into an m times larger POMDP (Chatterjee, Chmelik, and Davies 2016; Junges et al. 2018), and then search for a memoryless policy

in this larger POMDP. Chatterjee et al. (Chatterjee, Chmelik, and Davies 2016) include a slight variation to this unfolding, allowing smaller-than-memoryless policies by enforcing the same action over various observations.

3.2 Incremental encoding of the winning region

We avoid the following restrictions of the one-shot approach. (1) In order to increase the likelihood of finding winning policies, we do not restrict ourselves to (very) small-memory policies, and (2) we do not have to fix a maximal rank k . These modifications allow us to find more winning policies, without guessing hyper-parameters. As we do not need to fix the belief-state, those parts of the winning region that are easy to find for the solver are encountered first. This optimism of finding small and easy strategies first is beneficial for the performance, but does not restrict the generality.

Idea. The key idea is that we iteratively add short-cuts that represent known winning policies. We find a winning policy σ for some belief states in the first iteration, and then add a fresh action to all (original) POMDP states: This action leads to a REACH state, if the state is part of a winning belief-support under policy σ , and an AVOID state otherwise. Adding this action does not change winning regions in the POMDP (see Lemma 1), but extends the belief support states that may win via a memoryless policy. Instead of actually adjusting the POMDP, we realize this idea directly on the encoding. We find winning states based on a solution, and instead of adding actions, we allow the solver to decide following individual policies from each observation.

Example. Consider the small Cheese-POMDP (Littman, Cassandra, and Kaelbling 1995) in Fig. 1(left) with $\text{REACH} = \{10\}$ and $\text{AVOID} = \{9, 11\}$. From belief support $B = \{6, 8\}$ there is no memoryless winning policy—In states $\{6, 8\}$ we have to go north, which prevents us from going south in state 7. However, we can find a memoryless winning policy for $\{1, 5\}$, see Fig. 1 (center). If we add shortcuts, we can now find a memoryless winning policy for $B = \{6, 8\}$, depicted in Fig. 1(right).

Encoding. First, we use unbounded (real) variables rather than Boolean variables for the ranking (Wimmer et al. 2014). This relaxation avoids the growth of the encoding and admits arbitrarily large ranks with a fixed-size encoding into difference logic. This logic is an extension to propositional logic that can be checked using a satisfiability-modulo-theories (SMT) solver (Barrett et al. 2009). We use a data structure Win such that $\text{Win}(z)$ encodes all winning belief supports with observation z . Internally, the data structure stores maximal winning belief supports as bit-vectors. By construction, for every $B \in \text{Win}(z)$ a winning region exists, i.e., conceptually, there is a shortcut-action leading to REACH .

Our encoding represents an observation-based policy that can decide to take a shortcut, which means that it follows a previously computed winning policy from there (using Lemma 1). In addition to $A_{z,\alpha}$ and C_s from the previous encoding, we use the following variables: The ranking of state s is now encoded using a real-valued R_s . Furthermore,

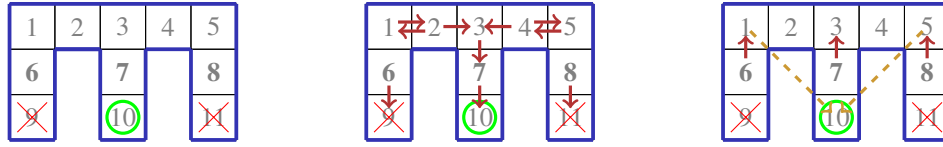


Figure 1: Example: States are cells, actions are moving in the cardinal directions (if possible), and observations are the directions with adjacent cells, e.g., the boldface states 6, 7, 8 share an observation.

the policy takes shortcuts in states s where D_s is true. For each observation, we must take the same shortcut, referred to by a positive integer-valued index I_z . The policy may decide to *switch*, that is, to follow a shortcut *after* taking an action starting in a state with observation z . If F_z is true, the policy takes some action from z -states, we take a shortcut. Finally, a variable $U_z \in \mathbb{B}$ encodes if the policy is winning in a belief support that is not yet in $\text{Win}(z)$.

The conjunction of (6)–(13) yields the encoding $\Phi_{\mathcal{P}}^{\varphi}(\text{Win})$:

$$\bigwedge_{z \in \Omega} \bigvee_{\alpha \in \text{EnAct}(z)} A_{z,\alpha} \wedge \bigwedge_{s \in \text{AVOID}} \neg C_s \wedge \neg D_s \quad (6)$$

$$\bigwedge_{\substack{s \in S \\ \alpha \in \text{EnAct}(s) \\ z = \text{obs}(s)}} \left(C_s \wedge A_{\text{obs}(s),\alpha} \wedge \neg F_{z(s)} \rightarrow \bigwedge_{s' \in \text{post}_s(\alpha)} C_{s'} \right) \quad (7)$$

$$\bigwedge_{\substack{s \in S \\ \alpha \in \text{EnAct}(s) \\ z = \text{obs}(s)}} \left(C_s \wedge A_{z,\alpha} \wedge F_z \rightarrow \bigwedge_{s' \in \text{post}_s(\alpha)} D_{s'} \right) \quad (8)$$

Similar to before, we select at least one action and avoid states should not be reached (6). Reached states are closed under the transitive closure, however, only if we do not switch to taking a shortcut (7). Furthermore, we mark the states reached after switching (8)—we need to select a shortcut for these states.

$$\bigwedge_{s \in S} D_s \rightarrow I_{\text{obs}(s)} > 0 \wedge \bigwedge_{z \in \Omega} I_z \leq |\text{Win}(z)| \quad (9)$$

$$\bigwedge_{\substack{z \in \Omega \\ 0 \leq i \leq |\text{Win}(z)|}} \bigwedge_{\substack{s \in S \setminus \text{Win}(z)_i \\ \text{obs}(s) = z}} I_z \neq i \vee \neg D_s \quad (10)$$

If we reach a state s after switching, then we must pick a shortcut (9). This shortcut must correspond to a policy that is winning for belief support $\{s\}$, based on our knowledge stored in Win_z . Equivalently, we cannot pick a strategy that is not winning for $\{s\}$: In every state, we either do not follow a shortcut or the associated policy is (not not) winning (10).

$$\bigwedge_{s \in S} C_s \rightarrow \left(\bigvee_{\alpha \in \text{EnAct}(s)} \left(A_{\text{obs}(s),\alpha} \wedge \left(\bigvee_{s' \in \text{post}_s(\alpha)} R_s > R_{s'} \right) \right) \vee F_{z(s)} \right) \quad (11)$$

The ranking function is updated: Either we have a successor state with a lower rank (as before, but with real-valued ranks), or we follow a shortcut—which either leads to the target or

Algorithm 1 Naive construction of winning regions

Input: POMDP \mathcal{P} , reach-avoid specification φ ,

Output: Winning region encoded in Win

$\text{Win}(z) \leftarrow \{s \in \text{REACH} \mid \text{obs}(s) = z\}$ for all $z \in \Omega$

$\Phi \leftarrow \text{Encode}(\mathcal{P}, \varphi, \text{Win})$ ▷ Create encoding (6)–(13).

while $\exists \nu$ s.t. $\nu \models \Phi$ **do** ▷ Call an SMT solver

$\text{Win}(z) \leftarrow \text{Win}(z) \cup \{B \mid s \in B \text{ iff } \nu(C_s)\}$ for all $z \in \Omega$

$\Phi \leftarrow \text{Encode}(\mathcal{P}, \varphi, \text{Win})$

to a violation of the specification (11).

$$\bigvee_{z \in \Omega} U_z \wedge \bigwedge_{\substack{z \in \Omega \\ \text{Win}(z) = \emptyset}} \left(U_z \leftrightarrow \bigvee_{\substack{s \in S \\ \text{obs}(s) = z}} C_s \right) \quad (12)$$

$$\bigwedge_{\substack{z \in \Omega \\ \text{Win}(z) \neq \emptyset}} \left(U_z \leftrightarrow \bigvee_{X \in \text{Win}(z)} \bigwedge_{\substack{s \in S \setminus X \\ \text{obs}(s) = z}} \neg C_s \right) \quad (13)$$

Finally, we ensure extending the winning region with at least one belief support. For an observation for which we have not found a winning belief support yet, finding any policy from any state with this observation updates the winning region (12). For other observations, it means finding a winning policy for a belief support that is not subsumed by a previous one (13), see also Lemma 1.

Naive algorithm. Algorithm 1 starts with a winning region consisting of reach states. We encode the problem as above and use an SMT solver to find a new winning policy extending the winning region, and iterate until we find no further policy. We update $\text{Win}(z)$ if the solver indicates that the winning region is extended. In each iteration, Win contains a winning region. When we find no more policies on the (conceptually) extended POMDP, we terminate.

Theorem 2. *If $\nu \models \Phi_{\mathcal{P}}^{\varphi}(\text{Win})$, then $B_z = \{s \mid \nu(C_s) = \text{true}, \text{obs}(s) = z\}$ is a winning belief support. Thus, in any iteration, Algorithm 1 computes a winning region.*

The algorithm always terminates because the set of winning regions is finite while it does not necessarily find the maximal winning region. Formally, the winning region is the greatest fixpoint and we iterate from below. However, iterating from above requires to reason that none of the doubly-exponentially many policies is winning for a particular belief support state; whereas our approach profits from finding simple strategies early on. Unfolding of memory as discussed earlier also makes this algorithm complete, yet suffering from the same blow-up. A main advantage is that the algorithm often avoids the need for unfolding when searching for a winning policy or large winning regions.

Algorithm 2 Incremental construction of winning regions

Input: POMDP \mathcal{P} , reach-avoid specification φ
Output: Winning region encoded in Win
 $\text{Win}(z) \leftarrow \{s \in \text{REACH} \mid \text{obs}(s) = z\}$ for all $z \in \Omega$
 $\text{Win} \leftarrow \text{GraphPreprocessing}(\text{Win})$
 $\Phi_{\text{fix}} \leftarrow \text{Encode}_{\text{fix}}(\mathcal{P}, \varphi, \text{Win})$ \triangleright Create encoding (6)–(10),(11)
 $\Phi_{\text{inc}} \leftarrow \text{Encode}_{\text{inc}}(\mathcal{P}, \varphi, \text{Win})$ \triangleright Encode (12), (13)
while $\exists \nu$ s.t. $\nu \models \Phi$ **do** \triangleright Call an SMT solver, fix ν
 do \triangleright Extend policy
 $\Phi_{\nu} \leftarrow \bigwedge \{A_{z,\alpha} \mid \nu(U_z) \wedge \nu(A_{z,\alpha})\}$ \triangleright Part. fix policy
 while $\exists \nu$ s.t. $\nu \models \Phi_{\text{fix}} \wedge \Phi_{\text{var}} \wedge \Phi_{\nu}$ \triangleright Call SMT, fix ν
 $\text{Win}(z) \leftarrow \text{Win}(z) \cup \{B \mid s \in B \text{ iff } \nu(C_s)\}$ for all $z \in \Omega$
 $\text{Win} \leftarrow \text{GraphPreprocessing}(\text{Win})$
 $\Phi_{\text{fix}} \leftarrow \Phi_{\text{fix}} \wedge \text{Encode}_{(9),(10)}(\mathcal{P}, \varphi, \text{Win})$ \triangleright Update: (9),(10)
 $\Phi_{\text{inc}} \leftarrow \text{Encode}_{\text{inc}}(\mathcal{P}, \varphi, \text{Win})$ \triangleright Encode (12), (13)

Graph-based preprocessing. To reduce the number of SMT invocations, we employ polynomial-time graph-based heuristics. The first step is to find all states that violate the specification, and make them absorbing. Then, we iteratively search for more winning observations. An observation z is winning, if the belief-support $\{s \mid \text{obs}(s) = z\}$ is winning. We start with a previously determined winning region W . We iteratively update W by adding states $S' = \{s \mid \text{obs}(s) = z\}$ for some observation z , if there is an action α such that from every $s \in S'$, taking that action leads to the current winning region within one step, i.e., $\sum_{s' \in W} \mathbf{P}(s, \alpha, s') = 1$.

Optimized algorithm. We improve the naive algorithm along the following four lines to obtain Algorithm 2¹: First, before updating the winning region with a policy, we aim to extend the policy as much as possible, i.e., we want to find more states with the same observation that are winning under the same policy. Therefore, we fix the variables for action choices that yield a new winning policy, and search whether we can extend this policy by finding more states and actions that are compatible with the policy. Second, we observe that large parts of the encoding stay intact, and use an incremental approach in which we first push all the constraints from the POMDP onto the stack, then all the constraints from the winning region, and finally a constraint that asks for progress. After we found a new policy, we pop the last constraint from the stack, add new constraints regarding the winning region (notice that the old constraints remain intact), and push new constraints that ask for extending the winning region to the stack. We refresh the encoding periodically to avoid unnecessary cluttering. Third, further constraints (1) make the usage of shortcuts more flexible—we allow taking shortcuts either immediately or after the next action, and (2) enable an even more incremental encoding. These constraints yield a faster convergence of the algorithm. Fourth, we add the graph-preprocessing discussed above during the outer iteration.

4 Empirical Evaluation

We evaluate our iterative approach to find winning policies or large winning regions, as introduced in Sect. 3.2, against our

¹Some tweaks are abstracted, we refer to the implementation.

adaption and implementation of the one-shot approach (Chatterjee, Chmelik, and Davies 2016), see Sect. 3.1.

Setting. We implemented the one-shot algorithm and our iterative algorithm, on top of the model checker STORM (Dehnert et al. 2017) and the SMT solver Z3 (de Moura and Bjørner 2008). We use a MacBook Pro MV962LL/A, a single core, no randomization, and never exceed a 4GB memory limit. The time-out (TO) is 15 minutes.

Set-up. We consider two variants of the iterative algorithm. (1, for finding **fixpoints**): The optimized algorithm as described above. (2, for finding a policy from the **initial** state): The algorithm as before, but any outer iteration starts with an SMT-check to see whether we find a policy covering the initial states. We realize the latter by fixing (temporarily) the C_s -variables. In the first iteration, this configuration and its resulting policy closely resemble the one-shot approach.

Baseline. We compare with the one-shot algorithm and apply our novel graph-based preprocessing to identify more winning observations. There are two settings: (1) We (manually, a-priori) search for the **optimal parameters**: each instance for the smallest amount of memory possible, and for the smallest maximal rank (subject to being a multiplicative of five) that yields a result. Guessing parameters as an “oracle” is hard and time-consuming. Therefore, we investigate (2) the performance of the one-shot algorithm by **fixing the parameters** to two memory-states and a maximal rank of 30. These parameters provide results for most benchmarks.

Benchmarks. Our benchmarks involve agents operating in $N \times N$ grids, inspired by, e.g., (Svorenová et al. 2015; Chatterjee, Chmelik, and Davies 2016; Smith and Simmons 2004; Dietterich 1998; Brockman et al. 2016).

While our approaches work for general POMDPs, we focus on grid worlds to enable a suitable visualization, see Fig. 2 for video stills of simulating the following benchmarks. *Rocks* is a variant of *rock sample*. The grid contains two rocks which are either valuable or dangerous to collect. To find out with certainty, the rock has to be sampled from an adjacent field. The goal is to collect a valuable rock, bring it to the drop-off zone, and not collect dangerous rocks. *Refuel* concerns a rover that shall travel from one corner to the other, while avoiding an obstacle on the diagonal. Every movement costs energy and the rover may recharge at recharging stations to its full battery capacity E . It receives noisy information about its position and battery level. *Evade* is a scenario where a robot needs to reach a destination and evade a faster agent. The robot has a limited range of vision (R), but may scan the whole grid instead of moving. A certain safe area is only accessible by the robot. *Avoid* is a related scenario where a robot shall keep distance to patrolling agents. These agents move with uncertain speed, yielding partial information about their position, but the robot may exploit their predefined routes. Details on *Intercept* and *Obstacle* are in the supplements.

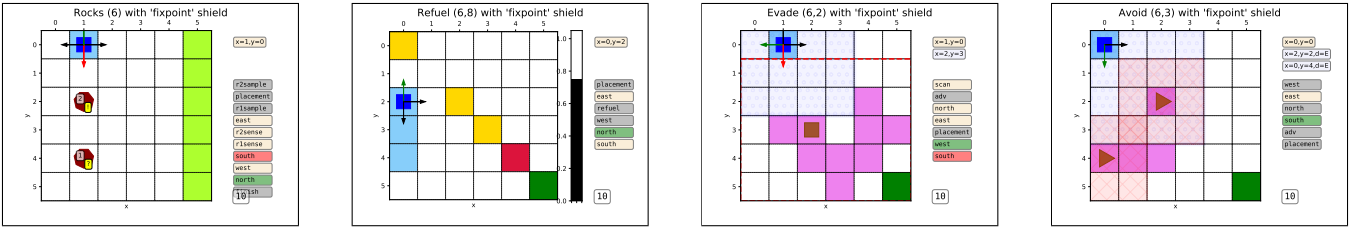


Figure 2: Video stills from simulating a shielded agent on four different benchmarks. Videos can be found online.

		Inst.	Rocks (N)		Refuel (N,E)		Evade (N,R)		Avoid (N,R)		Intercept (N,R)		Obstacle (N)		
			4	6	6,8	7,7	6,2	7,2	6,3	7,4	7,1	7,2	6	8	
		$ S $	331	816	270	302	4232	8108	5976	13021	4705	4705	37	65	
		#Tr	3484	7292	1301	1545	28866	57570	14373	33949	18049	18049	224	421	
		$ \Omega $	65	74	36	35	2202	4172	3300	8584	2002	2598	4	4	
		$ S^b $	3.5E5	7.7E25	5.6E14	7.4E19	1.1E8	4.4E11	1.1E15	2.9E17	6.4E10	2.7E9	1.1E9	2.9E17	
incremental	fixpoint	Time	19	753	6	3	142	613	167	745	116	86	2	30	
		#Iter.	36	284	40	30	4	6	3	4	8	8	68	150	
		#solve	1702	13650	1023	528	681	1129	629	1027	1171	976	839	4291	
		$ W $	3.5E5	7.7E25	1.2E11	2.1E8	1.0E8	4.2E11	1.1E15	2.9E17	9.2E4	2.9E4	4.1E7	3.8E14	
initial	initial	Time	17	226	2	2	49	576	10	40	11	2	<1	<1	
		#Iter.	29	65	2	4	1	1	1	1	2	1	10	12	
		#solve	1215	2652	62	80	1	1	1	1	81	1	114	229	
		$ W $	4.4E4	1.8E13	8.4E6	3.7E4	5.0E7	1.0E11	3.7E5	6.9E10	6.2E3	2.1E3	4.1E5	4.5E9	
1-shot	opt	Time	120	TO	2	<1	12	270	22	53	8	1	1	195	
		Mem,k	2,10	?	2,15	2,15	1,20	1,30	1,30	1,25	2,10	1,10	6,10	5,50	
1-shot	fix	opt	Time	TO	TO	11	37	TO	TO	TO	TO	28	18	N/A	N/A

Table 1: For each benchmark instance (columns), we report the name and relevant characteristics: the number of states ($|S|$), the number of transitions (#Tr, the edges in the graph described by the POMDP), the number of observations ($|\Omega|$), and the number of belief support states ($|S^b|$). For the computation of winning regions, we provide the run time (Time, in seconds), the number of outer iterations (#Iter.) in Alg. 2, and the number of invocations of the SMT solver (#solve), and the approximate size of the winning region ($|W|$). We then report these numbers when searching for a policy that wins from the initial state. For the 1-shot method, we provide the time for the optimal parameters (on the next line)—TOs reflect settings in which we did not find any suitable parameters, and the time for the preset parameters (2,30), or N/A if no policy can be found with these parameters.

Results for Problem 1. Tab. 1 details the numerical benchmark results. The iterative algorithm finds winning policies for the **initial** state *without guessing parameters* and is often *faster* versus the one-shot approach with an oracle providing **optimal** parameters, and significantly faster versus the one-shot approach with reasonably **fixed** parameters. In more detail: *Rocks* shows that the model can handle large numbers of iterations, solver invocations, and winning regions. The iterative nature significantly outperforms the one-shot approach. The iterative approach does also scale to larger models, see e.g., *Avoid*. *Refuel* shows the large sensitivity of the one-shot method on the lookahead (going from 15 to 30 increases the runtime), while *Evade* shows the sensitivity to the memory (from 1 to 2). In contrast, the iterative approach does not rely on user-input, yet delivers comparable performance on *Refuel* or *Avoid*. It suffers slightly on *Evade*, where the one-shot approach has a reduced overhead.

Results for Problem 2. Winning regions obtained from running iteratively to a **fixpoint** are significantly larger than from running them until an **initial** winning policy is found (cf. the table), but requires some extra computational effort. If we let a *shielded agent* move randomly through the grid-worlds,

we see on the videos (Fig. 2) that the larger winning regions indeed translate to more permissiveness, that is, freedom to move for the agent. The shield is correct by construction, thus all runs indeed never reach an avoid state, and eventually reach the target (albeit after many steps). The latter is not true for the unshielded agents.

Data availability. Videos, source code, model files, and logs of the experiments are available open-source on github.com/sjunges/shielding-POMDPs.

5 Conclusion

We provided an iterative approach to find POMDP policies that satisfy almost-sure reachability specifications. The superior scalability is demonstrated on a string of benchmarks. Furthermore, this approach allows to shield agents in POMDPs and guarantee that any exploration of an environment satisfies the specification, without needlessly restricting the freedom of the agent. While we demonstrate the effectiveness of our approach in terms of random exploration of an environment, we plan to investigate a tight interaction with state-of-the-art reinforcement learning.

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