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Event-Triggered Observer-Based Fuzzy Control for Coal-Fired Power Generation Systems Based on Singularly Perturbed Theory

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ABSTRACT The fuzzy controller design method for coal-fired power generation systems (CFPGSs) based on the singular perturbation theory is proposed in this paper. By analyzing the characteristics of two-time-scale, nonlinearity and input saturation for CFPGSs and using the singular perturbation theory and fuzzy control theory, the CFPGSs are modeled as fuzzy singularly perturbed systems with actuator saturation. In addition, considering the factors of the unavailable information on some state variables (such as inlet steam mass flow) and the limited bandwidth of network, an event-triggered observer-based controller structure is constructed. Then, by establishing an Lyapunov-Krasovskii functional based on the parameter ε and the upper bound of time-delay, a design method of event-triggered observer-based anti-windup controller is proposed, such that the closed-loop systems are asymptotically stable. And, the ill-conditioned numerical problems and Zeno phenomenon are avoided in the design process. Finally, a simulation verifies the effectiveness of the proposed method, that is, the system outputs (electric power and main steam pressure) tend to the reference value.

INDEX TERMS Event-triggered mechanism, T-S fuzzy control, coal-fired power generation systems, singularly perturbed theory, observer-based control.

I. INTRODUCTION

NORMAL power supply is an indispensable part of real work and life. The energy and power production are closely related. As China is rich in coal resources, coal consumption accounts for more than 60% of the total energy consumption according to the data before 2017 [1]. Global energy consumption is expected to grow by 28% over the next 20 years. Even if the annual growth rate of renewable energy reaches the 2.3% of the expected demand, fossil fuels will still account for 77% of global energy consumption after 20 years. It is predicted that China's coal-fired power generation will account for about 70.31% of the total power generation and will maintain long-term stability by 2040 [2], [3]. It can be seen that coal-fired power generation is still in a leading position in combination with the characteristics of the current stage and future development.

For the coal-fired power generation systems (CFPGSs) with the characteristics of large time-delay and strong coupling, many experts and scholars have analyzed and discussed the CFPGSs, and obtained valuable results [4]–[7], [9]–[11], such as fuzzy control scheme, predictive control scheme, coordinated control scheme, etc. The method of dead-time compensation feedback linearization is used to give a nonlinear control scheme to achieve the stability and tracking performance of the closed-loop system [6]. Based on the system model of energy conservation, a gain scheduling PI control scheme is proposed in [7]. Most of these results are based on the transfer function model, which is not conducive to analyze the complex characteristics of nonlinear dynamic systems, such as multivariate, large time-delay, input saturation and so on. In addition, with the development of digital technology, the update signal of the system is often the discrete measurement data obtained by sampling. And the choice of sampling period affects the system performance. If the sampling period becomes larger, the important information is less, which may lead to the instability of the system. If the sampling period becomes smaller, the number of transmission is larger, such that the burden of network transmission is increased and the situation of measurement data loss is easily increased. How to select an appropriate sampling period and construct an appropriate mechanism to filter the data to ensure the system performance are problems worthy of discussion. Considering CFPGSs in the network environment, the problem of proposing an efficient and reliable control strategy to meet the demand of power supply is needed to be further studied.

CFPGS is a kind of large-scale system, which transforms fuel chemical energy into electrical energy. Because it contains many interconnected time-delay nonlinear subsystems, the whole systems have the characteristics of high dimensionality and nonlinearity. Since T-S fuzzy model combines the flexibility of fuzzy logic theory and the rigorous mathematical analysis tools into a unified framework, it has been extensively studied and successfully applied to approximate a wide class of nonlinear control systems [8], [9]. In addition, CFPGSs include the coal-fired heat generation process and the heat transfer power generation process. Compared with the dynamic characteristics of the coal-fired process, the dynamic of the heat transfer process changes faster, which shows that the systems have the two-time-scale characteristic. If the fast dynamic characteristic is ignored, directly using the feedback control method proposed for normal system to analyze and design the two-time-scale system is easy to lead to ill-conditioned numerical problem [8]–[11]. The controller design method based on singular perturbation theory can solve this problem [12]–[15]. The systems with two-time-scale characteristic are described as singularly perturbed systems (SPSs), which contain a singular perturbation parameter ε characterizing the degree of separation of fast and slow dynamics [16]–[18]. Recently, in order to save the network resources [19], the problem of event-triggered observer-based controller design for SPSs with actuator saturation remains as an open area.

First, considering the linear model of CFPGSs at a certain equilibrium point, the characteristics of two-time-scale, input saturation and nonlinearity of CFPGSs are analyzed. Using singular perturbation theory and fuzzy theory, CFPGSs are described as T-S fuzzy SPSs with actuator saturation. Second, for the situation that the incomplete measurements and the limited bandwidth of networked control systems, an event-triggered mechanism (ETM) is introduced to filter, discard and describe the incomplete characteristic of the sampled-data. And an anti-windup (AW) control model based on event-triggered observer is constructed. Furthermore, by constructing an Lyapunov-Krasovskii functional (LKF), a control strategy based on incomplete measurement information is proposed for the controlled systems, which avoids ill-conditioned numerical problem and Zeno phenomenon. In addition, the proposed scheme can only use incomplete measurement information to complete the control task, which effectively saves the network resources. Finally, the effectiveness of the proposed method is verified by simulation, that is, the proposed method can ensure that the actual states of the system tend to the expected value. The main contributions are as follows:

(1) By fully analyzing the complex characteristics of CFPGSs, T-S fuzzy SPSs with actuator saturation are established, which is conducive to design the desired controller by constructing a proper Lyapunov function.

(2) For the networked control systems, by co-designing the ETM, observer, AW compensator and controller, a design scheme is obtained, which avoids ill-conditioned numerical problem, Zeno phenomenon, and efficiently reduce the frequency of triggering.

(3) By modeling the CFPGSs as T-S fuzzy SPSs with actuator saturation, the proposed control scheme can apply for a class of T-S fuzzy saturated SPSs to deal with the controller design problem based on partial measurements.

The remainder of this paper is structured as follows. In Section II, considering the incompletely measurable state variables and input saturation of CFPGSs, the actual CFPGSs with two-time-scale characteristic are modeled as T-S fuzzy SPSs with actuator saturation. In order to reduce the network burden, Section III introduces an event detector including the state information, which can filter and discard the sampled-data. And a controller design scheme based on the event-triggered observer is proposed for the constructed model. The control objective is to ensure that the outputs (electric power and main steam pressure) tend to the reference index by using incomplete measurement information to adjust the boiler demand and the throttle opening position. In Section IV, the effectiveness of the control scheme is verified by a simulation.

Notations: For a matrix X, X^+ denotes the Moore-Penrose pseudo-inverse matrix of X, $\mathrm{H}(X)$ is used to describe $X + X^T$. And the symmetric element of X is replaced by the symbol $*$. If not emphasized, matrices are assumed to be with appropriate dimensions.

II. MODELING T-S FUZZY SINGULARLY PERTURBED SYSTEMS

According to the working principle of CFPGSs, the systems can be divided into two parts: coal-fired heat generation and electric energy conversion, which are shown in FIGURE 1.

From FIGURE 1, we can get that the coal in the form of large lumps is converted to fine powder through the coal pulverizer. And the fine powdered coal is conveyed through the conveyor to the boiler and mixed with the air for combustion. Then, the heat produced by combustion makes the water evaporate. And the steam is generated. The steam pressure is used to drive the turbine to rotate through the throttle valve, such that the heat energy is converted into mechanical energy. Next, the mechanical energy is transmitted to the power generation system by the transmission system. And the generator converts the mechanical energy into electrical energy, which finally integrates into the power grid [2], [3].

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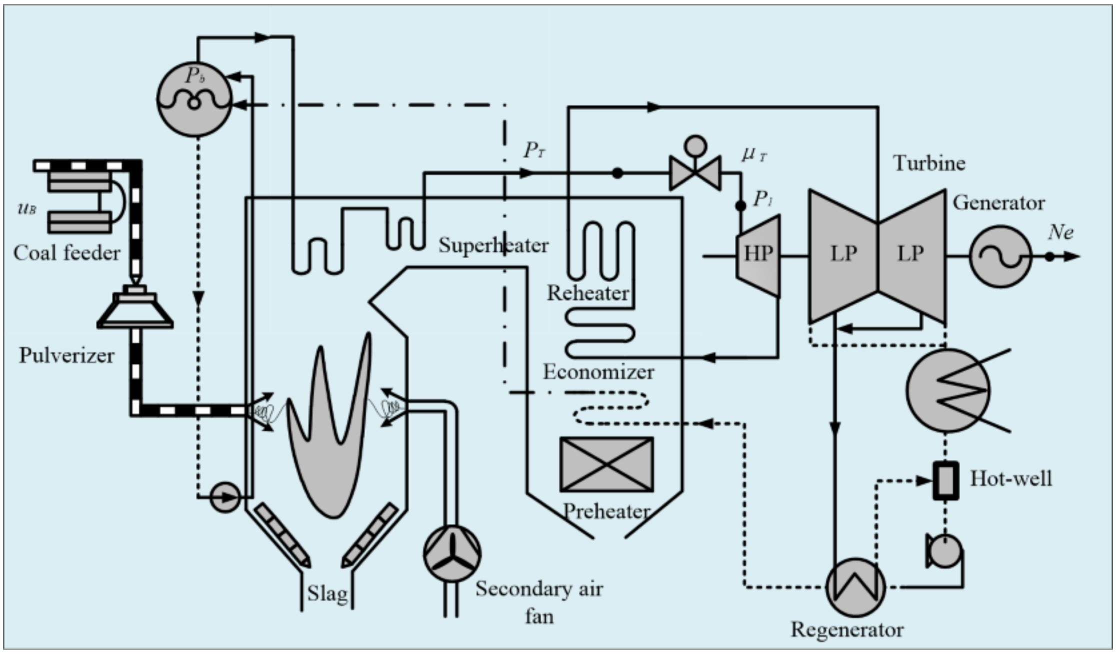

FIGURE 1. The schematic of CFPGSs.

For modeling the CFPGSs, the following assumptions are given [7]:

(1) In steam and water circuits, the temperature is well controlled, whose variation is small. Thus, the influence of temperature on pressure and power can be ignored;

(2) The coal quality variation can be ignored;

(3) The level of drum remains unchanged or can be adjusted rapidly;

(4) The fluid pressure is uniformly distributed in one lumped tube, and the frictional resistance is concentrated at the outlet of the tube;

(5) The pressure in the governing stage is proportional to the product of main steam pressure and the throttle valve opening.

According to the fundamental mass and energy balance law, the dynamic characteristics of CFPGS are modeled as follows [7]

$$\begin{cases} \dot{q}_f = \frac{1}{c_0}\left[u_B - q_f\right] \\ \dot{D}_b = \frac{1}{c_5}\left[2.46 q_f^{1.230} - D_b\right] \\ \dot{p}_b = \frac{1}{c_6}\left[D_b - 42.51 p_b^{0.956}\sqrt{p_b - p_T}\right] \\ \dot{p}_T = \frac{1}{c_7}\left[42.51 p_b^{0.956}\sqrt{p_b - p_T} - D_T\right] \\ \dot{p}_1 = \frac{1}{c_1}\left[0.0083\mu_T p_T - p_1\right] \\ \dot{D}_T = \frac{1}{c_2}\left[74.74 p_1 - D_T\right] \\ N_e = 0.86 D_T^{0.852} \end{cases} \tag{1}$$

where the state variables $q_f, D_b, p_b, p_T, p_1, D_T$ respectively represent mass flow rate of the coal entering into the furnace (t/h), steam generation rate (t/h), drum pressure (MPA), main steam pressure (MPA), pressure in governing stage(MPA), inlet steam mass flow rate (t/h). The output N_e represents electric power (MW), the control input u_B and μ_T represent the boiler demand (t/h) and the throttle opening position (%), which meet the constraints $0 \le u_B \le 150,\ 0 \le \mu_T \le 100$, $c_0, c_1, c_2, c_5, c_6, c_7$ are inertia constants, where the values are shown in TABLE 1 [7]

TABLE 1. The parameters of CFPGS (1)

Constant	c_0	c_1	c_2	c_5	c_6	c_7
Value	22	5	5	380	4057	5101

Then, the model (1) is rewritten by the following nonlinear system

$$\begin{cases} \dot{x}_1 = \frac{1}{22}\left[u_1 - x_1\right] \\ \dot{x}_2 = \frac{1}{380}\left[2.46 x_1^{1.230} - x_2\right] \\ \dot{x}_3 = \frac{1}{4057}\left[x_2 - 42.51 x_3^{0.956}\sqrt{x_3 - x_4}\right] \\ \dot{x}_4 = \frac{1}{5101}\left[42.51 x_3^{0.956}\sqrt{x_3 - x_4} - x_6\right] \\ \dot{x}_5 = \frac{1}{5}\left[0.0083 u_2 x_4 - x_5\right] \\ \dot{x}_6 = \frac{1}{5}\left[74.74 x_5 - x_6\right] \end{cases} \tag{2}$$

The outputs of the system (2) are

$$\begin{cases} z_1 = 0.86 x_6^{0.852} \\ z_2 = x_4 \end{cases}$$

where

$$\begin{aligned} x &= \begin{bmatrix} x_1^T & x_2^T & x_3^T & x_4^T & x_5^T & x_6^T \end{bmatrix}^T \\ &= \begin{bmatrix} q_f^T & D_b^T & p_1^T & p_T^T & p_b^T & D_T^T \end{bmatrix}^T \end{aligned}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_B \\ \mu_T \end{bmatrix},\ z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} N_e \\ p_T \end{bmatrix}$$

Choose the following reference states

$$x_r = \begin{bmatrix} 122.6 & 911.5 & 17.6 & 15.7 & 12.2 & 911.5 \end{bmatrix}^T$$

$$z_r = \begin{bmatrix} 285.9 & 15.7 \end{bmatrix}^T,\ u_r = \begin{bmatrix} 122.6 & 93.5 \end{bmatrix}^T$$

Using Taylar series approximation to linearize the system (2), we can obtain the following linear model

$$\begin{cases} \dot{\hat{x}}_1 = \frac{1}{22}\left[\hat{u}_1 - \hat{x}_1\right] \\ \dot{\hat{x}}_2 = \frac{1}{380}\left[9.14\hat{x}_1 - \hat{x}_2\right] \\ \dot{\hat{x}}_3 = \frac{1}{4057}\left[\hat{x}_2 - 288.6\hat{x}_3 + 239.2\hat{x}_4\right] \\ \dot{\hat{x}}_4 = \frac{1}{5101}\left[288.6\hat{x}_3 - 239.2\hat{x}_4 - \hat{x}_6\right] \\ \dot{\hat{x}}_5 = \frac{1}{5}\left[0.78\hat{x}_4 - \hat{x}_5 + 0.13\hat{u}_2\right] \\ \dot{\hat{x}}_6 = \frac{1}{5}\left[74.74\hat{x}_5 - \hat{x}_6\right] \end{cases} \tag{3}$$

The outputs are

$$\begin{cases} \hat{z}_1 = 0.2672\hat{x}_6 \\ \hat{z}_2 = \hat{x}_4 \end{cases}$$

where $\hat{x} = x - x_r$, $\hat{z} = z - z_r$, $\hat{u} = u - u_r$, $-122.6 \le \hat{u}_1 \le 27.4$, $-93.5 \le \hat{u}_2 \le 6.5$.

The system (3) is rewritten as follows

$$\begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u} \\ \hat{z} = \hat{C}_c\hat{x} \end{cases} \tag{4}$$

where

$$\hat{A} = 10^{-3} \times \begin{bmatrix} -45.4 & 0 & 0 & 0 & 0 & 0 \\ 24.1 & -2.6 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & -71.1 & 58.95 & 0 & 0 \\ 0 & 0 & 56.57 & -46.89 & 0 & -0.196 \\ 0 & 0 & 0 & 155.2 & -200 & 0 \\ 0 & 0 & 0 & 0 & 14948 & -200 \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0.0454 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.026 \\ 0 & 0 \end{bmatrix}$$

$$\hat{C}_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.2672 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues of matrix A for the linear system (4) are calculated as -0.0026, -0.0073, -0.0454, $-0.1302 + 0.0368i$, $-0.1302-0.0368i$, -0.2504. These six eigenvalues can be divided into two clusters: one cluster includes the first three eigenvalues, which are relatively closed to the origin to represent the slow state of the system; another cluster includes the latter three eigenvalues, which are relatively far away from the origin to represent the fast state of the system. We can get the singular perturbation parameter $\varepsilon = 0.0073/0.1301 \approx 0.05 \ll 1$. It can be seen that the CFPGSs have the two-time-scale characteristic.

The aim of this paper is to ensure the output (electric power) tracking reference value by controlling the boiler demand and the throttle opening position. That is to say, when the system (4) is asymptotically stable, the actual output tends to the reference output of the system.

For the nonlinear model of CFPGSs, a T-S fuzzy SPS with input saturation is described by the following fuzzy model

$$\begin{aligned} &\text{Plant rule } i:\\ &\text{IF}\\ &v_1(t) \text{ is } M_{i1}, v_2(t) \text{ is } M_{i2}, \cdots, v_g(t) \text{ is } M_{ig}\\ &\text{THEN}\\ &\begin{cases} E(\varepsilon)\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i\text{sat}(\hat{u}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases}\\ &\text{for } i = 1, 2, \cdots, r \end{aligned} \tag{5}$$

where r is the number of IF-THEN rules, M_{ij}, $(i = 1, \cdots, r, j = 1, \cdots, r)$ are fuzzy sets, $v_1(t), \cdots, v_g(t)$ are premise variables, A_i, B_i and C_i represent known real matrices

$$E(\varepsilon) = \begin{bmatrix} I_{3\times3} & 0 \\ 0 & \varepsilon I_{3\times3} \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} \hat{x}_1^T & \hat{x}_2^T & \hat{x}_4^T & \hat{x}_3^T & \hat{x}_5^T & \hat{x}_6^T \end{bmatrix}^T$$

$$\hat{u}(t) = \begin{bmatrix} \hat{u}_1(t-d) \\ \hat{u}_2(t) \end{bmatrix},\ \hat{y} = \begin{bmatrix} \hat{x}_5^T & \hat{x}_3^T & \hat{z}_1^T & \hat{z}_2^T \end{bmatrix}^T$$

The aforementioned model can be extended to a class of T-S fuzzy SPSs with input saturation, where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector, $\hat{u}(t) \in \mathbb{R}^p$ is the control input, $\hat{y}(t) \in \mathbb{R}^q$ is the measurement output, $E(\varepsilon) = \text{diag}\{I_{n_1}, \varepsilon I_{n_2}\} \in \mathbb{R}^{n\times n}$ with $n = n_1 + n_2$.

III. OBSERVER-BASED FUZZY CONTROLLER DESIGN

Denote $v(t) = [v_1(t), \cdots, v_g(t)]^T$ and $w_i(v(t)) = \prod_{j=1}^{p} M_{ij}(v_j(t))$, where $M_{ij}(v_j(t))$ is the grade of membership of $v_j(t)$ in M_{ij}. In this paper, it is assumed that $w_i(v(t)) \ge 0$, $\sum_{i=1}^{r} w_i(v(t)) > 0$.

Let $\mu_i(v(t)) = \frac{w_i(v(t))}{\sum_{i=1}^{r} w_i(v(t))}$, we have $\mu_i(v(t)) \ge 0$, $\sum_{i=1}^{r} \mu_i(v(t)) = 1$. Define $\mu_i(v(t))$ as μ_i.

Through singleton fuzzifier, product inference, and center-average defuzzifier, the global dynamics of T-S fuzzy SPS (5) with actuator saturation is described as follows

$$\begin{cases} E(\varepsilon)\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i\left[A_i\hat{x}(t) + B_i\text{sat}(\hat{u}(t))\right] \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i\left[C_i\hat{x}(t)\right] \end{cases} \tag{6}$$

Considering an event detector placed between the sampler and the observer in networked control system, we can judge whether an event is triggered or not by an ETM, which means that whether the current sampled-data is transmitted or not. When a data transmission is completed, the detector will judge the triggering condition at each sampling instant. To keep the signals continuous, a zero-order holder (ZOH) is embedded.

From the scheme shown in FIGURE 2, once a condition is satisfied, the current sampled-data is transmitted to the observer. The triggering instants are denoted by $\{t_k\}_{k=0}^{\infty}$

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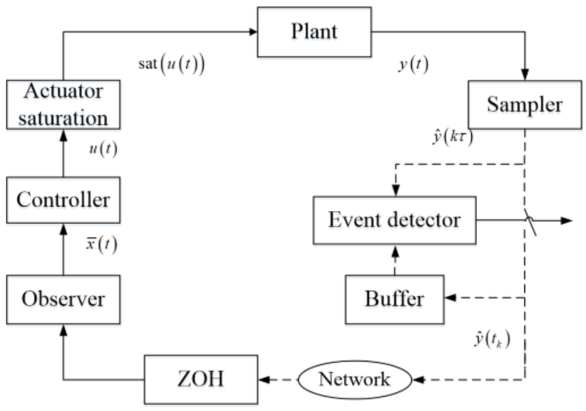

FIGURE 2. The fuzzy control scheme based on the event-triggered observer.

satisfying $t_k < t_{k+1}$ and $t_0 = 0$. For the current triggering instant t_k, the next triggering instant t_{k+1} is determined by the following ETM

$$t_{k+1}=t_k + \min_{k\tau} \left\{ k\tau \left| e_{etm}^T (t) \Omega e_{etm} (t) \geq f(t) \right. \right\} \tag{7}$$

where $f(t) = \delta e_y^T (t_k + k\tau) \Omega e_y (t_k + k\tau)$, $e_{etm}(t) = e_y (t_k + k\tau) - e_y (t_k)$, $e_y (t_k) = \hat{y} (t_k) - \bar{y} (t_k)$, $\hat{y}(t_k)$ is the measurement output at triggering instant t_k, Ω is a symmetric positive definite matrix, $\tau > 0$, $\delta > 0$.

Remark 1: The ETM (7) is constructed based on the sampled-data. Obviously, the interval between two consecutive triggering instants satisfies $t_{k+1} - t_k \geq \tau > 0$, which shows that the ETM (7) can avoid Zeno phenomenon. In addition, the parameter δ in the ETM (7) is given in advance within an allowed range. It should be noted that the scalar δ impacts the system performance and the communication. If δ becomes larger, the feedback information is less, which may lead to the instability of the system. If δ becomes smaller, the number of triggering is larger, such that the burden of network transmission is increased. By making a tradeoff within the allowed range, the parameter δ is proposed to ensure the system performance and reduce the frequency of triggering. If $\delta = 0$, the ETM (7) becomes a time-triggered mechanism.

The fuzzy controller based on the event-triggered observer is designed as follows

Controller rule i :
IF
$v_1(t)$ is $M_{i1}, v_2(t)$ is $M_{i2}, \cdots, v_g(t)$ is M_{ig}
THEN

$$\begin{cases} E(\varepsilon)\dot{\bar{x}}(t) = A_i \bar{x}(t) + B_i \hat{u}(t) + E_{ci}\psi(\hat{u}(t)) \\ \qquad + L_i (\hat{y}(t_k) - \bar{y}(t_k)) \\ \hat{u}(t) = K_{xi}\bar{x}(t) + K_{di}\bar{x}(t-d) \\ t_k \leq t < t_{k+1} \end{cases} \tag{8}$$

for $i = 1, \cdots, r$

where $\psi(\hat{u}(t)) = [\text{sat}(\hat{u}(t)) - \hat{u}(t)]$, K_{xi}, K_{di} are controller gain matrices, L_i is observer gain matrix, and E_{ci} is AW compensator gain matrix.

Because the controller rules are the same as the plant rules, the global fuzzy controller is given as follows

$$\begin{cases} E(\varepsilon)\dot{\bar{x}}(t) = \sum_{i=1}^{r} \mu_i [A_i \bar{x}(t) + E_{ci}\psi(\hat{u}(t)) \\ \qquad + B_i \hat{u}(t) + L_i (\hat{y}(t_k) - \bar{y}(t_k))] \\ \hat{u}(t) = \sum_{i=1}^{r} \mu_i [K_{xi}\bar{x}(t) + K_{di}\bar{x}(t-d)] \\ t_k \leq t < t_{k+1} \end{cases} \tag{9}$$

Define $\tau(t) = t - t_k - k\tau$. Obviously, the time-varying delay $\tau(t) \in (0, \tau]$ is piecewise continuous with $\dot{\tau}(t) = 1$ for $t \neq t_k$, and $\dot{\tau}(t)$ do not exist for $t = t_k$.

Let $e(t) = \hat{x}(t) - \bar{x}(t)$, from (6) and (9), we can obtain the following error system

$$\begin{aligned} E(\varepsilon)\dot{e}(t) = \sum_{i=1}^{r} \mu_i [&A_i e(t) + L_i C_i e(t_k) \\ &+ (B_i - E_{ci})\psi(\hat{u}(t))] \\ t_k \leq t < t_{k+1} \end{aligned} \tag{10}$$

Combining (6) and (10), the closed-loop system can be obtained

$$\begin{aligned} \tilde{E}(\varepsilon)\dot{\tilde{x}}(t) = \sum_{\substack{i=1 \\ j=1}}^{r} \mu_i \mu_j \Big[&\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{\psi i}\psi(\hat{u}(t)) \\ &+ B_{ei} e_{etm}(t) + \tilde{B}_{dij}\tilde{x}(t-d) \\ &+ \tilde{L}_i \tilde{x}(t-\tau(t)) \Big] \end{aligned} \tag{11}$$

where

$$\tilde{x}(t) = \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}, \; \tilde{E}(\varepsilon) = \begin{bmatrix} E(\varepsilon) & 0 \\ * & E(\varepsilon) \end{bmatrix}$$

$$\tilde{A}_{ij} = \tilde{A}_i + \tilde{B}_i \tilde{K}_{xj}, \; \tilde{B}_{dij} = \tilde{B}_i \tilde{K}_{dj}$$

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_i \end{bmatrix}, \; \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \; B_{ei} = \begin{bmatrix} 0 \\ L_i \end{bmatrix}$$

$$\tilde{L}_i = \begin{bmatrix} 0 & 0 \\ 0 & -L_i C_i \end{bmatrix}, \; \tilde{B}_{\psi i} = \begin{bmatrix} B_i \\ B_i - E_{ci} \end{bmatrix}$$

$$\tilde{K}_{dj} = \begin{bmatrix} K_{dj} & -K_{dj} \end{bmatrix}, \; \tilde{K}_{xj} = \begin{bmatrix} K_{xj} & -K_{xj} \end{bmatrix}$$

In order to derive the results of our study, the necessary lemmas are proposed.

Lemma 1: [20] For symmetric matrices G_1, G_2 and a scalar $\varepsilon_0 > 0$, if

$$G_1 \geq 0$$

$$G_1 + \varepsilon_0 G_2 > 0$$

hold, then

$$G_1 + \varepsilon G_2 > 0, \forall \varepsilon \in (0, \varepsilon_0]$$

Lemma 2: [20] If there exist a scalar $\varepsilon_0 > 0$ and matrices P_i $(i = 1, \cdots, 3)$ with $P_i = P_i^T (i = 1, 2)$ satisfying

$$P_1 > 0 \tag{12}$$

$$\begin{bmatrix} P_1 & \varepsilon_0 P_3^T \\ * & \varepsilon_0 P_2 \end{bmatrix} > 0 \tag{13}$$

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then, we have

$$E(\varepsilon)P(\varepsilon) = P^T(\varepsilon)E(\varepsilon) > 0, \forall \varepsilon \in (0, \varepsilon_0] \tag{14}$$

where $P(\varepsilon) = \begin{bmatrix} P_1 & \varepsilon P_3^T \\ P_3 & P_2 \end{bmatrix}$.

Lemma 3: [21] For any diagonal positive-definite matrix $\Lambda \in \mathbb{R}^{p\times p}$, the nonlinearity $\psi(v) = \mathrm{sat}(v) - v$ satisfies the following inequality

$$\psi^T(v)\Lambda(\psi(v) + w) \le 0, \quad \forall v, w \in \mathcal{S}(v_0)$$

where $\mathcal{S}(v_0) = \{v, w \in \mathbb{R}^p \,|{-v_0} \le v - w \le v_0\}$, and $v_0 \in \mathbb{R}^p$ is given.

Lemma 4: [22] Given a matrix $W > 0$, then, for all continuous function ω in $[a, b] \to \mathbb{R}^n$, the following inequality holds

$$\begin{aligned} &(b-a)\int_a^b \omega(s)^T W\omega(s)\,ds \\ &\ge \left(\int_a^b \omega(s)\,ds\right)^T W\left(\int_a^b \omega(s)\,ds\right) + 3\Xi^T W\Xi \end{aligned}$$

where $\Xi = \int_a^b \omega(s)\,ds - \frac{2}{b-a}\int_a^b\int_a^s \omega(r)\,drds$.

Lemma 5: [23] Given real matrices T_1 and T_2 of appropriate dimensions, for any positive-definite matrix G, we can get

$$T_1T_2 + {T_2}^T{T_1}^T \le T_1G{T_1}^T + {T_2}^TG^{-1}T_2$$

Theorem 1: For given scalars $\varepsilon_0 > 0$, $\tau > 0$, if there exist symmetric matrix P_2, matrices P_3 X_i, Y_{xi}, Y_{di}, G_i, M, N, and positive-definite symmetric matrices P_1, $\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}$, S, Ω, Q_i $(i = 1, \cdots, 5)$, such that LMIs (12)-(13) and the following inequalities hold

$$\begin{bmatrix} P_1 & * & * \\ 0 & P_1 & * \\ M_{j1(h)} & M_{j1(h)} & 1 \end{bmatrix} \ge 0 \tag{15}$$

$$\begin{bmatrix} P_1 & * & * & * & * \\ \varepsilon_0 P_3 & \varepsilon_0 P_2 & * & * & * \\ 0 & 0 & P_1 & * & * \\ 0 & 0 & \varepsilon_0 P_3 & \varepsilon_0 P_2 & * \\ M_{j1(h)} & \varepsilon_0 M_{j2(h)} & M_{j1(h)} & \varepsilon_0 M_{j2(h)} & 1 \end{bmatrix} \ge 0 \tag{16}$$

$$\begin{bmatrix} \Psi_{ii}(0) & * & * \\ \Delta_i(0) & \Delta_{4i} & * \\ \Delta_{1ii}(0) & \Delta_{2i}(0) & \Delta_3(0) \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} \Psi_{ij}(0) + \Psi_{ji}(0) & * & * \\ 2\Delta_i(0) & 2\Delta_{4i} & * \\ \Delta_{1ij}(0) + \Delta_{1ji}(0) & 2\Delta_{2i}(0) & 2\Delta_3(0) \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \Psi_{ii}(\varepsilon_0) & * & * \\ \Delta_i(\varepsilon_0) & \Delta_{4i} & * \\ \Delta_{1ii}(\varepsilon_0) & \Delta_{2i}(\varepsilon_0) & \Delta_3(\varepsilon_0) \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} \Psi_{ij}(\varepsilon_0) + \Psi_{ji}(\varepsilon_0) & * & * \\ 2\Delta_i(\varepsilon_0) & 2\Delta_{4i} & * \\ \Delta_{1ij}(\varepsilon_0) + \Delta_{1ji}(\varepsilon_0) & 2\Delta_{2i}(\varepsilon_0) & 2\Delta_3(\varepsilon_0) \end{bmatrix} < 0 \tag{20}$$

where

$$\Psi_{ij}(\varepsilon) = \begin{bmatrix} \varphi_{11ij} & \varphi_{12ij} & \varphi_{13i} & \bar{N} & \varphi_{15ij} & R_1 & R_2 \\ * & -Q_5 & Q_5 & 0 & -\tilde{Y}_{dj}^T & 0 & 0 \\ * & * & -Q_1 - Q_5 & Q_1 & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & -R_1 - \tau R_2^T & -R_2 - \tau R_3 \\ * & * & * & * & -2S & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & R_3 + \frac{6}{\tau}Q_2 \\ * & * & * & * & * & * & -\frac{12}{\tau^2}Q_2 \end{bmatrix}, \Delta_{1ij}(\varepsilon) = \begin{bmatrix} \tilde{A}_{ij}\tilde{P}(\varepsilon) & \tilde{B}_{dij}\tilde{P}(\varepsilon) & \varphi_{13i} & 0 & \tilde{B}_{\psi i}S & 0 & 0 \\ -\tau N & 0 & 0 & \tau N & 0 & 0 & 0 \\ 0 & 0 & \tilde{C}_i\tilde{P}(\varepsilon) & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta_i(\varepsilon) = \begin{bmatrix} \tilde{X}_i & \underbrace{0\cdots 0}_{6} \end{bmatrix}, \Delta_{2i}(\varepsilon) = \begin{bmatrix} \tilde{X}_i \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta_3(\varepsilon) = \begin{bmatrix} \tau\bar{Q}_4 - \mathrm{H}\left(\tilde{P}(\varepsilon)\right) & 0 & 0 \\ * & -\tau\bar{Q}_4 & 0 \\ * & * & -\delta^{-1}\Omega^{-1} \end{bmatrix}$$

$$\varphi_{11ij} = \mathrm{H}\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right) + \tau^2 Q_2 - \bar{N}, \bar{N} = \mathrm{H}\left(N\tilde{E}(\varepsilon)\right)$$

$$\varphi_{12ij} = \tilde{B}_{dij}\tilde{P}(\varepsilon), \tilde{P}(\varepsilon) = \mathrm{diag}\{P(\varepsilon), P(\varepsilon)\}$$

$$\varphi_{13} = \begin{bmatrix} 0 & 0 \\ 0 & -X_i \end{bmatrix}, \Delta_{4i} = C_i^+\Omega^{-1}C_i^{+T} - \mathrm{H}(P(\varepsilon))$$

$$\varphi_{15ij} = \begin{bmatrix} B_iS - Y_{xj}^T + E(\varepsilon)M_j^T \\ B_iS - G_i + Y_{xj}^T + E(\varepsilon)M_j^T \end{bmatrix}$$

$$\varphi_{44} = -Q_1 - Q_3 - \bar{N}, \varphi_{66} = \mathrm{H}(R_2) - 4Q_2$$

$$\tilde{A}_{ij}\tilde{P}(\varepsilon) = \begin{bmatrix} A_iP(\varepsilon) + B_iY_{xj} & -B_iY_{xj} \\ 0 & A_iP(\varepsilon) \end{bmatrix}$$

$$\tilde{B}_{\psi i}S = \begin{bmatrix} B_iS \\ B_iS - G_i \end{bmatrix}, \tilde{B}_{dij}\tilde{P}(\varepsilon) = \begin{bmatrix} B_iY_{dj} & -B_iY_{dj} \\ 0 & 0 \end{bmatrix}$$

$$\tilde{Y}_{dj}^T = \begin{bmatrix} Y_{dj}^T \\ -Y_{dj}^T \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} 0 & C_i \end{bmatrix}, \tilde{X}_i = \begin{bmatrix} 0 & X_i \end{bmatrix}$$

$$\bar{Q}_4 = \tilde{P}(\varepsilon)Q_4\tilde{P}^T(\varepsilon), M_j = \begin{bmatrix} M_{j1} & M_{j2} \end{bmatrix}$$

$$h \in [1, p], \; i, \; j = 1, \cdots, r, \; 1 \le i < j \le r$$

Then, for any $\varepsilon \in (0, \varepsilon_0]$, the closed-loop system (11) is asymptotically stable within $\Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right) = \left\{\tilde{x}\,\middle|\,\tilde{x}^T\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x} \le 1\right\}$. The event-triggered observer-based AW fuzzy controller gain matrices are as follows

$$K_{xi} = Y_{xi}P^{-1}(\varepsilon), K_{di} = Y_{di}P^{-1}(\varepsilon)$$

$$L_i = X_i P^{-1}(\varepsilon) C_i^{+}, E_{ci} = G_i S^{-1}$$

In addition, the set $\Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right)$ is an estimate of the domain of attraction.

Proof 1: Let $v = \hat{u}$ and $w = \tilde{M}_j \tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x} + \hat{u}$. Then by Lemma 3, the nonlinear term $\psi\left(\tilde{K}\tilde{x}\right)$ satisfies

$$\begin{array}{l} \psi^T(\hat{u})\Lambda\left(\psi(\hat{u}) + \tilde{M}_j\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x} + \hat{u}\right) \le 0 \\ \forall \tilde{x} \in S(\rho) \end{array} \tag{21}$$

where Λ is an arbitrary diagonal positive-definite matrix, $S(\rho) = \left\{\zeta \middle| -\rho \le \tilde{M}_j\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x} \le \rho\right\}$ and $\rho = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$.

From Lemma 1 and LMIs (15)-(16), for any $\varepsilon \in (0, \varepsilon_0]$, we have

$$\begin{bmatrix} \tilde{P}^T(\varepsilon)\tilde{E}(\varepsilon) & * \\ \tilde{M}_{j(h)}\tilde{E}(\varepsilon) & 1 \end{bmatrix} \ge 0, h \in [1,p], j = 1, \cdots, r \tag{22}$$

Pre- and post-multiplying the inequality (22) by $\operatorname{diag}\left\{ \tilde{P}^{-T}(\varepsilon) \quad I \right\}$ and its transpose, respectively, we have

$$\begin{bmatrix} \tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon) & * \\ \tilde{M}_{j(h)}\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon) & 1 \end{bmatrix} \ge 0, h \in [1,p], j = 1, \cdots, r$$

which implies that for $\forall \varepsilon \in (0, \varepsilon_0]$, it holds that

$$\tilde{P}^{-T}(\varepsilon)\tilde{E}(\varepsilon)\tilde{M}_{j(h)}^T\tilde{M}_{j(h)}\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon) \le \tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)$$

Then, for any $\tilde{x}(t) \in \Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right)$, we have

$$\tilde{x}^T(t)\tilde{P}^{-T}(\varepsilon)\tilde{E}(\varepsilon)\tilde{M}_{j(h)}^T\tilde{M}_{j(h)}\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x}(t) \le 1$$

which means $\Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right) \subseteq S(\rho)$.

According to Lemma 2, LMIs (12)-(13) ensures that inequality (14) holds, which is equivalent to

$$\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon) = \tilde{P}^{-T}(\varepsilon)\tilde{E}(\varepsilon) > 0, \forall \varepsilon \in (0, \varepsilon_0]$$

Choose an ε-dependent LKF

$$V(\tilde{x}(t)) = V_1(\tilde{x}(t)) + V_2(\tilde{x}(t)) + V_3(\tilde{x}(t)) + V_4(\tilde{x}(t))$$

where

$$\begin{aligned} V_1(\tilde{x}(t)) = {} & \tilde{x}^T(t)\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x}(t) \\ & + \eta^T(t)\tilde{P}^{-T}(\varepsilon)\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}\tilde{P}^{-1}(\varepsilon)\eta(t) \end{aligned}$$

$$\begin{aligned} V_2(\tilde{x}(t)) = {} & \int_{t-\tau}^{t-\tau(t)} \phi^T(s)\tilde{P}^{-T}(\varepsilon)Q_1\tilde{P}^{-1}(\varepsilon)\phi(s)\,ds \\ & + \int_{t-\tau}^{t-\tau(t)} \tilde{x}^T(s)\tilde{P}^{-T}(\varepsilon)Q_3\tilde{P}^{-1}(\varepsilon)\tilde{x}(s)\,ds \end{aligned}$$

$$\begin{aligned} V_3(\tilde{x}(t)) = {} & \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \tilde{x}^T(s)\bar{Q}_2\tilde{x}(s)\,ds d\theta \\ & + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\tilde{x}}^T(s)\tilde{E}^T(\varepsilon)Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(s)\,ds d\theta \end{aligned}$$

$$V_4(\tilde{x}(t)) = \int_{t-d}^{t-\tau(t)} \phi^T(s)\tilde{P}^{-T}(\varepsilon)Q_5\tilde{P}^{-1}(\varepsilon)\phi(s)\,ds$$

$$\eta^T(t) = \begin{bmatrix} \int_{t-\tau}^{t}\tilde{x}^T(s)\,ds & \int_{t-\tau}^{t}\int_{t-\tau}^{\theta}\tilde{x}^T(s)\,ds d\theta \end{bmatrix}$$

$$\phi(s) = \tilde{x}(s) - \tilde{x}(t - \tau(t)), \ \bar{Q}_2 = \tilde{P}^{-T}(\varepsilon)Q_2\tilde{P}^{-1}(\varepsilon)$$

Computing the time derivative of LKF along the trajectories of system (11), it holds that

$$\begin{aligned} \dot{V}_1(\tilde{x}(t)) = {} & 2\dot{\tilde{x}}^T(t)\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon)\tilde{x}(t) \\ & + 2\dot{\eta}^T(t)\tilde{P}^{-T}(\varepsilon)\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}\tilde{P}^{-1}(\varepsilon)\eta(t) \end{aligned}$$

$$\begin{aligned} \dot{V}_2(\tilde{x}(t)) = {} & -\phi(t)^T\tilde{P}^{-T}(\varepsilon)Q_1\tilde{P}^{-1}(\varepsilon)\phi(t) \\ & -\tilde{x}(t-\tau)^T\tilde{P}^{-T}(\varepsilon)Q_3\tilde{P}^{-1}(\varepsilon)\tilde{x}(t-\tau) \end{aligned}$$

$$\begin{aligned} \dot{V}_3(\tilde{x}(t)) = {} & \tau^2\tilde{x}^T(t)\bar{Q}_2\tilde{x}(t) - \tau\int_{t-\tau}^{t}\tilde{x}^T(s)\bar{Q}_2\tilde{x}(s)\,ds \\ & + \tau\dot{\tilde{x}}^T(t)\tilde{E}^T(\varepsilon)Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(t) \\ & - \int_{t-\tau}^{t}\dot{\tilde{x}}^T(s)\tilde{E}^T(\varepsilon)Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(s)\,ds \end{aligned}$$

$$\dot{V}_4(\tilde{x}(t)) = -\phi^T(t-d)\tilde{P}^{-T}(\varepsilon)Q_5\tilde{P}^{-1}(\varepsilon)\phi(t-d)$$

In order to deal with the integral term, we use Lemma 4 to obtain

$$\begin{aligned} & -\tau\int_{t-\tau}^{t}\tilde{x}^T(s)\bar{Q}_2\tilde{x}(s)\,ds \\ & \le -3\Xi^T\tilde{P}^{-T}(\varepsilon)Q_2\tilde{P}^{-1}(\varepsilon)\Xi \\ & \quad -\left[\int_{t-\tau}^{t}\tilde{x}(s)\,ds\right]^T\bar{Q}_2\left[\int_{t-\tau}^{t}\tilde{x}(s)\,ds\right] \end{aligned} \tag{23}$$

where $\Xi = \left[\int_{t-\tau}^{t}\tilde{x}(s)\,ds - \int_{t-\tau}^{t}\int_{t-\tau}^{\theta}\tilde{x}(s)\,ds d\theta\right]$.

Then, we introduce any matrix N of appropriate dimensions satisfying the following equation

$$\begin{aligned} & 2{\eta_1}^T(t)\widehat{N}\tilde{E}(\varepsilon) \\ & \quad \left[\tilde{x}(t) - \tilde{x}(t-\tau) - \int_{t-\tau}^{t}\dot{\tilde{x}}(s)\,ds\right] = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} & \tau\eta_1^T(t)\widehat{N}Q_4^{-1}\widehat{N}^T\eta_1(t) \\ & \quad - \int_{t-\tau}^{t}\eta_1^T(t)\widehat{N}Q_4^{-1}\widehat{N}^T\eta_1(t)\,ds = 0 \end{aligned} \tag{25}$$

where $\eta_1 = \begin{bmatrix} x^T(t) & x^T(t-\tau) \end{bmatrix}^T$, $\widehat{N} = \begin{bmatrix} -\hat{N} \\ \hat{N} \end{bmatrix}$, $\hat{N} = \tilde{P}^{-T}(\varepsilon)N\tilde{P}^{-T}(\varepsilon)$.

According to the ETM (7), the following inequality holds within the triggering interval

$$\begin{aligned} & \delta e(t-\tau(t))^T C_i^T\Omega C_i e(t-\tau(t)) \\ & - e_{etm}^T(t)\Omega e_{etm}(t) > 0 \end{aligned} \tag{26}$$

Combining (21), (23)-(26), we have

$$\begin{aligned} & \dot{V}(\tilde{x}(t)) \\ & \le \zeta^T(t)\hat{\Psi}_{ij}(\varepsilon)\zeta(t) + \tau\eta_1^T(t)\widehat{N}Q_4^{-1}\widehat{N}^T\eta_1(t) \\ & \quad + \tau\dot{\tilde{x}}^T(t)\tilde{E}^T(\varepsilon)Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(t) \\ & \quad - \int_{t-\tau}^{t}[\eta_2]^T Q_4^{-1}[\eta_2]\,ds \\ & \le \zeta^T(t)\hat{\Psi}_{ij}(\varepsilon)\zeta(t) + \tau\eta_1^T(t)\widehat{N}Q_4^{-1}\widehat{N}^T\eta_1(t) \\ & \quad + \tau\dot{\tilde{x}}^T(t)\tilde{E}^T(\varepsilon)Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(t) \end{aligned} \tag{27}$$

where

$$\begin{aligned} \zeta^T(t) = \big[& \tilde{x}^T(t) \quad \tilde{x}^T(t-d) \quad \tilde{x}^T(t-\tau(t)) \\ & \tilde{x}^T(t-\tau) \quad \psi^T(u(t)) \quad \int_{t-\tau}^{t}\tilde{x}^T(s)\,ds \\ & \int_{t-\tau}^{t}\int_{t-\tau}^{\theta}\tilde{x}^T(s)\,ds d\theta \quad e_{etm}^T(t) \big] \end{aligned}$$

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$$\eta_2 = Q_4\tilde{E}(\varepsilon)\dot{\tilde{x}}(s) + \widehat{N}^T\eta_1(t)$$

$$\hat{\Psi}_{ij}(\varepsilon) = \begin{bmatrix} \hat{\varphi}_{11ij} & \hat{\varphi}_{12ij} & \hat{\varphi}_{13ij} & \hat{\varphi}_{14} & \hat{\varphi}_{15ij} & \hat{R}_1 & \hat{R}_2 & \hat{\varphi}_{18i} \\ * & -\hat{Q}_5 & \hat{Q}_5 & 0 & -\tilde{K}_{dj}^T\Lambda & 0 & 0 & 0 \\ * & * & \hat{\varphi}_{33i} & \hat{Q}_1 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\varphi}_{44} & 0 & -\hat{R}_1-\tau\hat{R}_2^T & -\hat{R}_2-\tau\hat{R}_3 & 0 \\ * & * & * & * & -\mathrm{H}(\Lambda) & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\varphi}_{66} & \hat{R}_3+\frac{6}{\tau}\hat{Q}_2 & 0 \\ * & * & * & * & * & * & -\frac{12}{\tau^2}\hat{Q}_2 & 0 \\ * & * & * & * & * & * & * & \Omega \end{bmatrix}$$

$$\hat{\varphi}_{11ij} = \mathrm{H}\left(\tilde{P}^{-T}(\varepsilon)\tilde{A}_{ij} - \hat{N}\tilde{E}(\varepsilon)\right) + \tau^2\hat{Q}_2$$

$$\hat{\varphi}_{12ij} = \tilde{P}^{-T}(\varepsilon)\tilde{B}_{dij}, \hat{\varphi}_{33i} = \delta\tilde{C}_i^T\Omega\tilde{C}_i - \hat{Q}_1 - \hat{Q}_5$$

$$\hat{\varphi}_{13} = \begin{bmatrix} 0 & 0 \\ 0 & -P^{-T}(\varepsilon)L_iC_i \end{bmatrix}$$

$$\hat{\varphi}_{14} = \mathrm{H}\left(\hat{N}\tilde{E}(\varepsilon)\right), \hat{\varphi}_{18i} = P^{-T}(\varepsilon)L_i$$

$$\hat{\varphi}_{15ij} = \tilde{P}^{-T}(\varepsilon)\begin{bmatrix} B_i - Y_{xj}^T\Lambda + E(\varepsilon)M_j^T\Lambda \\ B_i - G_i\Lambda + Y_{xj}^T\Lambda + E(\varepsilon)M_j^T\Lambda \end{bmatrix}$$

It can be seen from Lemma 5 that for any $\varepsilon \in (0, \varepsilon_0]$, the following inequality holds

$$\tau\bar{Q}_4 - \mathrm{H}\left(\tilde{P}(\varepsilon)\right) \geq \tau^{-1}Q_4^{-1}$$

$$C_i^+\Omega^{-1}C_i^{+T} - \mathrm{H}(P(\varepsilon)) \geq P^T(\varepsilon)C_i^T\Omega C_iP(\varepsilon)$$

According to Lemma 1 and Schur complement, for any $\varepsilon \in (0, \varepsilon_0]$, LMIs (17)-(20) imply that

$$\begin{bmatrix} \bar{\varphi}_{11ij} & \bar{\varphi}_{12ij} & \bar{\varphi}_{13ij} & \bar{\varphi}_{14} & \bar{\varphi}_{15ij} & R_1 & R_2 & \bar{\varphi}_{18ij} \\ * & \bar{\varphi}_{22ij} & \bar{\varphi}_{23ij} & 0 & \bar{\varphi}_{25ij} & 0 & 0 & \bar{\varphi}_{28ij} \\ * & * & \bar{\varphi}_{33i} & Q_1 & \bar{\varphi}_{35i} & 0 & 0 & \bar{\varphi}_{38ij} \\ * & * & * & \bar{\varphi}_{44} & 0 & -R_1-\tau R_2^T & -R_2-\tau R_3 & 0 \\ * & * & * & * & \bar{\varphi}_{55i} & 0 & 0 & \bar{\varphi}_{58i} \\ * & * & * & * & * & \varphi_{66} & R_3+\frac{6}{\tau}Q_2 & 0 \\ * & * & * & * & * & * & -\frac{12}{\tau^2}Q_2 & 0 \\ * & * & * & * & * & * & * & \bar{\varphi}_{88i} \end{bmatrix} < 0 \tag{28}$$

where

$$\bar{\varphi}_{11ij} = \varphi_{11ij} + \tau N^T\bar{Q}_4^{-1}N + \tau\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{A}_{ij}\tilde{P}(\varepsilon)$$

$$\bar{\varphi}_{12ij} = \varphi_{12ij} + \tau\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{B}_{dij}\tilde{P}(\varepsilon)$$

$$\bar{\varphi}_{13ij} = \varphi_{13ij} + \tau\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\varphi_{13i}$$

$$\bar{\varphi}_{14} = \bar{N} - \tau N^T\bar{Q}_4^{-1}N$$

$$\bar{\varphi}_{15ij} = \varphi_{15ij} + \tau\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{B}_{\psi i}S$$

$$\bar{\varphi}_{18ij} = \tilde{X}_i^T + \tau\left(\tilde{A}_{ij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{X}_i$$

$$\bar{\varphi}_{22ij} = \tau\left(\tilde{B}_{dij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{B}_{dij}\tilde{P}(\varepsilon) - Q_5$$

$$\bar{\varphi}_{23ij} = Q_5 + \tau\left(\tilde{B}_{dij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\varphi_{13i}$$

$$\bar{\varphi}_{25ij} = \tau\left(\tilde{B}_{dij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{B}_{\psi i}S - \tilde{Y}_{dj}^T$$

$$\bar{\varphi}_{28ij} = \tau\left(\tilde{B}_{dij}\tilde{P}(\varepsilon)\right)^T\bar{Q}_4\tilde{X}_i, \bar{\varphi}_{35i} = \tau(\varphi_{13i})^T\bar{Q}_4\tilde{B}_{\psi i}S$$

$$\bar{\varphi}_{38i} = \tau(\varphi_{13i})^T\bar{Q}_4\tilde{X}_i, \bar{\varphi}_{44} = \varphi_{44} + \tau N^T\bar{Q}_4^{-1}N$$

$$\bar{\varphi}_{55i} = \tau\left(\tilde{B}_{\psi i}S\right)^T\bar{Q}_4\tilde{B}_{\psi i}S - 2S, \bar{\varphi}_{58i} = \tau\left(\tilde{B}_{\psi i}S\right)^T\bar{Q}_4\tilde{X}_i$$

$$\bar{\varphi}_{33i} = -Q_1 - Q_5 + \delta\tilde{P}^T(\varepsilon)\tilde{C}_i^T\Omega\tilde{C}_i\tilde{P}(\varepsilon)$$

$$\bar{\varphi}_{88i} = \tau\left(\tilde{X}_i\right)^T\bar{Q}_4\tilde{X}_i - P^T(\varepsilon)C_i^T\Omega C_iP(\varepsilon)$$

Pre- and post-multiplying the inequality (28) by

$$\mathrm{diag}\left\{\underbrace{P^{-T}(\varepsilon)\cdots P^{-T}(\varepsilon)}_{4}, S^{-T}, P^{-T}(\varepsilon), P^{-T}(\varepsilon), \left(P^{-1}(\varepsilon)C_i^+\right)^T\right\}$$

and its transpose, respectively, we have

$$\dot{V}(\tilde{x}(t)) < 0 \tag{29}$$

Then, integrating both sides of the inequality (29) from 0 to ∞ yields

$$V(\tilde{x}(t)) < V(\tilde{x}(0)) < 1$$

This shows that the state trajectory of the closed-loop system (11) starting from the set $\Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right)$ remains in it. The set $\Omega\left(\tilde{E}(\varepsilon)\tilde{P}^{-1}(\varepsilon), 1\right)$ is an estimate of the domain of attraction for the closed-loop system (11). This completes the proof.

Remark 2: By adopting a Wirtinger integral inequality and free-weighting matrices, we get the event-triggered observer-based controller design method for fuzzy SPSs with input saturation. It is worth noting that the nonlinearity can be solved by using proper matrix inequalities and Schur complement. And, the existence of parameter ε and saturation factor causes the computation complexity in the process of controller design. Considering the method in [24], [25] and modifying the matrix $P(\varepsilon)$, an ε-dependent fuzzy/piecewise Lyapunov function can be constructed to design the controller. For this method, the number of decision variables will be increased, but the conservatism may be reduced. We will consider it in our future work.

Remark 3: For SPSs with input saturation, the existence of AW compensator makes the separation principle fail. A control strategy of designing the ETM, observer, controller and AW compensator simultaneously is proposed. The limitation of this control strategy is that the proposed algorithm is more complex than the one based on the separation principle. The obtained method is reduced to solving a set of LMIs, which can be solved efficiently.

TABLE 2. Number of decision variables and lines of LMIs in Theorem 1

	Decision variables $\mathfrak{D}$	Lines $\mathfrak{L}$
LMIs in Theorem 1	$\frac{11(n^2+n)+p^2+p+q^2+q}{2}+9n^2+2n+rn(3p+q)$	$rp(2n1+2n+2)+r(r+1)(7n+p+q)$

Remark 4: The proposed LMI conditions can be solved in polynomial time by specialized algorithms with complexity proportional to $\mathfrak{D}=\mathcal{L}^3\mathfrak{A}$, where $\mathfrak{L}$ and $\mathfrak{D}$ denote the numbers of lines and decision variables of LMIs, respectively. In TABLE 2, the numbers of lines and decision variables of LMIs in Theorem 1 are presented.

IV. SIMULATION STUDY

Considering the fuzzy system (5), the dynamic model of nonlinear system (2) is expressed as the following T-S fuzzy model

Plant Rule 1 :
IF
$x_1(t)$ is about 122.6, $x_3(t)$ is about 17.6 ,
$x_4(t)$ is about 15.7, $x_6(t)$ is about 911.5
THEN

$$\begin{cases} E(\varepsilon)\dot{x}(t)=A_1x(t)+B_1\text{sat}(u(t)) \\ y(t)=C_1x(t) \end{cases}$$

Plant Rule 2 :
IF
$x_1(t)$ is about 78.9, $x_3(t)$ is about 14.9
$x_4(t)$ is about 14.01, $x_6(t)$ is about 531.89
THEN

$$\begin{cases} E(\varepsilon)\dot{x}(t)=A_2x(t)+B_2\text{sat}(u(t)) \\ y(t)=C_2x(t) \end{cases}$$

where the membership functions of the fuzzy sets are chosen as follows

$$\mu_1=\frac{|x_1(t)|}{43.7}\cdot\frac{|x_3(t)|}{1.69}\cdot\frac{|x_4(t)|}{2.7}\cdot\frac{|x_6(t)|}{1379.61}$$

$$A_1=\begin{bmatrix} -\frac{1}{22} & 0 & 0 & 0 & 0 & 0 \\ \frac{9.14}{380} & \frac{-1}{380} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-239.2}{5101} & \frac{288.6}{5101} & 0 & \frac{-1}{5101} \\ 0 & \frac{0.1}{8114} & \frac{23.92}{8114} & \frac{-28.86}{8114} & 0 & 0 \\ 0 & 0 & 0.008 & 0 & -0.01 & 0 \\ 0 & 0 & 0 & 0 & 0.7474 & -0.01 \end{bmatrix}$$

$$A_2=\begin{bmatrix} \frac{-1}{22} & 0 & 0 & 0 & 0 & 0 \\ \frac{8.26}{380} & \frac{-1}{380} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-298.08}{5101} & \frac{332.12}{5101} & 0 & \frac{-1}{5101} \\ 0 & \frac{0.1}{8114} & \frac{29.808}{8114} & \frac{-33.212}{8114} & 0 & 0 \\ 0 & 0 & 0.005 & 0 & -0.01 & 0 \\ 0 & 0 & 0 & 0 & 0.7474 & -0.01 \end{bmatrix}$$

$$C_1=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2894 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2894 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1=\begin{bmatrix} 1/22 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.001 \\ 0 & 0 \end{bmatrix}, B_2=\begin{bmatrix} 1/22 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.0012 \\ 0 & 0 \end{bmatrix}$$

For given $\varepsilon_0=0.05$, $d=43$, $\tau=0.2$, $\delta=0.1$, solving LMIs in Theorem 1, we can obtain the following observer-based controller gain matrix and the parameter of ETM (7) simultaneously

$$K_1=-10^{-6}\times\begin{bmatrix} 634 & 30.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 97.5 & 1.6 & 158.7 & 0.2 \end{bmatrix}$$

$$L_1=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0315 & -0.0004 & -0.0008 & 0.0101 \\ -0.0153 & -0.0001 & 0.0001 & 0.005 \\ -0.0021 & 0 & 0 & 0.0007 \\ 0.0029 & 0 & -0.001 & -0.0006 \end{bmatrix}$$

$$E_{c1}=10^{-4}\times\begin{bmatrix} 274 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1377 & 32 & 2 & -1 \end{bmatrix}^T$$

$$K_2=10^{-4}\times\begin{bmatrix} -31 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -484 & -16 & -1718 & 1 \end{bmatrix}$$

$$L_2=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1426 & -0.0008 & 0.0012 & 0.0477 \\ -0.0097 & -0.0001 & 0 & 0.0032 \\ -0.0017 & 0 & 0 & 0.0006 \\ 0.0021 & 0 & -0.0011 & -0.0008 \end{bmatrix}$$

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$$E_{c2} = 10^{-4} \times \begin{bmatrix} 275 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -990 & 19 & 4 & -35 \end{bmatrix}^T$$

$$\Omega = 10^{-3} \begin{bmatrix} 3.8017 & 0.0089 & -0.0346 & 0.0559 \\ * & 0.0422 & -0.001 & 0.0015 \\ * & * & 0.014 & 0.0014 \\ * & * & * & 0.0443 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 8.953 & 0.033 & 0 \\ * & 9.0471 & 0.0002 \\ * & * & 13.2512 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.0001 & 0 \\ 0.0686 & 4.2721 & 0.036 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.6938 & 0.4612 & -0.0071 \\ * & 196.9925 & -0.55 \\ * & * & 0.0851 \end{bmatrix}$$

Under the condition $\varepsilon = 0.05$, the release instants and the triggering time intervals are presented in FIGURE 3. And FIGURE 4 shows the state trajectories of the controlled system.

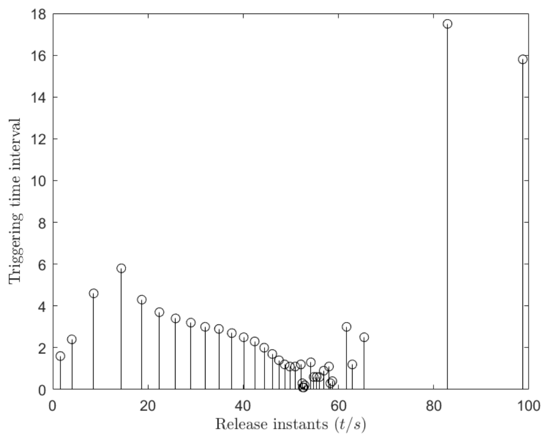

FIGURE 3. The triggering time intervals.

It can be seen from FIGURE 3 that the designed ETM can effectively reduce the frequency of data transmission, such that the network resources are saved and the calculation burden in the controller design process is reduced. Compared with the method based on continuous feedback data, applying ETM to filter sampled-data artificially can complete the data update and feedback control based on less measurement information of the system.

Meanwhile, it can be seen from FIGURE 4 that the fuzzy controller (9) based on incomplete measurement information and state estimator can guarantee that the system (5) is asymptotically stable. That is to say, for the case of incomplete measurement information, the obtained control scheme ensures that the main steam pressure and the output electric power of CFPGSs tend to the reference value by controlling the boiler demand and the throttle opening position, which means that the proposed method achieves the expected control goal.

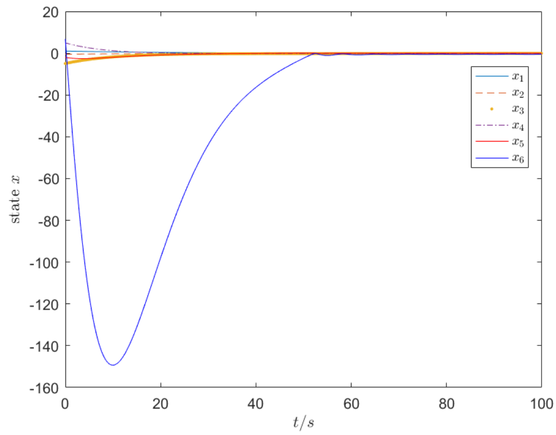

FIGURE 4. Trajectories of the controlled plant.

V. CONCLUSION

Through analyzing the structure and operation characteristics of CFPGSs, we summarize the characteristics of two-time-scale, input delay, input saturation and nonlinearity for the controlled systems. Using the singular perturbation theory and fuzzy control theory, the actual CFPGSs are modeled as T-S fuzzy SPSs with actuator saturation. Because of the actual situation that the system has the feature of incomplete measurement information, an observer-based AW controller model is proposed. In addition, an ETM is introduced to reduce the number of data transmission. By constructing an ε-dependent LKF, a design method of AW controller based on event-triggered observer is proposed, which effectively saves the network resources and ensures that the system can achieve steady state under the condition of incomplete measurement information. Finally, the simulation results show that the feedback controller based on partial measurement information can achieve the aim that the outputs (electric power and main steam pressure) of CFPGSs tend to the reference value, which shows that the proposed control scheme is effective.

By constructing a proper LKF, designing a dynamic ETM to further reduce the communication resources and proposing a less conservative design method for systems will be considered in our future work.

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