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Principle of Self-Support (PSS) and Its Extensions With Fractional Calculus and Event-Triggered Scheme

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ABSTRACT As an ingenious concept which broadens the traditional error-feedback mechanism, the principle of self-support (PSS) is reviewed in this article with two extensions. Instead of regarding a control signal as an input variable or a cause of the control processes, PSS emphasized the independence and significance of the control signal in the whole dynamics. Moreover, due to nonlocal and history-dependent properties for fractional-order operators, many processes with memory and heredity can be well modeled by fractional-order systems, a pioneering work on the fractional-order generalized PSS (FOGPSS) is introduced. Meanwhile, motivated by the event-triggered schemes that enjoy benefits of reduced sampling efforts and better resource utilization, an event-triggered controller based on PSS is further presented. Finally, some future research directions are discussed in conclusion.

INDEX TERMS Principle of self-support, fractional-order systems, event-triggered control, finite-time practical tracking.

I. INTRODUCTION

As an ingenious concept, a new theoretical framework by introducing the principle of self-support (PSS) was established by Novaković in [1] to emphasize the independence of control signals. It has been theoretically proved and widely applied in robot control synthesis [2], robust tracking control for robots [3], [4], kinematic control of robots [5], inverse kinematics problem [6]. First of all, let us here briefly review the definition of PSS, which is based on the following qualitative characteristics for each phenomenon's existence:

(i) Self-existence: $p = p$. Each phenomenon is an entity with its own being and nature, such as thing, single element, unit, process, etc. It exists as something in itself, not as any other thing.

(ii) Existence as a whole: $p : \{W\}$. Each phenomenon exists as a whole. It is or has a wholeness which includes all other phenomena. "Whatever comes into existence, always comes as a whole."

(iii) Existence in a whole: $W : \{p, \ldots, \}$. No phenomenon exists entirely alone because each is a part of other phenomena.

In fact, these three points can be gathered in the following expression, which is adopted as a symbolical description of PSS.

$$\underset{\text{phenomenon}}{p} : \left\{ \underset{\substack{\text{phenomenon's}\\ \text{own being}}}{p}, \underset{\text{whole}}{W} \right\} \tag{1}$$

From this new point of view, the following natural phenomena can be regarded as self-support systems. On the one hand, as introduced in [7], [8], owing to global warming, the movement of melting ice changes the regional climate and then leads the sea levels to rise, but instead, the rising sea surface can also affect the ice movement, as seen in Fig. 1.

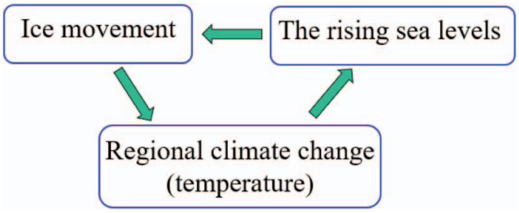

FIGURE 1. Interaction between the ice movement and the rising sea levels.

FIGURE 2. Uroboros.

So they are reciprocally cause and effect, as they are interrelated, interact and constitute an integral whole (self-support as a whole).

On the other hand, the "self-referential" expression (a description about describing itself), for which the best representative seems to be a medieval paradox [9], the Uroboros, the archetype of a "vicious circle" formed by a snake, biting its own tail, looped in a circle in the following Fig. 2. Then how could we distinguish the beginning and the end? How to make it clear the relationship between the cause and the effect? On the basis of PSS idea, it is thus in some way a self-supported system.

Now, let us clarify how to solve control problems through the detailed implementation of PSS, and how should we think about the relationship between the control signal u and the specific controlled plant under the PSS conceptual framework? As we all know, an original control purpose, in general, is to design a strategy to achieve simple, clear and effective control effect. However, it should be stated at once that a crucial difference between "describing a problem" and "solving a problem" is undoubtedly the fact that for modeling (and simulating) purposes one may (or even has to) build a complex expression, while in solving a problem one should tend to build expressions (algorithms) which are as simple as possible. Therefore, do we really need explicit systems' models? The answer is: for simulation purposes — maybe, for problem solving — not at all.

Coincidentally, PSS emphasizes the fundamental importance of autonomy, it points out that although there exist some (simple or complicated) connections between data in a practical system, there is no need to incorporate them in solution of the considered problem. In fact, only measured data, not any functional relationships between them, is available for controller design. Therefore, some existing control methodologies which need to evaluate an exact mathematical model firstly and then incorporate that in a control law, will definitely increase computational complexity and reduce its versatility.

However, the critical idea of PSS for obtaining an effective solution is to look at the control signal u as a concentration of the whole dynamics, it points out that u should be regarded as an independent variable, and needs to be viewed as a variable with the "equal rights" to the others, that is to say, the control signal u should not be regarded as a so-called input variable or a cause of the control processes, for example, the thermostat is just as controlled by the temperature in the room as it controls that temperature. In other words, the (changed) temperature is (at least partly) caused by the result of its causal influence upon the thermostat. All in all, control is not absolute.

Based on the essences mentioned above, once accepted PSS as a way of thinking, it becomes the basis for creative discovery and design because it offers an abundance of possible extensions. In the following discussions, we will show how does PSS work, and introduce its detailed design procedure and some further investigations. In fact, compared with previous papers which are presented from the engineering standpoint, this article is intended to review and develop PSS from a theoretical point of view. The main contributions of this article are summarized as follows: (i) Combined the basic idea of PSS with fractional-order operators [10], [11] to introduce an extended PSS. (ii) Owing to the fact that event-triggered mechanism can reduce the control updates while providing a satisfactory performance [12]–[14], an event-triggered controller based on PSS is firstly proposed in this article.

The remainder of the paper is organized as follows. In Section II, an important theorem for explaining PSS is presented and, for better understanding and clarity, derived, proved and illustrated. Section III reviews the generalized work of PSS with regards to fractional calculus. Moreover, an event-triggered controller based on PSS is investigated in Section IV. Finally, Section V concludes this article.

II. AN IMPORTANT THEOREM IN PSS

It is well known that a robotic system is a typical mechatronics system that integrates mechanical, electrical and electronic elements. By using these elements properly (actuators, position/rate sensors, force/torque sensors and processors for data input/output, etc.), the dynamic systems (manipulators, mobile robots, etc.) can be controlled to perform some specific tasks. In the following, a robotic system will be chosen to be an example to illustrate the expression (1) in detail, and introduce a fundamental theorem in PSS.

In fact, a kinematic control signal u in robotic systems can be regarded as a self-supported variable:

(i) Self-existence: the control signal u really exists as an entity, it has its natural physical meaning.

(ii) Existence as a whole: the control signal u could be viewed as a concentration of the notion (phenomenon) of dynamics (whole) in the following model, as including in itself the other single phenomenon such as inertia, Coriolis forces, friction, etc.

$$D(q)\ddot{q} + d(q, \dot{q}) = u, \tag{2}$$

where $q = q(t) \in \mathbb{R}^n$ is the joint coordinates vector; $u = u(t) \in \mathbb{R}^n$ is the control signal; $D(q) = D^T(q) \in \mathbb{R}^{n\times n}$ is the positive definite nonsingular "inertia" matrix; $d(q, \dot{q})$ is the vector function grouping the Coriolis, centrifugal, gravitational and friction forces.

(iii) Existence in a whole: the control signal u is just a part of a greater system — a robot, which could be defined as made up of: mechanical parts, energy inputs, signal paths, etc.

Thus, a control signal in (2) could be regarded as a self-supported variable:

$$\underset{\substack{\text{control}\\ \text{signal}}}{u} : \left\{ \underset{\substack{\text{control signal as a}\\ \text{physical phenomenon}}}{u}, \underset{\substack{\text{robotic}\\ \text{system}}}{W} \right\} \tag{3}$$

The expression (3) is then suitable not only for describing a robotic system but also easy to design the corresponding controller. Moreover, it is held that the fundamental objective of control is to design a suitable approach such that the controlled system can track the reference system with a desired level of accuracy. Now, the desired tracking set in a robotic system is given as follows:

$$S_d : \begin{cases} \dot{e}_i = -\alpha_i \cdot \left(e_i - e_{fi}\right), \\ \left|e_{fi}\right| \le p, \end{cases} \tag{4}$$

where $e_{fi} = e_{fi}(t)$ stands for the final errors (or on-line tracking quality), p and α_i are arbitrarily chosen small positive scalars to represent the desired tracking precision (tracking precision in the case of ideal measurements), the desired error decay rate (convergence rate), respectively.

However, in practice, it is hard to get the exact values of the desired variables, so based on the given system (2), we will introduce a more practical way to design the control law which can be applied to the available model rather than the exact model.

For the system (2), it is supposed that the measured joint positions and velocities can be obtained as follows:

$$\tilde{q}(t) = q(t) + w_1(t), \quad \tilde{\dot{q}}(t) = \dot{q}(t) + w_2(t), \tag{5}$$

where $\tilde{q}(t)$ and $\tilde{\dot{q}}(t)$ are available for feedback, $w_1(t),\ w_2(t) \in \mathbb{R}^n$ represent measurement uncertainties and satisfy the following conditions:

$$|w_{1i}(t)| \le c_1, \quad |w_{2i}(t)| \le c_2, \quad \forall\ i \in [1, n]. \tag{6}$$

Let $q_d(t) \in \mathbb{R}^n$ be the desired position, then $e(t) = q_d(t) - q(t)$ is the ideal error, $\tilde{e}(t) = q_d(t) - \tilde{q}(t)$ stands for the measured error. Combining with (5), it is easy to get

$$\tilde{e}(t) = e(t) - w_1(t), \quad \tilde{\dot{e}}(t) = \dot{e}(t) - w_2(t). \tag{7}$$

By the help of above measured errors $\tilde{e}(t)$ and $\tilde{\dot{e}}(t)$, the following theorem gives a methodology for achieving practical tracking (with the settling time τ_s) based on PSS.

Theorem 1: To exhibit practical tracking with the following error requirements:

$$\varepsilon_a(t) = \left\{ e(t): e_i^2(t) \le {e_{i0}}^2 \cdot \exp(-\alpha_i \cdot t) \right\}, \tag{8}$$

$$\varepsilon_f(t) = \left\{ e(t): |e_i(t)| \le 2\left(p + c_1 + \frac{c_2}{\alpha_i} \right) \right\}, \tag{9}$$

where $\varepsilon_a(t)$ denotes the admitted instantaneous errors for $t \in [0, \tau_s]$ and $\varepsilon_f(t)$ denotes admissible final instantaneous errors for $t \in [\tau_s, +\infty)$, e_{i0} is the value of e_i at $t = 0$. It is sufficient to design the following feedback control law:

$$u(t) = B \cdot \left[\tilde{\dot{e}}(t) + A \cdot \tilde{e}(t) \right], \tag{10}$$

where A and B are diagonal matrices, that is, A=diag$\{\alpha_i\}$, $B =$ diag$\{\beta_i\}$, $i = 1, \ldots, n$, whose elements α_i, β_i are positive scalars to be determined and β_i satisfying

$$\beta_i \ge \frac{|u_i(t)|}{\alpha_i \cdot p} \tag{11}$$

to guarantee the practical tracking of the system (2).

Proof: Construct the following Lyapunov function:

$$V(t) = \frac{1}{2} e^T(t) \cdot e(t) = \sum_i^n V_i(t) = \frac{1}{2} \sum_{i=1}^n e_i^2(t), \tag{12}$$

it is easy to get that $V(t) \ge 0$, then along the trajectory of (7), together with (10), we have

$$\begin{aligned} \dot{V}_i(t) &= e_i(t) \cdot \dot{e}_i(t) \\ &= e_i(t) \left[\tilde{\dot{e}}_i(t) + w_{2i}(t) \right] \\ &= e_i(t) \left\{ \frac{u_i(t)}{\beta_i} - \alpha_i \cdot [e_i(t) - w_{1i}(t)] + w_{2i}(t) \right\} \\ &\le e_i(t) \left\{ \frac{u_i(t) \cdot \alpha_i \cdot p}{|u_i(t)|} + \alpha_i \cdot w_{1i}(t) + w_{2i}(t) - \alpha_i \cdot e_i(t) \right\} \\ &\le |e_i(t)| \left\{ \frac{|u_i(t)| \cdot \alpha_i \cdot p}{|u_i(t)|} + \alpha_i \cdot c_1 + c_2 - \alpha_i \cdot |e_i(t)| \right\} \\ &= |e_i(t)| (\alpha_i \cdot p + \alpha_i \cdot c_1 + c_2) - \alpha_i e_i^2(t), \end{aligned} \tag{13}$$

it is equivalent to

$$\dot{V}_i(t) + \alpha_i V_i(t) \le |e_i(t)| (\alpha_i \cdot p + \alpha_i \cdot c_1 + c_2) - \frac{1}{2} \alpha_i |e_i(t)|^2. \tag{14}$$

On the one hand, if $|e_i(t)| > 2(p + c_1 + \frac{c_2}{\alpha_i})$, then we can get

$$\dot{V}_i(t) + \alpha_i V_i(t) < 0, \tag{15}$$

which means for $i \in [1, n]$, $e_i(t)$ is exponential convergent, and finally lies in the region (9) in a finite time with the desired error decay rate α_i.

On the other hand, it is easy to obtain from

$$\begin{aligned} u_i(t) &= \beta_i \cdot \left[\dot{\tilde{e}}_i(t) + \alpha_i \cdot \tilde{e}_i(t)\right] \\ &= \beta_i \cdot \{\dot{e}_i(t) - w_{2i}(t) + \alpha_i \cdot [e_i(t) - w_{1i}(t)]\}, \quad (16) \end{aligned}$$

to get that $\dot{e}_i(t) = \frac{u_i(t)}{\beta_i} - \alpha_i \cdot e_i(t) + \alpha_i \cdot w_{1i}(t) + w_{2i}(t)$, hence, once $|e_i(t)| \leq 2(p + c_1 + \frac{c_2}{\alpha_i})$, we can get

$$\begin{aligned} |\dot{e}_i(t)| &\leq \frac{|u_i(t)| \cdot \alpha_i \cdot p}{|u_i(t)|} + \alpha_i \cdot |e_i(t)| + \alpha_i \cdot c_1 + c_2 \\ &\leq \alpha_i \cdot p + 2\alpha_i \cdot (p + c_1 + \frac{c_2}{\alpha_i}) + \alpha_i \cdot c_1 + c_2 \\ &= 3\alpha_i \cdot \left(p + c_1 + \frac{c_2}{\alpha_i}\right). \quad (17) \end{aligned}$$

Therefore, both $|e_i(t)|$ and $|\dot{e}_i(t)|$ are all upper bounded. In summary, the designed algorithm based on PSS guarantees that the time evolution of $|e_i(t)|$ will enter into the prescribed region of (8) and (9) in a finite time. ■

Remark 1: In this algorithm, the designed controller (10) completely circumvents system model, so it is model-free because the gains β_i in (11) are synthesized on the basis of available information concerning the control signal only, not related to any parameters of the model itself.

Remark 2: In fact, the theory of PSS is not intended to produce control algorithms, it tends to explore a new and radically different conceptual framework in control theory, and its simple structure embraces other algorithms. Meanwhile, it should be mentioned that the PSS actually broadens the definition of error-feedback principle, because the design of controller is not only related to tracking errors but also includes a relation between the control signal and its own self, so it forms a powerful tool for on-line control.

III. GENERALIZED FRACTIONAL-ORDER PSS

Fractional-order calculus, as a natural generalization of classical integer-order calculus, describes the dynamic behavior of complex systems and solves mathematical problems more accurately [15]. Moreover, due to the fact that it possesses the properties of nonlocal and history-dependent [16], many processes with memory and heredity can be better characterized by using fractional-order models [17]–[22].

Motivated by the aforementioned fractional-order operators' theoretical significance and wide applications in numerous circuit systems, it is significant and, in fact, necessary to extend PSS to the fractional-order world, which is called fractional-order generalized principle of self-support (FOGPSS) in [23].

Generally speaking, there are three commonly used definitions of the fractional-order differential operator, i.e., the Grunwald-Letnikov derivative denoted by ${}^{GL}_{t_0}D^{\alpha}_t$, the Riemann-Liouville derivative denoted by ${}^{RL}_{t_0}D^{\alpha}_t$, and the Caputo definitions derivative denoted by ${}^{C}_{t_0}D^{\alpha}_t$. The Caputo fractional-order operator is adopted here due to the practicality of the interpretations for initial conditions that it allows, and this expression can also be interpreted in the frequency domain through the Laplace or Fourier transform, since several applications in system and process engineering are analyzed in this domain.

Definition 1: Given a non-integer order $\alpha > 0$, the uniform formula of a fractional integral for an integrable function $f(t)$ is defined as

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} \mathrm{d}\tau, \quad (18)$$

where ${}_{t_0}D_t^{-\alpha}$ represents the fractional integral of order α on $[t_0, t]$ and $\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} \mathrm{d}t,\ s \in \mathbb{C},\ Re(s) > 0$ is Euler's gamma function.

Definition 2: Given a non-integer order $\alpha > 0$, the Caputo fractional derivative of function $f(t)$ is given by

$$ {}^{C}_{t_0}D^{\alpha}_t f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{1+\alpha-n}} \mathrm{d}\tau, \quad (19)$$

where n is the first integer larger than α, that is, $n - 1 < \alpha \leq n, n \in \mathbb{Z}^+$.

In particular, when $0 < \alpha < 1$, then Caputo fractional derivative is defined as

$$ {}^{C}_{t_0}D^{\alpha}_t f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} \mathrm{d}\tau, \quad (20)$$

where ${}^{C}_{t_0}D^{\alpha}_t$ represents Caputo fractional derivative of order α on $[t_0, t]$.

Before introducing the detailed FOGPSS, some useful lemmas and inequalities should be given at first.

Lemma 1 [24]: Let $x(t) \in \mathbb{R}^n$ be a differentiable vector-value function. Then, for $t \geq 0$, the following inequality holds

$$ \frac{1}{2} {}^{C}_{0}D^{\alpha} \left[x^T(t)x(t)\right] \leq x^T(t) {}^{C}_{0}D^{\alpha} x(t), \quad (21)$$

where $\alpha \in (0, 1)$.

Lemma 2 [25]: Consider the following fractional-order system

$$ {}^{C}_{0}D^{\alpha} x(t) = f(t, x), \quad 0 < \alpha < 1, \quad (22)$$

let $x = 0$ be an equilibrium point for the system (22) and if there exists a Lyapunov function $V(t, x(t))$ and class-$\mathcal{K}$ functions κ_i $(i = 1, 2, 3)$ satisfying

$$ \kappa_1(\|x\|) \leq V(t, x(t)) \leq \kappa_2(\|x\|),$$

and

$$ {}^{C}_{0}D^{\beta}_t V(t, x(t)) \leq -\kappa_3(\|x\|),$$

where $\beta \in (0, 1)$. Then the origin of system (22) is asymptotically stable.

In [23], by a similar technique as the integer-order PSS in Theorem 1, they gave a generalized theorem to present the

FOGPSS under the following assumptions for a first-order model

$$\dot{x}(t) = -a_p x(t) + b_p u(t) + d(x, t), \tag{23}$$

where $x(t)$, $u(t) \in \mathbb{R}$ are the state and control input, respectively. a_p, $b_p > 0$ are bounded uncertain parameters (constants), $d(x, t)$ is the external disturbance signal. Let $x_d(t)$ be the desired reference trajectory, $x_e(t) = x_d(t) - x(t)$ is the tracking error.

Assumption 1: The position of x_d to be tracked is not directly available, but it moves within a known bounded region with a constrained velocity, i.e., $|x_d(t)| \leq b_1$, $|\dot{x}_d(t)| \leq b_2$, where $b_1, b_2 > 0$ are known constants.

Assumption 2: There exist positive constants $\underline{a}$, $\bar{a}$, $\underline{b}$, $\bar{b}$, $\bar{d}$ for the considered system (25), such that for all x and t, it holds that

$$\underline{a} \leq |a_p| \leq \bar{a}, \ \ \underline{b} \leq |b_p| \leq \bar{b}, \ \ |d(x, t)| \leq \bar{d}. \tag{24}$$

Assumption 3: It is assumed that the error measurement $x_e(t)$ can be denoted by

$$\tilde{x}_e(t) = x_e(t) -_0 D_t^{-\alpha} w(t), \ \ 0 < \alpha < 1, \tag{25}$$

where the estimated error function $\omega(t) \in L_1[a, b]$ satisfies that

$$|\omega(t)| \leq c_1, \ \ \left|{}_0 D_t^{-\alpha} w(t)\right| \leq c_2. \tag{26}$$

Theorem 2: Under the given assumptions 1-3, for the considered system (25), taking the FOGPSS feedback law

$$u(t) = \bar{\beta}\tilde{s}(t), \tag{27}$$

where $\bar{\beta}$ is a designed parameter satisfying

$$\bar{\beta} > \frac{u_{\max}}{\delta\varepsilon_0} > 0, \tag{28}$$

where $\delta > 0$, $u_{\max}$ is also a designed parameter which satisfies that $|u(t)| \leq u_{\max}$, and $\tilde{s}(t)$ is the fractional-order estimated error feedback signal

$$\tilde{s}(t) = {}_0^C D_t^{\alpha} \tilde{x}_e(t) + \delta\tilde{x}_e(t), \tag{29}$$

then the real tracking error $x_e(t)$ will be driven into $D_{\varepsilon_0} \triangleq \{x_e(t){:}|x_e(t)| \leq \frac{\delta c_2 + c_1}{\delta} + \varepsilon_0\}$.

Proof: Choose a Lyapunov function $V(t) = \frac{1}{2}x_e^2(t)$, taking the fractional-order derivative of $V(t)$ and using Lemma 1 yields

$${}_0^C D_t^{\alpha} V(t) = {}_0^C D_t^{\alpha}\left(\frac{1}{2}x_e^2(t)\right) \leq x_e(t) {}_0^C D_t^{\alpha} x_e(t), \tag{30}$$

then the following processes are similar to the proof of integer-order ones in Theorem 1, to eventually obtain that ${}_0^C D_t^{\alpha} V(t) < 0$. Hence, based on Lemma 2, the tracking error $x_e(t)$ will asymptotically converge to the given region D_{ε_0}. ■

Remark 3: Note that the given theorem on the practical tracking for fractional-order systems mainly concentrated on fractional order α belonging to $0 < \alpha < 1$. In practice, there exist numerous fractional-order models described by fractional order α lying in $1 < \alpha < 2$, for example, super-diffusion and power electronics. However, it should be mentioned that in fact, the stability analysis for fractional-order system with order $1 < \alpha < 2$ is totally different from the corresponding condition in case of $0 < \alpha < 1$, therefore, future work will extend to FOGPSS for fractional-order systems with order $1 < \alpha < 2$.

Remark 4: As authors mentioned in their paper, the biggest contribution of [23] is to bring up the new design idea about fractional order research framework for the first time. There indeed exist some drawbacks in the design of FOGPSS, for example, the design parameter $\bar{\beta}$ in (30) is chosen as a constant, so the given controller (29) is similar to the traditional fractional-order PD controller. However, β_i in the original algorithm (11) for the integer-order PSS is time-varying, there is no doubt that the time-varying parameters are the main differences between PSS and normal error-feedback control. Therefore, to reduce the conservatism and avoid the occurrence of algebraic loop, how to implement more reasonable algorithms for β_i is one of the main directions of our further research.

IV. EVENT-TRIGGERED CONTROL BASED ON PSS

In recent years, there have emerged great interests in utilizing event-triggered schemes to deal with control issues such as state feedback control [26], output feedback control [27], [28] and sliding mode control [29]. To a great extent, it is related to the fact that event-triggered control can significantly reduce data transmission, communication costs, and relevant computation burden while maintaining a satisfactory level of control performance [30]. Different from the traditional time triggered approach which is over-provisioning, event-triggered control is a superior and aperiodic control strategy, it only updates the control task and transmits the control signal when a pre-specified mathematical inequality is violated [31]. In this way, not every system signal will be transmitted to the controller design. Thus, there is no doubt that the event-based control scheme is closer to the behavior of people [32].

Owing to these advantages, relevant studies on the event-triggered control have achieved fruitful results [33]–[35]. However, it should be noted that most of existing triggered conditions in the aforementioned papers are based on the internal models and explicit system parameters, to design state-feedback event-triggered controllers and guarantee systems' stability and the desired performance. However, few papers have addressed event-triggered control for the trajectory tracking issues when the internal system dynamics are unknown, which appeals to us to utilize the basic idea of PSS, to present a novel event-triggered mechanism based on the tracking error, and design a suitable feedback controller by utilizing less model information.

Consider a class of nonlinear continuous-time systems in the following form

$$\begin{cases} \dot{x}(t) = f(x) + g(x)u(t), \ \ \forall t > 0, \\ x(0) = x_0, \end{cases} \tag{31}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \longrightarrow \mathbb{R}^{n\times m}$ are nonlinear functions.

The main control objective in this section is to design a feedback controller $u(t)$ based on the measured tracking error $e(t) \triangleq x_d(t) - x(t)$ and its event-triggered mechanism, such that the trajectory of the system (31) practically tracks the specified continuously bounded reference signal $x_d(t)$ for a prescribed tracking accuracy $p > 0$ in a finite time, which means that the tracking error can be driven into a prescribed bounded feasible region $D_p \triangleq \{e(t) : |e_i(t)| \leq p,\ i = 1, 2, \ldots, n\}$.

As we mentioned before, in order to save resources, the event-triggered controller will work with a sampled version of the measured error obtained at the triggering instants, that is, the controller is dynamically updated only when, as we shall see later, the triggering condition is satisfied.

For that reason, one needs to introduce a sampled-data component that is characterized by a monotone increasing sequence of sampling instants (broadcast release times) $\{t_k\}_{k=0}^{\infty}$, where t_k is the $(k+1)-$th sampling instant. Without loss of generality, it is assumed that the first triggered instant is given by $t_0 = 0$, so it satisfies that $0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$. For simplicity, it is assumed that in this article, the sampled-data systems have zero task delays and there is no Zeno phenomenon on the event-triggered sampling process.

In general, an event-triggered scheme includes a monitor and a sampler. Firstly, the monitor detects the difference between the current sampling error $e(t)$ and the latest transmitted sampling error signal $e(t_k)$,

$$\widetilde{e}(t) = e(t) - e(t_k),\ \ \forall t \in \big[t_k,\ t_{k+1}\big), \tag{32}$$

as well as the gap between $\dot{e}(t)$ and the value of $\dot{e}(t)$ at instant t_k, which is denoted as $\dot{e}(t_k)$,

$$\widetilde{\dot{e}}(t) = \dot{e}(t) - \dot{e}(t_k),\ \ \ \forall t \in \big[t_k,\ t_{k+1}\big), \tag{33}$$

then it checks if the relationship between $\widetilde{e}(t)$ and $\widetilde{\dot{e}}(t)$ violates the following condition, to decide whether an event is triggered or not,

$$\|A\widetilde{e}(t)\| + \big\|\widetilde{\dot{e}}(t)\big\| < \sigma\|Ae(t)\|, \tag{34}$$

where σ is a designed positive threshold constant. A is the diagonal matrix which has been given in Theorem 1.

Secondly, once the inequality (34) is violated, the current tracking error will be sampled immediately and then transmitted to the controller, and held by a zero-order holder (ZOH) until the next triggering. Thus, it is easy to get the next triggered instant t_{k+1} can be induced by

$$t_{k+1} = \inf\big\{t > t_k \big| \|A\widetilde{e}(t)\| + \big\|\widetilde{\dot{e}}(t)\big\| \geq \sigma\|Ae(t)\|\big\}. \tag{35}$$

By utilizing condition (35), the problem of practical tracking to be addressed in this section is to combine with the framework of PSS, to develop an event-based controller for tracking a given trajectory within a desired ultimate bound.

Remark 5: Note that once an event is triggered at $t = t_k$, the gaps $\widetilde{e}(t)$ and $\widetilde{\dot{e}}(t)$ will be reset to zero at the same time and start to grow in a new round until the next event happens.

The following theorem gives a methodology for achieving practical tracking (with the settling time τ_s) based on an event-triggered controller.

Theorem 3: Under the event-triggered mechanism (35) with $\sigma \in (0, \frac{1}{2})$, taking an error-feedback controller as follows

$$u(t) = u(t_k) = B[\dot{e}(t_k) + Ae(t_k)],\quad t \in \big[t_k, t_{k+1}\big), \tag{36}$$

where $A = \mathrm{diag}\{\alpha_i\}$, $B = \mathrm{diag}\{\beta_i\}$, $i = 1, 2, \ldots, n$, and β_i are selected to satisfy

$$\beta_i \geq \frac{2|u_i(t)|}{(1 - 2\sigma)\alpha_i \cdot p}, \tag{37}$$

then the system (31) can achieve practical tracking, and the tracking error $e(t)$ satisfies

$$\varepsilon_a(t) = \Big\{e(t) : e_i^2(t) \leq {e_{i0}}^2 \cdot \exp(-\alpha_i \cdot t)\Big\}, \tag{38}$$

$$\varepsilon_f(t) = \{e(t) : |e_i(t)| \leq p\}, \tag{39}$$

where $p,\ \alpha_i,\ \varepsilon_a(t),\ \varepsilon_f(t)$ are the same with those given in Theorem 1.

Proof: Construct the following Lyapunov function

$$V(t) = \frac{1}{2}e^T(t)\cdot e(t) = \frac{1}{2}\sum_{i=1}^{n} e_i^2(t), \tag{40}$$

one can obtain that $V(t) \geq 0$ and its time derivative as follows

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n} e_i(t)\cdot \dot{e}_i(t)\\ &= \sum_{i=1}^{n} e_i(t)\big\{\dot{e}_i(t_k) + \widetilde{\dot{e}}_i(t)\big\}\\ &= \sum_{i=1}^{n} e_i(t)\left\{\frac{u_i(t)}{\beta_i} - \alpha_i e_i(t_k) + \widetilde{\dot{e}}_i(t)\right\}\\ &= \sum_{i=1}^{n} e_i(t)\left\{\frac{u_i(t)}{\beta_i} - \alpha_i[e_i(t) - \widetilde{e}_i(t)] + \widetilde{\dot{e}}_i(t)\right\}\\ &= \sum_{i=1}^{n} e_i(t)\left\{\frac{u_i(t)}{\beta_i} - \alpha_i e_i(t) + \alpha_i\widetilde{e}_i(t) + \widetilde{\dot{e}}_i(t)\right\}.\end{aligned} \tag{41}$$

Combining the triggered scheme (35) and the designed condition (37), and considering a single $i-$th element, it yields $V(t) = \sum_i^n V_i(t)$ and $V_i(t) = \frac{1}{2}e_i^2(t)$, one has

$$\begin{aligned}&\dot{V}_i(t) + \alpha_i V_i(t)\\ &\leq |e_i(t)|\left\{\frac{|u_i(t)|}{\beta_i} - \frac{1}{2}\alpha_i|e_i(t)| + \big|\alpha_i\widetilde{e}_i(t) + \widetilde{\dot{e}}(t)\big|\right\}\\ &< |e_i(t)|\left\{\frac{1}{2}(1 - 2\sigma)\alpha_i\cdot p - \frac{1}{2}\alpha_i|e_i(t)| + \sigma\alpha_i|e_i(t)|\right\}.\end{aligned} \tag{42}$$

On the one hand, if $|e_i(t)| > p$, then we can get

$$\dot{V}_i(t) + \alpha_i V_i(t) < 0, \tag{43}$$

which means $V_i(t) < V_i(0) \cdot \exp(-\alpha_i \cdot t)$, then it is easy to obtain from the form of V_i to get that

$$e_i^2(t) \leq {e_{i0}}^2 \cdot \exp(-\alpha_i \cdot t), \tag{44}$$

thus it is showed by the Lyapunov stability theorem that the tracking errors will exponentially converge to the region (39) in a finite time, and $|e_i(t)|$ is guaranteed to be upper bounded by $|e_{i0}| \cdot \exp(-\frac{1}{2}\alpha_i \cdot t)$,

On the other hand, once $e_i(t)$ lies in the region (39), it ensures $|e_i(t)| \leq p$. Meanwhile, under the controller of (36) with respect to the constrained condition (37), $|\dot{e}(t)|$ is also upper bounded as follows

$$\begin{aligned} |\dot{e}_i(t)| &= \left|\widetilde{\dot{e}}_i(t) + \dot{e}_i(t_k)\right| \\ &\leq \left|\widetilde{\dot{e}}_i(t)\right| + |\dot{e}_i(t_k)| \\ &= \left|\widetilde{\dot{e}}_i(t)\right| + \left|\frac{u_i(t)}{\beta_i} - \alpha_i e_i(t_k)\right| \\ &\leq \left|\widetilde{\dot{e}}_i(t)\right| + \left|\frac{u_i(t)}{\beta_i}\right| + \alpha_i |e_i(t) - \widetilde{e}_i(t)| \\ &\leq \alpha_i |\widetilde{e}_i(t)| + \left|\widetilde{\dot{e}}_i(t)\right| + \left|\frac{u_i(t)}{\beta_i}\right| + \alpha_i |e_i(t)| \\ &< \sigma\alpha_i |e_i(t)| + \left(\frac{1}{2} - \sigma\right)\alpha_i p + \alpha_i |e_i(t)| \\ &\leq \frac{3}{2}\alpha_i p. \end{aligned} \tag{45}$$

Through the above analysis, it has been proved that $|e_i(t)|$ and $|\dot{e}_i(t)|$ are all upper bounded. Thus, it concludes that based on PSS and the proposed event-triggered scheme, the designed feedback controller (38) can ensure that the tracking error $e(t)$ is driven into the pre-specified bounded feasible region D_p in a finite time. ■

Remark 6: It should be pointed out that the parameter δ has a great influence on the event-triggered instants, that is the different values of threshold constant δ in the triggering condition (35) will lead to different event-triggered frequencies. Generally speaking, a smaller σ corresponds to a shorter event-triggered period. In addition, if $\sigma = 0$, the event-triggered controller (38) is returned to a traditional continuous feedback controller, and the selection of β_i in (39) is consistent with the traditional condition in PSS without measurement uncertainties.

Remark 7: In this section, we are mainly concerned with how to use less information or weaker conditions, to guarantee the tracking error to satisfy a given tracking accuracy after a finite time, which addresses the practical tracking issue. Hence, it makes the designed controller more realistic in practical applications.

Remark 8: Compared with the traditional asymptotically stable control laws, which require that the convergence rate is at best exponential with infinite settling time, the control laws with finite-time convergence are more desirable. However, in

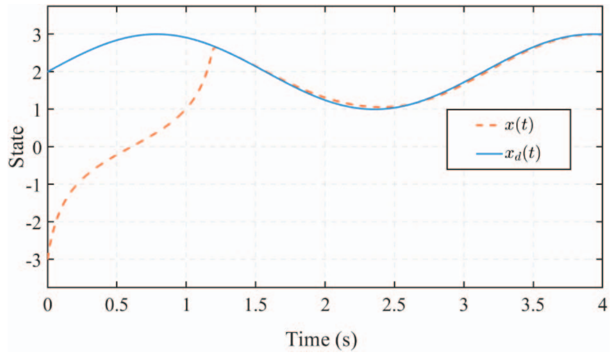

FIGURE 3. State responses of $x(t)$ and $x_d(t)$.

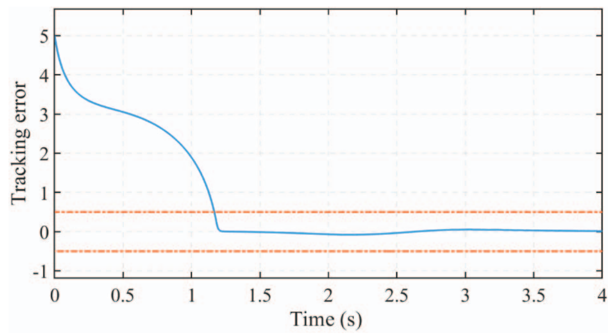

FIGURE 4. Tracking error $e(t)$.

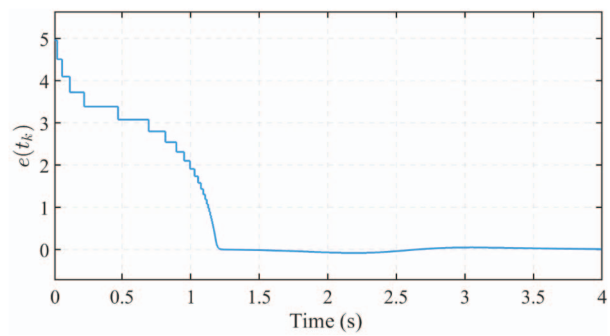

FIGURE 5. Triggered signal $e(t_k)$.

Theorem 3, we just build a basic framework for finite-time tracking issue based on PSS and event-triggered mechanism, the corresponding characterization between the finite-time tracking settling time and stability performance needs to be investigated in our following research.

In the following, we illustrate the proposed event-triggered controller on the following nonlinear system, require the controlled system

$$\dot{x}(t) = 2\cos(x) + 0.15x^2 u(t), \tag{46}$$

to track the desired trajectory given by

$$x_d(t) = 2 + \sin(2t). \tag{47}$$

The initial value of system (46) is chosen as $x(0) = -3$, besides, in the following simulations, other relevant parameters are given as $\sigma = 0.1$, $\alpha = 80$, $p = 0.5$, $|u|_m = 20$.

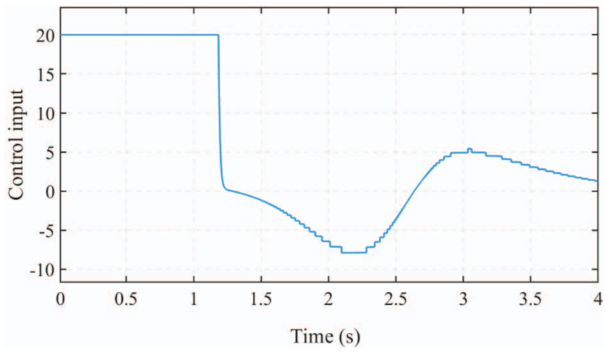

FIGURE 6. Control input $u(t)$.

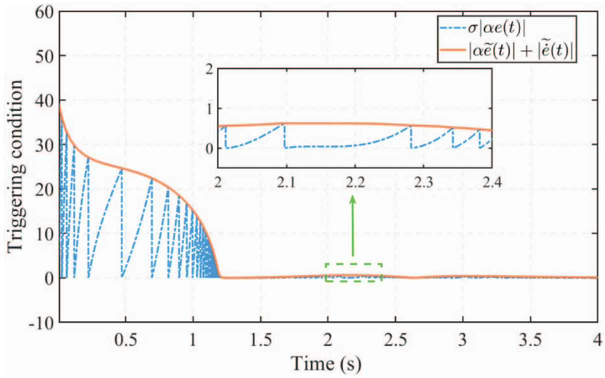

FIGURE 7. The evolution of $|\alpha\widetilde{e}(t)|+|\widetilde{e}(t)|$ along with its bound $\sigma|\alpha e(t)|$.

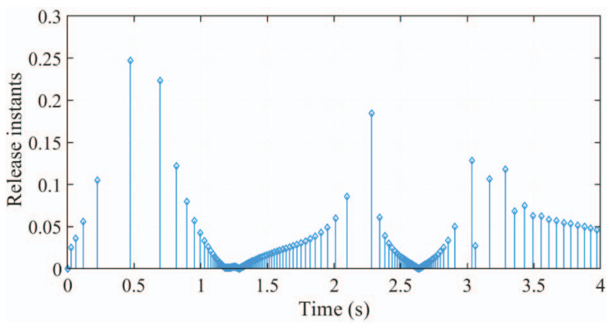

FIGURE 8. Inter-execution periods under the event-triggered mechanism.

Through the proposed event-triggered mechanism in (35) and the derived conditions in Theorem 3, the Simulink model can be constructed to present the following figures.

Obviously, from Fig. 3 and Fig. 4, it is easy to demonstrate that under the event-triggered feedback controller, the trajectory of tracking error $e(t)$ can be driven into the prescribed small bounded region in a finite time. Meanwhile, under the implementation of ZOH, the signal of the event-triggered sampling error $e(t_k)$ and the corresponding control input $u(t)$ are illustrated in Fig. 5 and Fig. 6, respectively.

Furthermore, the evolution of $|\alpha\widetilde{e}(t)|+|\widetilde{e}(t)|$ along with its boundary $\sigma|\alpha e(t)|$ is shown in Fig. 7. As we can see, once an event is triggered, which means $|\alpha\widetilde{e}(t)|+|\widetilde{e}(t)|$ exceeds a prescribed bound, the error norm will be reset to zero immediately and start to grow in a new round until it reaches the next triggered condition, thus it always satisfies the relationship $|\alpha\widetilde{e}(t)|+|\widetilde{e}(t)|\le\sigma|\alpha e(t)|$.

In addition, the Fig. 8 depicts the inter-sampling instants under the event-triggered scheme (37), it is easy to observe that the event-triggered mechanism is updated aperiodically, so there is no doubt that it can significantly reduce the sampling numbers and computation resources.

V. CONCLUSION AND FUTURE WORK

In this article, the basic theoretical framework of PSS was reviewed firstly. After that, a theorem about FOGPSS was introduced. Besides, due to the fact that the event-triggered mechanism can significantly reduce the sampling numbers and computation resources, an event-triggered feedback controller based on PSS for practical tracking was investigated accordingly. Although PSS has been proposed for more than 25 years, further work and development of this idea have not been extended effectively, motivated by the wide applications of fractional calculus in a great amount of circuits and systems, future research should be concentrated in the following directions.

1. As we mentioned in Remark 4, an outstanding characteristic of the original PSS is the time-varying property for the selection of gain β_i, therefore, we will denote to designing the improved FOGPSS by reducing the conservatism of selection gains β_i in further research. For example, in general, it is simplest to choose constant gains β_i which based on the constrained bounds of $u_i(t)$:

$$u_i(t)=\beta_i\cdot\left[\dot{\tilde{e}}_i(t)+\alpha_i\cdot\tilde{e}_i(t)\right], \tag{48}$$

with

$$\beta_i=\frac{|u_i(t)|_m}{\alpha_i\cdot p}, \tag{49}$$

where $|u_i(t)|_m$ denotes some suitable, large enough values of the control signal, for simplicity, an "absolute" maximum c_3 can be adopted here, which means $|u_i(t)|\le c_3,\ \forall\ i\in[1,n]$. For example, in practical applications, there always exist maximal available torque or voltage of the actuators.

Returning to the given procedures of FOGPSS, we can see that in Theorem 2, the authors used this algorithm to get the desired upper bound $u_{\max}$ in (29) for $|u(t)|$:

$$|u|=\left|\frac{\dot{x}+a_px-d(x,t)}{b_p}\right|\le\frac{|\dot{x}|+\bar{a}|x|+\bar{d}}{\underline{b}}\triangleq u_{\max}. \tag{50}$$

However, instead of using this form, it is better to adopt the time-varying gains $\beta_i(t)$ form for implementation purpose, which based on the previous values $u_i(t-T)$ and the bounds of $\dot{u}_i(t)$:

$$u_i(t)=\beta_i(t)\cdot\left[\dot{\tilde{e}}_i(t)+\alpha_i\cdot\tilde{e}_i(t)\right], \tag{51}$$

with

$$\beta_i(t)=\frac{|u_i(t-T)|+c_4\cdot T}{\alpha_i\cdot p}, \tag{52}$$

where c_4 denotes for $i \in [1, n]$, it satisfies that $|\dot{u}_i(t)| \leq c_4$. Obviously, in this improved algorithm, a smaller T will lead to less conservative results.

Meanwhile, it is worthwhile considering the generalized case of variables' internal coupling, in this case, further research about controller design based on non-diagonal matrices A and B will also be valuable and interesting.

2. As a software realization of event-triggered control, the self-triggered scheme will also be considered, because it possesses a more reasonable condition. As introduced in most event-triggered papers, the self-triggered controller is the same as the one in the event-triggered scheme, while the next task triggered instant is predicted only depending on the last triggered data $e(t_k)$, instead of detecting real-time error $e(t)$. For example, we can also design an improved event condition for (37) to achieve practical tracking for the system (33) as follows

$$t_{k+1} = \inf\left\{t > t_k \big| \|(1+\sigma)A\widetilde{e}(t)\| + \left\|\dot{\widetilde{e}}(t)\right\| \geq \sigma \|Ae(t_k)\|\right\}.$$

Our future works will focus on quantitative comparisons between the event-triggered, self-triggered and periodic-triggered controllers, and detailed analysis of Zeno behavior to avoid infinite triggering and sampling issues.

3. On the other hand, switched systems, which can be viewed as an important kind of hybrid dynamical systems, are consisted of several separated subsystems and a logical rule that specifies which subsystem will be activated along the system trajectory at each instant of time. Moreover, when a switched system contains at least one fractional-order subsystem, it will be called a switched fractional-order system [36]–[39], which can be characterized by

$$ {}_{0}^{C}D_t^{\alpha} x(t) = A_{\sigma(t)} x(t), \tag{53}$$

where $0 < \alpha < 1$, $x(t) \in \mathbb{R}^n$ is the system state vector, $A_{\sigma(t)} \in \mathbb{R}^{n\times n}$ are known constant matrices. Besides, for a given constant $T > 0$, the function $\sigma(t) : [0, T] \longrightarrow S = \{1, 2, \ldots, N\}$ (N denotes the total number of subsystems, it is assumed that $N \geq 2$ to avoid trivial cases) is the switching signal, which is assumed to be a piecewise constant function of time t and right-continuous at the switching instants, and when $t \in [t_k, t_{k+1})$, we say that the i_k-th subsystem is active.

However, to date, to the best of our knowledge, switched systems involving the Caputo fractional derivative have not been studied suitably due to the negligence of memory for fractional-order operators. In most existing papers, they all take the terminal value of the last interval as the initial value of the next interval for the switching-type fractional-order system. In fact, even when $t \in [t_k, t_{k+1})$, the Caputo derivative in this interval should be ${}_{0}^{C}D_t^{q}$ not ${}_{t_k}^{C}D_t^{q}$. It means for $t \in [t_k, t_{k+1})$, the correct form for this subsystem should be

$$ {}_{0}^{C}D_t^{\alpha} x(t) = A_{k+1} x(t). \tag{54}$$

Therefore, to overcome the above mentioned drawbacks in these papers and avoid misleading of switched fractional-order systems, some innovative works have been derived by us recently. In the next step, we will apply PSS to this special kind of hybrid system to design the corresponding sub-controller for every subsystem, and give the practical stability analysis of the considered systems.

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