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Application of ZigZag Decoding in Frameless ALOHA

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ABSTRACT As ZigZag decoding (ZD) can resolve a collision between two packets, the combination of ZD and successive interference cancellation (SIC) is expected to provide a lower packet loss rate (PLR) and higher average throughput than the conventional frameless ALOHA. Therefore, in this paper, we considered applying ZD in frameless ALOHA. A straightforward implementation is first considered while deriving the exact theoretical expression for the PLR, and implementation results show higher PLR and lower throughput performance than in the conventional scheme. In addition, we propose a sophisticated implementation called enhanced ZigZag decodable frameless ALOHA (E-ZDFA), where the transmission probability is dynamically increased to improve the throughput by enhancing the chances for unretrieved users to transmit. The computer simulations revealed that E-ZDFA achieves higher throughput performance than the original frameless ALOHA and the state-of-the-art irregular-repetition-slotted ALOHA. Moreover, E-ZDFA achieves a lower error floor than frameless ALOHA, enabling the prevention of user silence, i.e., no transmission of the packet during the protocol.

INDEX TERMS Frameless ALOHA, successive interference cancellation, ZigZag decoding.

I. INTRODUCTION

The advances in wireless communications allow not only high-speed communication but also high connectivity, resulting in the emergence of the Internet of Things (IoT) [1]. In particular, the number of users connected to the network is very large in the IoT scenario, so the interest in *grant-free access* is high [2]. As users autonomously transmit their packets in the case of grant-free access, e.g., ALOHA [3], dealing with packet collision is crucial. Coded ALOHA schemes, where interslot (time domain) successive interference cancellation (SIC) is installed, are well-known candidates for solving packet collision as they achieve high-throughput performance comparable with that of time division multiple access (TDMA) [4], [5]. Coded ALOHA uses the concept of codes-on-graphs, such as low density parity check (LDPC) codes [6]. While the bipartite graph of LDPC codes describes the relationship between codeword bits and parity checks, the bipartite graph of coded ALOHA depicts the relationship between transmitted packets (or transmitting users) and received packets (or time slots). Liva [4] proposed irregular-repetition-slotted ALOHA (IRSA), where each user determines the number of retransmissions by using the probability mass function called *degree distribution*, which is theoretically optimized to maximize the peak-throughput performance. In [5], IRSA was generalized into coded slotted ALOHA (CSA), where data packets are encoded by error-correcting codes in advance and then divided into multiple blocks to be transmitted; IRSA is considered as a CSA scheme using repetition codes. The degree distribution of IRSA was optimized again in [5], allowing larger maximum degree of users than that achieved in [4], and the optimized IRSA was theoretically shown to achieve a throughput of 0.977.

As grant-free access accommodates a large number of users with fluctuating demands, the number of contending users is expected to fluctuate. Although IRSA is known to attain high throughput performance, as shown in [4] and [5], the base station (BS) is required to suitably select the number of time slots, i.e., frame length prior to the transmission of users, to achieve the designed throughput performance. If the frame length is not appropriate, the throughput performance is degraded from the designed value; a shorter frame results in an overloaded situation where transmitted packets frequently

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collide, whereas a longer frame leads to unnecessary time slots. To mitigate this limitation, frameless ALOHA was proposed in [7] by introducing the idea of rateless codes [8] into coded ALOHA, while IRSA is based on LDPC codes. In frameless ALOHA, users decide whether to transmit packets at each slot based on the given transmission probability. The receiver attempts to retrieve the received packets through SIC until a sufficiently large number of packets is retrieved. Transmission is terminated when the number of retrieved users attains the given threshold so that the frame length is determined *a posteriori*. Such a *frameless* structure allows for the adaptive determination of the appropriate frame length so that the designed throughput performance is always achieved. Furthermore, as the optimization of the degree distribution for IRSA [4], [5] is based on density evolution analysis, which presumes an infinitely large number of users, the optimized degree distribution would yield lower throughput performance than the designed performance when the number of users is not very large; the number of active users in a practical network is fluctuating and fractional due to sporadic demand. At this point, as frameless ALOHA designs the transmission probability instead of the degree distribution itself, the resulting degree follows the binomial distribution, which can be reproduced even when the number of users is not so large. Hence, frameless ALOHA can be considered as good a candidate for a random access scheme for networks with practically large number of users.

The SIC process of coded ALOHA, including frameless ALOHA, is identical to belief propagation (BP) decoding over binary erasure channels, i.e., a peeling decoder, so analytical tools for codes-on-graphs are available for coded ALOHA. While the peeling decoder can only start decoding from *degree-1 check nodes*, i.e., parity checks which include only one bit of a codeword, more powerful decoders that can start decoding from check nodes with higher degree have been proposed. For instance, Olmos *et al.* [9] proposed a tree-structure expectation propagation (TEP) decoder, where a check node with two unknown codeword bits, i.e., *a degree-2 check node*, can be used as a starting point of decoding. The TEP decoder was shown to yield a better bit-error-rate performance than the BP decoder. However, the TEP decoder requires the structure of the bipartite graph, and this requirement cannot be fulfilled in frameless ALOHA because it continues probabilistic transmission where the bipartite graph is constructed on the fly. Notably, the result of the TEP decoder showed that the idea of directly resolving collided packets improves decoding performance.

For multiple access, ZigZag decoding (ZD) has been proposed to demodulate two colliding packets as a solution to the hidden terminal problem [10]. In ZD, colliding users are requested to immediately retransmit their packets by the receiver so as to receive two colliding packets. Then, if the two packets are received with different delays, data packets can partially be demodulated from each received packet. The demodulated part can be used to cancel collision in part in the other colliding packet; as a result, two colliding packets can be retrieved in a ZigZag manner. As the TEP decoder improves the decoding performance of LDPC codes, ZD is considered to improve the packet-retrieval performance of frameless ALOHA.

Oinaga *et al.* [11] proposed ZigZag decodable coded slotted ALOHA (ZDCSA), where ZD is straightforwardly introduced in IRSA. The authors compared ZDCSA with conventional IRSA and showed that ZDCSA achieves better throughput performance when the number of users is *moderately* large, i.e., 10^3. However, they also pointed out that the asymptotic throughput of ZDCSA is lower than that of IRSA. This is because ZD requires packet retransmission, resulting in the degradation of throughput performance. Moreover, as IRSA determines the number of retransmissions and slots in which each user transmits its packet in advance, the latter slots are used for transmission of already-retrieved users, for which the throughput performance is also degraded.

To this end, this paper proposes ZigZag decodable frameless ALOHA (ZDFA), where ZD is introduced in frameless ALOHA [7]. First, a straightforward implementation of ZD in frameless ALOHA is discussed, where it is revealed that the asymptotic throughput performance degrades along with the ZDCSA scenario [11] because of additional time slots. Then, we exploited the frameless nature to propose a sophisticated implementation called *enhanced ZDFA (E-ZDFA)*, where the transmission probability is dynamically increased to enhance the chances for unretrieved users to transmit. Simultaneously, users retrieved via ZD or received without collision, i.e., retrieved upon being received, are acknowledged by the receiver through two-bit feedback to stop retransmission in following slots. The error floor of E-ZDFA was theoretically derived in this study, confirming that E-ZDFA has the potential to retrieve more users than the original frameless ALOHA. Computer simulations were conducted, and they show that a suitably chosen transmission probability leads to improved throughput performance of the frameless ALOHA and IRSA with the degree distribution obtained in [5]. The contributions of this study are summarized as follows:

- We discuss the straightforward implementation of ZD in frameless ALOHA by deriving an exact theoretical representation of PLR.
- To improve throughput performance, we propose a novel protocol named E-ZDFA, where ZD is applied in frameless ALOHA with some modification.
- We numerically show that E-ZDFA outperforms conventional schemes in terms of average throughput and PLR. Moreover, E-ZDFA is revealed to achieve higher throughput performance in practical scenarios also.

The remainder of this paper is organized as follows. In Section II, the system model is described. Conventional frameless ALOHA protocol, which our proposals are based on, is explained in Section III. We discuss the application of ZD in frameless ALOHA and present the proposed E-ZDFA in Section IV. E-ZDFA is then numerically evaluated in

Section V. In Section VI, we conclude this paper and suggest some future research directions.

II. SYSTEM MODEL

Our model is comprised of a network with N transmitting users and a common BS. Each user has its own packet at the beginning of the communication, and new packets are supposed to not be generated (no backlogging). Throughout the study, a noise-free channel was considered, where the transmitted packet can be successfully retrieved by the receiver without collision. Packets that collide are considered to be lost because for mathematical tractability reasons, the capture effect [12] is not considered. As this model is considered to be the worst-case scenario, it provides a lower-bound of the throughput performance in practical situations, where the capture effect would be available.

Moreover, the BS was assumed to be able to distinguish the following conditions for each time slot:

(a) No users have transmitted.
(b) Only one user has transmitted, i.e., the time slot is a singleton.
(c) Two users have transmitted and collided.
(d) Three or more users have transmitted and packets have collided.

In particular, the BS should detect condition-(c) so that ZD can be operated. This was realized by supposing that each packet contains a unique word that identifies the transmitter. The BS can detect the collision of two packets by calculating the correlation between the received packet and unique words [10], and the BS then acknowledges the collision of two packets when the correlation has two peaks. This assumption is practical because each transmitted packet should contain information indicating the transmitter of the packet.

Time slots have two types of *subslots*: uplink subslot (US) and downlink subslot (DS); these are demonstrated in Fig. 1. Users transmit their packets in the US, and the BS broadcasts a feedback signal to the following DS. A detailed explanation about feedback signals from the BS is provided in the subsequent sections. Upon transmission, slots are organized into a *frame*.

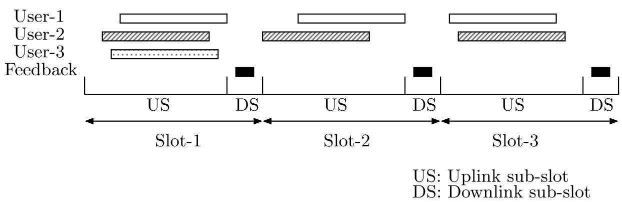

FIGURE 1. Illustration of time slots and subslots.

III. FRAMELESS ALOHA PROTOCOL

This section briefly describes the conventional frameless ALOHA protocol. Transmission probability p is given to the users as follows:

$$p = \frac{G}{N}, \tag{1}$$

where G is the *target degree*, which indicates the number of users simultaneously transmitting in one time slot.

By using probability p, each user independently decides whether to transmit a packet in every time slot. The probabilistic transmission of users continues until the BS retrieves a sufficiently large number of users, i.e., the transmission is terminated when the sufficiently large number of packets ($\lfloor \alpha N \rfloor$) is retrieved, using a given threshold $\alpha \in (0, 1]$, where $\lfloor \cdot \rfloor$ denotes the floor function. To this end, the BS uses one-bit feedback to inform users if the frame is continued as follows:

- The frame is ended as the desired PLR has been achieved at the BS.
- The frame is continued as desired PLR has not been achieved yet.

Note that $\alpha = 1$ implies that the transmission continues until all the users are retrieved. However, this setting causes a large delay because of the probabilistic transmission; this is because the PLR performance has an error floor caused by the probability that the user never transmits during the frame [7]. Further, as the PLR achieved when the transmission is terminated is considered to be close to the error floor, reducing the error floor would lead to higher throughput performance. Moreover, a lower error floor allows the BS to use a larger α with smaller degradation of throughput.

The received packets may contain collisions of packets, so the BS uses SIC to resolve the collision and retrieve the collided packets. The SIC for frameless ALOHA is equivalent to the peeling decoder for LDPC codes [6] and can be described as follows:

(i) Retrieve the transmitted packets from singleton slots, which are assumed to be empty.
(ii) Subtract the packets from all the received signals comprising the packets.

After step (ii), some collided packets become singletons, and the above-mentioned operations are repeated until all the singleton slots vanish. To execute step (ii), each packet was assumed to include information indicating the time slot in which it was transmitted. Note that the retrieved packets might be included in future received packets. To this end, if each user identification (ID) is used as the seed for a random-number generator for choosing time slots to transmit while sharing the same random number generator, then the receiver can determine all future transmissions and subtract signals from all the received packets [13].

IV. ZDFA

This section explains the application of ZD in frameless ALOHA. Next, we discuss a simple implementation, where ZD is straightforwardly introduced into frameless ALOHA, and we propose a sophisticated implementation in which the transmission probability is dynamically increased.

A. ZD

To perform ZD, when a collision of two users is detected, the BS requires the users to retransmit immediately in the next

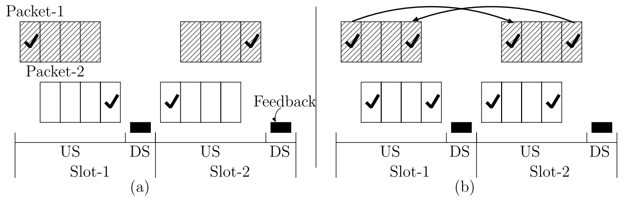

FIGURE 2. **Illustration of ZD. (a) Two packets collide in slot-1 and are retransmitted in the next slot. The first segment of packet-1 and the last segment of packet-2 are free of collision in slot-1; thus, they are retrievable. (b) By canceling the retrieved segments, the third segment of packet-2 becomes free of collision in slot-1, while the second segment of packet-2 becomes free of collision in slot-2.**

time slot. Throughout this study, we assumed that packets collide with *segment-wise* delay and the possible back-off is segment-wise, where the slot is supposed to be further divided into segments. Fig. 2 depicts how ZD proceeds with each packet consisting of four segments. Upon detecting a collision of two packets, the BS broadcasts a feedback signal, which requires the colliding users to immediately retransmit the packets. Then, users who have transmitted the packets retransmit their packets in the next slot, while other users refrain from transmitting packets. Fig. 2-(a) shows that the first segment of packet-1 (gray colored) and the last segment of packet-2 (white colored) are received without collision. If the difference between the arrival of two packets is different in two received packets, as shown in the figure, then the retrieved segment of packet-1 is collided in the second slot, where the BS can cancel a retrieved segment from slot-2, as shown in Fig. 2-(b). Upon canceling segments, new segments become collision-free, and the same procedure is iterated. The cancellation can be performed if packets are received with different delays in two slots. We simply modeled ZD as a random variable without considering the actual arrival timing. The two retransmitted packets are retrieved with probability ω; however, ZD fails to retrieve both packets with probability $(1-\omega)$.

B. STRAIGHTFORWARD IMPLEMENTATION

Let us first consider a straightforward implementation of ZD in frameless ALOHA, which is termed *ZDFA*. In this scenario, users operate in the same manner as in the original frameless ALOHA, except for the requested retransmission caused by the feedback signal from the BS. In ZDFA, the BS uses a two-bit feedback signal to inform users of the following three conditions:

- The frame is ended as the desired PLR has been achieved at the BS.
- The frame is continued as the desired PLR has not been achieved.
- Two packets have collided, and the corresponding users are required to retransmit.

When the BS requests the colliding users to retransmit their packets upon detecting the collision of two packets, the colliding users immediately retransmit their packets in the next time slot, while the other users refrain from transmitting. When the slot ends, all the users restart the probabilistic transmission.

1) THEORETICAL ANALYSIS FOR PLR

Let us consider the theoretical expression for the PLR of ZDFA. Transmission of users can be depicted via bipartite graph, as shown in Fig. 3. The bipartite graph consists of *variable nodes*, *observation nodes*, and edges between these two kinds of nodes. Variable and observation nodes correspond to transmitted packets and time slots, respectively, and an edge denotes that the packet of the connected variable node is transmitted in the slot of the connected observation node. Moreover, the *degree* of a node is defined as the number of edges connected to the node; the degree of a variable node shows the number of times the corresponding user has transmitted during the frame, and the degree of an observation node indicates the number of users that have transmitted in the slot. For the sake of visibility, the additional slot for ZD is omitted in the graph, as the additional slots always appear after two packets collide.

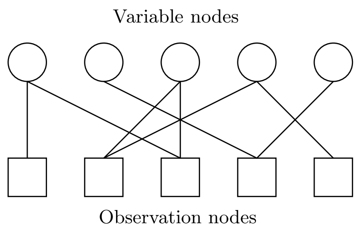

FIGURE 3. **Illustration of bipartite graph of ZDFA; the requested retransmission due to ZD is omitted.**

Similar to the original frameless ALOHA, density evolution [6] can be used to analyze the asymptotic performance. By using dummy variable x and denoting the number of time slots by T, let us define the *node-perspective* degree distribution of variable and observation nodes as

$$L(x) \triangleq \sum_{k=0}^{T} L_k x^k, \tag{2}$$

and

$$R(x) \triangleq \sum_{k=0}^{N} R_k x^k, \tag{3}$$

respectively.

The coefficient of degree distributions with index k corresponds to the probability that the node has degree-k. As users transmit their packets with probability p, we have

$$L_k = \binom{T}{k} p^k (1-p)^{T-k}, \tag{4}$$

and

$$R_k = \binom{N}{k} p^k (1-p)^{N-k}, \tag{5}$$

where the required retransmission of users for ZD is ignored.

The node-perspective degree distribution yields an *edge-perspective* degree distribution as

$$\lambda(x) \triangleq \sum_{k=1}^{T} \lambda_k x^{k-1} = \frac{L'(x)}{L'(1)}, \tag{6}$$

and

$$\rho(x) \triangleq \sum_{k=1}^{N} \rho_k x^{k-1} = \frac{R'(x)}{R'(1)}, \tag{7}$$

where $\cdot'$ denotes the derivative of a function.

Then, according to [7], given the number of time slots T, the PLR performance of the original frameless ALOHA can be given by

$$p_{\mathrm{e}}(T) = L(1-\rho(1-x_l^{(T)})), \tag{8}$$

where $x_l^{(T)}$ denotes the probability that the edge is connected to the variable node of an unretrieved user at the l-th iteration and is given by

$$\begin{aligned} x_l^{(T)} &= \lambda(1-\rho(1-x_{l-1}^{(T)})) \\ &= \lambda\left(1-\rho_1-\sum_{k=2}^{N}\rho_k(1-x_{l-1}^{(T)})^{k-1}\right). \end{aligned} \tag{9}$$

Regarding ZDFA, additional packet retrieval through ZD should be considered. Note that $(1-\rho(1-x_{l-1}^{(T)}))$ in (9) gives the probability that the edge is connected to the observation nodes corresponding to the colliding slots. In the original frameless ALOHA, BS can resolve the collision only when slots become singletons. In contrast, in ZDFA, BS can also resolve the collision when slots contain collision of two packets. This modification can be reflected in the analysis, yielding

$$\begin{aligned} x_l^{(T)} = \lambda\Bigg(1-\rho_1-\rho_2\Big(\omega+(1-\omega)(1-x_{l-1}^{(T)})\Big) \\ -\sum_{k=3}^{N}\rho_k(1-x_{l-1}^{(T)})^{k-1}\Bigg), \end{aligned} \tag{10}$$

and if $\omega = 1$, i.e., ZD always succeeds[1], (10) is simplified to

$$x_l^{(T)} = \lambda\left(1-\rho_1-\rho_2-\sum_{k=3}^{N}\rho_k(1-x_{l-1}^{(T)})^{k-1}\right). \tag{11}$$

In (10) and (11), additional slots dedicated to ZD for immediate retransmission are implicitly ignored, so the theoretical PLR curve shows an earlier waterfall region than is seen in exact PLR performance. Therefore, a penalty for the additional slots should be included. If there are T independent slots in which users can perform probabilistic transmission, the average number of resulted slots including required retransmission is calculated as $(T+TR_2)$, where R_2 is the probability that the observation node has degree-2. Then, the PLR performance of ZDFA can be given by

$$\begin{aligned} p_{\mathrm{e}}^{(\mathrm{ZD})}&(T+TR_2) \\ &= L\left(1-\rho_1-\rho_2-\sum_{k=3}^{N}\rho_k(1-x_l^{(T)})^{k-1}\right), \end{aligned} \tag{12}$$

or if there are T time slots in total (including required retransmission), we have

$$\begin{aligned} p_{\mathrm{e}}^{(\mathrm{ZD})}&(T) \\ &= L\left(1-\rho_1-\rho_2-\sum_{k=3}^{N}\rho_k(1-x_l^{\left(\frac{T}{1+R_2}\right)})^{k-1}\right). \end{aligned} \tag{13}$$

2) THROUGHPUT PERFORMANCE OF ZDFA

This study focused on the achievable throughput performance of ZDFA. By using (13), the throughput performance at the T-th slot, namely $S(T)$, is defined as

$$S(T) \triangleq \frac{N(1-p_{\mathrm{e}}^{(\mathrm{ZD})}(T))}{T}. \tag{14}$$

The throughput performance (and consequently PLR performance) of ZDFA is determined by the target degree, which is optimized to maximize throughput performance. According to [14], the average throughput performance of frameless ALOHA can be maximized by finding the optimal target degree that maximizes the *peak* throughput. At the peak point, the corresponding PLR should be less than the given threshold. It is worth noting that the retrieval of all the users would result in a large delay because of the probabilistic transmission [7]. In this study, the optimization policy used in [14] was followed, and the optimal target degree for ZDFA was obtained using the following optimization problem:

$$\max_{G} \ \sup_{T} S(T) \tag{15}$$

$$\text{s.t.} \quad p_{\mathrm{e}}^{(\mathrm{ZD})}(T^*) \le 1-\alpha, \tag{16}$$

where $T^* = \arg\sup_T S(T)$, and α denotes the threshold on the fraction of retrieved users.

By using brute-force search over G, $G = 3.76$ was revealed to achieve the highest throughput performance, where the corresponding peak throughput was 0.856, while the original frameless ALOHA with the optimal target degree of $G = 3.09$ achieved a peak throughput of 0.867 [14]. This result reveals that straightforward implementation of ZD in frameless ALOHA causes degradation of the throughput performance. Fig. 4 depicts the PLR performance of ZDFA with $G = 3.76$. For comparison, the PLR performance of the original frameless ALOHA with $G = 3.76$ is shown. We confirmed that our derived analysis (12) coincides with the result of computer simulations, verifying that the theoretical analysis provides the exact PLR performance of ZDFA. The PLR performance of frameless ALOHA should have an error floor caused by the probability that the user never transmits during the frame. Hence, for frameless ALOHA, a higher

[1] Probability ω can be regarded as 1 if the number of segments is sufficiently large.

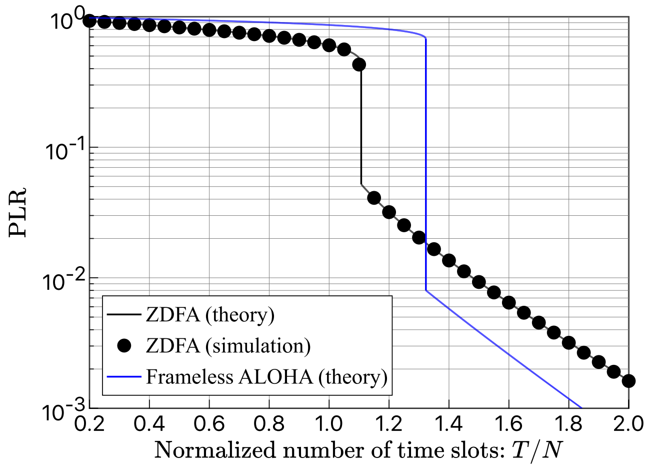

FIGURE 4. Comparison of PLR performance of ZDFA and the original frameless ALOHA. The target degree of $G = 3.76$ is used for all curves, and $N = 10^4$. ZDFA has a higher error floor than frameless ALOHA.

transmission probability results in a lower error floor. However, in Fig. 4, ZDFA shows a higher error floor than the original frameless ALOHA, while ZDFA with $G = 3.76$ uses the same transmission probability as frameless ALOHA with $G = 3.76$. This is because ZDFA uses some slots for the required retransmission to perform ZD, where other users are prohibited to transmit. In other words, even if the frame length is T, users do not always have T chances to transmit their packets, which results in a high error floor. Although the occurrence of a waterfall in ZDFA is earlier than in frameless ALOHA, a higher error floor limits the number of retrieved users, so the resulting throughput performance is lower than that of frameless ALOHA. In other words, lowering the error floor of ZDFA is an obvious solution to the degradation of throughput performance.

C. E-ZDFA

To solve the aforementioned problem of lower throughput performance in ZDFA as compared to frameless ALOHA, we propose *E-ZDFA*, which lowers the error floor by utilizing additional one-bit feedback from the BS. E-ZDFA exploits three additional features: *retransmission canceling*, *transmission probability updating*, and *predictive-canceling*. Specifically, the BS uses a feedback signal when a single user has been retrieved from a collision-free slot or two users have been retrieved through ZD. With the feedback, the retrieved user can acknowledge that its own packet has been successfully retrieved. Then, the user stops retransmitting in the following slots to suppress collision. Simultaneously, other users also acknowledge that one (or two) user(s) has (have) been retrieved, and the number of contending users decreases; if the feedback indicating the retrieval of the transmitted packet is received after receiving the feedback requiring retransmission, the number of retrieved packets is considered to be two, as the BS broadcasts the feedback requiring retransmission upon detecting collision between two packets. Otherwise, only one user is retrieved via collision-free reception. Next, transmission probability in E-ZDFA is dynamically updated, i.e., increased to encourage transmission of other users. Moreover, as the BS knows when the retrieved packets are transmitted, it is able to cancel retrieved packets as soon as the packet arrives, thus performing ZD. The three aforementioned features work together to lower the error floor and exploit higher throughput performance than the original frameless ALOHA.

1) FEEDBACK SIGNAL UTILIZATION

In E-ZDFA, the BS broadcasts a feedback signal to users when the transmitted packets are retrieved via ZD as well as when only one packet is received and retrieved. It is worth noting that the feedback signal does not specify which user is retrieved, so the feedback only requires one additional bit; regardless of whether the transmitted packet(s) is (are) retrieved. Users who have acknowledged that their packets are retrieved cancel their retransmission in the following slots (*retransmission canceling*). Note that in the original frameless ALOHA, retransmission canceling neither improves throughput performance nor does it lower the error floor because the SIC and retransmission canceling are interchangeable. However, in case of ZD, retransmission canceling should be performed first because it plays an important role in lowering error floor and increasing throughput by retrieving more users.

Owing to retransmission canceling, the channel load decreases as the frame size increases, and users are able to transmit more frequently. Hence, E-ZDFA dynamically increases the transmission probability of users (*transmission probability updating*). Note that all users are able to know the number of users who have stopped retransmission by observing the feedback from the BS. For example, when the feedback declaring the correct reception occurs immediately after the retransmission request, the number of users can be determined to be two. Let $N_{\mathrm{ret}}^{(T)}$ denote the number of users acknowledged as retrieved by the BS up to the T-th slot. Then, instead of (1), the transmission probability at the T-th slot is dynamically updated as

$$p^{(\mathrm{EZ})}(T) = \frac{G}{N - N_{\mathrm{ret}}^{(T)}}. \tag{17}$$

Conventional frameless ALOHA also uses a feedback signal to indicate the end of a frame; the feedback signal only requires one bit. In E-ZDFA, the BS broadcasts a feedback signal to inform users of the following four conditions:

- The transmitted packet(s) is (are) retrieved.
- Two packets have collided, and the corresponding users are required to retransmit.
- The frame is ended as the desired PLR has been achieved at the BS.
- The frame is continued as the desired PLR has not been achieved.

Therefore, the feedback signal of E-ZDFA only requires at most two bits, which can be neglected for evaluating throughput performance.

2) PREDICTIVE CANCELING OF PACKETS

Although the feedback signal can stop retransmission of retrieved users, users retrieved via SIC do not stop retransmission as they cannot acknowledge whether their packets have been retrieved. However, the BS can predict when retrieved packets are transmitted because each packet contains the information indicating the time slots in which the packet is transmitted. This feature allows the BS to cancel retrieved packets as soon as a packet is received (*predictive-canceling*).

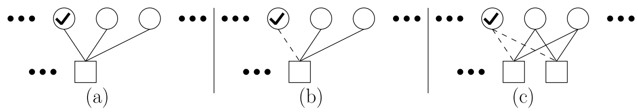

FIGURE 5. Illustration of predictive-canceling. (a) Three users transmit simultaneously, and one has already been retrieved via SIC. (b) The BS can cancel the packet as it knows that the packet will be transmitted in this slot. The slot can be regarded as a degree-2 slot. (c) The BS requires the users to retransmit packets immediately to perform ZD.

Fig. 5 illustrates predictive-canceling. In Fig. 5-(a), three users transmit in a time slot, and one of them has already been retrieved through SIC. As the BS knows that the slot contains the retrieved packet, the BS cancels the packet, as shown in Fig. 5-(b). Then, the slot becomes degree-2 so that the BS broadcasts the second feedback requiring the *three* users to immediately retransmit the packets in the next slot. After canceling the retrieved packet from the received packet, the BS performs ZD on these two slots, as shown in Fig. 5-(c). Suppose that ZD has succeeded and the BS broadcasts the fourth feedback; users can guess that two packets are retrieved via ZD and increase $N_{\mathrm{ret}}^{(T)}$ by two.

3) ERROR FLOOR ANALYSIS FOR E-ZDFA

Owing to the retransmission canceling and transmission probability updating, the degree distribution of E-ZDFA varies dynamically as the number of slots increases. As a result, it is difficult to track the exact behavior of degree distributions; hence, density evolution cannot be straightforwardly applied in E-ZDFA. A simple yet informative approach to analyze the system is to derive the error floor of E-ZDFA. Frameless ALOHA protocols have an error floor because of the probabilistic transmission, and the error floor is calculated by considering the probability that the user never transmits during the frame. A higher error floor yields an unstable system, where non-negligible fraction of users are not retrieved upon terminating the frame. At this point, our proposed E-ZDFA dynamically updates the transmission probability and is expected to have a lower error floor than the original frameless ALOHA. Therefore, instead of the exact PLR analysis, we theoretically derived the error floor of E-ZDFA by analyzing $N_{\mathrm{ret}}^{(T)}$.

Let us define $N^{(T)} \triangleq N - N_{\mathrm{ret}}^{(T)}$ as the number of contending users at the T-th slot. Hence, the probability that k users transmit at the k-th slot, namely $R_k^{(T)}$, is given by

$$R_k^{(T)} = \binom{N^{(T)}}{k} \left(p^{(\mathrm{EZ})}(T)\right)^k \left(1 - p^{(\mathrm{EZ})}(T)\right)^{N^{(T)}-k}. \tag{18}$$

The probability of a single user being retrieved from collision-free reception is $R_1^{(T)}$, and the probability that two users are retrieved via ZD is $\omega R_2^{(T)}$. Moreover, the probability that retrieved packets are included in a received signal should be considered. Thus, the density evolution analysis for ZDFA was used to estimate the number of retrieved packets. Let us denote the approximated PLR at the T-th slot by $\beta(T)$, which is defined as in (13) using density evolution. When the slot has degree-k, the probability that $(k-1)$ edges have been retrieved so that the remaining packet can be retrieved is

$$\binom{k}{1} \beta(T) \left(1 - \beta(T)\right)^{k-1}, \tag{19}$$

and similarly, the probability of two packets remaining is

$$\binom{k}{2} \left(\beta(T)\right)^2 \left(1 - \beta(T)\right)^{k-2}. \tag{20}$$

Then, $N_{\mathrm{ret}}^{(T)}$ is calculated as

$$\begin{aligned} N_{\mathrm{ret}}^{(T)} \approx\ & N_{\mathrm{ret}}^{(T-1)} + R_1^{(T)} + 2\omega R_2^{(T)} \\ & + \sum_{k=3}^{N^{(T)}} R_k^{(T)} \left(\binom{k}{1} \beta(T) \left(1 - \beta(T)\right)^{k-1} \right. \\ & \left. + 2\omega \binom{k}{2} \left(\beta(T)\right)^2 \left(1 - \beta(T)\right)^{k-2} \right), \end{aligned} \tag{21}$$

where $N_{\mathrm{ret}}^{(T)}$ is updated while the inequality

$$\frac{N_{\mathrm{ret}}^{(T)}}{N} + \left(1 - \beta(T)\right) \leq 1 \tag{22}$$

is satisfied. Furthermore, if $N_{\mathrm{ret}}^{(T)}/N + (1 - \beta(T)) > 1$, $N_{\mathrm{ret}}^{(T)}$ is not updated and the same transmission probability is used in following slots.

By using (21), the transmission probability at the T-th slot can be theoretically calculated. Finally, the error floor for the PLR performance of E-ZDFA is given by

$$p_{\mathrm{e,LB}}(T + T\bar{R}_2^{(T)}) = \prod_{t=1}^{T} (1 - p^{(\mathrm{EZ})}(t)), \tag{23}$$

or equivalently

$$p_{\mathrm{e,LB}}(T) = \prod_{t=1}^{\frac{T}{1+\bar{R}_2^{(T)}}} (1 - p^{(\mathrm{EZ})}(t)), \tag{24}$$

where $\bar{R}_2^{(T)}$ is the average fraction of additional slots occurring within the T original slots and is given by

$$\bar{R}_2^{(T)} = \frac{\sum_{t=1}^{T} R_2^{(t)}}{T}. \tag{25}$$

V. NUMERICAL EXAMPLES

A. THROUGHPUT AND PLR PERFORMANCE

We evaluated the achievable throughput performance of E-ZDFA. As the derivation of theoretical expressions for PLR (and throughput) of E-ZDFA is difficult, computer simulations were used instead to seek the target degree that maximizes the average throughput. In order to investigate the achievable throughput, the number of users should be sufficiently large. To this end, we have supposed that there exists $N = 10^4$ users. By using brute-force search, the target degree of $G = 3.32$ yielded the highest throughput of 0.929, which outperforms the peak throughput of conventional frameless ALOHA (0.867).

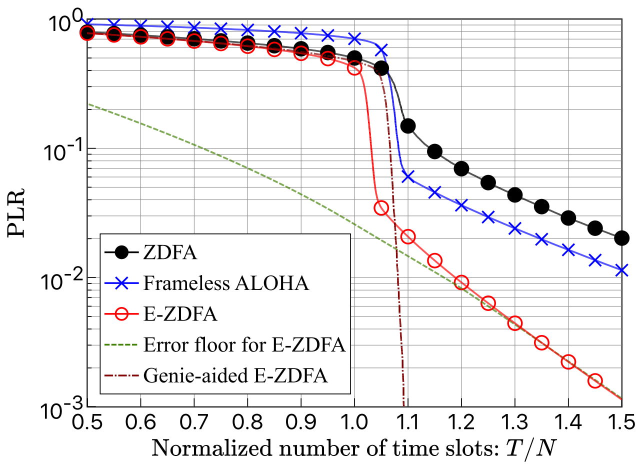

FIGURE 6. PLR performance of E-ZDFA. E-ZDFA, frameless ALOHA, and ZDFA use target degrees of $G = 3.32$, $G = 3.09$, and $G = 3.32$, respectively. The number of users is 10^4. Genie-aided E-ZDFA is the scenario where the BS is able to specify all retrieved users so that all such users can halt their retransmission and other users can increase the transmission probability by taking the number of retrieved users into account.

Fig. 6 depicts the PLR performance of E-ZDFA with $G = 3.32$. For comparison, PLR performance of ZDFA (without additional feedback) with $G = 3.32$ and the original frameless ALOHA with the optimized target degree $G = 3.09$ are also depicted. Note that the PLR curves in Fig. 6 are obtained through computer simulations, which did not include theoretical error floor analysis. The comparison of E-ZDFA and ZDFA showed that transmission probability updating lowers the error floor while showing an earlier waterfall region. Moreover, E-ZDFA is observed to have a lower error floor than conventional frameless ALOHA owing to the additional features. Notably, the PLR of E-ZDFA approaches the theoretical error floor given in (24) with an increasing number of time slots, and the error floor achieves lower PLR than in ZDFA and frameless ALOHA.

While E-ZDFA dynamically updates the transmission probability by implicitly informing users of the number of retrieved users through feedback signals, more users might be retrieved via SIC. Hence, a natural question that may arise here is whether the system can obtain higher gain than E-ZDFA or not if users can perfectly know the retrieved users. To this end, let us consider *genie-aided* E-ZDFA, where users can perfectly know the remaining number of users and update their own transmission probability based on that while all retrieved users halt their retransmission. Note that the transmission probability of (17) may become larger than 1 when the number of unretrieved users is smaller than G. In order to avoid this problem, the following equation is used to calculate the probability instead of (17):

$$p^{(\mathrm{EZ})}(T) = \begin{cases} \dfrac{G}{N - N_{\mathrm{ret}}^{(T)}} & \text{if } N - N_{\mathrm{ret}}^{(T)} > G \\ \dfrac{G}{\lceil G \rceil + 1} & \text{otherwise,} \end{cases} \tag{26}$$

where $\lceil \cdot \rceil$ denotes the ceiling function.

The PLR curve of the genie-aided E-ZDFA has also been plotted in Fig. 6. Genie-aided E-ZDFA does not exhibit the error floor since it allows all the users to transmit at least once thanks to the probability update with perfect knowledge of remaining users. This however results in the degradation of throughput performance; while the throughput of E-ZDFA is 0.929, genie-aided E-ZDFA achieves a throughput of only 0.919. As the transmission probability of genie-aided E-ZDFA increases faster than that of E-ZDFA, more users would collide, delaying the waterfall region while degrading throughput. Moreover, in order to realize the genie-aided feedback, the length of feedback signal must be longer than that of E-ZDFA so that all users can be specified, and longer feedback causes lower throughput in practice. From these aspects, our proposed E-ZDFA achieves a good trade-off between throughput and complexity of additional feedback, as a throughput performance comparable to that of the genie-aided version can be achieved with an only two-bit feedback signal.

B. COMPARISON WITH STATE-OF-THE-ART APPROACHES

E-ZDFA automatically determines the suitable frame length due to the frameless structure and achieves the designed throughput performance for any number of users. This section shows the comparison of E-ZDFA with a state-of-the-art coded ALOHA scheme, namely IRSA with degree distribution derived in [5], in terms of average throughput. In this study, the degree distribution for IRSA is set as

$$\begin{aligned} L(x) = {} & 0.494155x^2 + 0.159085x^3 + 0.107372x^4 \\ & + 0.070336x^5 + 0.045493x^6 + 0.019898x^7 \\ & + 0.024098x^{11} + 0.008636x^{12} + 0.005940x^{13} \\ & + 0.008749x^{15} + 0.002225x^{18} + 0.001261x^{20} \\ & + 0.002607x^{22} + 0.008092x^{23} + 0.002287x^{24} \\ & + 0.012274x^{25} + 0.002530x^{26} + 0.003094x^{27} \\ & + 0.002558x^{28} + 0.005891x^{29} + 0.013419x^{30}, \end{aligned} \tag{27}$$

where the peak throughput is 0.977.

With this degree distribution, each user decides the number of retransmissions and then selects time slots to transmit the packet from the frame. Therefore, while E-ZDFA and frameless ALOHA automatically determine the frame length, IRSA requires the BS to determine the frame length *prior to transmission*. Moreover, the suitable frame length varies with the number of users, and the BS should obtain the appropriate frame length before the beginning of every frame. In the evaluation, for each number of users, the frame length that yields the highest throughput is obtained via brute-force search; the throughput performance of IRSA is maximized at every point. Although this setting is a bias in favor of IRSA, our proposed E-ZDFA still achieves higher throughput performance than IRSA for a practical number of users, i.e., $N \leq 10^3$.

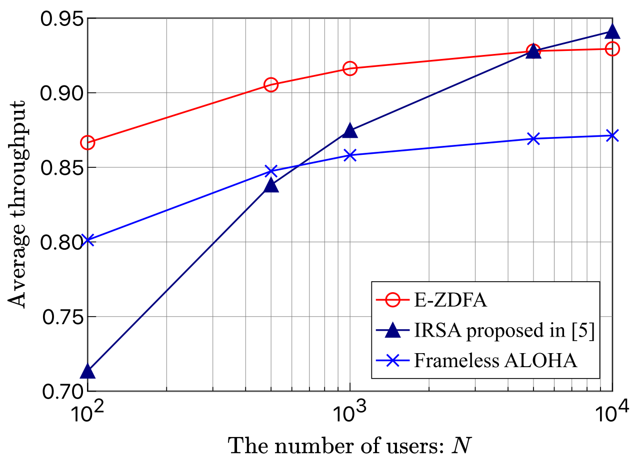

FIGURE 7. Throughput performance versus the number of users. E-ZDFA and frameless ALOHA use the target degrees of $G = 3.32$ and $G = 3.09$, respectively, while IRSA uses the optimized degree distribution of (27). IRSA has the highest throughput performance when $N = 10^4$, while the throughput of IRSA degrades as the number of users decreases. E-ZDFA achieves higher throughput performance than IRSA with a practically large number of users.

Fig. 7 shows the throughput performance against the number of users in the network. When $N = 10^4$, IRSA gives the highest throughput performance as it asymptotically exhibits a throughput of 0.977. However, when the number of users decreases, the throughput of IRSA is worse than that of E-ZDFA, even for a suitable frame length. The original frameless ALOHA also outperforms IRSA for $N \leq 500$. This is because the degree distribution of IRSA is optimized so that the peak throughput is maximized in the asymptotic setting, where numbers of users and time slots are infinite. Therefore, if the number of users is not sufficiently large so that the degree distribution in the graph is not typical, the performance significantly degrades. In contrast, E-ZDFA and frameless ALOHA show less degradation of throughput than IRSA as the degree of frameless ALOHA schemes follows a binomial distribution, which has a gentler slope than the optimized degree distribution of (27). Therefore, frameless ALOHA schemes can reproduce the designed degree distribution even when the number of users is not very large; thus, the achieved throughput performance is close to the designed performance. In other words, E-ZDFA and frameless ALOHA are more suitable for multiple access in the network with fluctuating demands than IRSA. Furthermore, owing to the use of ZD, E-ZDFA always achieves higher throughput than the original frameless ALOHA.

C. EFFECT OF THRESHOLD ON PLR

Let us consider how the threshold on PLR, namely α, affects the throughput of E-ZDFA by comparing it with that of frameless ALOHA. Recall that frameless ALOHA protocols are terminated when $\lfloor \alpha N \rfloor$ packets are retrieved at the BS. Owing to the probabilistic transmission of users, a larger α is considered to require a larger number of time slots, which would degrade throughput. At this point, as E-ZDFA achieves a lower error floor than the original frameless ALOHA, E-ZDFA is expected to be more robust against the increase of α than frameless ALOHA.

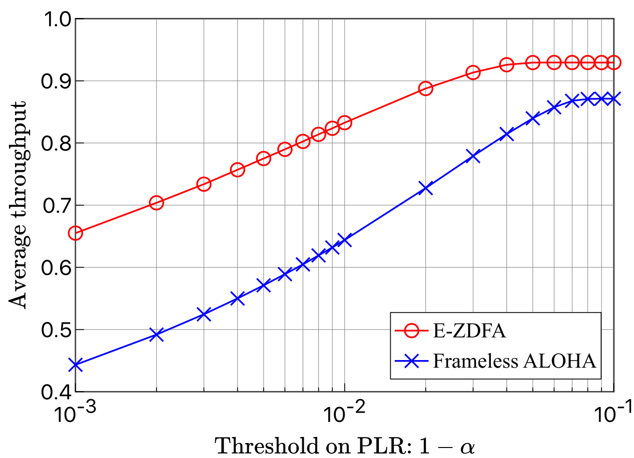

FIGURE 8. Throughput performance versus the threshold on PLR, namely $(1 - \alpha)$. E-ZDFA and frameless ALOHA use target degrees of $G = 3.32$ and $G = 3.09$, respectively, and the number of users is 10^4. E-ZDFA achieves higher throughput performance for arbitrary values of $(1 - \alpha)$.

Fig. 8 shows the throughput performance of E-ZDFA with different values of α, where the horizontal axis corresponds to the required PLR, namely $(1 - \alpha)$, and the number of users is set to $N = 10^4$, as in Fig. 6 also. The threshold on PLR can be regarded as the *guaranteed PLR*, as the transmission continues until the PLR achieves threshold. From the figure, E-ZDFA is shown to achieve higher throughput than the original frameless ALOHA for all α. Moreover, the degradation of throughput according to the increase of α (decrease of $1 - \alpha$) is suppressed as E-ZDFA achieves a lower error floor than frameless ALOHA, as discussed earlier. Thus, the result confirms that E-ZDFA is capable of achieving higher throughput than frameless ALOHA, while achieving arbitrary PLR.

D. THROUGHPUT PERFORMANCE IN A PRACTICAL SCENARIO

Finally, we evaluated the throughput performance of E-ZDFA in a practical scenario, where ZD fails to retrieve packets with positive probability $(1 - \omega)$. As the traffic in the IoT is supposed to be sporadic [15], the actual users would be a subset of all users in the network, and the number of active users should be less than the number of all users. Then, let us consider a relatively smaller network than so far discussed (e.g., $N = 10^3$) with the threshold on PLR set to $\alpha = 0.8$. The difference between the arrivals of packets is realized by the random selection of back-off patterns by users. Hence, the probability of failure of ZD is obtained through the number of possible patterns. Specifically, if the number of back-off patterns is denoted by CW, then the probability is calculated as

$$1 - \omega = \frac{1}{\mathrm{CW}}. \tag{28}$$

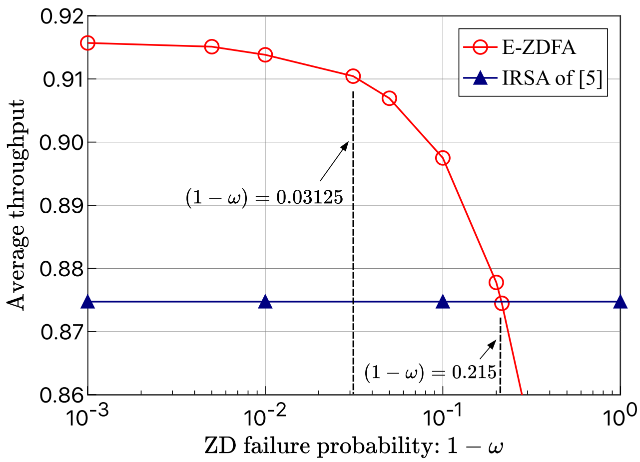

FIGURE 9. Throughput performance versus the probability of failure of ZD, namely $(1-\omega)$ with $N = 10^3$ users. E-ZDFA uses a target degree of $G = 3.32$, and IRSA uses a degree distribution of (27). In order for E-ZDFA to outperform IRSA, the required probability is 0.215.

Fig. 9 shows the throughput performance of E-ZDFA for various values of $(1-\omega)$. For comparison, IRSA with the degree distribution from [5] is also depicted. E-ZDFA can be observed to outperform IRSA when $(1-\omega) \geq 0.215$, and the corresponding value of CW is approximately 4.65. Therefore, to outperform the conventional IRSA, only five back-off patterns are required. In particular, it was addressed by Gollakota and Katabi [10] that CW is initialized to 32 in standard 802.11, resulting in $(1-\omega) = 1/32 = 0.03125$. As shown in the figure, $(1-\omega) = 0.03125$ yields a throughput of 0.91. Therefore, even in a practical scenario, our proposed E-ZDFA outperforms the state-of-the-art IRSA.

VI. CONCLUSION AND FUTURE WORKS

In this paper, we investigated the application of ZD in frameless ALOHA and revealed that the straightforward application of ZD causes a higher error floor than in the original frameless ALOHA, thus resulting in lower throughput performance. To deal with the error floor problem, we proposed E-ZDFA, which utilizes two-bit feedback. This additional feature allows E-ZDFA to achieve a lower error floor and higher throughput performance than state-of-the-art IRSA and frameless ALOHA. Moreover, E-ZDFA was confirmed to be robust against the requirement of PLR.

We would like to conclude this paper by proposing some future works. Although the error floor of E-ZDFA was theoretically analyzed in this study, this cannot be used to optimize the target degree while the density evolution cannot be applied to E-ZDFA directly due to the non-stationarity caused by retransmission canceling and transmission updating. The analytical expression of either peak or average throughput remains as our future work. Moreover, effects of a physical layer were not considered in this study. However, different received power among users may enhance the retrieval probability, as shown in [12], [16], which is known as the *capture effect*. The design of E-ZDFA considering power-domain control also remains as our future work.

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