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# Real-Time Solution to Quadratically Constrained Quadratic Programs for Predictive Converter Control

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**Abstract:** This paper considers the real-time implementation of MPC tailored to voltage source converters with inductive-capacitive filter. Previous work has shown that the nonlinear and non-convex MPC problem can be equivalently formulated as a convex quadratically constrained quadratic program (QCQP). We develop two tailored algorithms based on the OSQP and HPIPM solvers to efficiently solve this QCQP. As the aforementioned solvers do not support quadratic constraints, we extend them so that they can solve QCQPs. We provide numerical comparison between the proposed methods and state-of-the-art solvers and show that our solvers are suitable for embedded applications.

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**Keywords:** Predictive control, Embedded systems, Inverters, Efficient algorithms, Operator splitting method, Riccati-based interior-point method.

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## 1. INTRODUCTION

With the development of renewable energy, the modern power system is undergoing significant change with ever increasing presence of grid-connected power converters (Parvez et al., 2016). Model predictive control (MPC) has been applied for a wide range of converter control applications (Vazquez et al., 2016), and has shown to improve both dynamic performance and efficiency of converters, see *e.g.*, Spudić and Geyer (2017); Richter et al. (2016); Hokayem et al. (2014); Almér et al. (2013, 2015).

Application of MPC requires the solution of optimization problems in real time. The fast control frequencies of converters mean that the real-time optimization has to be performed at high speed. This is a challenging problem which has been the focus of a large body of work (Ferreau et al., 2017). A recent theoretical result has shown that for certain power converter topologies, the non-convex MPC problem can be equivalently reformulated as a convex quadratically constrained quadratic program (QCQP) with a certain structure (Almér et al., 2020). This result motivates further research into fast real-time optimization of QCQP, which is carried out in the present paper.

The solution to MPC problems can be computed with iterative optimization methods and solvers, *e.g.*, Richter et al. (2012); Domahidi et al. (2013, 2012). First-order methods exploit first-order information of the problem to iteratively compute an optimal solution. Operator splitting techniques are well-known first-order methods that formulate the optimization problem as a problem of find-

ing a zero of the sum of monotone operators (Lions and Mercier, 1979; Douglas and Rachford, 1956). The alternating direction method of multipliers (ADMM) is a particular operator splitting method, which has been studied extensively (Gabay and Mercier, 1976; Glowinski, 1984; Eckstein and Bertsekas, 1992; Boyd et al., 2011). Due to its computationally cheap iterations and good practical convergence behavior, ADMM is well suited to embedded applications with limited computing resources wherein high accuracy solutions are typically not required due to noise in the data.

Very recently, a novel operator splitting method for quadratic programs based on ADMM was proposed (Stellato et al., 2020; Banjac et al., 2017, 2019). The method exploits a new splitting requiring the solution to a quasi-definite linear system. The method makes no assumptions on the problem data other than convexity. Furthermore, it can provide primal and dual infeasibility certificates if the problem is infeasible. Besides, warm starting can be easily integrated to reduce the number of iterations and if the problem matrices are time-invariant, the matrix factorization can be cached and reused multiple times to improve the computation time.

Another important class of optimization methods are interior-point methods. They are second-order methods for constrained optimization. An important method in this class is the Mehrotra predictor-corrector method (Mehrotra, 1992), a primal-dual interior-point method which is widely used in practical implementations due to the good performance across various problems (Wright, 1997).

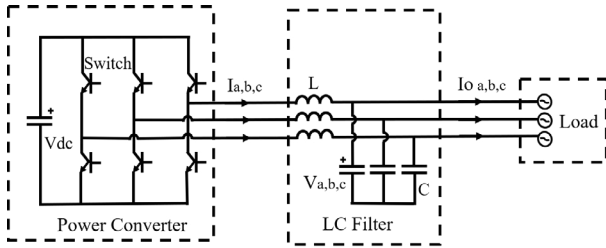


Fig. 1. Two level three-phase voltage source converter with LC filter

Recently, based on previous work (C.V.Rao et al., 1998), a predictor-corrector method for linear MPC addressed on a Riccati factorization was proposed (Frison and Jørgensen, 2013). An efficient implementation exists in the HPIPM package (Frison et al., 2014), which significantly improves performance for small to medium scale problems compared to other implementations of interior-point methods. An important part of the speedup comes from an optimized linear algebra implementation called BLASFEO (Frison et al., 2018).

This paper explores the real-time implementation of MPC tailored to voltage source converters with inductive-capacitive filter. Based on the previous work (Almér et al., 2020), where an equivalent formulation of MPC as a QCQP, we develop two tailored algorithms based on the OSQP and HPIPM solvers to efficiently solve this QCQP. As the aforementioned solvers do not support quadratic constraints, we extend them so that they can solve QCQPs. Besides, we provide numerical comparison between the proposed methods and state-of-the-art solvers and show that our solvers are suitable for embedded applications.

The content of the present paper is as follows. We discuss the problem setup and review the specific MPC problem in Section 2. Section 3 presents the main result of this paper: two efficient algorithms to determine the optimal solution. In Section 4, we implement the proposed method and illustrate the results by simulations. Finally, we conclude the paper.

## 2. BACKGROUND

### 2.1 System Model

The configuration of a voltage source converter interconnected to the load via an inductive-capacitive (LC) filter is presented in Fig. 1. The system consists of the LC filter, the load (modeled as an independent current source), and the switching stage.

The state of the LC filter is described in stationary  $abc$  frame with state vector and dynamics

$$x := [\sqrt{L}I_a \ \sqrt{L}I_b \ \sqrt{L}I_c \ \sqrt{C}V_a \ \sqrt{C}V_b \ \sqrt{C}V_c]^T$$

$$\dot{x}(t) = A_c x(t) + B_c s(t)V_{dc} + F_c I_o(t) \quad (1)$$

where  $I_p$  are the inductor currents,  $V_p$  are the capacitor voltages,  $p \in \{a, b, c\}$ .  $L$ ,  $C$  and  $V_{dc}$  are the filter inductance, capacitance and (constant) DC link voltage respectively.  $I_o = [I_{oa}, I_{ob}, I_{oc}]^T \in \mathbb{R}^3$  are three-phase load currents, and  $s = [s_a, s_b, s_c]^T \in \{-1, 1\}^3$  represents the

state of the converter switches. Expressions for the system matrices  $A_c$ ,  $B_c$ ,  $F_c$  can be found in (Almér et al., 2020).

### 2.2 MPC Formulation

The MPC problem of controlling (1) is nonlinear and non-convex, since the control input is confined to a discrete set. However, it was shown in (Almér et al., 2020), that under the assumption that the LC filter resonance frequency  $c = 1/\sqrt{LC}$  satisfies  $cT_s \leq \pi$ , where  $T_s$  is the sampling time, the nonlinear MPC problem can be equivalently formulated as a QCQP. The switches  $s_k$  are firstly expressed in terms of duty cycle and phase shift, then they are transformed into control input  $v_{p,k}$  in another convex set via a bijective mapping. Denote the discretized state and load current  $x_k = x(kT_s)$  and  $I_{o,k} = I_o(kT_s)$  respectively.

The QCQP takes the following form

$$\begin{aligned} \min_{x_k, v_k} \quad & \sum_{k=1}^{N_p} (x_k - r_{\text{ref},k})^T Q_c (x_k - r_{\text{ref},k}) \\ \text{s.t.} \quad & x_{k+1} = Ax_k + FI_{o,k} + Mv_k, \quad k \in 0, \dots, N_p - 1 \\ & x_0 = x_{\text{init}} \\ & \|v_{p,k} - r_1\|^2 \leq R^2 \\ & \|v_{p,k} - r_2\|^2 \leq R^2 \\ & k \in 0, \dots, N_p - 1, \quad p \in a, b, c \end{aligned} \quad (2)$$

where  $Q_c$  is a positive definite matrix,  $N_p$  is the prediction horizon,  $r_{\text{ref},k} \in \mathbb{R}^6$  is the reference signal,  $v_k = [v_{a,k}, v_{b,k}, v_{c,k}]^T \in \mathbb{R}^6$ ,  $r_1 = [0, 0.5]^T \in \mathbb{R}^2$ ,  $r_2 = 0.5[\sin(cT_s), -\cos(cT_s)]^T \in \mathbb{R}^2$  and  $R = 0.5$ . Expressions for the system matrices  $A$ ,  $F$ ,  $M$  can be found in (Almér et al., 2020).

## 3. PROPOSED ALGORITHMS

In this section, we introduce two algorithms based on the solvers OSQP and HPIPM for solving MPC problem in (2) efficiently.

### 3.1 Operator Splitting Method for QCQP

Recent research based on a particular version of ADMM, proposed in (Stellato et al., 2020; Banjac et al., 2017), shows that ADMM is effective for solving quadratic programs. Defining the variables of problem (2) as

$$x = [x_1, \dots, x_{N_p}, v_0, \dots, v_{N_p-1}]^T \in \mathbb{R}^{12N_p},$$

the MPC problem (2) can be written into the following form:

$$\begin{aligned} \min_x \quad & \frac{1}{2}x^T Px + q^T x \\ \text{s.t.} \quad & \bar{A}_1 x = b \\ & \bar{A}_2 x \in B \end{aligned}$$

where

$$P = 2 \begin{bmatrix} I_{N_p} \otimes Q_c & \\ & I_{N_p} \otimes \mathbf{0}_{6 \times 6} \end{bmatrix}$$

$$q = -2 \begin{bmatrix} Q_c r_{\text{ref},1} \\ \vdots \\ Q_c r_{\text{ref},N_p} \\ \mathbf{0}_{6N_p \times 1} \end{bmatrix}, \quad b = \begin{bmatrix} Ax_0 + FI_{o,0} \\ FI_{o,1} \\ \vdots \\ FI_{o,N_p-1} \end{bmatrix}$$

**Algorithm 1** Operator splitting method for QCQP.

**Require:** Initial points  $x_0, z_0, y_0$  and parameters  $\rho > 0, \sigma > 0, \alpha \in (0, 2)$

**repeat**

$$(\tilde{x}_{k+1}, \tilde{z}_{k+1}) \leftarrow \underset{A\tilde{x}=\tilde{z}}{\operatorname{argmin}} \left\{ \frac{1}{2} \tilde{x}^T P \tilde{x} + q^T \tilde{x} + \frac{\sigma}{2} \|\tilde{x} - x_k\|_2^2 + \frac{\rho}{2} \|\tilde{z} - z_k + \frac{1}{\rho} y_k\|_2^2 \right\}$$

$$x_{k+1} \leftarrow \alpha \tilde{x}_{k+1} + (1 - \alpha) x_k$$

$$z_{k+1} \leftarrow \Pi_{\mathcal{C}}(\alpha \tilde{z}_{k+1} + (1 - \alpha) z_k + \frac{1}{\rho} y_k)$$

$$y_{k+1} \leftarrow y_k + \rho(\alpha \tilde{z}_{k+1} + (1 - \alpha) z_k - z_{k+1})$$

$$k \leftarrow k + 1$$

**until** termination condition is satisfied

$$\bar{A}_1 = \begin{bmatrix} \mathbf{I}_6 & & & -M & & \\ -A & \mathbf{I}_6 & & & -M & \\ & & \ddots & & & \ddots \\ & & & -A & \mathbf{I}_6 & & \\ & & & & & & -M \end{bmatrix}_{6N_p \times 12N_p}$$

$$\bar{A}_2 = \begin{bmatrix} \mathbf{0}_{6N_p \times 6N_p} & \mathbf{I}_{6N_p} \\ \mathbf{0}_{6N_p \times 6N_p} & \mathbf{I}_{6N_p} \end{bmatrix}$$

$$\mathbf{B} := B_1 \times \dots \times B_1 \times B_2 \times \dots \times B_2 \subseteq \mathbb{R}^{6N_p + 6N_p}$$

$$B_1 := \{v \in \mathbb{R}^2 \mid \|v - r_1\| \leq R^2\}$$

$$B_2 := \{v \in \mathbb{R}^2 \mid \|v - r_2\| \leq R^2\}.$$

and where  $\otimes$  and  $\times$  denote the Kronecker and Cartesian products, respectively.  $\mathbf{I}_m$  is identity matrix and  $\mathbf{0}_{m \times n}$  is zero matrix.

Note that the problem above can be written in the following form:

$$\begin{aligned} \min_x \quad & \frac{1}{2} x^T P x + q^T x \\ \text{s.t.} \quad & \bar{A} x \in \mathcal{C} \end{aligned}$$

where

$$\bar{A} = \begin{bmatrix} \bar{A}_1 \\ \bar{A}_2 \end{bmatrix} \quad \text{and} \quad \mathcal{C} := \{b\} \times \mathbf{B}.$$

The formulation above matches the one in (Stellato et al., 2020, §1.1), and thus we can employ the same splitting method, which is summarized in Algorithm 1. In Algorithm 1, the operator  $\Pi_{\mathcal{C}}$  denotes the Euclidean projection onto the set  $\mathcal{C}$ . Since  $\mathcal{C}$  is given as a Cartesian product of multiple sets, we can evaluate  $\Pi_{\mathcal{C}}(z)$  by computing projections onto the singleton  $\{b\}$  and the balls  $B_1$  and  $B_2$ . The projection onto the ball  $B_i$  has the following closed-form solution:

$$\Pi_{B_i}(v) = \begin{cases} v & \|v - r_i\| \leq R \\ \frac{v - r_i}{\|v - r_i\|} R + r_i & \text{otherwise} \end{cases},$$

which is illustrated in Fig. 2. The termination conditions for Algorithm 1 can be found in (Stellato et al., 2020, §3.4).

### 3.2 Riccati-Based Interior-Point Method for QCQP

An alternative approach to solve the quadratically constrained MPC problem in (2) is to extend the Riccati-based interior-point method proposed in (Frison and Jørgensen, 2013; Frison et al., 2014). Consider the MPC problem in (2), we define the decision variables  $w = [u_0, x_1, u_1, \dots, u_{N_p-1}, x_{N_p}]^T \in \mathbb{R}^{12N_p}$ , dual variable  $\pi = [\pi_1, \pi_2, \dots, \pi_{N_p}]^T \in \mathbb{R}^{6N_p}$  associated with equality constraints and  $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_{6N_p}]^T \in \mathbb{R}^{6N_p}$  associated with

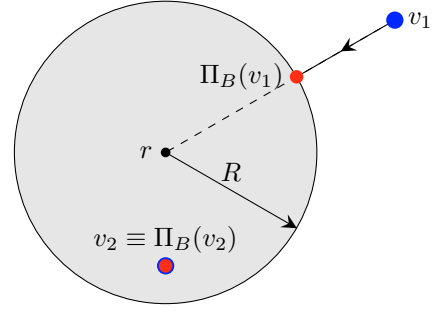


Fig. 2. The Euclidean projection onto the ball  $B$  centered at  $r$  and with radius  $R$ .

quadratic constraints. Furthermore,  $t = [t_1, t_2, \dots, t_{6N_p}]^T \in \mathbb{R}^{6N_p}$  are introduced as slack variables. The relaxed KKT conditions read as

$$\mathbb{F}(w, \pi, \lambda, t) = \begin{bmatrix} (\tilde{H} + C_\lambda)w - \tilde{A}^T \pi + g \\ -\tilde{A}w + b \\ \|v_{a,0} - r_1\|^2 - R^2 + t_1 \\ \|v_{a,0} - r_2\|^2 - R^2 + t_2 \\ \|v_{b,0} - r_1\|^2 - R^2 + t_3 \\ \|v_{b,0} - r_2\|^2 - R^2 + t_4 \\ \vdots \\ \|v_{c,N_p-1} - r_1\|^2 - R^2 + t_{6N_p-1} \\ \|v_{c,N_p-1} - r_2\|^2 - R^2 + t_{6N_p} \\ T\Lambda e \end{bmatrix} = 0 \quad (3)$$

where we define the following quantities:

$$\tilde{H} + C_\lambda = 2 \begin{bmatrix} \mathcal{K}_1 & & & \\ & Q_c & & \\ & & \ddots & \\ & & & \mathcal{K}_{N_p} & \\ & & & & Q_c \end{bmatrix}_{12N_p \times 12N_p}$$

$$\mathcal{K}_i = \begin{bmatrix} \sum_{j=1}^2 \lambda_{6(i-1)+j} \mathbf{I}_2 \\ \sum_{j=3}^4 \lambda_{6(i-1)+j} \mathbf{I}_2 \\ \sum_{j=5}^6 \lambda_{6(i-1)+j} \mathbf{I}_2 \end{bmatrix}$$

$$g = -2 \begin{bmatrix} \mathcal{L}_1 \\ Q_c r_{\text{ref},1} \\ \vdots \\ \mathcal{L}_{N_p} \\ Q_c r_{\text{ref},N_p} \end{bmatrix}, \quad b = \begin{bmatrix} Ax_0 + FI_{o,0} \\ FI_{o,1} \\ \vdots \\ FI_{o,N_p-1} \end{bmatrix}$$

$$\mathcal{L}_i = \begin{bmatrix} \lambda_{6(i-1)+1} r_1 + \lambda_{6(i-1)+2} r_2 \\ \lambda_{6(i-1)+3} r_1 + \lambda_{6(i-1)+4} r_2 \\ \lambda_{6(i-1)+5} r_1 + \lambda_{6(i-1)+6} r_2 \end{bmatrix}, \quad i = 1, 2, \dots, N_p$$

$$\tilde{A} = \begin{bmatrix} -M & \mathbf{I}_6 & & & \\ & -A & -M & \mathbf{I}_6 & \\ & & & \ddots & \ddots \\ & & & & -A & -M & \mathbf{I}_6 \end{bmatrix}_{6N_p \times 12N_p}$$





**Algorithm 3** Backward Riccati for solving KKT system.**Require:** KKT system and  $N_p$ ;Set  $\beta_{N_p} \leftarrow \text{Chol}(\mathbf{P})$ **for**  $n = N_p - 1 \rightarrow 1$  **do** $\Sigma \leftarrow \beta_{n+1}^T [M \ A]$ **if**  $n = 0$  **then** $\mathbf{S}_0 = \mathbf{0}_{6 \times 6}, \mathbf{Q}_0 = 2Q_c$ **end if** $\mathcal{M} \leftarrow \Sigma^T \Sigma + \begin{bmatrix} \mathbf{R}_n & \mathbf{S}_n \\ \mathbf{S}_n^T & \mathbf{Q}_n \end{bmatrix}$  $\begin{bmatrix} \Theta_n^n \\ L_n^T \beta_n \end{bmatrix} \leftarrow \text{Chol}(\mathcal{M})$ **end for**Set  $p_{N_p} \leftarrow \tilde{q}_{N_p}$ **for**  $n = N_p - 1 \rightarrow 0$  **do** $l_n \leftarrow (\Theta_n^T)^{-1} (\tilde{s}_n + M^T (\beta_{n+1}^T \beta_{n+1} \tilde{b}_n + p_{n+1}))$  $p_n \leftarrow \tilde{q}_n + A^T (\beta_{n+1}^T \beta_{n+1} \tilde{b}_n + p_{n+1}) - L_n^T l_n$ **end for****for**  $n = 0 \rightarrow N_p - 1$  **do** $\Delta u_n \leftarrow -(\Theta_n^T)^{-1} (L_n \Delta x_n + l_n)$  $\Delta x_{n+1} \leftarrow A \Delta x_n + M \Delta u_n + \tilde{b}_n$  $\Delta \pi_{n+1} \leftarrow L_{n+1}^T L_{n+1} \Delta x_{n+1} + p_{n+1}$ **end for**

Table 1. Simulation settings.

System parameters	Values
Inductance of coil $L$	0.736 p.u.
Capacitance $C$	3.336 p.u.
Nominal DC voltage $V_{dc}$	0.727 p.u.
Nominal load current amplitude $I_o$	0.591 p.u.
Sampling (and Control) period $T_s$	310 $\mu$ s
Prediction horizon $N_p$	2
Penalties in $Q_c$ for inverter current error	1/30
Penalties in $Q_c$ for capacitor voltage error	10/30

or directly set it to a constant value  $\tau_k \in [0.9, 1]$ . We record the averaged number of iterations for high accuracy solution over 100 random initial points. The results are shown in Fig. 3, the choice of adaptive  $\tau_k$ , *i.e.*,

$$\tau_k = 1 - 0.01e^{-(k-1)}, \quad k = 1, 2 \dots \quad (9)$$

speeds up convergence.

Next, we perform the comparison with other commercial interior point solvers GUROBI 7.5.1, MOSEK 8.0 and open-source solvers SDPT3 4.0 and ECOS. We execute all the methods with default settings. The fast gradient method for real-time MPC (Richter et al. (2012)) is not considered here since it requires approximation of the ellipsoidal constraints by boxes and cannot obtain exact solution.

For each solver, the averaged computation time is computed for 1000 random initial points under different accuracies. As shown in Fig. 4, the proposed operator splitting method and Riccati method are suitable for the case of low-accuracy applications like embedded control (*e.g.* tolerances of  $10^{-3}$  to  $10^{-4}$  in Fig. 4) and outperform the competing solvers, while Riccati method performs better than all others when a high accuracy is required.

Finally, warm-starting can significantly improve computation times in MPC. In particular, we initialize the iterates

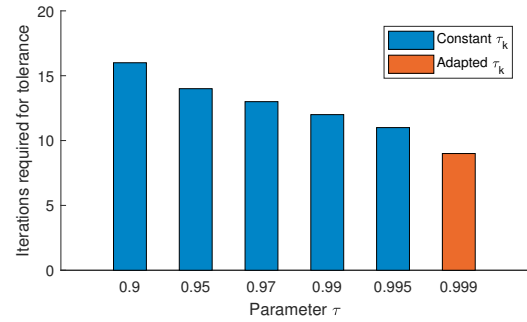


Fig. 3. The number of iterations reaches tolerance with different values of  $\tau$ .

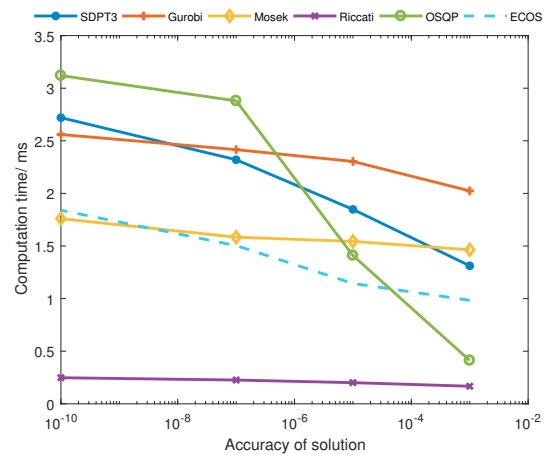


Fig. 4. Computation time vs solution accuracy for proposed methods and solvers.

of our methods to the solution of the previous problem and compute the averaged computation time. The obtained speedup is shown in Fig. 5.

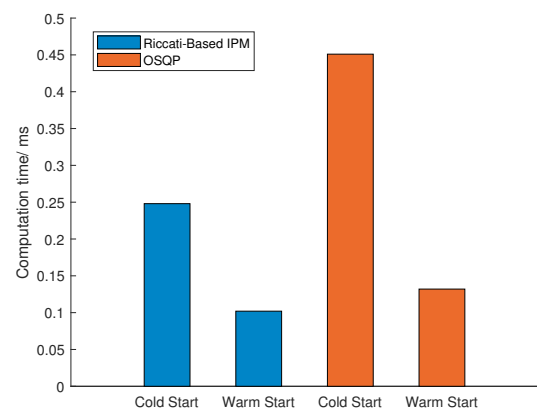


Fig. 5. Comparison of warm-start performance given low accuracy.

## 5. CONCLUSION

Motivated by a new formulation of a power electronics MPC problem, we presented two tailored optimization algorithms for real-time control using QCQP. In the ADMM framework, inspired by a novel operator splitting method, the quadratic constraints and the linear dynamics are

reformulated into an easy-to-project set. The other formulation based on the primal-dual interior-point method uses a Riccati-type factorization to solve the KKT system efficiently. We present several simulation results of an MPC tracking problem and show that our tailored QCQP algorithms outperform off-the-shelf solvers. In a future study, techniques like preconditioning and heuristic methods could be integrated into OSQP to further reduce the number of iterations. Furthermore, a real-world embedded control setup is envisaged.

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