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GROUP ROBOTICS (M GINI AND F AMIGONI, SECTION EDITORS)

# Recent Advances in Formations of Multiple Robots

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## Abstract

**Purpose of Review** Formation control is a canonical problem in multi-robot systems, which focuses on the ability of a group of robots to travel in coordination through an area, while maintaining a certain shape or a particular behavior. The robot groups vary in their communication, computation, and sensing capabilities. Moreover, the formation control task itself may have various objectives. These divergences force the use of different models for controlling the formation and for analyzing the task performance. In this paper, we describe the formation control problem and survey recent advances focusing on aspects of maintaining a formation by a group of robots distinguished by the means of analysis.

**Recent Findings** Various approaches may be applied for the sake of formation maintenance, whereas each approach possesses a different perspective in regard with formation control. Recent research focuses on *combining* those approaches, due to their applicability regarding certain scenarios. For instance, consensus-based control and collision avoidance are usually intertwined together for the sake of reaching a consensus in a manner which is collision-free. Furthermore, machine learning (ML)–based methods for navigating a robot team through unknown complex environments can be incorporated, where the robot team aims to reach a goal position while avoiding collisions and maintaining connectivity. Moreover, recent approaches focus on developing new mechanisms or adapt existing ones for formation control for tolerating limitations in sensing, communication, and coordination, preferably distributively while providing performance guarantees.

**Conclusion** Such combined approaches yield that the means of analysis, which can be applied to each one separately, can also be utilized in an intertwined manner, and thus provide us with novel methods for preserving formation. Whereas some approaches were vastly investigated (e.g., consensus-based formation control) and need to be adapted to distributed imperfect settings, others still require further insight for unveiling brand new architectures and tools (e.g., ML-based formation control).


**Keywords** Formation control · Multi-robot systems · Multi-agent systems · Swarm robotics · Cooperative control · Motion planning · Collision avoidance · Stability analysis · Graph theory · Machine learning · Deep learning · Task allocation

## Introduction

The problem of formation control is long examined in the literature of multi-robot systems. In nature, traveling in formation has significant advantages, providing individuals with protection from predators (e.g., fish forming a bait ball) and natural forces (e.g., birds flying in windy conditions), as well as enabling them to perform otherwise impossible tasks or performing them more efficiently (e.g., carrying food). Stemming from the study of such phenomena, the problem of formation control has been vastly examined in the literature on multi-robot systems. For robots, operating in formation improves the efficiency, robustness, and feasibility of missions like exploration [1], transportation [2], and containment [3]. This has led to research on formation control as a canonical problem and on its potential adaptation to other applications, yielding thousands of papers in this subject in the past decade, as well as numerous surveys. This paper summarizes key ideas in the research on formation control from recent years.

This article belongs to the Topical Collection on *Group Robotics*

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The canonical formation control problem is roughly composed of two different sub-problems, each attracting great interest of its own: generating a formation (referred here as *Formation Creation*) and maintaining it (referred to as *Formation Maintenance*). Although both are entwined and in some cases inseparable, they may be profoundly different in their goal, the tools to achieve them, and their analysis. In the formation creation problem, the goal is to have the robots arrange about a certain shape, where they may be required to optimize some criteria during the arrangement or determine a shape that best admits to some characteristics. In the formation maintenance problem, the goal is to have the robots travel in a connected formation through an environment—either in a predefined shape and while minimizing deviation from it, or simply in a connected shape. One basic requirement shared between all variants of the formation control problem is that the robots travel while avoiding collisions with their peers as well as obstacles in the environment.

In the formation creation problem, the focus is usually on the aspects of determining which formation to create, and/or how to position the robots about it. Determining a *best* formation refers to choosing a shape that best suits the robots, the environment, and the mission. The analysis of a possible solution depends on the specific criteria in hand. For example, how the placement of the robots optimize their resilient to failures [4] or survivability of threats [5]. Best arranging robots about a given shape usually regards to minimizing the time to complete the rearrangement [6, 7], or, for simple robots with limited communication, computation, or sensing capabilities, it is examined whether formation (pattern) creation can be guaranteed and under which conditions [8–10]. The two aspects may be combined such that a formation is chosen in a way that optimizes the process of spreading the robots about it [11, 12]. The formation creation problem gained recent interest from the modular robotics community, which are concerned with the question of how to bring the autonomous parts of the robots to gather in a desired configuration while minimizing cost or maximizing data acquisition [13, 14].

The central problem in formation control, and thus the main focus of this paper, is the formation maintenance problem, which boils down to one question: deciding how to calculate the next position of a robot as part of the formation. The answer to this question depends heavily on the decision-maker's knowledge on the robots and on the environment. If all the knowledge is concentrated in some centralized unit (either within the formation or external to it), then this unit can decide for each robot where to go next in order to maintain the formation (or transfer a reference point to each robot for calculating their next position relative to that point). In the very opposite case, each robot holds no global information about its peers or the environment, and thus decision-making must be encapsulated in the robot itself (thus decentralized) and depends solely on its local sensing. A whole continuum lies in between these two extremities, allowing the robots to act in different centralized or decentralized control schemes, suitable for the mission and the robots' capabilities. With that being said, the selection of all papers discussed throughout the entire paper was due to their prominence in terms of contributions with respect to other papers in the realm of formation control.

## Formation Control Classification

The fundamental approaches to the formation control problem have been vastly expanded in previous studies. In this section, we provide the main classifications of such approaches.

### Formation Control Coordination Schemes

Coordination within a formation of robots have been historically characterized by Beard et al. [15]. They have divided the means for multi-robot coordination to three main approaches: *leader-following*, *behavior-based*, and *virtual structure*. Although there are plenty of studies that combine ideas from these three approaches (starting from the work by Beard et al. themselves [15]), they are still referred to as the three basic ways for controlling a formation also today.

In the **leader-follower** approach, the formation is led by one leader, the *global leader*, that navigates it through the environment. This approach brings a flexibility of control based on the capabilities and the knowledge of the formation members: if they can communicate with the leader or sense it directly, then they compute their next move based on their location relative to it. Otherwise, each robot will follow one or more *local leaders* (which may or may not be the global leader), where the description of the local leaders is formed in a *control graph*, a directed graph in which each node represents a robot in the formation, and an edge from a vertex $u$ to $v$ indicates that the robot $u$ follows $v$ in the formation. Each robot must have a path leading to the global leader. Determining the edges of the control graph is one of the key points in leader-follower formations, where this usually involves graph-theory based methods.

In the **behavior-based** approach, each robot determines its actions based on a combination of several behaviors (or forces), for example, obstacle avoidance and formation keeping, which is determined by the local information each robot possesses on its peers and the environment, sometimes also combined with some global knowledge. This method is commonly used in swarm robotics, implemented in a completely distributed manner, where each robot chooses simple actions based only on very limited local sensing, allowing the formation to easily scale to large groups of robots (similar to a flock of birds).

The **virtual structure** approach is meant for cases in which full information about the formation is collected by one centralized source. According to this approach, the formation is treated as a single structure, thus dynamics planning is computed for this structure, and the control law for each robot is then derived from the general plan and monitored by this centralized unit.

### Sensing Capability and Interaction Topology

Classifying various approaches to the formation control problem, in terms of sensing capabilities and the interaction topology, concerns with the following questions: what variables are *sensed* and what variables are *controlled* in an active manner for the sake of attaining the desired (or undesired) formation. Where the types of sensed variables determine the *sensing capability* of the agents, the types of controlled variables are necessarily affecting the *interaction topology*. Based on this observation, Oh et al. [16] suggest a classification of the existing formation control schemes into three main approaches: *position*-, *displacement*-, and *distance*-based.

In the **position-based** control, a global coordinate system is assumed, and each agent senses its own *local* position with respect to it. The desired formation is thus characterized in regards to the global coordinate system, where each agent acts to reaching its corresponding position in the final formation. This approach requires strong coordination capabilities between the formation members.

In the **displacement-based** control, each agent is capable of sensing its *relative* positions of its neighbors, with respect to a global coordinate system. Accordingly, the desired formation is specified by the desired displacements. As opposed to the position-based control, neither any knowledge regarding the global coordinate system nor the agents' local positions are required. However, the agents ought to know the global coordinate system's orientation.

In the **distance-based** control, the desired formation is provided by inter-agent distances, which are controlled in an active manner. In contrast to the displacement-based control, it is not a necessity for the agents to acquire a certain alignment with each other's local coordinate system. However, agents are assumed to have the capability of sensing relative position of their neighbors with respect to their own local coordinate system.

Each of the mentioned types of control above consists of its own benefits and deficiencies. Where the position-based control is the most beneficial with respect to the interaction topology, its downside is the necessary knowledge and tight coordination required in the system. On the contrary, distance-based control is the exact opposite regarding those terms. In accordance, displacement-based balances between both characteristics.

## Formation Control by Analysis

We distinguish between different researches for maintaining multi-robot formations by the means of analysis. Methods for guaranteeing the formation is kept can be applied in the three main approaches described previously in Section "Formation Control Coordination Schemes", and each focuses on a different perspective of the formation control. Note that the research fields may be combined. Many researches consider various approaches to the formation maintenance problem at hand, due to their being highly analogous to each other when regarding certain scenarios. Swarm robotics is an obvious example, although a combination might also be applied to other approaches as well. For instance, consensus-based control and collision avoidance are usually intertwined together for the sake of reaching a consensus in a manner which is collision-free.

In Table 1, we present a concise comparison of the vast research on the mentioned topic, while concentrating on the following parameters: the relevant research fields, the formation control approach, the sensing capabilities, the information assumption, and the means of analysis proposed in each work. The papers are listed in the order of first section appearance and are tagged with additional fields in which they engage.

### Consensus-Based Formation Control

*Consensus-based* control focuses on the type of information available to the robots on their peers and the environment, and how those should be shared and integrated in order to decide on the next move of the robot and guarantee a stable consensus (thus stable formation).

One way for controlling a formation for all three approaches of formation control when regarding a consensus-based control is modeling the system by first- or second-order dynamics [17]. This method is based on the fact that each robot in the formation keeps an information state that is updated based on its locals neighborhood. If the information-flow topology forms a spanning tree, then the robots are guaranteed to reach and maintain a consensus. The core of the method is in choosing the information that will yield consensus. The means of analysis utilized in consensus-based control vary, and will briefly discussed in the paragraphs which follow.

Recent developments in consensus-based formation control concentrate on determining when and how consensus

**Table 1** Comparison of the recent advances regarding formation control

| Field | Paper | Control approach | Sensing and info. | Means of analysis |
|---|---|---|---|---|
| Section "Consensus-Based Formation Control" | Aditya et al. [21] | Behavior-based | Position-based | Consensus control proposed by Listman et al. [22]; the extended Kalman filter with non-linear systems; Taylor expansion; Jacobian matrix |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking, Model Predictive Formation Control" | Kuriki and Namerikawa [34] | Leader-follower | Position-based | Constrained Optimization; Trajectory Tracking; Model Predictive Control (MPC) |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking" | Alonso-Mora et al. [35, 36] | Behavior-based | Position-based | Sequential Convex Programming; Local and Global path planning |
| Section "Consensus-Based Formation Control" | Arranz et al. [23] | Behavior-based | Noisy signals estimation | Gradient-Ascent (GA) or Newton–Raphson-like ascent (NRA) algorithms |
| Section "Consensus-Based Formation Control" | Antonelli et al. [25] | Behavior-based | Displacement-based | Lyapunov analysis; Jacobian of the task; State observer |
| Section "Consensus-Based Formation Control" | Aranda et al. [33] | Centralized; behavior-based | Vision-based | A visual information obtained by UAVs |
| Sections "Consensus-Based Formation Control, Formation Control in Swarm Robotics" | Deng et al. [37] | Virtual structure; leader-follower | Displacement-based | Two-Layer Nearest Neighbor Information (TNNI) |
| Section "Consensus-Based Formation Control" | Goodwine [26] | Behavior-based | Viscoelastic model [27, 28] | Gamma function; Grünwald-Letnikov derivative; Asymmetric control structure |
| Section "Consensus-Based Formation Control" | Habibi et al. [29] | Leader-follower | Position-based | Multi-Hop Communications [30]; A distributed, tree-based algorithm, built on the Extreme-Comm algorithm [31] |
| Sections "Consensus-Based Formation Control, Rigid Formation" | Montijano et al. [32] | Leader-follower | Vision-based; homography-based | 3-D distributed control law; Sensor fusion algorithm |
| Sections "Consensus-Based Formation Control, Rigid Formation, Optimization-Based Formation Tracking" | Rahimi et al. [38] | Virtual leader-follower | Error-based (both position and velocity) | Back-Stepping synchronized controller [39]; Cross coupling synchronization |
| Sections "Consensus-Based Formation Control, Rigid Formation" | Burns et al. [40] | Persistent leader-follower | Distance-based | Derivation from rigidity circuits; Building larger formations by merging smaller ones; Recursive construction through trilateration graphs |
| Sections "Consensus-Based Formation Control, Rigid Formation" | Chen et al. [41] | Behavior-based | Distance-based | Potential functions; Gradient-descent control law; Lyapunov analysis; Coordinate transformation technique |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking, Model Predictive Formation Control" | Abichandani et al. [42] | Behavior-based | Distance-based | A real-time Receding Horizon Mixed Integer Nonlinear Programming (RH-MINLP) based optimization; a connected Mobile AdHoc Network (MANET) framework |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking" | Otte et al. [43] | Behavior-based | UDP-based comuunication | Any-Com intermediate solution sharing algorithm (Any-Com ISS) [44]; A modified Any-Time Rapidly Exploring Random Tree (RRT) algorithm |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking" | Pan et al. [45] | Leader-follower | Virtual springs-based | The K-neighbor model [46]; The K-spring model topologies' network |

**Table 1** (continued)

| Field | Paper | Control approach | Sensing and info. | Means of analysis |
|---|---|---|---|---|
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls" | Cao et al. [47] | Docking | Distance- and displacement-based | The discrete-time Lasalle's invariance principle [48]; The diminishing persistent excitation idea [49–51]; The adaptive radius assignment idea [52] |
| Sections "Consensus-Based Formation Control, Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls" | He et al. [53] | Evader-follower | Local observation-based | The linear quadratic regulator method |
| Sections "Consensus-Based Formation Control, Formation Control in Swarm Robotics" | Hauri et al. [54] | Behavior-based | Position- and distance-based | Reynolds' Model [55]; The implementation offered by Olfati-Saber [56]; Shape-steering rule; The velocity mapping method [57]; A goal-directed approach; Shaped flocking |
| Sections "Consensus-Based Formation Control, Formation Control in Swarm Robotics" | Jia and Wang [58] | Leader-follower | Position- and distance-based | A distributed cohesive flocking algorithm (which is based on potential functions); LaSalle–Krasovskii invariance principle [48] |
| Sections "Consensus-Based Formation Control, Coalition Formation" | Gunn and Anderson [59] | Centralized; role-based | Position-based | The Role Check process; Logic; Task lists; Exhaustive task assignment |
| Sections "Rigid Formation, Adaptive Estimators of Formation Controls" | Cai et al. [60] | Persistent leader-follower | Distance-based | Rigid graph theory; Back-Stepping control technique; Adaptive controller which accounts for parametric uncertainty; Lyapunov analysis; Euler-Lagrange-like dynamic model |
| Section "Machine Learning" | Derhami and Momeni [61] | Actor-critic-based | Distance-based | Fuzzy Actor-Critic Reinforcement Learning (FACRL); Multi Agent Fuzzy Reinforcement Learning (MAFRL) |
| Sections "Machine Learning, Formation Control in Swarm Robotics" | Khan et al. [62] | Centralized; Markovian behavior-based | Position-based | Graph Convolutional Networks (GCNs); Normalized graph Laplacian; Graph Policy Gradients (GPG) algorithm |
| Sections "Machine Learning, Adaptive Estimators of Formation Controls" | Jiang et al. [63] | Centralized; local behavior-based | Local observation-based (no inter-robot communication) | Deep Neural Network (DNN); Convolutional Neural Network (CNN); Fully-Connected (FC) network; Euclidean loss; Adam optimizer |
| Sections "Machine Learning, Optimization-Based Formation Tracking" | Lin et al. [64] | Centralized; Markovian actor-critic-based | Local observation-based | Partially Observable Markov Decision Process (POMDP); Proximal Policy Optimization (PPO) [65], which is a deep reinforcement learning (DRL)-based method |
| Sections "Machine Learning, Optimization-Based Formation Tracking, Model Predictive Formation Control, Adaptive Estimators of Formation Controls" | Xiao et al. [66] | Leader-follower | Position-based | A neural-dynamic optimization-based *non-linear model predictive control* (NMPC); *Separation-Bearing-Orientation Scheme* (SBOS) for regular leader–follower formation; *Separation-Distance Scheme* (SDS) for obstacle avoidance; *Quadratic Programming* (QP); *Primal-Dual Neural Network* (PDNN) with parallel capability |
| Section "Optimization-Based Formation Tracking" | Desai et al. [67]; Kaminka et al. [68] | Leader-follower | Separation-bearing control | An unweighted directed graph and Monitoring multi-graphs (respectively). |
| Sections "Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls" | Yoo and Park [69, 70] | Leader-follower | Separation-bearing control | A unified error transformation strategy [69]; A new non-linear error function [70]; Lyapunov analysis |
| Sections "Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls" | Dai et al. [71] | Leader-follower | Distance-based | The geometric obstacle avoidance control method (GOACM); An adaptive tracking control algorithm |
| Sections "Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls, Formation Control in Swarm Robotics" | Lee et al. [72] | Leader-follower | Distance-based | Model Predictive Control (MPC); Receding Horizon Particle Swarm Optimization (RHPSO); A dynamic coevolving particle swarm optimization algorithm |

**Table 1** (continued)

| Field | Paper | Control approach | Sensing and info. | Means of analysis |
|---|---|---|---|---|
| Section "Optimization-Based Formation Tracking" | Benzerrouk et al. [73] | Behaviour-based; virtual structure | Relative position-based | A RCC algorithm which is derived from auction sales activities; An obstacle avoidance controller based on the limit cycle methods [74–76]; Lyapunov analysis |
| Sections "Optimization-Based Formation Tracking, Adaptive Estimators of Formation Controls" | Choi et al. [77] | Evader-follower | Position- and distance-based | Unmanned Underwater Vehicles (UUVs); Time Difference of Arrival (TDOA); The Maximum Likelihood (ML) algorithm; The natural Frenet-Serret frame [78] |
| Section "Optimization-Based Formation Tracking" | Liu et al. [79] | Leader-follower | Position- and distance-based | $l2$-norm Least Square (l2-LS) algorithm; Dynamic Window Least Square (DWLS) dynamic window least square algorithm (DWLS) |
| Sections "Optimization-Based Formation Tracking, Model Predictive Formation Control" | Nascimento et al. [80] | Leader-follower | Position- and distance-based | Non-linear Model Predictive Formation Control (NMPFC) while using potential functions; A modified $A^*$ path planning algorithm |
| Sections "Optimization-Based Formation Tracking, Model Predictive Formation Control" | Peng et al. [81] | Leader-follower | Asynchronous clock | The receding horizon control principle (also referred to as model predictive control (MPC)); ACADO tool [82]; An auxiliary acceleration term; Syn points |
| Sections "Optimization-Based Formation Tracking, Formation Control in Swarm Robotics" | Xu et al. [83] | Behaviour- and classification-based | Local observation-based | Classification-based searching for initial formation |
| Sections "Optimization-Based Formation Tracking, Formation Control in Swarm Robotics" | Vásárhelyi et al. [84] | Behaviour- and classification-based | Distance-based | Oscillation-free flocking algorithms; Smooth functions; Slack in potential valleys; Special over-damped dynamics; A repulsive distance-based potential; A global attraction; "GPS-vision" based swarm |
| Sections "Optimization-Based Formation Tracking, Formation Control in Swarm Robotics" | Dang et al. [85] | Leader-follower | Distance-based | Distance-based attractive/repulsive force fields; Positive gain factors; Relative position vector; Relative velocity vector between each robot and each virtual node; Lyapunov analysis |
| Sections "Optimization-Based Formation Tracking, Coalition Formation" | Huang et al. [86] | Behaviour-based | Position-based | A task-priority control approach for human-robot collaboration system; Robots Self-determination Task Design |
| Section "Adaptive Estimators of Formation Controls" | Shen et al. [87] | Leader-follower | Displacement-based | State feedback linearization method; An adaptive proportional integral derivative (*PID*) algorithm; Lyapunov analysis |
| Section "Formation Control in Swarm Robotics" | Gallardo et al. [88] | Centralized; virtual leader-follower | Distance-based | A centralized ROS Master; real-time Video from a Drone; HSV algorithm; Potential functions |
| Section "Formation Control in Swarm Robotics" | Li et al. [89] | Finite state machine behaviour | Local observation-based | A progressive formation approach |
| Section "Coalition Formation" | Dutta et al. [90] | Coalition formation | Similarity-based | A clustering-based coalition formation methodology [91]; Cooperative Game Theory; A weighted complete graph modeled by [92]; A region growing process; Approximate graph partitioning; 0-1 integer linear program |
| Section "Coalition Formation" | Ge et al. [93] | Cluster formation | Position-based | An aperiodic sampled-data Cluster Formation Protocol (CFP); Lyapunov analysis |

Each paper is tagged by its relevant research field, as well as its formation control approach, assumed sensing capabilities and information assumption. We also present the utilized means of analysis of each study can be guaranteed when the robots operate in imperfect conditions. Peng et al. [18] convert the formation control problem to an information consensus problem for non-holonomic robots for converging to a desired formation and maintaining it, assuming that the dynamics of the systems is not entirely known to all robots. Wei et al. [19] examine consensus in second-order multi-robot systems under communication delays and noisy measurements, and prove that if the maximum time delay of all robots is bounded, consensus can be maintained. Wang et al. [20] examine the problem of consensus in leader-follower systems in systems suffering from time-delayed, noisy communication, showing mean square consensus conditions by several formation control schemes.

Such imperfect conditions sometimes arise the need for utilizing estimation methods in various scenarios. Aditya et al. [21] aim to estimate the movements trajectory of these robots. They first make use of the consensus control proposed by Listman et al. [22], which controls the robots to a meeting point (rendezvous). The estimation of the robots' movements is done by a method called *the extended Kalman filter*, which extends the original Kalman filter to work with non-linear systems. Arranz et al. [23] deal with the problem of estimating in a collaborative way the gradient and the Hessian matrix of an unknown signal via noisy measurements collected by a group of robots. They also address the problem of noisy measurements and their effect on the gradient and Hessian estimation. Since the noise error increases rapidly for decreasing formation radius, they propose a sensible choice for the radius. Consequently, to validate the quality of the proposed strategy, they numerically compare it with an alternative least-squares-based solution [24]. Antonelli et al. [25] focus on a distributed controller–observer schema for tracking control of centroid and formation of a multi-robot system with first-order dynamics. They design, for each robot, a *state observer* providing an estimate for the overall system state, which asymptotically converges to a collective state.

Goodwine [26] investigates fractional-order modeling for multi-robot coordinated control problems, even when the individual robots and interconnections have the usual integer-order dynamics. They study a specific system with a topological structure for the interactions motivated by a *viscoelastic model* [27, 28], which can be described with mechanical models constructed of elastic springs obeying Hook's law and viscous dashpots obeying Newton's law of viscosity. Recalling that a generalization of the factorial function is the gamma function, it is used as a generalization of the derivatives in the fractional-order modeling. Afterwards, the *Grünwald-Letnikov derivative*, which allows one to take the derivative a non-integer number of times, is used to give an approximation for the resulting fractional-order differential equations. This work puts forth the fractional-order model as a useful tool in the multi-robot control area, which has been relatively unstudied.

Habibi et al. [29] are interested in techniques for the sake of multi-robot transportation. They aim to address situations where global sensing, communication, and geometry are not readily available or too costly to implement. They use *multi-hop communications* [30] to exchange local information and local geometry in order to cooperate with other robots, avoiding the need for a shared global coordinate frame. They use a *distributed*, *tree-based algorithm* to estimate the centroid, in which each robot builds a tree rooted on itself, extending to all robots responsible for transporting an object. Each robot needs to relay the trees for every other robot, which requires each robot to maintain a list of those trees, that is built with the *Extreme-Comm algorithm* [31], in which each robot constructs a list of all the robots' hard-coded IDs, and then selects its task based on its relative position in this list.

Montijano et al. [32] present a fully distributed solution to drive a team of robots to reach a desired formation in the absence of an external positioning system that localizes them. They propose a 3D distributed control law, designed at a kinematic level, that uses two simultaneous consensus controllers: one to control the *relative orientations between robots*, and another for the *relative positions*. For applying the controller to aerial robots, they also deal with the problem of estimating the relative orientations and positions using a *vision sensor*. Thus, they employ a novel *sensor fusion algorithm* to estimate the relative pose of the robots using on-board cameras and information from the *inertial measurement unit* (IMU). The overall idea is to compute a *rectification homography* with the information given by the IMU in such a way that the features are observed as if the robots' roll and pitch are equal to zero.

As opposed to other studies, Aranda et al. [33] propose a new vision-based control method to drive a set of robots moving on the ground plane to a desired formation. This *visual* information is obtained by aerial cameras carried by unmanned aerial vehicles (UAVs) acting as control units, whose motion needs to be controlled to ensure that the visibility of the ground robots is maintained. Using a current image of a subset of the ground robots and a reference one, each camera computes a transformation that creates a set of desired image points, from which it defines desired motion objectives, which are transmitted to the robots. Then, each robot computes its actual control input using this information, received from one or multiple sources.

## Rigid Formation

*Rigidity* in formation maintenance concerns with the conditions under which multi-robots systems can perform as one body, while retaining persistence after the loss of any one edge in the formation. In various scenarios, agents should perform as a single (rigid) body, for accomplishing several tasks. In such cases, agents are not capable of achieving a certain mission on their own (such as the purpose of transporting goods [2]), and thus they are required to maintain a fixed shape and minimize the deviation from it.

The *Back-Stepping control method*, which was proposed to control a class of non-linear systems by Kokotovic in 1992 [39], may be utilized in several variations of rigid formation. Back-Stepping is a recursive and *Lyapunov-based* approach, which addresses several significant problems, such as time-varying formation and tracking error minimization. More precisely, suppose the system can be defined as follows [94]:

$$\dot{\eta} = f(\eta) + g(\eta)\varepsilon, \quad \dot{\varepsilon} = u \tag{1}$$

where $[\eta^T \quad \varepsilon] \in \mathbb{R}^{n+1}$ is the vector of states and $u \in \mathbb{R}$ is the control signal. If $f$ is a smooth function in its domain, then the goal is to design a state feedback controller which stabilizes the origin (i.e., $\eta = \varepsilon = 0$).

Rahimi et al. [38] aim at designing a decentralized controller, based on a synchronization signal in the presence of time-varying formation. For special applications of rescue and surveillance, a set of agents, consisting of both unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs), are considered. They utilize a combination of the *Back-Stepping control method* and a *cross coupling synchronization*. Intertwined together, each agent receives the errors of its neighbors in a coupling way in the feedback for the sake of producing its control signal.

Cai et al. [60] extend the graph rigidity-based formation control framework to planar multi-robotic vehicles with dynamics, while assuming the vehicles parameters are subjected to uncertainty. Using an interactions undirected graph, they model the problem as a distance-based formation control problem. The key tools for the proposed solution were *rigid graph theory* and the *Back-Stepping control technique*. The latter enabled to rigorously embed a high-level, single-integrator-based control law into an *actuator-level adaptive controller* that accounts for parametric uncertainty in the vehicle dynamics, while ensuring asymptotic convergence of the inter-vehicle distance errors to zero using a *Lyapunov* analysis.

Burns et al. [40] focus on persistent leader-follower formations, which maintain local distance constraints in order to preserve the global shape of the formation after the failure of sensors used to maintain constraints. In particular, they deal with collective transport, defined as the transportation of an object by a group of robots which are not physically attached to each other, what one robot could not carry on its own, the group can move together. They deal with the problem by defining *redundant* persistence which, in the same way that a rigidity circuit remains rigid after the loss of any one edge, retain persistence after the loss of any one edge in the formation.

Chen et al. [41] present a distributed control law for a group of three differential-drive robots to maintain a desired rigid formation with a common desired velocity. They first define the problem using an undirected interaction topology for a group of three agents with the single-integrator model, under the assumptions that the initial positions of the robots are not collinear. With a set of *potential functions* that have their minimums at the desired distances, a *gradient-descent control* law is derived. While employing this control law, a *Lyapunov-based* approach is utilized to minimize the formation separation error and prove that each robot reaches the desired velocity. The control law is extended to a group of non-holonomic robots by using a coordinate transformation technique while considering the input saturation non-linearity.

## Machine Learning

*Machine learning* (ML) control aims at characterizing various policies, which shall be obtained by the agents in the formation, through the use of machine learning methods. The three basic machine learning paradigms of supervised learning, unsupervised learning and reinforcement learning are widely utilized for the sake of formation creation and maintaining it. Numerous variations of those paradigms are utilized in the literature, and will briefly discussed in this section.

*Fuzzy actor-critic reinforcement learning* (FACRL) is a combination of fuzzy system as function approximation and classic *actor-critic* method. There are two fuzzy systems, one for generation of action (called **actor**), and other for estimation of value function (called **critic**). This architecture is applied in condition that the defined reinforcement signal is the same for all the agents, and due to the fact that each agent is in a different state, their related critic function will also be different, whereby the agent's policy will be improved. Derhami and Momeni [61] propose a method which is called *multi-agent fuzzy reinforcement learning* (MAFRL). Since classic RL requires a discretization of the action space, a *linear interpolation based action selection* is applied for generating a continuous action selection method. The utilized selection method improved the results from the view point of average distance passed by the agents as well as from the view point of system resistance against failure, although it decreased the convergence speed.

*Deep learning* is a machine learning paradigm, which allows computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction. Jiang et al. [63] consider the problem of deep learning robot control policies to achieve multi-robot formation from the robot's local observation without inter-robot communication. During the training phase, the *deep neural network* (DNN) is trained by model-based methods to generate analytic decentralized control laws with full state measurement. The DNN is composed of a three-layered *convolutional neural network* (CNN) and a *fully connected* (FC) network to approximate the control policy function. They define the loss function for each iteration of the *minibatch learning* as the *Euclidean loss*. They also use a centralized, gradient-descent-based training algorithm with Adam optimizer to learn the optimal parameters of the DNN.

Khan et al. [62] consider the problem of learning policies to control a large number of homogeneous robots, while exploiting the underlying graph structure among the robots and the team and environment are assumed to be Markovian. Inspired by *convolutional neural networks* (CNNs), they make use of a new architecture called *graph convolutional networks* (GCNs), which utilize a bank of graph filters. A GCN actually exploits the underlying interactions graph which represents the swarm of robots. Similar to standard reinforcement learning, they execute these policies in the environment, collect a centralized reward, and use policy gradients [95] to update the weights of policy network. They call this algorithm *Graph Policy Gradients* (GPG). They show that their proposed method is able to scale better than existing reinforcement methods that employ fully connected networks. More importantly, they show that by using their locally learned filters they are able to zero-shot transfer policies trained on just three robots to over hundred robots.

Some researches combine both FACRL and Deep Learning, which were mentioned earlier. Lin et al. [64] introduce a novel deep reinforcement learning (DRL)–based method for navigating a robot team through unknown complex environments, where the geometric centroid of the robot team aims to reach the goal position while avoiding collisions and maintaining connectivity. The multi-robot navigation problem is formulated as a *partially observable Markov decision process* (POMDP). They propose a centralized policy learning, which employs an *actor-critic-based DRL algorithm*. Once this algorithm and a decentralized executing paradigm are applied, a decentralized policy can be derived. In this paper, the recently proposed *proximal policy optimization* (PPO) [65], which is a state-of-the-art DRL algorithm, guarantees stable performance improvement during training.

## Optimization-Based Formation Tracking

The concept of collision avoidance is usually discussed alongside formation tracking. Collision avoidance schemes are usually designed by a control framework for achieving formation, while maintaining a safe inter-agent distance. For their sake, the most frequent approach is modeling the problem at hand as an *optimization problem*, which aims at minimizing the tracking error. Furthermore, optimization problems with inequality constraints can significantly increase the computation time compared to problems without those constraints. To address this problem, some studies incorporate penalty terms to indirectly express the inequality constraints. Readers could refer to [96], which pays special attention to the collision avoidance problem.

Given a set of target formation shapes, Alonso-Mora et al. [35] aim to optimize the parameters (such as position, orientation, and size) of the multi-robot formation in a neighborhood of the robots. Their approach relies on convex and non-convex optimization methods to obtain the locally optimal state of the formation. If all the robots first agree on a convex obstacle-free region, and then compute a target formation therein, then a formation in collision with an obstacle would not appear. Then, the robots optimize, via *sequential convex programming*, the formation parameters. Their proposed method is intended for local motion planning, and thus deadlocks may arise. To avoid deadlocks, their method can be employed in combination with a global planner, in a manner similar to the work on centralized formation control by Alonso-Mora et al. [36]. As opposed to [34], here, the computation is carried out by each robot in the formation and does not rely heavily on the leader; thus, it can scale more easily to large teams.

Desai et al. [67] defined a control graph as an unweighted directed graph (digraph) whose vertices are the robots in the formation. They show that a formation can be stably maintained if the control graph implies each robot (except a single leader) maintains its bearing (angle) and separation (distance) with respect to one other robot (target). This type of formation control is known as SBC (separation-bearing control). In light of their work, Kaminka et al. [68] tackle this problem by the use of *monitoring multi-graphs*, from which directed trees are induced. Each such tree is an optimal control graph for a given task (e.g., message passing, formation maintenance), with respect to a given criteria (e.g., sensing costs, individual position error). Such multi-graphs distinguish between different sensing configurations of robots. That is, they compactly represent all possible SBC control graphs for a given placement of robots.

Dai et al. [71] describe a switching formation strategy for multi-robots with velocity constraints to avoid and cross such obstacles. Using the *geometric obstacle avoidance control method* (GOACM), the leader robot is responsible for planning a safe path and guiding the follower robots. The follower robots switch into an obstacle avoidance formation from the predefined formation, by calculating the new desired distances and desired angles. Furthermore, with a robot priority model, the proposed strategy also solves the collision problem between the follower robots, while minimizing the robot's trajectory and velocity tracking errors.

Benzerrouk et al. [73] deal with the navigation in formation of a multi-robot system (MRS) in a fully reactive way without any motion path planning. The obstacle avoidance controller proposed by Benzerrouk et al. is based on the limit cycle methods [74–76]. They proposed to combine the virtual structure and behavior-based approaches, due to the drawbacks that each approach has separately. The achieved task is thus divided into two behavior patterns: *attraction to a dynamic target* and *obstacle avoidance*. In the first basic task, it is a cost minimization function which helps each robot to choose this target. Their method is actually inspired by *auction sales activities*, which allow a task allocation for the MRS. They propose a *Relative Cost Coefficient (RCC) algorithm* which is derived from this kind of strategies. By comparing RCCs of the same target, robots negotiate and decide whether to take a specific target or give it up to another robot.

Choi et al. [77] consider a team of unmanned underwater vehicles (UUVs) such that each UUV measures the arrival time of sound signal generated from the evader. The purpose of the UUV team is to intercept the underwater evader based on the arrival time measurements. *Time Difference of Arrival* (TDOA) was utilized to localize an evader generating a signal and algorithm to estimate the evader position. The *Maximum Likelihood* (ML) algorithm is used for the sake of estimating the true evader position based on sensing measurements. The proposed approach only requires that the leader measures the relative position of a follower using proximity sensors. Thus, to enable this control, the communication link between the leader and a follower must be established. In accordance, they acknowledge that the maximum formation size is limited by the maximum communication range between the leader and a follower.

Liu et al. [79] study the problem of constructing topology graph that represents magnitude of robots interaction via observing trajectories. They transform the inference problem into a linear regression problem. The optimal estimation of Perron matrix that contains the interaction profile is derived using *l2-norm least square algorithm* (*l*2-LS). Considering the link failure and creation, they propose a novel *dynamic window least square algorithm* (DWLS) to identify dynamic changing topology. This is the first time to consider the topology inference problem of multi-robot formation control systems via external observation, which requires no prior knowledge of system dynamics.

Otte et al. [43] deal with finding solutions to the multi-robot path-planning (navigation) problem that have guarantees on completeness, are robust to communication failure and incorporate varying team size. They extend their previously proposed algorithm, *Any-Com intermediate solution sharing algorithm* (Any-Com ISS) [44], which is used to minimize computational complexity per robot and maximize communication bandwidth between team-members. Consequently, each team is formed into a distributed computer that utilizes surplus communication bandwidth to help achieve better solution quality and to speed-up consensus time.

Pan et al. [45] investigate the path planning problem and formation controls which ensure the robots' collaborative working. Based on the *K-neighbor model* [46], they propose the *K-spring model topologies' network*, according to the number of virtual springs connected to each robot. The leader robot is connected to all followers through virtual springs. In order to solve the local minima using this approach, a virtual target search method is designed. They thus propose a novel approach for the multi-robot local path planning in the predefined formation, which can ensure that the robots maintain the original formation during the path planning. Furthermore, compared with the artificial potential field method, the proposed method can deal with complex and diverse obstacles without any priori knowledge of the working environment. The proposed method is also capable of resolving the *multi-robot* path planning, whereas the artificial potential field method can be solely utilized for the *single robot* path planning.

## Model Predictive Formation Control

*Model Predictive Formation Control* (MPFC) refers to a class of control algorithms which utilize an explicit process model to predict the future response of an entity (such as a robot), and is widely investigated in the settings of collision avoidance and formation tracking. It is also referred to as *receding horizon control* or *moving horizon control*.

Kuriki and Namerikawa [34] propose a cooperative formation control strategy with collision-avoidance capability for a multi-unmanned aerial vehicle (UAV) system using decentralized MPFC and consensus-based control. Specifically, each UAV regarded as a decoupled subsystem solves a local sub-problem, which is a local optimization problem, only for its own plan. Lee et al. [72] propose a novel MPFC based on *receding horizon particle swarm optimization* (RHPSO) for formation control of non-holonomic mobile robots by incorporating collision avoidance and control input minimization and guaranteeing asymptotic stability. They suggest a dynamic coevolving particle swarm optimization algorithm that under some conditions can guarantee stability of the system.

Abichandani et al. [42] address the problem of decentralized, outdoor formation coordination with multiple quadcopters. The coordination of motion of multiple robotic vehicles in a shared workspace so that they avoid collisions is known as *Multi-Vehicle Path Coordination* (MVPC). In previous work, they reported on simulation and experimental validation of a real-time *Receding Horizon Mixed Integer Non-linear Programming* –based optimization framework that achieves decentralized MVPC under communication constraints [97–99]. Here, they extend their work to autonomous outdoor quadcopters by explicitly enforcing spatial formation constraints, while maintaining a connected *Mobile AdHoc Network* (MANET). The optimization problem aims to minimize the total distance (arc length) between their current location and the goal position over the entire receding horizon.

Nascimento et al. [80] propose a solution to the obstacle avoidance problem in multi-robot systems when applied to active target tracking—*a non-linear model predictive formation control* (NMPFC). They present an approach which uses potential functions as terms of the NMPFC. These terms penalize the proximity with mates and obstacles. A strategy to avoid singularity problems with the potential functions using a modified $A^*$ path planning algorithm was then introduced.

Peng et al. [81] deal with the leader–follower formation control problem for a group of networked non-holonomic mobile robots that are subject to bounded time-varying communication delays and an asynchronous clock. Based on MPFC and *ACADO tool* [82], they implement a fully distributed unified control framework to address the leader–follower formation control problem.

Xiao et al. [66] offer a neural-dynamic optimization-based *non-linear MPC* (NMPC) method for controlling leader–follower mobile robots formation. The leader–follower robots contain two models: *separation-bearing-orientation scheme* (SBOS) for regular leader–follower formation and *separation-distance scheme* (SDS) for obstacle avoidance. Regarding the obstacles in the environments, a control strategy is proposed for two-robot formations which includes SBOS leader–follower model and SDS collision avoidance model. After deriving the formation-error kinematics of both SBOS and SDS, a constrained *quadratic programming* (QP) can be obtained by transforming the MPC method. Then, over a finite-receding horizon, the QP problem can be solved by utilizing the *primal-dual neural network* (PDNN) with parallel capability.

### Adaptive Estimators of Formation Controls

*Adaptive estimators* are widely used in order to decrease uncertainties in the system, due to their nature of approximating the unknown variables in a recursive manner. Such estimators consider an estimate of the parameters at hand (e.g., desired position), and are then dynamically adjusted so as to minimize the measured formation errors.

Cao et al. [47] study the problem of distance-based relative docking of a single robot and the distance-based spatiotemporal cooperative formation control problem for multiple robots. They design an adaptive estimator for estimating the desired docking position with distance measurements from the range sensor and self-displacements from the odometer. Furthermore, the convergence of the system employing the proposed controller is proved via the discrete-time Lasalle's invariance principle [48] provided that the triggering positions satisfy some mild condition. While generalizing this problem to spatiotemporal cooperative formation control, the previous controller is extended to a *cooperative adaptive estimator*, which is consensus-based and can communicate with and measure its distance to neighbors, but not necessarily to the landmark. This extension is done by applying the diminishing persistent excitation idea [49–51] or the adaptive radius assignment idea in [52].

Regarding Shen et al. [87], their objective is to investigate a formation controller design for the non-holonomic mobile robots system, where the leader robot's velocity information including linear velocity and angular velocity is not available for the follower robot. They first use the *state feedback linearization method* to obtain the ideal control law. Then, they propose an adaptive proportional integral derivative (*PID*) algorithm, which in each step utilizes the output of the former ideal control law for dynamically adjusting the PID parameters according to the formation errors. By employing the proposed adaptive PID algorithm, it is shown that the leader–follower formation can be achieved without knowing the leader robot's velocities.

Considering Dai et al. [71], for solving the non-holonomic navigation problem, given that each robot correctly tracks its waypoints, the formation for the system can be correctly formed and maintained. An *adaptive tracking control algorithm* for the kinematic part is designed based on [100]. This adaptive control algorithm is used to minimize the robot's trajectory and velocity tracking errors.

Some studies consider Unmanned Underwater Vehicles (UUVs). According to Choi et al. [77], which was mentioned earlier, the UUVs chase an evader while preserving a *spherical formation*. A leader is located at the center of spherical the formation, while all other UUVs are called followers. If the distance between the evader and the UUVs is much longer than the formation size of UUVs (far-field evader), then the estimation diverges. Thus, the formation size is controlled adaptively to assure the convergence of the evader estimation.

He et al. [53] introduce a cooperative formation control system with static and dynamic target rounding up, using small-scaled underwater spherical robots. Given the environmental disturbances in practical underwater scenarios, an adaptive control algorithm is designed to control the underwater spherical robot, which has good robustness and adaptability. The *linear quadratic regulator method* was adopted to plan the trajectories of three underwater spherical robots, and these three robots rounding up the target in the form of arc.

Yoo and Park [70] present a distributed *connectivity-preserving* formation tracking scheme which is designed by deriving only a *non-linearly* transformed formation error without defining any potential functions. Even though this approach sheds light upon the problem at hand, it still lacks the full solution in two aspects: their proposed non-linear error transformation cannot deal with the collision avoidance problem among the robots, and the same communication ranges of the robots (i.e., the undirected graph topology) are only considered. In [70], they develop a unified error transformation strategy. A new non-linear error function different from [69] is derived to deal with additional control objectives besides the distributed formation tracking, namely, connectivity preservation and collision avoidance. The unified formation strategy using the proposed error function is recursively established to design *approximation-based adaptive local tracking controllers*.

## Formation Control in Other Settings

The fundamental approaches to the formation control problem might be applicable in other settings as well. In this section, we briefly discuss such applications in the settings of both *swarm robotics* and *coalition formation*.

### Formation Control in Swarm Robotics

*Swarm robotics* is a specific research field of multi-robot systems which is highly motivated by natural phenomena, where a large number of robots are controlled in a cooperative manner. The individual robots in the swarm have extremely low computational, sensing and communication capabilities, thus require the use of simple coordination mechanisms.

Deng et al. [37] analyze the limitations of existing algorithms for large-scale mobile robot swarm formation and consensus control problems. They introduce a distributed control architecture and the extended consensus algorithm with *two-layer nearest neighbor information* (TNNI). It is shown that convergence is reached if the directed graph of the system contains a swarm of directed spanning trees. Combined with the distributed control architecture, the entire control system can change the number of formation robots arbitrarily.

Xu et al. [83] extend the behavior-based navigation method to swarm robot systems and focus on two kinds of important formation control problems: *initial formation* and *formation control while avoiding obstacles*. The overall behavior conducted by a robot is the combination of several sub-behaviors, that is, moving to the goal, avoiding obstacles, wall-following, avoiding robot, and formation keeping. Based on the information detected from the surrounding environment, the robot chooses the proper behaviors. For the initial formation problem, they present an improved algorithm named *classification-based searching for initial formation*. This algorithm is suitable for large-scale robot formation where the number of robots is more than one hundred. Based on the initial state of the robots, the algorithm will classify robots to different types and use different methods and processing sequences to handle these types, which can significantly reduce the time complexity.

In Vásárhelyi et al. [84], a flocking algorithm had to be found that is not sensitive to large delays in terms of stability, that is, in which delays do not generate undesired oscillations. They propose the *first* outdoor *"GPS-vision"-based swarm* of ten autonomous flying robots with decentralized hardware, self-control, and stable self-organization capabilities for flocking, target tracking, and formation flights. Their methods combine several techniques, such as using smooth functions instead of sharp ones, using slack in potential valleys or using special over-damped dynamics.

Dang et al. [85] consider a swarm of robots and their mission is to track a moving target in two-dimensional space, under influence of the dynamic and noisy environments. Moreover, robots must also automatically escape the obstacles in order to continue to track the moving target with their swarm. Firstly, surrounding the virtual nodes, *distance-based attractive force fields* are created to drive the free robots towards the desired positions. They present a formation control law, which utilizes the positive gain factors, the relative position vector, the relative velocity vector between each robot and each virtual node. Additionally, a *damping term* is also utilized. The stability analysis based on the *Lyapunov* approach is given. Furthermore, a new combination of *rotational force field* and *repulsive force field* in designing an obstacle avoidance controller allows the robot to avoid and escape the convex and non convex obstacle shapes.

Gallardo et al. [88] deal with formation control of a collection of vehicles (both UAVs and UGVs) using a virtual leader-follower approach. They employ Robot Operating System (ROS), a robust message passing infrastructure, to pass all of the navigation data mentioned above back and forth between each sub-process that may need this data. Their Central Command Station, running the *ROS Master*, can be executed on any piece of hardware that is running ROS. Each agent and a Drone communicates with the ROS Master through Wireless Network. The ROS Master gets real-time video from the Drone, identifies and tracks each agent from the video feed using the *HSV algorithm*, and calculates the forces between the agents based on their proposed algorithm. This force value is then translated into linear and angular data to the agents, so that they can move in formation. Additionally, the HSV algorithm also calculates a virtual leader position for the formation, which is essentially the geometric center of the formation. In this formation control algorithm, potential functions between the agents are used to implement group cohesion and separation.

Hauri et al. [54] describe a system that takes real-time user input to direct a robot swarm. The goal of this work is to allow a non-expert user to represent drawings and animations with a robot swarm. They incorporate the traditional flocking approach defined by Reynolds [55], and use the implementation offered by Olfati-Saber [56]. They augment the traditional steering rules by formulating an additional steering rule called the shape-steering rule, which has the effect of causing the robots to adopt a target shape. The preferred holonomic velocity, which defines a preferred velocity for a non-holonomic agent, is achieved using the velocity mapping method in [57]. The proposed shaped flocking method is then compared to a goal-directed approach, which executes a geometric analysis of a target shape, and explicitly computes the goal positions for robots within the shape.

Jia and Wang [58] study leader–follower cohesive flocking and formation flocking of multiple robotic fish swimming in the water surface under the guidance of only one leader with zero-value external input. Based on the combination of consensus protocol and potential function, a distributed cohesive flocking algorithm is designed for the robotic fish system consisting of one leader and several followers to structure a cohesive formation. The stability of the closed-loop system is analyzed by means of graph theory and the *LaSalle–Krasovskii invariance principle* [48], which addresses the difficulty of finding a negative definite derivative of a considered Lyapunov function for the sake of proving motion stability. They draw the swimming robotic fish by an extended unicycle model, whose geometrical center and mass center don't coincide.

Li et al. [89] address the problem of progressively deploying a swarm of robots to a formation defined as a point cloud, in a decentralized manner. They propose a progressive formation approach, which transforms a given point cloud into an acyclic directed graph (DAG). This graph is used by the control law to progressively form the target shape based only on local decisions. The problem is presented as a translation and rotation problem, while the behavior is represented as a finite state machine. This paper constitutes a first step towards the definition of behaviors for progressively deployed swarms, and it shows how a formation can gradually grow in time, with guaranteed convergence for the joining process.

## Coalition Formation

*Coalition formation* in the sense of formation control is usually considered when regarding the task allocation problem. The task allocation problem concentrates on the following question: how a set of $N$ robots can be optimally partitioned into $M$ coalitions to complete $M$ tasks without considering how each task may be subdivided among the robots in a single coalition.

Dutta et al. [90] study single-task multi-robot instantaneous (ST-MR-IA) task allocation problem, which is a well-known NP-hard problem. Their approach employs a clustering-based coalition formation methodology [91]. They present the problem from a cooperative game theoretic approach, while defining a similarity function $w$ between a pair of robots or a robot-task pair. That is, a higher value of $w$ indicates that the members of the pair of robots or the robot-task pair are "similar" and they should be in the same coalition, while a lower value of $w$ would mean that they are "dissimilar" and should be in different coalitions. With the goal of reducing the exponential complexity of finding the optimal coalition structure, they use the framework of Demaine [92] which models the set of robots and tasks as a weighted complete graph. Afterwards, this problem is given as a *0-1 integer linear program*, which minimizes the penalty. The coalition structure found by their proposed algorithm maximizes the cohesion quality, yet its social welfare (the sum over all coalitions' payoffs) is not taken into consideration. Thus, *a region growing process* is defined, in which a virtual ball (centered around a task) is grown in an iterative manner.

Ge et al. [93] study the problem of cluster formation control for a networked multi-agent system (MAS) in the simultaneous presence of aperiodic sampling and communication delays. By a proper modeling of aperiodic sampling and communication delays, an aperiodic sampled-data *cluster formation protocol* (CFP) is constructed such that the information exchanges among neighboring agents only occur intermittently at discrete instants of time. Furthermore, a discontinuous *Lyapunov* approach is developed to derive a design criterion on the existence of an admissible sampled-data CFP.

Gunn and Anderson [59] describe a framework for coordinating a changing collection of heterogeneous robots in complex and dynamic environments such as disaster zones. The framework includes facilities for team formation and management as well as facilities for task discovery and assignment. Their work assumes task allocation is generally **centralized** to a single robot. The minimum requirements expression for a task is a simple *Boolean expression* defining the attributes and corresponding values required for a robot to carry out a task. Similarly, suitability expressions are composed of **and** and **or** clauses, except the individual clauses are assigned a weight. In accordance, calculating a robot's suitability to fill a role involves summing up its suitability to carry out the tasks normally expected of that role. A robot's suitability is used for various purposes in their methodology. A team recognizes the failure of robots and responds by adjusting the roles of the remaining robots in a decentralized manner (this process is named Role Check). Upon encountering a robot from another team, a robot's suitability is used to determine whether it is suitable to perform team coordination activities on behalf of the overall team. Task lists are used by each robot in order to prioritize one task over another, whereas role-based task assignment is utilized.

Huang et al. [86] study the behavioral control of a group of autonomous robots under human intervention, and each agent is modeled by an single integrator. They design a behavior control under human intervention, where the highest priority task is the human-dominated task (human task), and all the robot self-determination tasks are turned into the second or even lower priority. The human operator could make an emergency decisions for the sake of helping the robot in complex environments. In such cases, when a robot encounters an obstacle, obstacle avoidance task is active and has the highest priority, unless human intervention occurs.

## Conclusion

In this paper, we presented a literature review on the recent advances regarding formation control for multi-robot systems, while concentrating on the means of analysis introduced for addressing this problem. For the sake of formation maintenance, we presented various approaches which may be used either separately or in a combined manner. Thus, we provided a comparison of the state-of-the-art means of analysis in different scenarios which are regarded as instances of the formation control problem.

At the inception of one's research in regard with the realm of formation control, choosing the most appropriate approach is of vital importance. The approach is strongly affected by the following four aspects (as depicted by Table 1): (1) the research field, (2) the formation control coordination scheme, (3) the sensing capabilities, and (4) the information assumption. As discussed earlier in this paper, the chosen approach determines the set of methods, which are available for researchers throughout their study, thus affecting the scope to which the research at hand can be expanded. Consequently, recent research focus on *combining* those approaches, due to their applicability regarding certain scenarios and for the sake of utilizing additional means of analysis.

Hence, once the four factors above are resolved, one should consider the possibility of forming such combinations as well, for the sake of enlarging the extent to which his research can reach. For choosing a specific approach comprising those combinations, a comparative analysis of approaches presented in the literature can be executed (based, among others, on the information provided in Table 1), so as to provide the researcher with suitable knowledge regarding both the benefits and the deficiencies encompassed by different techniques.

## Declarations

**Competing interests** The authors declare no competing interests.

**Human and Animal Rights and Informed Consent** This article does not contain any studies with human or animal subjects performed by any of the authors.

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