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A Unified Analysis on ARQ-Based Broadcast: A Road to the Optimum

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ABSTRACT This paper presents a unified framework for Automatic Repeat reQuest (ARQ)-based retransmission schemes in multi-user broadcast environments. Two perspectives, namely, coded packet design and the enhancement of the ARQ protocol, are used to establish the unified framework. Consequently, the combined use of index coding and ARQ incorporated with memory is demonstrated to be beneficial toward performance enhancement in terms of transmission efficiency and memory overhead. Specifically, the asymptotic and finite transmission efficiencies of the index-coded ARQ incorporated with memory are analyzed to be close to the optimum over a wide range of packet error probabilities and the number of users, which is validated numerical evaluations. Furthermore, the use of index coding is analyzed to be also favourable in terms of memory overhead in comparison with the memory ARQ. Thus, index-coded ARQ with appropriate use of memory is shown to be a reasonable option in a practical sense, which sheds light on the possibility that multi-user coded ARQ can play a crucial role for a reliable broadcast beyond unicast.

INDEX TERMS ARQ, broadcast, index coding, memory overhead, retransmission, transmission efficiency.

I. INTRODUCTION

From an information-theoretic perspective, there exist common and private messages for the transmission of information from one transmitter (Tx) to multiple receivers (Rxs) [1]. For the transmission of the common and private messages, broadcast and unicast are usually used, particularly in mobile communications. Unlike the unicast, the broadcast has a peculiarity in that it targets a large number of recipients. Therefore, it is necessary to increase reliability by reflecting this specificity, which is essential for mobile communication. For this reason, a representative broadcast coding scheme that emerged is a rateless coding [2], [3]. The rateless coding is leveraged for a reliable broadcast, beyond a sort of best-effort data transmission, in the application layer, and an Automatic Repeat reQuest (ARQ) is used mainly for a reliable unicast [4]. The ARQ is a practically attractive retransmission technique, as defined in the 3rd Generation Partnership Project (3GPP) standard specifications [5]. Therefore, if the ARQ performs reasonably in broadcast environments, its usefulness will be further winged.

However, the wall between different transmission technologies that provide two exclusive service areas has recently been broken. Specifically, several studies have explored the application of ARQ to broadcast service beyond unicast service. The application of ARQ is inherently not limited to unicast because ACK or Negative ACK (NACK) feedback from multiple Rxs is feasible through a feedback link, which is realized with an almost error-free channel. Another noteworthy discovery is that the transmission efficiency of the ARQ can be enhanced by using index coding, which is equivalent to network coding [6]. Depending on the target service, i.e., real-time or delay-tolerant service, various retransmission schemes have been developed with numerical validation. Table 1 depicts representative studies [7]–[20] conducted for achieving efficient retransmission in the framework of the ARQ protocol for point-to-point and point-to-multipoint (broadcast) communications in comparison with contributions from this study.

For validating our theoretic and numerical analysis on the ARQ based broadcast schemes, we also consider the Random

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TABLE 1. **A concise comparison of our work with the existing ARQ based broadcast schemes.**

Paper(s)	Perspective	Contents
[4], [7]	Classical ARQ protocols for point to point communications	· Go-back-N · Selective repeat ARQ and its enhancement · ARQ schemes in combination with a Forward Error Correction (FEC)
[8]– [12]	ARQ protocol improvement for broadcast	· Go-back-N for broadcast and its enhancement · Selective Repeat ARQ (SR-ARQ) for broadcast and its enhancement
[13]– [18]	Network (index) coding based ARQ design for broadcast	· Performance enhancement in the perspective of i. Asymptototic transmission efficiency ii. Finite transmission efficiency iii. Memory overhead
[19], [20]	Extension to secure communications	· Secure ARQ protocol design by incorporating network coding
Our work	Unifying framework for ARQ-based retransmission	· Analytic consideration on how to generate common messages for broadcast · Enhancement of transmission efficiency beyond the asymptotic optimality · Reduction of memory overhead of existing ARQ with a full memory

Linear Network Coding (RLNC) which achieves the optimal transmission efficiency [21]. It is noteworthy that the RLNC itself has been evolved to be feasible in a practical sense. Specifically, the development of RLNC for achieving not only reliability but also reasonable computational complexity, i.e., from block-based RLNC to the practical finite sliding window RLNC which does not require feedback. We refer to [22] and references therein.

The main contribution of this study is to provide a unified analysis on ARQ-based broadcast with the two Key Performance Indicators (KPIs). Specifically, the transmission efficiency and memory overhead at the Tx are considered and identified to validate efficient ARQ-based retransmission schemes, thereby enhancing their performance. Based on the unified framework with ARQ retransmission, a nearly optimal broadcast scheme has been devised as a by-product, which is shown to be Index Coded Automatic Repeat reQuest with Memory (IC-MARQ) [1] in the perspectives of the finite and asymptotic performance.

The remainder of this paper is organized as follows. In Section II, a system model is described, and ARQ-based retransmission schemes are analyzed based on technically feasible ingredients such as the index coding and ARQ with and without memory. More specifically, the perspectives of coded packet design and ARQ-based retransmission enhancement are explored in Section III. This is followed by the numerical validation presented in Section IV. Finally, the conclusion of the paper is outlined in Section V.

II. SYSTEM MODEL

A wireless channel is considered for a broadcast service that is assumed to be delay-tolerant such as bulk data transfer. The wireless channel consists of one Tx and K Rxs. For reliable packet transmission from one Tx, it is assumed that retransmission is feasible based on ACK or NACK feedback from each Rx, which is through the almost error-free channel. In this study, the wireless channel is assumed to be homogeneous, that is, the packet error probability $P^{[k]}$ at the k-th Rx is equal to P irrespective of k, where k ranges from 1 to K. The packet transmission is divided into two modes. One mode is the transmission of the original N packets, where N is considered to be sufficiently large. The other mode is retransmission for packet(s) such that at least one Rx fails to recover the packet(s). The ARQ protocol is operated in a consecutive manner, such that original packet transmission is completed, followed by the retransmission mode, where the number of retransmissions is greater than or equal to 1.

For any packet retransmission scheme, the transmission efficiency, one of the KPIs, is defined as

$$\Gamma = \frac{N_T}{N}, \tag{1}$$

where N_T is the total number of transmitted packets including $N_T - N$ retransmitted packets. To minimize the $N_T - N$ retransmitted packets, an index coding equivalent to network coding is considered. Instead of using uncoded packets, coded packets using index coding are considered to enhance transmission efficiency.

In this study, the retransmission schemes are considered under the unified framework of ARQ by incorporating coded packet design according to the packet error pattern from each Rx feedback. To understanding the mathematical performance analysis of retransmission, the following notations are used:

- $M^{[i]}(i = 1, 2, \cdots, K)$: Set of packets for which i Rxs decode the original N packets successfully and send ACK feedback.
- $M_j^{[i]}(j = 1, 2, \cdots, {}_K C_i)$: Disjoint subsets of $M^{[i]}$ denoting the j-th packet error pattern from the perspective of each Rx.
- For real-valued functions f and g, $f(x) = O(g(x))$ when α and x_0 exist such that $f(x) \leq \alpha \cdot g(x)$ for all $x \geq x_0$.

For example, when $K = 3$, all packet error patterns can be described as mentioned in Table 2.

In a wireless broadcast service environment, it is inevitable to confront packet errors occurred at each Rx. In this situation, the following are considered to guarantee reasonable transmission efficiency in the asymptotic as well as finite sense.

- Design of coded packet for retransmission
- Enhancement of ARQ-based retransmission

In the next two sections, two major technical issues are addressed using insights from mathematical analysis and implementation perspectives.

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[1] For further understanding on the IC-MARQ, please refer to the Section III.

TABLE 2. Packet error pattern when $K = 3$ [18].

Packet error pattern set $\left(M^{[i]}\right)$	$M^{[0]}$	$M^{[1]}$			$M^{[2]}$			$M^{[3]}$
Packet error pattern subset $\left(M_j^{[i]}\right)$	$M_1^{[0]}$	$M_1^{[1]}$	$M_2^{[1]}$	$M_3^{[1]}$	$M_1^{[2]}$	$M_2^{[2]}$	$M_3^{[2]}$	$M_1^{[3]}$
1st Rx	NACK	ACK	NACK	NACK	ACK	ACK	NACK	ACK
2nd Rx	NACK	NACK	ACK	NACK	ACK	NACK	ACK	ACK
3rd Rx	NACK	NACK	NACK	ACK	NACK	ACK	ACK	ACK

III. MULTI-USER CODED ARQ ANALYSIS

A. CODED PACKET DESIGN PERSPECTIVE

The prominent methodology to enhance transmission efficiency in the transmission mode is using coded packets in a broadcast environment to send common data to multiple Rxs [13]–[18].

On the other hand, there is always a trade-off between transmission efficiency and computational complexity to leverage the feasible coded packet. One reasonable way to achieve an appropriate level of trade-off is to consider a coded packet design for optimal retransmission from an asymptotic perspective. The following *Lemma* elaborates the steps to achieve asymptotically optimal transmission efficiency.

Lemma 1: For ARQ-based retransmission with K and P, index-coded packet using only the packet error pattern set of $M^{[K]}$ is optimal in terms of the transmission efficiency when $P < O\left(K^{-(1+\epsilon)}\right)$ for a sufficiently large K with $\epsilon > 0.6$

Proof: It follows from [18], and the specific coding scheme has been elaborated therein.

From *Lemma* 1, it is observed that the packet error pattern subset of $M_K^{[K-1]}$ is the dominant factor that is index-coded for achieving optimal transmission efficiency asymptotically. Then, one natural extension method is to consider multi-layered structures for index coding. Figure 1 describes the two-layered structure for index coding conceptually. When each Rx stores the successfully decoded packets and their corresponding sequence numbers, the feasibility of multi-layered index coding is guaranteed. However, the contribution of the multi-layer structure to the transmission efficiency is not significant.

This is because when L layered index coded structure is employed, the probability of occurrence of the most influential event associated with $M_K^{[K-1]}$ is proportional to

$$P^L (1-P)^L , \tag{2}$$

which becomes negligible when L increases due to P $(0 \leq P \leq 1/2)$. In this study, we focus on $L = 1$, which is a one-layered index coding structure.

Theorem 1: For a wireless channel with a broadcast service with packet error pattern set $M^{[i]}$'s $(i = 1, 2, \cdots, K)$ and their packet error pattern subsets $M_j^{[i]}$'s $(j = 1, 2, \cdots, {}_K C_i)$, the packet error message set to be index coded as a common data is only $M^{[K-1]}$.

Proof: For a given K, messages and Rx domains are denoted as $\mathtt{M}$ and $\mathtt{R}$, respectively. For a packet error pattern set of $M_j^{[i]}$, the lengths of $\mathtt{M}$ and $\mathtt{R}$ are given by ${}_K C_i$ and K, respectively. Then, i becomes the number of ACKs in the $\mathtt{R}$ domain. Consequently, the number of ACKs over $\mathtt{M}$ is given by

$$\frac{({}_K C_i) \cdot i}{K}. \tag{3}$$

From (3), it is evident that the number of ACKs is $K - 1$ over $\mathtt{M}$ domain only when $i = K - 1$, which corresponds to one common index coded data, i.e., $m_1^{[K-1]} \oplus m_2^{[K-1]} \oplus \cdots \oplus m_K^{[K-1]}$.

B. ARQ-BASED RETRANSMISSION ENHANCEMENT PERSPECTIVE

In the wireless broadcast service environment, the use of index coded packets for common messages is proved to be effective to achieve the asymptotic transmission efficiency, as described in *Lemma* 1. This transmission efficiency is closely linked to an ARQ retransmission scheme, especially in the finite range of the packet error probability (P). In the following theorem, the transmission efficiency of index-coded ARQ with $L = 1$ appears to be closely linked to ARQ retransmission, which remains a dominant factor along with leveraging coded packet transmission.

Lemma 2: For broadcast of N' packets with $N' \leq N$, the average number of total transmitted packets is given by

$$N' \cdot \Gamma_{\mathrm{ARQ}}. \tag{4}$$

Proof: The proof is straightforward when the definition of transmission efficiency is taken into account and thus is not provided.

There are a number of ARQ schemes of which the transmission efficiencies are different depending on the availability of memory for packet history management at one common Tx. In the following section, two typical ARQ schemes are considered from the perspective of the transmission efficiency.

Theorem 2: For a memory less ARQ with no packet history management, Γ is given by

$$\frac{1}{(1-P)^K}. \tag{5}$$

For Memory ARQ (MARQ) with packet history management, the following Γ is achieved.

$$\sum_{i=1}^{K} \frac{(-1)^{k-1}\binom{K}{k}}{1-P^K}. \tag{6}$$

Proof: In the case of a Tx without memory for packet history management, there is a retransmission whenever at

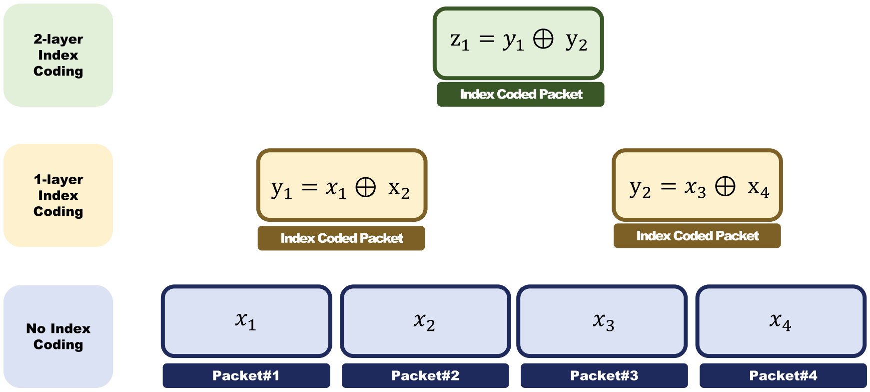

FIGURE 1. **Example: Multi-layered index coding when $L = 2$.**

least one Rx does not receive an original packet with probability of $1-(1-P)^K$. Therefore, the average number of total transmitted packets, including the retransmission is given by

$$\frac{N}{1-\left\{1-(1-P)^K\right\}} = \frac{N}{(1-P)^K}. \tag{7}$$

In the case of ARQ with memory for packet history management, the Tx can retransmit an original packet $m \in U$, only in the presence of at least one Rx that was not successful in decoding the original packet successfully during all the previous retransmission modes. Therefore, the average number of total transmitted packets are expressed as

$$N\sum_{t=1}^{\infty} t\left\{(1-P^t)^K-(1-P^{t-1})^K\right\}. \tag{8}$$

Intuitively, the t-th transmission is performed in the presence of at least one Rx with unsuccessful packet decoding until the $(t-1)$-th transmission. Using a power series with mathematical induction, the following is derived.

$$\begin{aligned} &N\sum_{t=1}^{\infty} t\left\{(1-P^r)^K-(1-P^{r-1})^K\right\} \\ &= N\sum_{i=1}^{K}\frac{(-1)^{k-1}\binom{K}{k}}{1-P^K}. \end{aligned} \tag{9}$$

Finally, by the definition of transmission efficiency in (1), the two transmission efficiencies of ARQ schemes, i.e., MARQ and ARQ with and without memory for packet history management are given by

$$\sum_{i=1}^{K}\frac{(-1)^{k-1}\binom{K}{k}}{1-P^K} \tag{10}$$

and

$$\frac{1}{(1-P)^K}, \tag{11}$$

respectively.

For notational convenience, (10) and (11) are denoted as Γ_{MARQ} and Γ_{ARQ}, respectively.

Theorem 3: For a given ARQ retransmission scheme with a transmission efficiency of Γ_{ARQ}, the following transmission efficiency is achieved.

$$1+\left\{1-(1-P)^K-(K-1)(1-P)^{K-1}P\right\}\cdot\Gamma. \tag{12}$$

Proof: For broadcasted original N packets, the set of all N packets are partitioned into $M^{[K]}$, $M^{[K-1]}$, and $U-\bigcup_{i=K-1}^{K} M^{[i]}$, where U is $\bigcup_{i=0}^{K} M^{[i]}$. The number of packets in the subsets of $M^{[K]}$, $M^{[K-1]}$, and $U-\bigcup_{i=K-1}^{K} M^{[i]}$ is given by

$$(1-P)^K N, \tag{13}$$

$$K(1-P)^{K-1}PN, \tag{14}$$

and

$$\left\{1-(1-P)^K-K(1-P)^{K-1}P\right\}N. \tag{15}$$

In (14), the number of packets can be reduced to

$$K(1-P)^{K-1}PN, \tag{16}$$

by leveraging index coding with $L = 1$.

$$(1-P)^{K-1}PN, \tag{17}$$

Using *Lemma* 2 with (13), (15), and (16), the average number of total transmitted packets including retransmission becomes

$$1+\left\{1-(1-P)^K-(K-1)(1-P)^{K-1}P\right\}\cdot\Gamma_{ARQ}. \tag{18}$$

Corollary 1: For a wireless broadcast service environment, index coded ARQ protocols[2] can achieve the following transmission efficiencies

$$1+\left\{1-(1-P)^K-(K-1)(1-P)^{K-1}P\right\}\cdot\Gamma_{\text{ARQ}} \tag{19}$$

and

$$\frac{1}{(1-P)^K}-\frac{(K-1)P}{1-P}, \tag{20}$$

respectively.

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[2] For brevity, index-coded ARQ with and without memory for packet history management will be denoted as IC-MARQ and IC-ARQ, respectively.

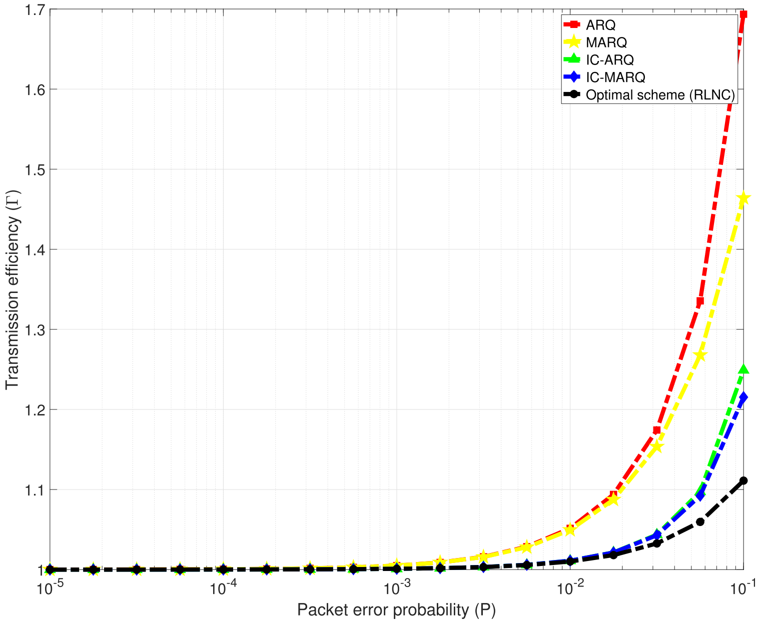

FIGURE 2. **Transmission efficiency with respect to the packet error probability when $K = 5$.**

Proof: It follows directly from *Theorem* 3 with some mathematical manipulations, which is omitted.

From evaluating (11) and (10) numerically, it is observed that the additional use of memory at the Tx can enhance transmission efficiency. Another non-trivial extension is as follows: even with a reduced memory at the transmitter, we can still guarantee further enhancement of the transmission efficiency in comparison to the memory ARQ in (10), which is achieved by taking advantage of coded packets for retransmission.

Theorem 4: For N packet ARQ-based transmission with full memory at the Tx, the use of index coding is always efficient, on an average, for storing transmitted packets in the retransmission mode, where, N is sufficiently large.

Proof: Consider the ARQ with full memory. Here, the number of packets that are correctly decoded at all Rxs is given by

$$(1-P)^K. \tag{21}$$

Therefore, the Tx needs to store $\left\{1-(1-P)^K\right\}\cdot N$ packets on an average, when N is sufficiently large. However, the number of packets to be stored can further be reduced to

$$\left\{1-(K-1)(1-P)^{K-1}P-(1-P)^K\right\}\cdot N. \tag{22}$$

This is achieved by leveraging the index coding with $L = 1$ over packets corresponding to $M^{[K-1]}$. From (22), the number of packets that do not need to be stored on the Tx, on an average, becomes

$$\left\{(K-1)(1-P)^{K-1}P+(1-P)^K\right\}\cdot N. \tag{23}$$

It is noted that the numerator in (23) has

$$\begin{aligned} KP-2P+1 &= KP-P+1-P && (24)\\ &\geq 1-P && (25) \end{aligned}$$

due to $K \geq 2$. Therefore,

$$\frac{(K-1)(1-P)^{K-1}P+(1-P)^K}{(1-P)^K} \geq 1, \tag{26}$$

which completes the proof.

Corollary 2: The memory overhead ratio during the retransmission mode is defined as

$$\frac{N'}{N}, \tag{27}$$

where N' is the number of packets to be stored for retransmission with $N' \leq N$, where N is the number of original packets. The memory overhead ratios of 0.6321 and 0.2642 are achieved when P goes to 0 with $K = 1/P$, where the former and latter correspond to memory ARQ and IC-ARQ with memory, respectively.

Proof: The proof is omitted because it is easily derived using *Theorem* 4 with some mathematical manipulations and the constant of

$$\begin{aligned} e &= \lim_{h\to 0}(1+h)^{1/h}\\ &\approx 2.7183. && (28) \end{aligned}$$

Remark 1: When P tends to 0, the number of Rxs for a broadcast service can be increased intuitively. Accordingly, the scaling of $K = 1/P$ is justified. Moreover, leveraging index coding can achieve efficiency in memory overhead, as shown in *Corollary* 2. Consequently, the use of index coding at the Tx during retransmission guarantees retransmission performance in terms of transmission efficiency as well as memory overhead. This increases the feasibility of implementation.

IV. NUMERICAL ANALYSIS

Based on the performance analysis in the previous section, ARQ-based retransmission schemes are evaluated using

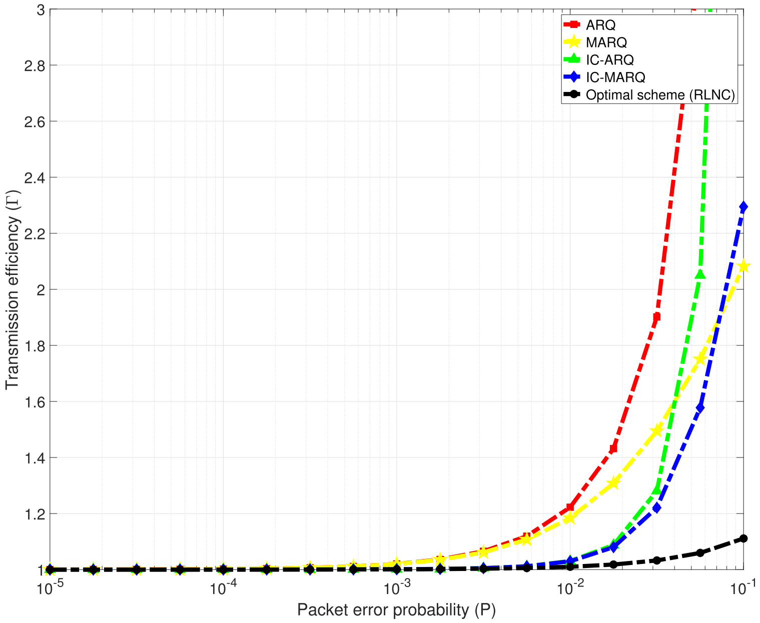

FIGURE 3. **Transmission efficiency with respect to the packet error probability when $K = 20$.**

TABLE 3. **Characterization of various ARQ-based retransmission schemes and their transmission efficiencies.**

Broadcast schemes	Transmission efficiency	Main characteristics
ARQ	$\Gamma_{\text{ARQ}} = 1/(1-P)^K$	· No packet history management of each Rx
ARQ with memory (MARQ)	$\Gamma_{\text{MARQ}} = \sum_{i=1}^{K} \frac{(-1)^{k-1}\binom{K}{k}}{1-P^K}.$	· Packet history management of each Rx
IC-ARQ	$\frac{1}{(1-P)^K} - \frac{(K-1)P}{1-P}$	· Use of coded packets as retransmitted packets · Use of memoryless ARQ for retransmission
IC-MARQ	$1 + \{1-(1-P)^K-(K-1)(1-P)^{K-1}P\} \cdot \Gamma_{\text{MARQ}}$	· Use of coded packets as retransmitted packets · Use of ARQ with memory for retransmission
Optimal scheme (Random Linear Network Coding)	$1/(1-P)$	· Non ARQ-based broadcast

a metric of transmission efficiency, which is described in Table 1. To enhance the transmission efficiency, the two perspectives considered are: coded packet design and ARQ-based retransmission. The contribution of the multi-layered ($L > 1$) index coding on the transmission efficiency is shown to be insignificant in coded packet design because P^L goes to 0 when $0 \leq P \leq 1/2$. In this context, the retransmission schemes are validated by leveraging the index coding with $L = 1$, by comparing it with typically used ARQ schemes. The enhancement of transmission efficiency in a wide range of packet error probabilities and the number of Rxs is guaranteed during the practical use of the ARQ-based retransmission scheme. This is covered in the next subsection.

A. TRANSMISSION EFFICIENCY WITH RESPECT TO THE PACKET ERROR PROBABILITY

Figs. 2 and 3 represents the transmission efficiency according to packet error probability. As the packet error probability tends to 0, it is observed that IC-ARQ and IC-MARQ are closer to the optimal scheme, i.e., RLNC. The first reason for this is that IC-ARQ has near optimal transmission efficiency when $P \leq O\left(K^{-(1+\epsilon)}\right)$, with sufficiently large K and $\epsilon > 0$ from (20). The second reason is that the IC-MARQ is always greater than or equal to IC-ARQ owing to more efficient ARQ-based retransmission by leveraging the packet history management of each Rx. The beneficial effect of index coding is noticeable by comparing ARQ (MARQ) with IC-ARQ (IC-MARQ).

One counterintuitive phenomenon is observed from Figs. 2 and 3. Specifically, the transmission efficiency of the MARQ is observed to be strictly greater than that of IC-MARQ in a wide range of packet error probabilities. Thus, when the packet error probability is not small, i.e., P is near 10^{-1}, the use of coded packets prevents the MARQ from enhancing the transmission efficiency. Note that this phenomenon is particularly pronounced when the number of Rxs (K) is large.

B. TRANSMISSION EFFICIENCY WITH RESPECT TO THE NUMBER OF RXS

Intuitively, as the number of Rxs increases, the number of retransmission increases, which is a common characteristic of all ARQ-based retransmission schemes.

On the other hand, the optimal scheme, i.e., RLNC achieves optimal transmission efficiency, which is a constant, at the cost of a considerable amount of computation. This is

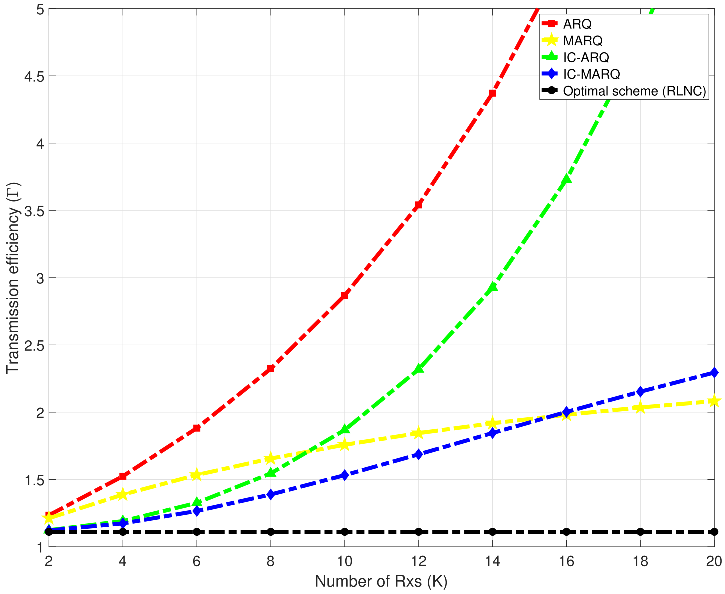

FIGURE 4. **Transmission efficiency with respect to the number of Rxs when $P = 10^{-1}$.**

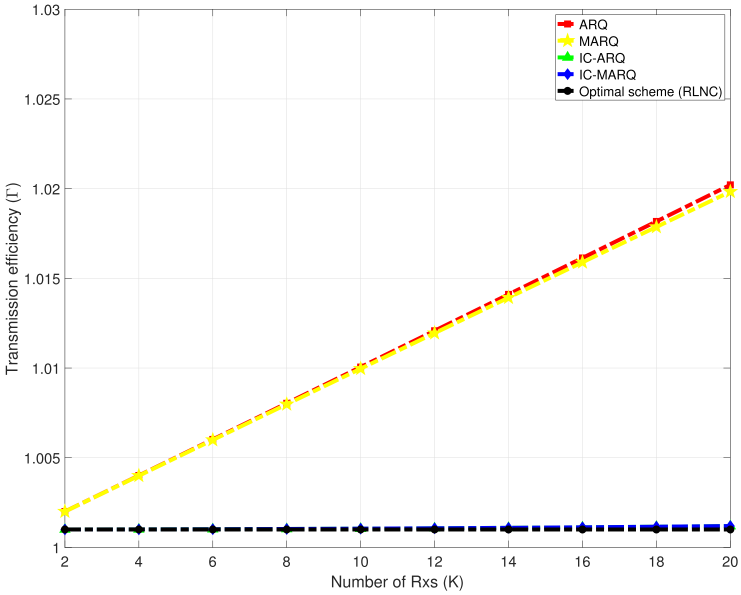

FIGURE 5. **Transmission efficiency with respect to the number of Rxs when $P = 10^{-3}$.**

related to a high decoding complexity over the Galois Field (GF) [23]. From Figs. 4 and 5, it is observed that as the packet error probability approaches 0, the contribution of packet history management with memory becomes negligible when index coding is leveraged, as observed from the comparison between IC-ARQ and IC-MARQ.

From the perspective of finite performance, IC-ARQ (IC-MARQ) outperforms ARQ (IC-ARQ), which is made feasible by coded packet design, called index coding. One counterintuitive phenomenon is the existence of a regime of packet error probabilities and the number of Rxs such that the index coding is ineffective, which is seen from Fig. 4. However, this phenomenon becomes insignificant as the packet error probability decreases to 0. In this asymptotic case, IC-ARQ and IC-MARQ are observed to be close to optimum.

V. CONCLUSION

In the quest for the optimum, which is the RLNC [21] for broadcast environments, it was shown that the combined use of coded packet design by leveraging index coding and ARQ protocol enhancement with packet history management was the beneficial approach in an individual case. Counter intuitively, there exist a regime of packet error probabilities and the number of Rxs such that simultaneous use of index coding and ARQ protocol incorporating memory for packet history management can sometimes collide, which is insignificant for all possible regimes of packet error probabilities and the number of Rxs. In addition, it was confirmed that the transmission efficiency of the IC-MARQ was close to the optimum when the packet error probability was close to 0, and the number of Rxs was not negligible. Based on KPIs such as the transmission efficiency and memory overhead, the ARQ-based retransmission scheme was shown to be a reasonable option based on asymptotic and finite perspective. Furthermore, when the packet error probability was close to 0, the simplified IC-ARQ with $L = 1$ was close to the optimum, even without the use of a sophisticated ARQ protocol, which was consistent with a previous study [18]. The RLNC as a non ARQ-based broadcast scheme can be considered as another crucial axis of reliable broadcast scheme. In this case, one of the critical technical issues can be to minimize an end-to-end delay, and in particular, the FEC design that achieves the minimum end-to-end latency from the viewpoint of delay transmission efficiency trade-off can become one of the critical issues to be handled in a practical sense [24], which can be a reasonable future research direction. As a by-product from the unified analysis on the ARQ-based broadcast, it was found that IC-MARQ is close to the optimum in the sense of transmission efficiency in a wide range of packet error probabilities and the number of users. Therefore, the broadcast service was observed to be achievable on a convincing level via ARQ, which highlights the possibility that ARQ can be truly used as a reliable broadcast beyond the unicast.

REFERENCES

[1] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York, NY, USA: Wiley, 1991.

[2] M. Luby, “LT code,” in *Proc. 43rd Ann. IEEE Symp. Found. Comput. Sci.*, Nov. 2002, pp. 271–282.

[3] A. Shokrollahi, “Raptor codes,” *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2551–2567, Jun. 2006.

[4] S. Lin, D. J. Costello, and M. J. Miller, “Automatic-repeat-request error-control schemes,” *IEEE Commun. Mag.*, vol. CM-22, no. 12, pp. 5–17, Dec. 1984.

[5] *Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Radio Link Control (RLC) Protocol Specification*, Standard 3GPP TS 36.322 v.16.0.0, 2020.

[6] M. Effros, S. E. Rouayheb, and M. Langberg, “An equivalence between network coding and index coding,” *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2478–2487, May 2015.

[7] E. Weldon, “An improved selective-repeat ARQ strategy,” *IEEE Trans. Commun.*, vol. C-30, no. 3, pp. 480–486, Mar. 1982.

[8] I. Gopal and J. Jaffe, “Point-to-multipoint communication over broadcast links,” *IEEE Trans. Commun.*, vol. C-32, no. 9, pp. 1034–1044, Sep. 1984.

[9] J. L. Wang and J. A. Silvester, “Optimal adaptive multireceiver ARQ protocols,” *IEEE Trans. Commun.*, vol. 41, no. 12, pp. 1816–1829, Dec. 1993.

[10] S. R. Chandran and S. Lin, “Selective-repeat-ARQ schemes for broadcast links,” *IEEE Trans. Commun.*, vol. 40, no. 1, pp. 12–19, Jan. 1992.

[11] W. S. Jeon and D. G. Jeong, “Improved selective repeat ARQ scheme for mobile multimedia communications,” *IEEE Commun. Lett.*, vol. 4, no. 2, pp. 46–48, Feb. 2000.

[12] J. He, K. R. Subramanian, L. Shang, and K.-K. Ma, “Analysis of a full-memory multidestination ARQ protocol over broadcast links,” *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1889–2001, Nov. 2001.

[13] M. Ghaderi, D. Towsley, and J. Kurose, “Reliability gain of network coding in lossy wireless networks,” in *Proc. 27th Conf. Comput. Commun. (IEEE INFOCOM)*, Apr. 2008, pp. 2171–2179.

[14] N. Lee, A. G. Dimakis, and R. W. Heath, Jr., “Index coding with coded side-information,” *IEEE Commun. Lett.*, vol. 19, no. 3, pp. 319–322, Mar. 2015.

[15] D. Nguyen, T. Tran, T. Nguyen, and B. Bose, “Wireless broadcast using network coding,” *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 914–925, Feb. 2009.

[16] K. Xu, W. Ma, L. Zhu, Y. Xu, Y. Gao, D. Zhang, and W. Xie, “NTC-HARQ: Network–turbo-coding based HARQ protocol for wireless broadcasting system,” *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4633–4644, Oct. 2015.

[17] J. Wang, K. Xu, Y. Xu, and D. Zhang, “Pseudo-systematic decoding of hybrid instantly decodable network code for wireless broadcasting,” *IEEE Wireless Commun. Lett.*, vol. 7, no. 5, pp. 840–843, Oct. 2018.

[18] S. W. Choi, “Index coded ARQ,” *Entropy*, vol. 22, no. 8, pp. 1–10, Aug. 2020.

[19] H. He and P. Ren, “Secure ARQ protocol for wireless communications: Performance analysis and packet coding design,” *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7158–7169, Aug. 2018.

[20] H. He, P. Ren, and X. Tang, “Joint network coding and ARQ design toward secure wireless communications,” *IEEE Trans. Commun.*, vol. 67, no. 5, pp. 3351–3362, May 2019.

[21] T. Ho, M. Médard, R. Koetter, D. R. Karger, M. Effros, J. Shi, and B. Leong, “A random linear network coding approach to multicast,” *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4413–4430, Oct. 2006.

[22] S. Wunderlich, F. Gabriel, S. Pandi, F. H. Fitzek, and M. Reisslein, “Caterpillar RLNC (CRLNC): A practical finite sliding window RLNC approach,” *IEEE Access*, vol. 5, pp. 20183–20197, 2017.

[23] R. Su, Q. T. Sun, and Z. Zhang, “Delay-complexity trade-off of random linear network coding in wireless broadcast,” *IEEE Trans. Commun.*, vol. 68, no. 9, pp. 5606–5618, Sep. 2020.

[24] X. Xu, Y. Zeng, Y. Li, and B. Vucetic, “Minimum-latency FEC design with delayed feedback: Mathematical modeling and efficient algorithms,” *IEEE Trans. Wireless Commun.*, vol. 19, no. 11, pp. 7210–7223, Nov. 2020.

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