

RELDEC: Reinforcement Learning-Based Decoding of Moderate Length LDPC Codes

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Abstract

In this work we propose RELDEC, a novel approach for sequential decoding of moderate length low-density parity-check (LDPC) codes. The main idea behind RELDEC is that an optimized decoding policy is subsequently obtained via reinforcement learning based on a Markov decision process (MDP). In contrast to our previous work, where an agent learns to schedule only a single check node (CN) within a group (cluster) of CNs per iteration, in this work we train the agent to schedule all CNs in a cluster, and all clusters in every iteration. That is, in each learning step of RELDEC an agent learns to schedule CN clusters sequentially depending on a reward associated with the outcome of scheduling a particular cluster. We also modify the state space representation of the MDP, enabling RELDEC to be suitable for larger block length LDPC codes than those studied in our previous work. Furthermore, to address decoding under varying channel conditions, we propose two related schemes, namely, agile meta-RELDEC (AM-RELDEC) and meta-RELDEC (M-RELDEC), both of which employ meta-reinforcement learning. The proposed RELDEC scheme significantly outperforms standard flooding and random sequential decoding for a variety of LDPC codes, including codes designed for 5G new radio.

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I. INTRODUCTION

Binary low-density parity-check (LDPC) codes are sparse graph-based channel codes. Due to their excellent error correcting performance for symmetric binary input channels [2], [3], they have recently been standardized for data communication in the 5G cellular new radio standard [4], [5]. Tanner graphs of LDPC codes are sparse bipartite graphs whose vertex sets are partitioned into check nodes (CNs) and variable nodes (VNs). Typically, iterative decoding on an LDPC Tanner graph is carried out via flooding: all CNs and VNs are updated simultaneously in each iteration [6]. In comparison, sequential decoding updates the nodes serially based on the latest messages propagated by their neighbors. Sequential scheduling problems seek to optimize the order of all CN (or VN) updates to improve the convergence speed and/or the decoding performance with respect to the flooding scheme [7], [8]. In this work, we improve the sequential decoding performance of LDPC codes using a novel reinforcement learning based scheme termed RELDEC (reinforcement learning-based decoding of moderate length LDPC codes), and its meta learning based variants, namely, agile meta-RELDEC (AM-RELDEC) and meta-RELDEC (M-RELDEC). These decoding schemes schedule groups (clusters) of CNs sequentially based on the learned scheduling order. In each scheduling instant, a cluster's neighbors are updated via flooding based on the latest messages propagated by its neighboring clusters.

A *node-wise scheduling* (NS) algorithm has been first proposed for sequential LDPC decoding in [9], where a single CN a is scheduled per decoding iteration based on its maximum residual. Here, a CN's residual is defined as the magnitude of the difference between two successive messages emanating from that CN to a neighboring VN. Our previous work in [10], [11] proposes a reinforcement learning-based NS (RL-NS) scheme which obviates the need for computing residuals. The NS algorithm is modeled as a finite Markov decision process (MDP) [12], where the Tanner graph is viewed as an m -armed slot machine with m CNs (arms), and an agent learns to schedule CNs that elicit the highest long-term expected reward. Long-term rewards are generated by an action-value function, where an action-value indicates how beneficial a particular choice of CN is for optimizing the CN scheduling order. The optimal scheduling order is the one that yields a codeword output by propagating the smallest number of CN to VN messages.

In [10], we have considered model-free reinforcement learning methods by (i) computing the Gittins index [13] of each CN, and (ii) utilizing Q-learning [14], [15], [12], respectively. In [11], in addition to model-free reinforcement learning, we have also considered a model-based RL-NS approach based on Thompson sampling [16].

Specifically, Q-learning is a Monte Carlo based reinforcement learning algorithm for computing optimal policies for MDPs by learning action-values [17], [18], [14]. However, for MDPs with a large state space cardinality, Q-learning may require tremendous computational effort. A multitude of methods for reducing the learning complexity have been proposed in the literature. For example, complexity may be reduced by partitioning the state space (see, *e.g.*, [19], [20]), imposing a state hierarchy (see, *e.g.*, [21]), or reducing dimensionality (see, *e.g.*, [22], [23]). In [24], [25], reinforcement learning is proposed for constructing polar codes. [24] focuses on belief propagation (BP) based polar code decoding and frames the factor graph selection problem as a multi-armed bandit problem. On the other hand, [25] frames the construction of polar codes as a maze traversing game where a chosen path in the maze corresponds to a unique polar code construction. Reinforcement learning has also been applied to hard decision-based iterative decoding in [18]. Furthermore, BP decoding of linear codes has been addressed in, *e.g.*, [26], [27], [28], which use deep learning-based on neural networks (NNs) to learn the noise on the communication channel. A deep learning framework based on hyper-networks is used for decoding short block length LDPC codes in [29].

In the machine learning field, a learning approach known as meta-learning has gained considerable attention in recent years. The term “meta-learning” first appeared in the field of educational psychology to refer to a system which is capable of taking control of its own learning [30]. Hence, meta-learning is also referred to as “learning to learn”. Meta-learning algorithms first accumulate experience on solving a variety of tasks, and then adapt for solving related but unseen tasks. During adaptation, the algorithm is expected to perform well on the new tasks with only a small number of training steps, and with a small amount of training data.

In [31] a model-agnostic meta-learning (MAML) algorithm is proposed for a wide range of problems including classification, regression, and reinforcement learning [31]. The MAML

scheme relies on gradient descent for model parameter optimization. Further, in [32], the authors proposed a meta-Q-learning (MQL) scheme which is also a gradient-based approach suitable for Q-learning based meta-learning. The application of meta-learning for solving problems related to communication systems is well known. For instance, MAML has been proposed for designing an optimized demodulator which quickly adapts to various channel conditions after being trained using a small number of pilot symbols [33], [34]. This demodulation scheme is suitable for an internet-of-things setting where the channel conditions may vary with each device. Moreover, MAML has been used for channel coding based on few pilot transmission in [35], where the focus is on learning a decoder for a fixed encoder via supervised learning. In comparison to our proposed meta RELDEC approaches, the meta-learning scheme in [35] is intended for a specific supervised learning problem that learns to correctly map a received noisy signal to the transmitted message. Further, the codeword length considered in [35] is substantially smaller than the codeword length considered in this work.

In the following, a sequential LDPC decoding algorithm is modeled as a finite MDP, where the code's Tanner graph is viewed as an environment with $\lceil m/z \rceil$ possible actions (cluster selections), where z is the cluster size and m is the number of CNs in the Tanner graph. Then, we apply RELDEC, AM-RELDEC and M-RELDEC, which all employ Q-learning, to learn an action-value function that determines how beneficial a particular choice of cluster is for optimizing the cluster scheduling policy. The learned policy is then incorporated in our sequential LDPC decoding algorithm for inference. Note that the state space size of the MDP, and hence the complexity of the RELDEC and meta-RELDEC schemes, can grow exponentially with the number of CNs, which can range in the hundreds for practical LDPC codes. Creating clusters of CNs reduces the state space cardinality of the MDP and significantly reduces the learning complexity. Given a cluster in the Tanner graph, let the *output* of that cluster at a particular iteration be the binary sequence resulting from hard-decisions on the posterior log-likelihood ratios (LLRs) computed by the (ordered) neighboring VNs. The state of the MDP in both our RELDEC and meta-RELDEC schemes are then given by the collection of all possible (cluster, cluster state) pairs.

To the best of our knowledge, reinforcement learning-based decoding has not previously been successfully applied to soft iterative decoding of moderate length LDPC codes in the open literature, apart from our previous work in [10], [11]. Our work also differs from the vast majority of works related to learning in channel decoding (including [18], [26], [27], [28], [29]) in that our decoder is not implemented via deep learning. Further, both of our RELDEC and meta-RELDEC schemes outperform standard BP decoding schemes in complexity by reducing the number of CN to VN message updates required for convergence. The two novel contributions of this work with respect to our previous paper [10], [11] are outlined below:

- One of the salient features of the proposed RELDEC and meta-RELDEC schemes is a significant reduction in the cardinality of the state space over our previous RL-NS scheme [10], [11]. In turn, this leads to a significant reduction in training complexity, which enables the decoding of much longer LDPC codes with block lengths of several hundred bits compared to maximally 196 bits in [10], [11]. Another distinct feature of our current work is that all clusters and their corresponding CNs are scheduled in *each* iteration, until a stopping condition or a maximum number of iterations is reached. This increases the amount of exploration for the reinforcement learning agent, leading to a much lower bit error rate (BER) in the error floor region as compared to our previous work, where *any* single CN can be scheduled in each iteration.
- In addition to estimating the action-value function using standard Q-learning, we propose novel Q-learning-based meta-learning schemes, *i.e.*, AM-RELDEC and M-RELDEC, for learning the cluster scheduling policy. In contrast to RELDEC and M-RELDEC, where the scheduling policy is fixed during decoding, the agility of AM-RELDEC allows for dynamic scheduling which can adapt to varying signal-to-noise ratios (SNRs) with only a small amount of additional training. Thus, AM-RELDEC is well suited for online training; for example, received pilot symbols can be leveraged as additional training examples.

The rest of the paper is organized as follows. Necessary background is given in Section II. In Section III, we explore how the CN scheduling policy is learned via RELDEC and then incorporated in our learning-based sequential decoding algorithm. Section IV discusses how this

policy is learned using AM-RELDEC and M-RELDEC. In Section V we discuss the experimental setup, and analyze numerical results by comparing the proposed learning-based decoding schemes to conventional LDPC decoders found in the literature. Section VI concludes the paper.

II. PRELIMINARIES

A. Low-density Parity-check Codes

An $[n, k]$ binary linear code is a k -dimensional subspace of \mathbb{F}_2^n , and may be defined as the kernel of a binary *parity-check matrix* $\mathbf{H} \in \mathbb{F}_2^{m \times n}$, where $m \geq n - k$. The code's *block length* is n , and *rate* is $(n - \text{rank}(\mathbf{H}))/n$. The *Tanner graph* of a linear code with parity-check matrix \mathbf{H} is the bipartite graph $G_{\mathbf{H}} = (V \cup C, E)$, where $V = \{v_0, \dots, v_{n-1}\}$ is a set of variable nodes (VNs) corresponding to the columns of \mathbf{H} , $C = \{c_0, \dots, c_{m-1}\}$ is a set of check nodes (CNs) corresponding to the rows of \mathbf{H} , and edges in E correspond to 1's in \mathbf{H} [36]. LDPC codes are a class of highly competitive linear codes defined via sparse parity-check matrices or, equivalently, sparse Tanner graphs [2]; they are amenable to low-complexity graph-based message-passing decoding algorithms, making them ideal for practical applications. BP iterative decoding, considered here, is one such algorithm.

In general, a (γ, k) -regular LDPC quasi-cyclic (QC) code is defined by a parity-check matrix with constant column and row weights equal to γ and k , respectively [37]. A (γ, p) array-based (AB) LDPC code is a type of QC code where p is prime, and its parity-check matrix possess a special structure [38]. In particular, for AB codes,

$$\mathbf{H}(\gamma, p) = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{I} & \dots & \mathbf{I} \\ \mathbf{I} & \sigma & \sigma^2 & \dots & \sigma^{p-1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \mathbf{I} & \sigma^{\gamma-1} & \sigma^{2(\gamma-1)} & \dots & \sigma^{(\gamma-1)(p-1)} \end{bmatrix}, \quad (1)$$

where σ^z denotes the circulant matrix obtained by cyclically left-shifting the entries of the $p \times p$ identity matrix \mathbf{I} by $z \pmod{p}$ positions. Notice that $\sigma^0 = \mathbf{I}$. In this work, *lifted* LDPC codes are obtained by replacing the non-zero (resp., zero) entries of the parity-check matrix with randomly

generated permutation (resp., all-zero) matrices.

Parity-check matrices of LDPC codes designed for 5G new radio (5G-NR) possess a QC structure which enables the nodes in its Tanner graph to be updated in parallel [39]. According to the third generation partnership project (3GPP) standard, 5G-NR LDPC codes can be obtained by lifting two types of QC base graphs, known as BG1 and BG2. Depending on the lifting factor, BG1 can be used for constructing LDPC codes with information bits ranging between 500 and 8448 bits, whereas the BG2 base matrix can be used for obtaining shorter codes with information bits ranging between 40 and 2560 bits [39], [40]. Since the amount of resources available for transmission can change dynamically in a cellular system, 5G-NR LDPC codes are required to have rate matching functionality to select an arbitrary amount of information bits for transmission. The rate of a 5G-NR LDPC code can be increased by puncturing CNs in the base graph. The BG1 (resp., BG2) matrix is used for rates between $1/3$ and $8/9$ (resp., $1/5$ and $2/3$) [39], [40].

B. Reinforcement Learning

In an reinforcement learning problem, an agent (learner) interacts with an environment whose *state space* can be modeled as a finite MDP [12]. The agent takes *actions* that alter the state of the environment and receives a *reward* in return for each action, with the goal of maximizing the total reward in a series of actions. The optimized sequence of actions is obtained by employing a policy which utilizes an *action-value function* to determine how beneficial an action is for maximizing the long-term expected reward. In the remainder of the paper, let $[[x]] \triangleq \{0, \dots, x-1\}$, where x is a positive integer. Suppose that an environment allows m possible actions, and let the random variable $A_\ell \in [[m]]$ with realization a represent the index of an action taken by the agent during learning step $\ell \in \{0, \dots, \ell_{\max} - 1\}$. Let S_ℓ with realization $s \in \mathbb{Z}$ represent the current state of the environment before taking action A_ℓ and let $S_{\ell+1}$ with realization s' represent a new state of the MDP after executing A_ℓ . Let a state space \mathcal{S} contain all possible state realizations. Also, let $R_\ell = R(S_\ell, A_\ell, S_{\ell+1})$ be the reward yielded at step ℓ after taking action A_ℓ in state S_ℓ which will yield state $S_{\ell+1}$.

Optimal policies for MDPs can be estimated via Monte Carlo techniques such as Q-learning. The estimated action-value function $Q_\ell(S_\ell, A_\ell)$ in Q-learning (also known as Q-function) represents the expected long-term reward achieved by the agent at step ℓ after taking action A_ℓ in state S_ℓ . To improve the estimation in each step, the action-value function is adjusted according to a recursion

$$Q_{\ell+1}(s, a) = (1 - \alpha)Q_\ell(s, a) + \alpha \left(R(s, a, s') + \beta \max_{a' \in [m]} Q_\ell(s', a') \right), \quad (2)$$

where s' represents the new state of the MDP after taking action a in state s , $0 < \alpha < 1$ is the *learning rate*, β is the *reward discount rate*, $Q_{\ell+1}(s, a)$ is a future action-value resulting from action a in the current state s [14], and ℓ is the total number of learning steps elapsed after observing the initial state S_0 in a learning episode¹. Note that an action in learning step ℓ is selected via an ϵ -greedy approach according to

$$a = \begin{cases} \text{selected uniformly at random w.p. } \epsilon \text{ from } \mathcal{A}, \\ \pi(s) \text{ selected w.p. } 1 - \epsilon, \end{cases} \quad (3)$$

where ϵ is the probability of exploration, \mathcal{A} is a set of all possible actions, and $\pi(s)$ is an agent's policy for taking an action in state s , given by

$$\pi(s) = \arg \max_a Q_\ell(s, a). \quad (4)$$

The goal of Q-learning is to find the optimal policy that maximizes the long-term expected reward in state s , given by

$$\pi^*(s) = \arg \max_a Q^*(s, a), \quad (5)$$

where $Q^*(s, a)$ is the true (and unknown) action-value for a given (s, a) pair.

¹A learning episode comprises all the Q-learning steps needed to learn the action-values corresponding to a single training example.

III. LEARNING THE SCHEDULING POLICY USING RELDEC

The proposed RELDEC scheme consists of a BP decoding algorithm in which the environment is given by the Tanner graph of the LDPC code, and the optimized sequence of actions, *i.e.*, the scheduling of individual clusters, is obtained using Q-learning. A single cluster scheduling step is carried out by sending messages from all CNs of a cluster to their neighboring VNs, and subsequently sending messages from these VNs to their CN neighbors. That is, a selected cluster executes one iteration of localized flooding in each decoding instant. Every cluster is scheduled exactly once within a single decoder iteration. Sequential cluster scheduling is carried out until a stopping condition is reached, or an iteration threshold is exceeded. The learning-based decoder relies on a cluster scheduling policy based on a learned action-value function.

The RELDEC learning framework for sequential decoding is shown in Fig. 1. The idea is that scheduling a cluster, represented by the blue squares (CNs), updates the state of the environment, namely the hard-decisioned beliefs associated to the blue VNs connected to the cluster. In return, the agent receives a reward which is commensurate with the proportion of correct hard decisions.

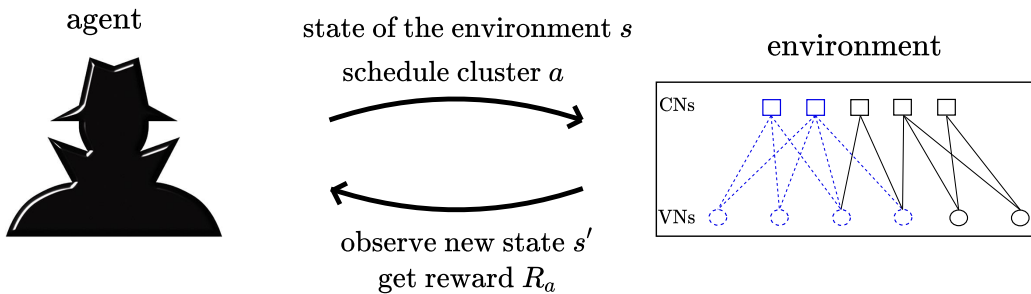


Fig. 1: Illustration of RELDEC’s learning framework. In each learning step, a fictitious agent schedules a cluster with index a when the environment state, based on hard-decisioned VN values, is s . Once an action is taken, the state of the environment changes from s to s' as the VN values are updated after scheduling, and the agent receives reward R_a that indicates the accuracy of the hard-decisions taken by the BP algorithm for each blue VN.

Let $\hat{\mathbf{x}}_a^{(\ell)} = [\hat{x}_{0,a}^{(\ell)}, \dots, \hat{x}_{l_a-1,a}^{(\ell)}] \in \{0, 1\}^{l_a}$ denote the state of the MDP after scheduling a cluster with index a during learning step ℓ , and let $s_a^{(\ell)} \in [[2^{l_a}]]$ refer to the index of a realization of $\hat{\mathbf{x}}_a^{(\ell)}$. Thus, $s_a^{(\ell)}$ also refers to the state of the MDP during learning step ℓ . Since the state space of the clusters are pairwise disjoint, set \mathcal{S} of our MDP contains $\sum_{a \in [[\lceil m/z \rceil]]} 2^{l_a}$ realizations of all

the cluster outputs $\hat{\mathbf{x}}_0^{(\ell)}, \dots, \hat{\mathbf{x}}_{\lceil m/z \rceil - 1}^{(\ell)}$, where a realization can be thought of as a (cluster, cluster state) pair. The action space is defined as $\mathcal{A} = \{ \lceil m/z \rceil \}$. Note that in the absence of clustering (*i.e.*, $z = m$), the state space cardinality would be 2^n , which for moderate length codes studied in this paper is prohibitively large. An example of a cluster-induced subgraph for the case $z = 2$, and the corresponding state vector $\hat{\mathbf{x}}_a^{(\ell)}$ is shown in Fig. 2.

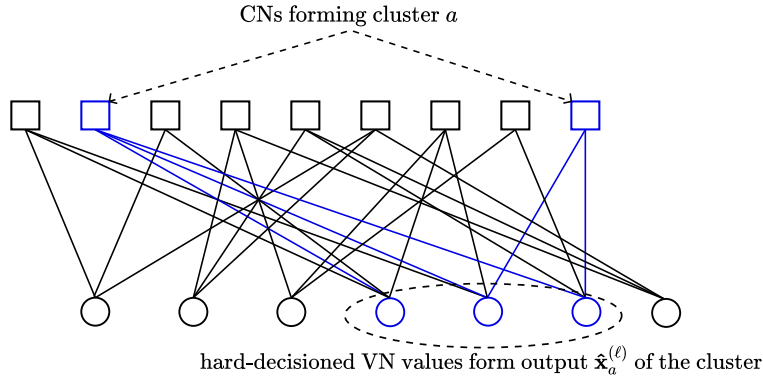


Fig. 2: Example of a cluster-induced subgraph, shown with blue squares (cluster consisting of 2 CNs), edges, and circles (VNs). The corresponding state of the cluster is $\hat{\mathbf{x}}_a^{(\ell)}$.

Let $\mathbf{x} = [x_0, \dots, x_{n-1}]$ and $\mathbf{y} = [y_0, \dots, y_{n-1}]$ represent the transmitted and the received words, respectively, where for each $v \in \llbracket n \rrbracket$, $x_v \in \{0, 1\}$ and $y_v = (-1)^{x_v} + z$ with $z \sim \mathcal{N}(0, \sigma^2)$. The posterior log-likelihood ratio (LLR) of x_v is expressed as $L_v = \log \frac{\Pr(x_v=1|y_v)}{\Pr(x_v=0|y_v)}$. Let $\hat{L}_\ell^{(v)} = \sum_{c \in \mathcal{N}(v)} m_{c \rightarrow v}^{(\ell)} + L_v$ be the posterior LLR computed by VN v during iteration ℓ , where $\mathcal{N}(v)$ denotes the set of neighboring CNs of VN v , $\hat{L}_0^{(v)} = L_v$, and $m_{c \rightarrow v}^{(\ell)}$ is the message received by VN v from neighboring CN c in iteration ℓ computed based on standard BP as

$$m_{c \rightarrow v}^{(\ell)} = 2 \operatorname{atanh} \prod_{v' \in \mathcal{N}(c) \setminus v} \tanh \left(\frac{m_{v' \rightarrow c}^{(\ell-1)}}{2} \right).$$

Here, $\mathcal{N}(c)$ denotes the set of neighboring VNs of c , and $m_{v \rightarrow c}^{(\ell)} = \sum_{c' \in \mathcal{N}(v) \setminus c} m_{c' \rightarrow v}^{(\ell)} + L_v$ is the message propagated from VN v to CN c . Moreover, let $\hat{L}_\ell^{(j,a)}$ be the posterior LLR computed during learning step ℓ by VN j in the subgraph induced by the cluster with index $a \in \{ \lceil m/z \rceil \}$. Hence, $\hat{L}_\ell^{(j,a)} = \hat{L}_\ell^{(v)}$ if VN v in the Tanner graph is also the j th VN in the subgraph induced by the cluster with index a .

Recall that in our prior work [11], the RL-NS algorithm scheduled a single CN a per decoding iteration based on its reward $\max_{v \in \mathcal{N}(a)} r_{a \rightarrow v}$, where $r_{a \rightarrow v}$ is the residual of CN a associated with the edge connecting to VN v , computed according to $r_{a \rightarrow v} \triangleq |m'_{a \rightarrow v} - m_{a \rightarrow v}|$. Here, $m_{a \rightarrow v}$ is the message sent by CN a to its neighboring VN v in the previous iteration, and $m'_{a \rightarrow v}$ is the message that CN a would send to VN v in the current iteration, if scheduled. Intuitively, the higher the residual of a CN, the further away that portion of the graph is from convergence. Thus, scheduling a CN with the highest residual (reward) leads to faster and more reliable decoding compared to the flooding scheme. Furthermore, in [11], the state-space of the MDP is given by the collection of all sequences representing quantized CN values within a cluster.

In contrast, in the proposed RELDEC scheme we consider a new state-space representation of the MDP which allows learned decoding of significantly longer block-length LDPC codes. In RELDEC, after scheduling cluster a during learning step ℓ , the state of the MDP associated with cluster a is given by its output $\hat{\mathbf{x}}_a^{(\ell)}$ that is obtained by taking hard decisions on the vector of posterior LLRs $\hat{\mathbf{L}}_{\ell,a} = [\hat{L}_\ell^{(0,a)} \dots \hat{L}_\ell^{(l_a-1,a)}]$, computed according to

$$\hat{x}_{j,a}^{(\ell)} = \begin{cases} 0, & \text{if } \hat{L}_\ell^{(j,a)} \geq 0, \\ 1, & \text{otherwise,} \end{cases} \quad (6)$$

where k_{\max} is the maximum CN degree of the cluster, and $l_a \leq k_{\max}z$ is the number of VNs adjacent to cluster a . We call $\hat{\mathbf{x}}_a^{(\ell)}$ the state of cluster a : it is comprised of the bits reconstructed by the sequential decoder after scheduling cluster a during iteration ℓ , *i.e.*, the state of the cluster is a sequence of hard-decision VN values associated with the cluster. The collection of signals $\hat{\mathbf{x}}_0^{(\ell)}, \dots, \hat{\mathbf{x}}_{\lceil m/z \rceil - 1}^{(\ell)}$ at the end of decoder iteration ℓ forms the entire state of the MDP associated with our RELDEC scheme.

During the learning phase, RELDEC informs the agent of the current state of the decoder and the reward obtained after performing an action (propagating messages from a cluster to its neighboring VNs). Based on these observations, the agent takes future actions, to enhance the total reward earned, which alters the state of the environment as well as the future reward. Given that the transmitted signal \mathbf{x} is known during the training phase, let $\mathbf{x}_a = [x_{0,a}, \dots, x_{l_a-1,a}]$ be

a vector containing the l_a bits of \mathbf{x} that are reconstructed in $\hat{\mathbf{x}}_a^{(\ell)}$ by cluster a . In each learning step ℓ , the reward R_a obtained by the agent after scheduling cluster a is defined as

$$R_a = \frac{1}{l_a} \sum_{j=0}^{l_a-1} \mathbb{1}(x_{j,a} = \hat{x}_{j,a}), \quad (7)$$

where $\mathbb{1}(\cdot)$ denotes the indicator function. Thus, the reward earned by the agent after scheduling cluster a is identical to the probability that the transmitted bits $x_{0,a}, \dots, x_{l_a-1,a}$ are correctly reconstructed. This new reward metric, resulting from the modification of the state-space representation, differs considerably from the maximum residual of the scheduled CN used as reward for RL-NS in [11].

The action-values learned by RELDEC are stored in a table with dimension $\sum_{a \in \lceil \lceil m/z \rceil \rceil} 2^{l_a} \times \lceil m/z \rceil$. In comparison, for the RL-NS scheme of [11], the learned action-values are stored in a table with dimension $\lceil m/z \rceil M^z \times \lceil m/z \rceil$, where $M = 4$ is the number of quantization levels used for quantizing the CN values, and M^z is the number of all possible sequences of quantized CN values associated with a cluster. Thus, for the MDP discussed in our previous work the state space cardinality, and hence the learning complexity, grows exponentially with z . Furthermore, a moderately large z (≥ 7) was chosen to ensure a small number of clusters, since there exist dependencies between clusters (*i.e.*, the state of one cluster may depend on the state of another cluster due to the presence of cycles) which Q-learning cannot take into account. However, due to the modification of the MDP in this work, even a choice of $z = 1$ provides considerable improvement in RELDEC's decoding performance in comparison to the flooding scheme. The size of the table is also significantly reduced, which in turn reduces RELDEC's learning complexity with respect to the RL-NS scheme of [11].

In the following, we discuss the learning approach used by RELDEC for obtaining the optimal CN scheduling policy for a given LDPC code. As the new state-space representation generates MDPs with moderately large state space size, we utilize standard Q-learning for determining the optimal cluster scheduling order, where the action-value, $Q_{\ell+1}(s_a^{(\ell)}, a)$, for choosing cluster a in state s_a is given by

$$Q_{\ell+1}(s_a^{(\ell)}, a) = (1 - \alpha)Q_\ell(s_a^{(\ell)}, a) + \alpha \left(R_a + \beta \max_{a' \in \llbracket \lceil m/z \rceil \rrbracket} Q_\ell(s_a^{(\ell)'}, a') \right). \quad (8)$$

The action-value function is recursively updated ℓ_{\max} times according to (8), where ℓ_{\max} is the maximum number of learning steps for a given channel output \mathbf{L} . In each learning step ℓ , cluster a is selected via an ϵ -greedy approach according to

$$a = \begin{cases} \text{selected uniformly at random w.p. } \epsilon \text{ from } \llbracket \lceil m/z \rceil \rrbracket, \\ \pi(s_a^{(\ell)}) \text{ selected w.p. } 1 - \epsilon, \end{cases} \quad (9)$$

where

$$\pi(s_a^{(\ell)}) = \arg \max_{a \in \llbracket \lceil m/z \rceil \rrbracket} Q_\ell(s_a^{(\ell)}, a). \quad (10)$$

For ties (as in the first iteration of Algorithm 1 for $\ell = 0$ and the initial \mathbf{L}), we choose an action uniformly at random from all the maximizing actions. During inference, the optimized cluster scheduling policy of standard Q-learning, $\hat{\pi}(s_{a_i}^{(I)})$, for scheduling the i th cluster during decoder iteration I is expressed as

$$\hat{\pi}(s_{a_i}^{(I)}) = \arg \max_{a_i \in \llbracket \lceil m/z \rceil \rrbracket \setminus \{a_0, \dots, a_{i-1}\}} \hat{Q}(s_{a_i}^{(I)}, a_i), \quad (11)$$

where $i \in \llbracket \lceil m/z \rceil \rrbracket$, and a_i indicates the cluster index to be scheduled at time instant i . Further, $\hat{Q}(s_{a_i}^{(I)}, a_i)$ represents the optimized action-value after training has been accomplished, which, as $\ell \rightarrow \infty$, approaches the optimal action-value $Q^*(s_{a_i}^{(I)}, a_i)$ [41], [12, Sec. 6.4]. The RELDEC scheme, which employs standard Q-learning, is shown in Algorithm 1. The input to this algorithm is a parity-check matrix \mathbf{H} and a set $\mathcal{L} = \{\mathbf{L}_0, \dots, \mathbf{L}_{|\mathcal{L}|-1}\}$ containing $|\mathcal{L}|$ realizations of \mathbf{L} over which Q-learning is performed, and the output is the optimized cluster scheduling policy $\hat{\pi}(s_{a_i}^{(I)})$. For each $\mathbf{L} \in \mathcal{L}$, the action-value function in (8) is recursively updated ℓ_{\max} times.

Once the optimized scheduling policy, $\hat{\pi}(s_{a_i}^{(I)})$, is learned using Algorithm 1, it is incorporated in our reinforcement learning-based sequential BP decoding scheme shown in Algorithm 2 to determine the optimized cluster scheduling order. The algorithm inputs are the soft channel

Algorithm 1: RELDEC

Input : set of channel information vectors \mathcal{L} , parity-check matrix \mathbf{H}
Output: optimized cluster scheduling policy $\hat{\pi}(s_a^{(I)})$

- 1 Initialization: $Q_0(s_a^{(0)}, a) \leftarrow 0$ for all $s_a^{(0)}$ and a
- 2 **for each** $\mathbf{L} \in \mathcal{L}$ **do**
- 3 $\ell \leftarrow 0$
- 4 $\hat{\mathbf{L}}_\ell \leftarrow \mathbf{L}$
- 5 determine initial states of all clusters using (6)
- 6 *// start of an episode*
- 7 **while** $\ell < \ell_{\max}$ **do**
- 8 select cluster a according to (9)
- 9 *// decode cluster via flooding*
- 10 **foreach** CN c in cluster a **do**
- 11 **foreach** VN $v \in \mathcal{N}(c)$ **do**
- 12 compute and propagate $m_{c \rightarrow v}^{(\ell)}$
- 13 **end**
- 14 **foreach** VN v in the subgraph of cluster a **do**
- 15 **foreach** CN $c \in \mathcal{N}(v)$ **do**
- 16 compute and propagate $m_{v \rightarrow c}^{(\ell)}$
- 17 **end**
- 18 $\hat{L}_\ell^{(v)} \leftarrow \sum_{c \in \mathcal{N}(v)} m_{c \rightarrow v}^{(\ell)} + L_v$ *// update posterior LLR*
- 19 **end**
- 20 *// determine cluster output*
- 21 **foreach** VN v in the subgraph of cluster a **do**
- 22 **if** $\hat{L}_\ell^{(v)} \geq 0$ **then**
- 23 $\hat{x}_{v,a}^{(\ell)} \leftarrow 0$
- 24 **end**
- 25 **else**
- 26 $\hat{x}_{v,a}^{(\ell)} \leftarrow 1$
- 27 **end**
- 28 **end**
- 29 determine index $s_a^{(\ell)'}$ of $\hat{\mathbf{x}}_a$
- 30 update R_a according to (7)
- 31 compute $Q_{\ell+1}(s_a^{(\ell)'}, a)$ according to (8)
- 32 $s_a^{(\ell+1)} \leftarrow s_a^{(\ell)'}$
- 33 $\ell \leftarrow \ell + 1$
- 34 **end**

information vector $\mathbf{L} = [L_0, \dots, L_{n-1}]$ comprised of LLRs and a parity-check matrix \mathbf{H} of the LDPC code, and $\hat{\mathbf{L}}_I = [\hat{L}_I^{(0)}, \dots, \hat{L}_I^{(n-1)}]$ is initialized using \mathbf{L} . The output is the reconstructed signal $\hat{\mathbf{x}}$ obtained after executing at most I_{\max} decoding iterations, or until the stopping condition shown in Step 32 is reached. Once decoding ends, we obtain the fully reconstructed signal estimate $\hat{\mathbf{x}} = [\hat{x}_0, \dots, \hat{x}_{n-1}]$. Note that the optimized scheduling policy in Step 9, obtained according to (11) in case of RELDEC, depends both on the graph structure and on the received channel values. In the next section, we discuss meta learning variants of RELDEC for obtaining this optimized scheduling policy.

Note that the learning-based sequential decoding scheme can be viewed as a sequential generalized LDPC (GLDPC) decoder when $z > 1$, where BP decoding of a cluster-induced subgraph is analogous to decoding a CN subcode of a GLDPC code. When $z = 1$, each cluster represents a single parity-check code, as is the case in a standard LDPC code. Since the full LDPC Tanner graph is connected and contains cycles, there exist dependencies between the messages propagated by the different clusters of the LDPC code. Consequently, the output of a cluster may depend on messages propagated by previously scheduled clusters. Thus, to improve reinforcement learning performance for $z > 1$, we ensure that the clusters are chosen to be as independent as possible. The choice of clustering is determined prior to learning using the cycle-maximization method discussed in [10], [11]: in short, clusters are selected to maximize the number of cycles in the cluster-induced subgraph to minimize inter-cluster dependencies.

IV. LEARNING THE SCHEDULING POLICY VIA META-RELDEC SCHEMES

In the section, we discuss the meta-variants of the RELDEC scheme in details. In comparison to MAML which utilizes gradient descent to optimize the model parameters such as the weights of a NN [31], our novel meta-reinforcement learning scheme presented below directly estimates the Q-function by minimizing a certain loss function in every learning step. In the case of sequential decoding, a meta-reinforcement learning task refers to learning the optimal CN scheduling order for a given LLR vector \mathbf{L} . We now define the following terms which we employ in the remainder of the paper.

Algorithm 2: Learning-based Sequential BP Decoding Scheme

Input : channel information \mathbf{L} , parity-check matrix \mathbf{H}
Output: reconstructed signal $\hat{\mathbf{x}}$

```

1 Initialization:
2    $I \leftarrow 0$ 
3    $m_{c \rightarrow v} \leftarrow 0$  // for all CN to VN messages
4    $m_{v \rightarrow c} \leftarrow L_v$  // for all VN to CN messages
5 if decoder iteration  $I < I_{\max}$  then
6   foreach cluster with index  $a_i$  do
7     Determine state  $s_{a_i}^{(I)}$ 
8   end
9   Incorporate an optimized cluster scheduling policy
10  foreach cluster with index  $a_i$  do
11    // decode cluster via flooding
12    foreach CN  $c$  in cluster  $a_i$  do
13      foreach VN  $v \in \mathcal{N}(c)$  do
14        compute and propagate  $m_{c \rightarrow v}^{(I)}$ 
15      end
16    foreach VN  $v$  in the subgraph of cluster  $a_i$  do
17      foreach CN  $c \in \mathcal{N}(v)$  do
18        compute and propagate  $m_{v \rightarrow c}^{(I)}$ 
19      end
20       $\hat{L}_I^{(v)} \leftarrow \sum_{c \in \mathcal{N}(v)} m_{c \rightarrow v}^{(I)} + L_v$  // update posterior LLR
21    end
22    // hard-decision step
23    foreach VN  $v$  in the subgraph of cluster  $a_i$  do
24      if  $\hat{L}_I^{(v)} \geq 0$  then
25         $\hat{x}_{v, a_i}^{(I)} \leftarrow 0$ 
26      end
27      else
28         $\hat{x}_{v, a_i}^{(I)} \leftarrow 1$ 
29      end
30    end
31     $i \leftarrow i + 1$ 
32  end
33  if  $\mathbf{H}\hat{\mathbf{x}} = \mathbf{0}$  then
34    break // stopping condition reached
35  end
36   $I \leftarrow I + 1$ 
37 end

```

- *Optimal global CN scheduling policy* $\pi^*(s_{a_i}^{(I)})$, defined as

$$\pi^*(s_{a_i}^{(I)}) \triangleq \arg \max_{a_i \in \{[m/z]\} \setminus \{a_0, \dots, a_{i-1}\}} Q^*(s_{a_i}^{(I)}, a_i), \quad (12)$$

where $Q^*(s_{a_i}^{(I)}, a_i)$ is the optimal global action-value function, and I is the decoder iteration during inference.

- *Optimal k -th local CN scheduling policy* $\pi_k^*(s_{a_i}^{(I)})$, defined as

$$\pi_k^*(s_{a_i}^{(I)}) = \arg \max_{a_i \in \{[m/z]\} \setminus \{a_0, \dots, a_{i-1}\}} Q^{(k)*}(s_{a_i}^{(I)}, a_i), \quad (13)$$

where $Q^{(k)*}(s_{a_i}^{(I)}, a_i)$ is the optimal k -th local action-value function, $k \in \{1, \dots, K\}$.

- *MDP instance*, defined as a tuple $(s_a^{(\ell)}, a, R_a, s_a^{(\ell)'})$ representing a single state transition of the MDP during learning step $\ell \in \{0, \dots, \ell_{\max} - 1\}$.
- *Mini-batch of MDP instances*: We define $\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L}) \triangleq \{(s_{a_0}^{(0)}, a_0, R_{a_0}, s_{a_0}^{(0)'})', \dots, (s_{a_{\ell_{\max}-1}}^{(\ell_{\max}-1)}, a_{\ell_{\max}-1}, R_{a_{\ell_{\max}-1}}, s_{a_{\ell_{\max}-1}}^{(\ell_{\max}-1)'})'\}$ as a mini-batch of ℓ_{\max} MDP instances obtained after taking ℓ_{\max} actions $a_0, \dots, a_{\ell_{\max}-1}$ according to policy $\pi(s_a^{(\ell)})$, $a \in \{[m/z]\}$, where $s_{a_0}^{(0)}$ is the initial state of the MDP after receiving \mathbf{L} during a learning episode.
- *Batch of MDP instances*: W.l.o.g., we define a batch $\mathcal{B}(\pi(s_a^{(\ell)}), \mathcal{L}) \triangleq \{\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L}_0), \dots, \mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L}_{|\mathcal{L}|-1})\}$ as a collection of $|\mathcal{L}|$ mini-batches, where \mathcal{L} is a set of LLR vectors.

Note that the Q-learning update procedure shown in (8) can also be written as

$$Q_{\ell+1}(s_a^{(\ell)}, a) = Q_{\ell}(s_a^{(\ell)}, a) + \alpha(U_{\ell}(s_a^{(\ell)}, a) - Q_{\ell}(s_a^{(\ell)}, a)), \quad (14)$$

where $U_{\ell}(s_a^{(\ell)}, a) = R_a + \beta \max_{a'} Q_{\ell}(s_a^{(\ell)'}, a')$, and $U_{\ell}(s_a^{(\ell)}, a) - Q_{\ell}(s_a^{(\ell)}, a)$ is the temporal difference (TD) error [12]. As learning iteration ℓ grows, $Q_{\ell+1}(s_a^{(\ell)}, a)$ approaches $Q^*(s_a^{(\ell)}, a)$, i.e., the meta-reinforcement learning scheme converges to the true Q-function $Q^*(s_a^{(\ell)}, a)$ [41], [12, Sec. 6.4]. Consequently, the TD error approaches 0 as $\ell \rightarrow \infty$. Suppose that meta-reinforcement learning is carried out on mini-batch $\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L})$ during a learning episode. At any given learning step ℓ , the meta-reinforcement learning algorithm computes a sum of squared TD errors

over this mini-batch, expressed as

$$\mathcal{L}(\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L})) = \sum_{(s_a^{(\ell)}, a, R_a, s_a^{(\ell)'}) \in \mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L})} (U_\ell(s_a^{(\ell)}, a) - Q_\ell(s_a^{(\ell)}, a))^2, \quad (15)$$

which is minimized as the agent takes an action via an ϵ -greedy policy in each learning step. Computing $\mathcal{L}(\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L}))$ allows us to terminate the meta-learning algorithm once the loss is below a certain threshold, making the adaptation phase fast and efficient. This is one of the salient features of the proposed meta-learning based decoding scheme which sets it apart from the baseline RELDEC scheme presented in Section III.

The goal of our meta-learning scheme is to learn the optimal action-values by observing all the MDP instances in $\mathcal{B}(\pi(s_a^{(\ell)}), \mathcal{L})$, where the optimal global action-value is expressed as

$$\begin{aligned} Q^*(s_a^{(\ell)}, a) &= \arg \min_{Q_\ell(s_a^{(\ell)}, a) \in \mathbb{R}} \mathbb{E}_{\mathbf{L} \in \mathcal{L}} [\mathcal{L}(\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L}))] \\ &= \arg \min_{Q_\ell(s_a^{(\ell)}, a) \in \mathbb{R}} \mathbb{E}_{\mathbf{L} \in \mathcal{L}} \left[\sum_{(s_a^{(\ell)}, a, R_a, s_a^{(\ell)'}) \in \mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L})} (U_\ell(s_a^{(\ell)}, a) - Q_\ell(s_a^{(\ell)}, a))^2 \right], \end{aligned} \quad (16)$$

where \mathbf{l} is a realization of \mathbf{L} , and the optimal k -th local action-value is expressed as

$$Q^{(k)*}(s_a^{(\ell)}, a) = \arg \min_{Q_\ell^{(k)}(s_a^{(\ell)}, a) \in \mathbb{R}} \mathbb{E}_{\mathbf{L}' \in \mathcal{L}_k} [\mathcal{L}(\mathcal{D}_k(\pi_k(s_a^{(\ell)}), \mathbf{L}'))], \quad (17)$$

where $\mathbf{L}' \in \mathcal{L}_k = \{\mathbf{L}_0^{(k)'}, \dots, \mathbf{L}_{|\mathcal{L}_k|-1}^{(k)'}\}$ is an LLR vector with realization \mathbf{L}' , $\mathcal{D}_k(\pi_k(s_a^{(\ell)}), \mathbf{L}')$ is a mini-batch of MDP instances for learning the k -th local policy

$$\pi_k(s_a^{(\ell)}) = \arg \max_a Q^{(k)}(s_a^{(\ell)}, a), \quad (18)$$

and $Q^{(k)}(s_a^{(\ell)}, a)$ represents the action-value function corresponding to the k -th policy. In the remainder of the paper, we use simplified notations \mathcal{B} and \mathcal{B}_k for the MDP batches $\mathcal{B}(\pi(s_a^{(\ell)}), \mathcal{L})$ and $\mathcal{B}_k(\pi_k(s_a^{(\ell)}), \mathcal{L}_k)$, respectively, as well as the simplified notations $\mathcal{D}_{\mathbf{L}}$ and $\mathcal{D}_{\mathbf{L}'}$ for $\mathcal{D}(\pi(s_a^{(\ell)}), \mathbf{L})$ and $\mathcal{D}_k(\pi_k(s_a^{(\ell)}), \mathbf{L}')$. Since the search over all possible action-values for a given $(s_a^{(\ell)}, a)$ pair in (16) and (17) is prohibitive, one workaround is to use standard Q-learning, which yields

optimized global and k -th local CN scheduling policies by iteratively minimizing empirical losses $\mathcal{L} = \frac{1}{|\mathcal{B}|}\mathcal{L}(\mathcal{B})$ and $\mathcal{L}_k = \frac{1}{|\mathcal{B}_k|}\mathcal{L}(\mathcal{B}_k)$, respectively, in each learning step.

In the following, a global policy update refers to learning the CN scheduling policy for a mixture of K possible SNRs using a training batch \mathcal{B} that contains $|\mathcal{B}|/K$ instances of the MDP corresponding to LLR vectors for a fixed SNR. On the other hand, a local policy update refers to learning the CN scheduling policy using batch \mathcal{B}_k of $|\mathcal{B}_k|$ MDP instances corresponding to LLR vectors for the k -th fixed SNR. In particular, the SNRs across the K local batches are distinct. Although the distribution of R_a may vary across tasks, we say that the tasks are related since the environment (the sequential BP decoder) does not change as learning progresses. We propose two distinct meta-reinforcement learning frameworks, each involving a bi-level policy optimization. Specifically, the first approach is a novel extension of the MAML scheme [31] to Q-learning, and the second approach follows the MQL scheme in [32].

- AM-RELDEC: First, the algorithm initializes a local policy $\pi_k(s_a^{(\ell)})$, $k \in \{1, \dots, K\}$, using a global policy $\pi(s_a^{(\ell)})$, and then optimizes the local policy by minimizing a sum of squared TD errors for the k -th batch \mathcal{B}_k of MDPs by taking actions according to an ϵ -greedy policy in each learning step. Second, the agent initializes a global policy $\pi(s_a^{(\ell)})$ using the learned local policies and then optimizes it by minimizing the squared TD error for a global batch \mathcal{B} . This procedure is repeated in every meta-learning iteration. Once learning ends, we obtain an optimized version of the global policy shown in (12), denoted as $\hat{\pi}(s_{a_i}^{(I)})$, and an optimized version of the k -th local policy shown in (13), denoted as $\hat{\pi}_k(s_{a_i}^{(I)})$.
- M-RELDEC: The agent first learns a global policy $\pi(s_a^{(\ell)})$ by minimizing the squared TD error for the global batch \mathcal{B} . Then, during the k -th adaptation phase, the agent initializes the k -th local policy $\pi_k(s_a^{(\ell)})$ using the learned global policy, and optimizes it by minimizing the squared TD error over the corresponding local batch \mathcal{B}_k by taking actions via an ϵ -greedy policy in each learning step. Once learning ends, we obtain $\hat{\pi}(s_{a_i}^{(I)})$ and $\hat{\pi}_k(s_{a_i}^{(I)})$.

The proposed AM-RELDEC scheme, which to the best of our knowledge has not been published in the open literature, is favorable for a wireless communications setting due to its agility: the global policy for CN scheduling can be quickly adapted online to any local policy corresponding

to a particular SNR (during the decoding phase). On the other hand, in case of M-RELDEC the meta-knowledge gained during the learning phase is not sufficient for online adaptation as the global policy is fixed and does not iteratively update with the local policies. The AM-RELDEC scheme is shown in Algorithm 3. It takes \mathcal{L} , \mathcal{L}_k and \mathbf{H} as inputs, and outputs an optimized global policy, $\hat{\pi}(s_{a_i}^{(I)})$, which is used as a starting point for optimizing the k -th local policy during online adaptation.

At the start of the meta-learning phase, the action-values corresponding to the k -th local policy are initialized using action-values corresponding to the global policy as shown in Step 5. The Q-learning error $\mathcal{L}(\mathcal{D}_{L'})$, where a mini-batch $\mathcal{D}_{L'} \subset \mathcal{B}_k$ contains MDP instances corresponding to a training sample L' , is updated after every $x \ll \ell_{\max}$ training steps in each learning episode which may result in an overall loss \mathcal{L}_k that is smaller than a threshold \mathcal{L}_{\min} . In this case we can move on to learning the next training example before all ℓ_{\max} learning steps are executed for the current training example. Furthermore, this modification helps to speed-up the adaptation phase for online learning as the same adaptation steps are used there (see Algorithm 5). As the agent repeatedly interacts with the environment by taking actions according to

$$a = \begin{cases} \text{selected uniformly at random w.p. } \epsilon \text{ from } \llbracket \lceil m/z \rceil \rrbracket, \\ \pi_k(s_a^{(\ell)}) \text{ selected w.p. } 1 - \epsilon, \end{cases} \quad (19)$$

and the action-value $Q^{(k)}(s_a^{(\ell)}, a)$ is iteratively updated in Step 28, $\pi_k(s_a^{(\ell)})$ is gradually optimized.

The global policy over a mixture of K SNRs is optimized in Steps 2-12 of Algorithm 4 (called from Step 31 of Algorithm 3). During the first step of the global policy optimization phase, the action-value function corresponding to global policy $\pi(s_a^{(\ell)})$ is obtained by computing the average action-values over all K local action-value functions. In each learning episode, global learning performs ℓ_{\max} training rounds as shown in Steps 4-9 of Algorithm 4. Thus, for the training example $L \in \mathcal{L}$, the cardinality of the corresponding mini-batch $\mathcal{D}_L \subset \mathcal{B}$ is ℓ_{\max} . As the agent repeatedly interacts with the environment, the global action-value $Q_{\ell+1}(s_a^{(\ell)}, a)$ is updated and $\pi(s_a^{(\ell)})$ is optimized by taking actions according to

Algorithm 3: AM-RELDEC

Input : set of LLR vectors \mathcal{L} , \mathcal{L}_k , parity-check matrix \mathbf{H}
Output: optimized global scheduling policy $\hat{\pi}(s_{a_i}^{(I)})$

- 1 $U_0(s_a^{(0)}, a) \leftarrow 0, Q_0(s_a^{(0)}, a) \leftarrow 0, \forall s_a^{(0)}, a, \mathcal{B} \leftarrow \emptyset$
- 2 **while** *not done* **do**
 - // meta-learning phase*
 - 3 $k \leftarrow 1$
 - 4 **while** $k \leq K$ **do**
 - // adapt to the k-th SNR*
 - 5 $\mathcal{B}_k \leftarrow \emptyset, Q^{(k)}(s_a^{(0)}, a) \leftarrow Q_0(s_a^{(0)}, a) \forall s_a^{(0)}, a$
 - // start of an episode*
 - 6 **for** *each new* $\mathbf{L}' \in \mathcal{L}_k$ **do**
 - 7 $\ell \leftarrow 0, \hat{\mathbf{L}}_\ell \leftarrow \mathbf{L}', \mathcal{D}_{\mathbf{L}'} \leftarrow \emptyset, \mathcal{L}_k \leftarrow 1$
 - 8 *determine initial states of all clusters using (6)*
 - 9 **while** $\mathcal{L}_k > \mathcal{L}_{\min}$ *and* $\ell < \ell_{\max}$ **do**
 - 10 *select cluster a according to (19)*
 - 11 *decode cluster induced sub-graph according to Steps 8-26 of Algorithm 1*
 - 12 *determine index $s_a^{(\ell)'}$ of $\hat{\mathbf{x}}_a$ via binary to decimal conversion*
 - 13 *update R_a according to (7)*
 - 14 $U_\ell(s_a^{(\ell)}, a) \leftarrow R_a + \beta \max_{a' \in \llbracket [m/z] \rrbracket} Q_\ell(s_a^{(\ell)'}, a')$
 - 15 *compute $Q_{\ell+1}(s_a^{(\ell)}, a)$ according to (8)*
 - 16 $s_a^{(\ell+1)} \leftarrow s_a^{(\ell)'}$
 - 17 $\mathcal{D}_{\mathbf{L}'} \leftarrow \mathcal{D}_{\mathbf{L}'} \cup (s_a^{(\ell)}, a, R_a, s_a^{(\ell)'})$
 - 18 **for** *every x new MDP instances in* $\mathcal{D}_{\mathbf{L}'}$ **do**
 - 19 $\mathcal{L}(\mathcal{D}_{\mathbf{L}'}) \leftarrow \sum_{(s_a^{(\ell)}, a, R_a, s_a^{(\ell)'}) \in \mathcal{D}_{\mathbf{L}'}} (U_\ell(s_a^{(\ell)}, a) - Q_\ell(s_a^{(\ell)}, a))^2$
 - 20 $\mathcal{B}_k \leftarrow \mathcal{B}_k \cup \mathcal{D}_{\mathbf{L}'}$
 - 21 $\mathcal{L}(\mathcal{B}_k) \leftarrow \mathcal{L}(\mathcal{B}_k) + \mathcal{L}(\mathcal{D}_{\mathbf{L}'})$
 - 22 $\mathcal{L}_k \leftarrow \frac{1}{|\mathcal{B}_k|} \mathcal{L}(\mathcal{B}_k)$ *// local error minimized as learning continues*
 - 23 **end**
 - 24 $\ell \leftarrow \ell + 1$
 - 25 **end**
 - 26 $Q_0(s_a^{(\ell)}, a) \leftarrow Q_\ell(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$
 - 27 **end**
 - 28 $Q^{(k)}(s_a^{(\ell)}, a) \leftarrow Q_0(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$ *// updates the k-th local policy*
 - 29 $k \leftarrow k + 1$
 - 30 **end**
 - 31 *perform Steps 1-13 of Algorithm 4 // global policy update*
 - 32 **end**

Algorithm 4: AM-RELDEC (continued from Step 31 of Algorithm 3)

```

1  $Q_0(s_a^{(0)}, a) \leftarrow \frac{1}{K} \sum_{k=1}^K Q^{(k)}(s_a^{(0)}, a) \forall s_a^{(0)}, a$  // initializes  $\pi(s_a^{(\ell)})$ 
   // start of an episode
2 for each new  $\mathbf{L} \in \mathcal{L}$  do
3    $\ell \leftarrow 0, \hat{\mathbf{L}}_\ell \leftarrow \mathbf{L}, \mathcal{D}_\mathbf{L} \leftarrow \emptyset$ 
4   while  $\ell < \ell_{\max}$  do
5     select cluster  $a$  according to (20)
6     repeat Steps 11-16 of Algorithm 3
7      $\mathcal{D}_\mathbf{L} \leftarrow \mathcal{D}_\mathbf{L} \cup (s_a^{(\ell)}, a, R_a, s_a^{(\ell)'})$ 
8      $\ell \leftarrow \ell + 1$ 
9   end
10   $\mathcal{L}(\mathcal{D}_\mathbf{L}) \leftarrow \sum_{(s_a^{(\ell_{\max})}, a, R_a, s_a^{(\ell_{\max})'}) \in \mathcal{D}_\mathbf{L}} (U_{\ell_{\max}}(s_a^{(\ell_{\max})}, a) - Q_{\ell_{\max}}(s_a^{(\ell_{\max})}, a))^2$ 
11   $\mathcal{B} \leftarrow \mathcal{B} \cup \mathcal{D}_\mathbf{L}$ 
12   $Q_0(s_a^{(\ell_{\max})}, a) \leftarrow Q_{\ell_{\max}}(s_a^{(\ell_{\max})}, a) \forall s_a^{(\ell_{\max})}, a$ 
13 end
14  $\mathcal{L} \leftarrow \frac{1}{|\mathcal{B}|} \mathcal{L}(\mathcal{B})$  // global error minimized as learning continues

```

$$a = \begin{cases} \text{selected uniformly at random w.p. } \epsilon \text{ from } [\lceil m/z \rceil], \\ \pi(s_a^{(\ell)}) \text{ selected w.p. } 1 - \epsilon, \end{cases} \quad (20)$$

leading to reduction of the global loss $\mathcal{L}(\mathcal{B})$. Once all meta-iterations, the number of executions of the while loop in Step 2, are done, Algorithm 3 generates an optimized global policy $\hat{\pi}(s_{a_i}^{(I)})$ that is used as a starting point for adapting the k -th local policy online.

Algorithm 5: AM-RELDEC (online adaptation phase)

```

Input : set of LLR vectors  $\mathcal{L}'_k$  obtained after channel estimation, parity-check matrix
           $\mathbf{H}$ , action-values  $Q_0(s_a^{(0)}, a)$  corresponding to optimized global policy
Output: optimized local scheduling policy  $\hat{\pi}_k(s_{a_i}^{(I)})$ 
1  $\mathcal{B}_k \leftarrow \emptyset, Q^{(k)}(s_a^{(0)}, a) \leftarrow Q_0(s_a^{(0)}, a) \forall s_a^{(0)}, a$ 
   // start of an episode
2 for each new  $\mathbf{L}' \in \mathcal{L}'_k$  do
3   | perform Steps 7-26 of Algorithm 3 // adapt to the current SNR with index  $k$ 
4 end
5  $Q^{(k)}(s_a^{(\ell)}, a) \leftarrow Q_0(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$  // updates the  $k$ -th policy of (18)

```

For online learning, we consider a wireless communication setting where the channel is

estimated accurately using pilot signals, based on which \mathbf{L}' is determined and fed to Algorithm 2. Then, in Step 9 of this algorithm, we invoke Algorithm 5. The input to Algorithm 5 includes a set of LLR vectors \mathcal{L}'_k , which contains \mathbf{L}' , and additional LLR vectors if available, based on the received pilots. The input also includes the action-values $Q_0(s_a^{(0)}, a)$, $\forall s_a^{(0)}, a$, corresponding to the optimized global policy learned using Algorithm 3. The output is an optimized CN scheduling policy for a new SNR $\hat{\pi}_k(s_{a_i}^{(I)})$ by following (11) according to

$$\hat{\pi}_k(s_{a_i}^{(I)}) = \arg \max_{a_i \in \{[m/z]\} \setminus \{a_0, \dots, a_{i-1}\}} Q^{(k)}(s_{a_i}^{(I)}, a_i), \quad (21)$$

which is incorporated in Step 9 of Algorithm 2 during decoding.

Since the global policy is optimized in an interactive manner along with the K local policies in Algorithm 3, it can be used to quickly adapt any new policy for the current channel condition using only a relatively small number of LLR vectors in \mathcal{L}'_k . This is achieved by initializing the local action-values $Q^{(k)}(s_a^{(0)}, a)$, $\forall s_a^{(0)}, a$, using the global values as shown in Step 1 of Algorithm 5. Thus, the online scheme obviates the need to store separate Q-tables for the local policies, which can be considerably large (around 350 mega-bytes for a single table) for the codes considered in this work, making AM-RELDEC suitable for practical wireless devices with limited storage space.

The M-RELDEC scheme is shown in Algorithm 6, which is a variant of the MQL algorithm of [32]. The input and outputs of M-RELDEC are identical to those of AM-RELDEC; however, unlike the latter, where both the local and global policies are updated in each meta-learning step, in M-RELDEC the global policy is learned only once during the meta-training phase, and the local policies are learned separately for each of the K batches during adaptation. Note that each local policy is initialized by the same global policy. To enable this initialization, the action-values learned for the global policy are stored as $\tilde{Q}(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$ in Step 5 and later used during adaptation in Step 12. Moreover, since the global policy does not adapt iteratively with local policies, M-RELDEC requires a large number of training instances to properly adapt to new local CN scheduling policies for unknown channel conditions, and hence is not suitable for online implementation.

Algorithm 6: M-RELDEC

Input : set of LLR vectors \mathcal{L} , \mathcal{L}_k , parity-check matrix \mathbf{H}
Output: optimized cluster scheduling policy $\hat{\pi}_k(s_{a_i}^{(I)})$ for the k -th SNR

- 1 $Q_0(s_a^{(\ell)}, a) \leftarrow 0, U_0(s_a^{(\ell)}, a) \leftarrow 0 \forall s_a^{(\ell)}, a$
- 2 $\mathcal{B} \leftarrow \emptyset$
- 3 perform Steps 2-13 of Algorithm 4 // meta-learning phase
// adaptation phase
- 4 $Q^{(k)}(s_a^{(\ell)}, a) \leftarrow Q_0(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a, k$
- 5 $\tilde{Q}(s_a^{(\ell)}, a) \leftarrow Q_0(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$
- 6 $k \leftarrow 1$
- 7 **while** $k \leq K$ **do**
 - // adapt to the k -th SNR
 - 8 $\mathcal{B}_k \leftarrow \emptyset$
 - 9 $Q_0(s_a^{(\ell)}, a) \leftarrow Q^{(k)}(s_a^{(\ell)}, a) \forall s, a$
 - 10 perform Step 6-27 of Algorithm 3
 - 11 $Q^{(k)}(s_a^{(\ell)}, a) \leftarrow Q_0(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$ // updates the k -th policy of (18)
 - 12 $Q_0(s_a^{(\ell)}, a) \leftarrow \tilde{Q}(s_a^{(\ell)}, a) \forall s_a^{(\ell)}, a$
 - 13 $k \leftarrow k + 1$
- 14 **end**

V. EXPERIMENTAL RESULTS

In this section, we compare the performances of our learning-based sequential decoding schemes with flooding (*i.e.*, all clusters are updated simultaneously per iteration), and a random sequential decoding scheme where the cluster scheduling order is randomly generated. We utilize each scheme for decoding a [384, 256]-WRAN irregular LDPC code (see [42]), a (3, 5) AB LDPC code of block length 500 bits, and a 5G-NR LDPC code. In the case of 5G-NR, the code is constructed by lifting the BG2 base matrix with dimensions 42×52 using an optimized lifting matrix obtained from the literature (see [39], [40]) with lifting factor 10, resulting in a 5G-NR LDPC code with block length 520 and a rate of approximately 1/5. The simulation of 5G-NR LDPC codes based on the BG1 matrix is beyond the scope of this work, as the graph contains several degree-19 CNs, which, once the code is lifted, renders both reinforcement and meta-reinforcement learning extremely computationally intensive. For all codes, the choice of block length is influenced by the run-time of the proposed learning schemes on our system. We both employ RELDEC in Algorithm 1 and its meta-variants AM-RELDEC and M-RELDEC in Algorithms 3-5 and 6, respectively.

Once learning is completed, the corresponding cluster scheduling policy for each code is incorporated in Step 9 of Algorithm 2, resulting in a RELDEC, AM-RELDEC, or M-RELDEC sequential decoding scheme for that code, depending on the chosen learning algorithm. In the case of RELDEC, the LLR vectors used for training are sampled uniformly at random over a range of K equally-spaced SNR values for a given code. Hence, there are $|\mathcal{L}|/K$ LLR vectors in \mathcal{L} for each SNR value considered. On the other hand, in case of AM-RELDEC and M-RELDEC, there are $|\mathcal{L}|/K$ (resp., $|\mathcal{L}_k|$) LLR vectors in \mathcal{L} (resp., \mathcal{L}_k) for each fixed SNR. For all learning schemes, we employ a reasonable choice of hyper-parameters leading to good decoding performance as follows, namely a learning rate of $\alpha = 0.1$, a reward discount rate of $\beta = 0.9$, a probability of exploration of $\epsilon = 0.6$, a maximum number of steps per learning episode of $\ell_{\max} = 50$, and a Q-learning loss threshold of $\mathcal{L}_{\min} = 1 \times 10^{-4}$. Moreover, we consider a cluster size of $z = 1$ and total number of SNR values of $K = 5$ for all codes. Specifically, we consider SNR values of 1, 2, 3, 4, and 5 (resp., 1, 1.5, 2, 2.5, and 3) dB for the WRAN (resp., AB and 5G-NR) LDPC code.

For RELDEC, we let $|\mathcal{L}| = 15000$, and for M-RELDEC, we consider $|\mathcal{L}| = 1000$ and $|\mathcal{L}_k| = 14000$. Further, we ensure that $1/K$ -th of the dataset for RELDEC contains LLR vectors of a fixed SNR; *i.e.*, RELDEC is only trained for a mixture of K SNR values. On the other hand, for AM-RELDEC, we obtain optimized local policies by performing 100 meta-iterations, where 99 of them are used for global-policy learning using Algorithm 3, and the last one is used for adaptation using Algorithm 5. We choose $|\mathcal{L}| = 7425$, $|\mathcal{L}_k| = 7425$, and thus 75 training examples are used for the learning the global and k -th local policy, respectively, in each meta-iteration. Furthermore, we consider two different sizes for the online adaptation training set, namely $|\mathcal{L}'_k| \in \{7, 75\}$. For $|\mathcal{L}'_k| = 75$ (resp., 7), the corresponding learning scheme is denoted as AM-RELDEC-75 (resp., as AM-RELDEC-7). Thus, for AM-RELDEC-75, the total amount of training data amounts to $|\mathcal{L}| + |\mathcal{L}_k| + |\mathcal{L}'_k| = 14925$, which is less than 15000 since the global policy is not adapted during the 100th meta-learning step (online learning). In contrast, for AM-RELDEC-7, the total amount of training data is $|\mathcal{L}| + |\mathcal{L}_k| + |\mathcal{L}'_k| = 14857$, which is slightly less than for AM-RELDEC-75 due to the reduced training set size used for online adaptation.

Note that the training set sizes $|\mathcal{L}|$ and $|\mathcal{L}|+|\mathcal{L}_k|$ are chosen to ensure that the dataset is large enough for accurate training without incurring too much computation time. Finally, we fix the maximum number of decoding iterations as $I_{\max} = 50$ for the AB and 5G-NR codes, but use $I_{\max} = 5$ for the WRAN code to enable a comparison with the hyper-network scheme of [29].

For both training and inference, we transmit all-zero codewords using BPSK modulation. Note that training with the all-zero codeword is sufficient as, due to the symmetry of the BP decoder and the channel, the decoding error is independent of the transmitted signal (see e.g. [43, Lemma 4.92]). For performance measures, we consider both the bit error rate (BER), given by $\Pr[\hat{x}_v \neq x_v]$, $v \in [[n]]$, and the frame error rate (FER), given by $\Pr[\hat{\mathbf{x}} \neq \mathbf{x}]$.

The BER vs. channel SNR, in terms of E_b/N_0 in dB, for the (3, 5) AB, the [520, 420] 5G-NR LDPC, and the [384, 256]-WRAN code using these decoding techniques are shown in Figs. 3(a), 4(a), and 5(a), respectively, where we have limited the plots to the more interesting moderate SNR regime. For all codes, AM-RELDEC-75 outperforms the other decoding schemes, including the state-of-the-art hyper-network decoder of [29] (in the case of the WRAN LDPC code) with a gain of around 0.5 dB for fixed BER, which shows the benefit of doing multiple meta-iterations for updating both the global and local CN scheduling policies. The FER vs. SNR performance shown in Figs. 3(b), 4(b), and 5(b) exhibits similar behavior. The AM-RELDEC-7 scheme, on average, outperforms the random sequential decoding scheme, which demonstrates the agility of AM-RELDEC for adapting local CN scheduling policies online using a very small number of training examples.

In Table I, we compare the average number of CN to VN messages propagated in the considered decoding schemes to attain the results in Figs. 3-5. We note that both RELDEC and AM-RELDEC-75, on average, generate a lower number of CN to VN messages when compared to the other decoding schemes, providing a significant reduction in message-passing complexity for moderate length LDPC codes. The number of CN to VN messages propagated by AM-RELDEC-7 and M-RELDEC are nearly identical to those of AM-RELDEC-75, and hence are not shown.

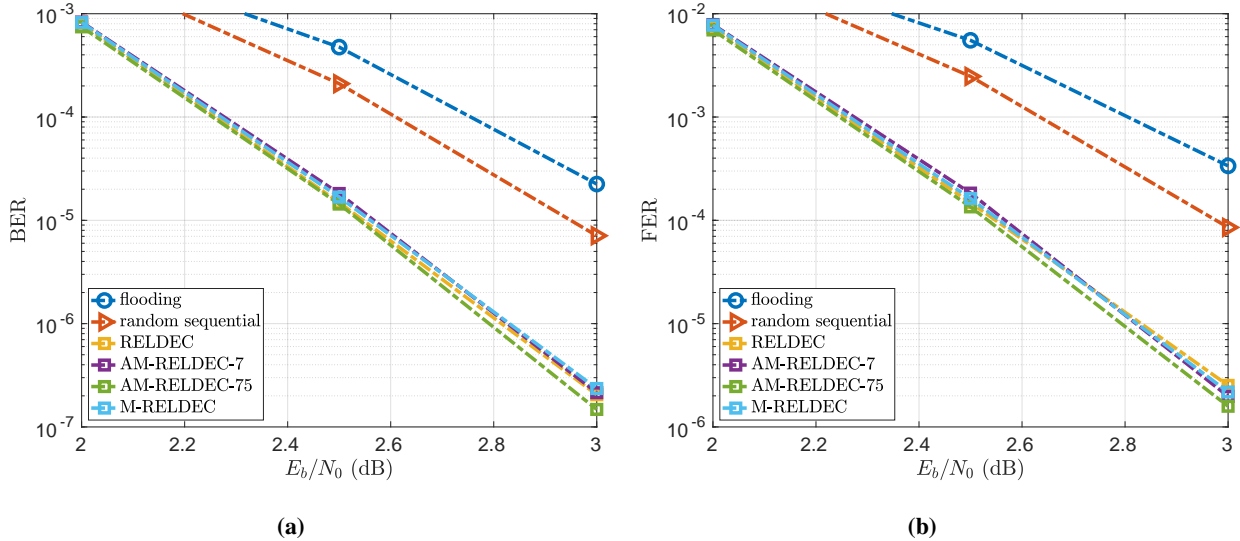


Fig. 3: BER and FER results using different BP decoding schemes for a $(3, 5)$ AB-LDPC code with block length $n = 500$ and $I_{\max} = 50$ decoder iterations for an AWGN channel.

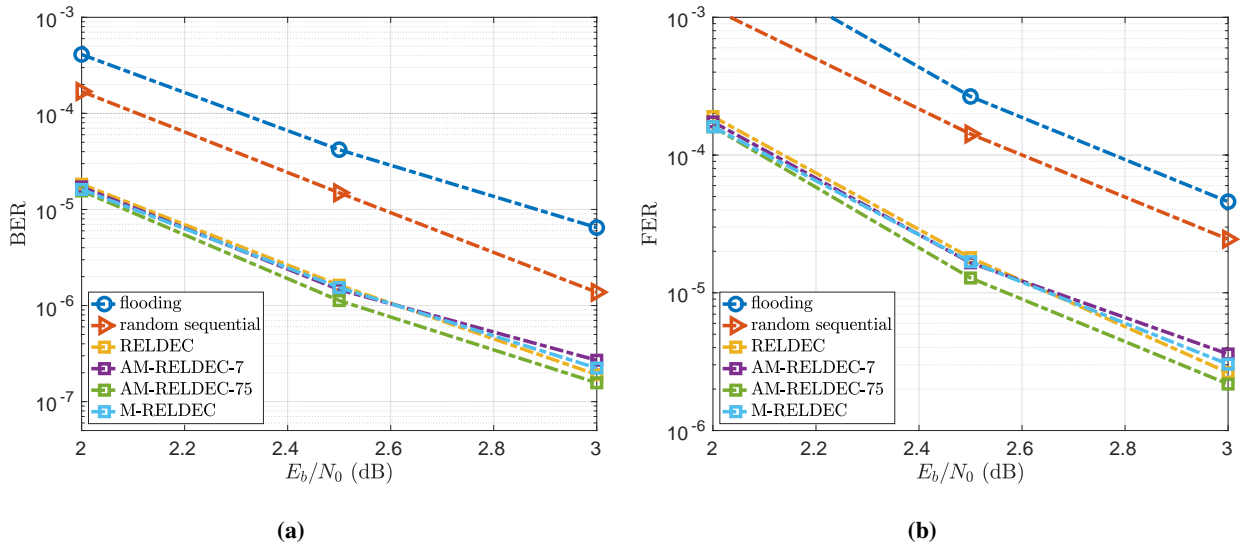


Fig. 4: BER and FER results using different BP decoding schemes for a $[520, 420]$ 5G-NR LDPC code and $I_{\max} = 50$ decoder iterations for an AWGN channel.

VI. CONCLUSION

We presented RELDEC, a novel reinforcement learning-based sequential decoding scheme proposed to optimize the scheduling of CN clusters for moderate length LDPC codes. In contrast to our previous work, the main contributions of this work include a new complexity-reduced

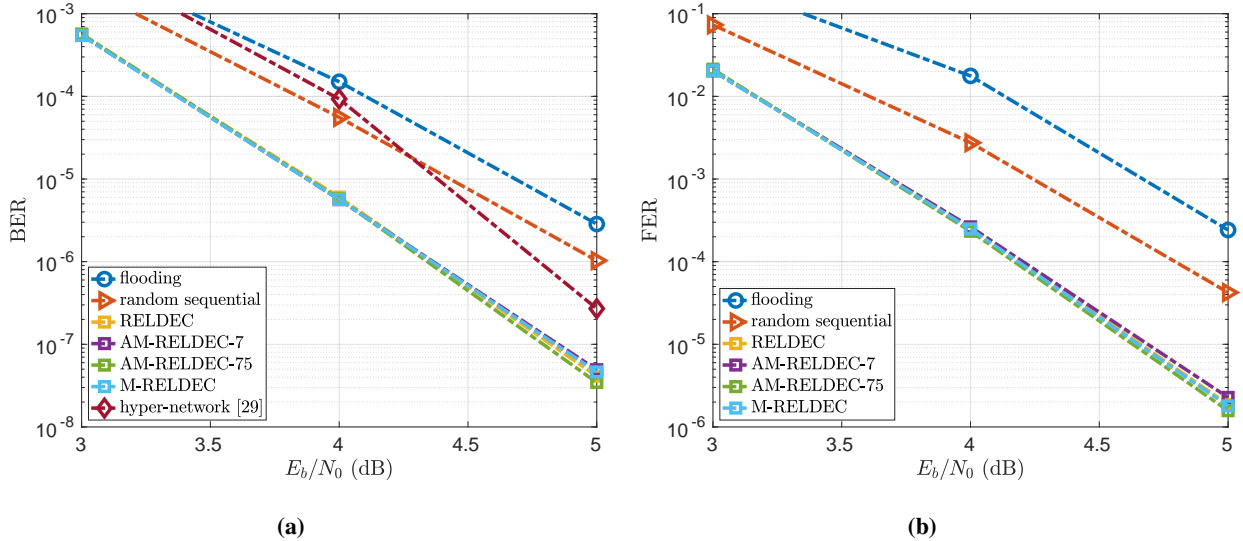


Fig. 5: BER and FER results using different BP decoding schemes for a $[384, 256]$ -WRAN LDPC code and $I_{\max} = 5$ decoder iterations for an AWGN channel. The BER results for the hyper-network scheme are taken from [29].

SNR (dB)	2	2.5	3	SNR (dB)	3	4	5
flooding	16409 (12752)	10742 (10745)	8123 (9491)	flooding	5171	3355	2184
random scheduling	10533 (7580)	6436 (6598)	4850 (5977)	random scheduling	3506	2207	1546
RELDEC	6601 (5771)	4821 (5131)	4028 (4619)	RELDEC	3193	2206	1584
AM-RELDEC-75	6440 (5763)	4725 (5128)	3956 (4615)	AM-RELDEC-75	3203	2206	1585

TABLE I: Average number of CN to VN messages propagated in various decoding schemes for the $(3, 5)$ AB $([520, 420])$ 5G-NR LDPC code, shown in the left, and the $[384, 256]$ -WRAN LDPC code, shown in the right, to attain the results shown in Figs. 3-5.

state space model built using the collection of possible outputs of individual clusters as well as a scheduling approach that updates all CN clusters sequentially within each decoder iteration. Furthermore, we propose novel meta-learning based sequential decoding schemes, namely AM-RELDEC and M-RELDEC to further improve the decoding performance with respect to RELDEC. The learning flexibility of AM-RELDEC allows the decoder to quickly adapt to changing channel conditions due to fading, making it well suited to error correction in wireless communications scenarios with moderate computational and memory complexity. Experimental results show that by learning the cluster scheduling order using RELDEC and its meta-learning counterparts, we can significantly outperform flooding and random scheduling schemes without any expensive computation of residuals as in previous sequential scheduling schemes. Our presented performance gains include lowering both BER and message-passing complexities.

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